On Quasi $Ng^{\#}-$ Open and Quasi $Ng^{\#}-$ Closed Mappings in Neutrosophic Topological Spaces

Pious Missier S[1], Babisha Julit R L[2]*, Martina Jency J[3]
[1]Department of Mathematics,
Don Bosco College of Arts and Science, KeelaEral, Thoothukudi-628 908, Tamil Nadu, India
[2]Department of Mathematics,
G. Venkataswamy Naidu College, Kovilpatti-628 502, Tamil Nadu, India
[3] Assistant Professor, Department of Mathematics,
Partician College of Arts and Science,
Chennai -600 020, Tamil Nadu, India

Emails: spmissier@gmail.com, babishajulitrl@gvncollege.edu.in, martinajency@gmail.com

Abstract

The aim of this article is to investigate a new kind of Neutrosophic open and closed mappings called quasi $Ng^{\#}-$ Open and quasi $Ng^{\#}-$ closed mappings and analyse their characterizations in Neutrosophic topological spaces with necessary examples.

Keywords: $\mathcal{N}g^{\#}-$ closed set; $\mathcal{N}g^{\#}-$ Open sets; $\mathcal{N}g^{\#}-$ open mapping; quasi $\mathcal{N}g^{\#}-$ open mapping; quasi $\mathcal{N}g^{\#}-$ closed mapping.

1 Introduction

Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache.[4] Neutrosophic set helps to study this indeterminacy in uncertain situations in real life problems.The concept of Neutrosophy has been developed into Neutrosophic topological spaces by Salama et.al.[11] in 2014. Many researchers have studied various topological spaces in accordance with Neutrosophy. Recently Pious Missier et.al.,[7–9] introduced the concept of $\mathcal{N}g^{\#}-$ closed sets, $\mathcal{N}g^{\#}-$ continuous functions, $\mathcal{N}g^{\#}-$ closed and $\mathcal{N}g^{\#}-$ open mappings, in Neutrosophic Topological Spaces. Further, Neutrosophic topology has been applied to the quasi open and closed mappings.

2 Preliminaries

Definition 2.1. [4]
A Neutrosophic set $\mathcal{NS}$ $\mathcal{A_N}$ is an object having the form $\mathcal{A_N} = \{\langle \lambda, \mu_{\mathcal{A_N}}(\lambda\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$. Here

1. $\mu_{\mathcal{A_N}}(\lambda)-$ degree of membership
2. $\sigma_{\mathcal{A_N}}(\lambda)-$ degree of indeterminacy
3. $\gamma_{\mathcal{A_N}}(\lambda)-$ degree of non-membership

A Neutrosophic set $\mathcal{A_N} = \{\langle \lambda, \mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ can be identified as an ordered triple $\langle \mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle$ in $]{-0}, 1{+}[$ on $\mathcal{X_N}$.

Definition 2.2. [11] For any two Neutrosophic sets $\mathcal{A_N} = \{\langle \lambda, \mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ and $\mathcal{B_N} = \{\langle \lambda, \mu_{\mathcal{B_N}}(\lambda), \sigma_{\mathcal{B_N}}(\lambda), \gamma_{\mathcal{B_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ we have

1. $\mathcal{A_N} \subseteq \mathcal{B_N} \iff \mu_{\mathcal{A_N}}(\lambda) \leq \mu_{\mathcal{B_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda) \leq \sigma_{\mathcal{B_N}}(\lambda)$ and $\gamma_{\mathcal{A_N}}(\lambda) \geq \gamma_{\mathcal{B_N}}(\lambda)$

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2. $\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}} = \langle \lambda, \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda) \wedge \mu_{\mathcal{B}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \wedge \sigma_{\mathcal{B}_{\mathcal{N}}}(\lambda)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \vee \gamma_{\mathcal{B}_{\mathcal{N}}}(\lambda)\rangle$
3. $\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}} = \langle \lambda, \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda) \vee \mu_{\mathcal{B}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \vee \sigma_{\mathcal{B}_{\mathcal{N}}}(\lambda)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \wedge \gamma_{\mathcal{B}_{\mathcal{N}}}(\lambda)\rangle$

Definition 2.3. [11] Let $\mathcal{A}_{\mathcal{N}} = \langle \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda), \gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda)\rangle$ be a $\mathcal{N}S$ on $\mathcal{X}_{\mathcal{N}}$, then the complement $\mathcal{A}_{\mathcal{N}}{}^c$ defined as

- $\mathcal{A}_{\mathcal{N}}{}^c = \{\langle \lambda, \gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda), 1 - \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda), \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda)\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$

Note that for any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}}$ and $\mathcal{B}_{\mathcal{N}}$,

- $(\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}})^c = \mathcal{A}_{\mathcal{N}}{}^c \cap \mathcal{B}_{\mathcal{N}}{}^c$
- $(\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}})^c = \mathcal{A}_{\mathcal{N}}{}^c \cup \mathcal{B}_{\mathcal{N}}{}^c$.

Definition 2.4. [11] A Neutrosophic topology $(\mathcal{NT})$ on a non-empty set $\mathcal{X}_{\mathcal{N}}$ is a family τ_N of Neutrosophic subsets in $\mathcal{X}_{\mathcal{N}}$ satisfies the following axioms:

1. $\mathbf{0}_{\mathcal{N}}, \mathbf{1}_{\mathcal{N}} \in \tau_N$
2. $\mathcal{R}_{N_1} \cap \mathcal{R}_{N_2} \in \tau_N$ for any $\mathcal{R}_{N_1}, R_{N_2} \in \tau_N$
3. $\bigcup \mathcal{R}_{N_i} \in \tau_N \quad \forall \quad \mathcal{R}_{N_i} : i \in I \subseteq \tau_N$

Here the empty set $\mathbf{0}_{\mathcal{N}}$ and the whole set $\mathbf{1}_{\mathcal{N}}$ may be defined as follows:

1. $\mathbf{0}_{\mathcal{N}} = \{\langle \lambda, 0, 0, 1\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$
2. $\mathbf{1}_{\mathcal{N}} = \{\langle \lambda, 1, 1, 0\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$

Definition 2.5. [11] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}_{\mathcal{N}}$. Then

1. $\mathcal{N}int(\mathcal{A}_{\mathcal{N}}) = \bigcup\{\mathcal{G}_{\mathcal{N}} : \mathcal{G}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{G}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) = \bigcap\{\mathcal{K}_{\mathcal{N}} : \mathcal{K}_{\mathcal{N}}$ is a $\mathcal{NCS}$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{K}_{\mathcal{N}}\}$ is called Neutrosophic closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.6. [5] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a neutrosophic $\mathcal{N}\alpha gCS$ if $\mathcal{N}\alpha cl(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{U}_{\mathcal{N}}$, whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{U}_{\mathcal{N}}$ and $\mathcal{U}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}_{\mathcal{N}}$. The complement of $\mathcal{N}\alpha gCS$ is $\mathcal{N}\alpha gOS$.

Definition 2.7. [7]
A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a Neutrosophic $g^{\#}-$ closed $(\mathcal{N}g^{\#}CS)$ if $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{Q}_{\mathcal{N}}$ whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{Q}_{\mathcal{N}}$ and $\mathcal{Q}_{\mathcal{N}}$ is $\mathcal{N}\alpha gOS$ in $\mathcal{X}_{\mathcal{N}}$. The complement of $\mathcal{N}g^{\#}CS$ is $\mathcal{N}g^{\#}OS$.

Definition 2.8. [10] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}_{\mathcal{N}}$. Then

1. $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \bigcup\{\mathcal{G}_{\mathcal{N}} : \mathcal{G}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}OS$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{G}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic $g^{\#}-$ interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \bigcap\{\mathcal{K}_{\mathcal{N}} : \mathcal{K}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}CS$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{K}_{\mathcal{N}}\}$ is called Neutrosophic $g^{\#}-$ closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.9. [8] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be $\mathcal{N}g^{\#}-$ continuous function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every neutrosophic closed set $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.10. [8] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be Neutrosophic $g^{\#}-$ irresolute function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every $\mathcal{N}g^{\#}CS$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.11. [10] A Neutrosophic Topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a $T_{\mathcal{N}}g^{\#}-$ space if every $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.12. [12] A function $f_N : (X, \tau_N) \longrightarrow (Y, \zeta_N)$ is called

1. Neutrosophic closed mapping (NCM) if $f_N(V_N)$ is a $\mathcal{NCS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NCS}$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
2. Neutrosophic open mapping (NOM) if $f_N(V_N)$ is a $\mathcal{NOS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NOS}$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.13. [9] Let $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be two Neutrosophic topological spaces. A mapping $f_N : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is called

1. $Ng^{\#}CM$ if $f_N(A_N)$ is $Ng^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NCS}\ \mathcal{A}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
2. $Ng^{\#}OM$ if $f_N(A_N)$ is $(Ng^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NOS}\ \mathcal{A}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
3. strongly $Ng^{\#}OM$ if $f_N(A_N)$ is $Ng^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $Ng^{\#}OS\ A_N$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
4. strongly $Ng^{\#}CM$ if $f_N(A_N)$ is $Ng^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $Ng^{\#}CS\ A_N$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

3 Quasi $\mathcal{N}g^{\#}-$ Open Mappings

Definition 3.1. A Neutrosophic mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be quasi $\mathcal{N}g^{\#}-$ open mapping (quasi- $\mathcal{N}g^{\#}OM$ for short) if $f_{\mathcal{N}}(\mathcal{V}_{\mathcal{N}})$ is a Neutrosophic open set in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{N}g^{\#}OS\ \mathcal{V}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Example 3.2. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.4, 0.6)\rangle, \langle m, (0.5, 0.4, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.2, 0.4, 0.7)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.2, 0.3, 0.5)\rangle, \langle m, (0.4, 0.3, 0.5)\rangle\}$,
$\mathcal{M}_{\mathcal{N}4} = \{\langle l, (0.1, 0.3, 0.6)\rangle, \langle m, (0.3, 0.2, 0.5)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and
$(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathcal{M}_{\mathcal{N}4}, \mathcal{M}_{\mathcal{N}4}^c, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces.
Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}4}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = l - 0.1$ and $f_{\mathcal{N}}(m) = m - 0.1$. Here $\mathcal{NOS}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}4}, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}\mathcal{OS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}$ both are $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ and $f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}1}) = \mathcal{M}_{\mathcal{N}3}, f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}2}) = \mathcal{M}_{\mathcal{N}4}$ both are $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore $f_{\mathcal{N}}$ is quasi- $\mathcal{N}g^{\#}OM$

Theorem 3.3. *Every quasi-* $\mathcal{N}g^{\#}OM$ *is* $\mathcal{NOM}$.

Proof. Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}OM$. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NOS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}OM$, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is Neutrosophic open mapping. □

Remark 3.4. The reverse implication of the above theorem need not be true.

Example 3.5. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.2, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.3, 0.3, 0.6)\rangle, \langle m, (0.4, 0.5, 0.6)\rangle\}$
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.4, 0.5, 0.6)\rangle, \langle m, (0.3, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic typologies on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = m$ and $f_{\mathcal{N}}(m) = l$. Here $\mathcal{NOS}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}, \mathcal{NOS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}\mathcal{OS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$. Hence $f_{\mathcal{N}}$ is Neutrosophic open mapping. Now $\mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}$ both are $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but $f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}1})$ is not $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore $f_{\mathcal{N}}$ is not quasi $\mathcal{N}g^{\#}-$ open mapping.

Theorem 3.6. *Every quasi-* $\mathcal{N}g^{\#}OM$ *is* $\mathcal{N}g^{\#}OM$.

Proof. Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}OM$. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NOS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}OM$, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Note that every $\mathcal{NOS}$ is $\mathcal{N}g^{\#}OS$. Therefore, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}OM$. □

Remark 3.7. Every $\mathcal{N}g^{\#}OM$ is need not be a quasi- $\mathcal{N}g^{\#}OM$.

Example 3.8. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.4, 0.3, 0.6)\rangle, \langle m, (0.5, 0.5, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.5, 0.5, 0.6)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$ are

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Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = m$ and $f_{\mathcal{N}}(m) = l$. Here $\mathcal{NCS}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{NCS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}\mathcal{OS}(\mathcal{X}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}\mathcal{CS}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping. Now $\mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}$ both are $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but $f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}1})$ is not $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore $f_{\mathcal{N}}$ is not quasi $\mathcal{N}g^{\#}-$ open mapping.

Theorem 3.9. *A Neutrosophic mapping* $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *is quasi* $\mathcal{N}g^{\#}-$ *open mapping if and only if for every* $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ *of* $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}int(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$.

Proof. Let $f_{\mathcal{N}}$ be a quasi $\mathcal{N}g^{\#}OM$. Now, we have $\mathcal{N}int(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{A}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}OS$. Hence, we get $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$. Since $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{NOS}$, $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}int(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$.
Conversely, assume that $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}) = f_{\mathcal{N}}(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}int(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ but $\mathcal{N}int(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$. Consequently, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}) = \mathcal{N}int(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$. That is $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore, $f_{\mathcal{N}}$ is quasi- $\mathcal{N}g^{\#}OM$.

□

Lemma 3.10. *If a Neutrosophic function* $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *is quasi-* $\mathcal{N}g^{\#}OM$ *then* $\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}^{-1}(\mathcal{N}int(\mathcal{A}_{\mathcal{N}}))$ *for every* $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ *of* $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Then $\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$ is a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ and $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, then $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))) \subseteq \mathcal{N}int(f_{\mathcal{N}}(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))) \subseteq \mathcal{N}int(\mathcal{A}_{\mathcal{N}})$. Thus $\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}^{-1}(\mathcal{N}int(\mathcal{A}_{\mathcal{N}}))$. □

Theorem 3.11. *A Neutrosophic mapping* $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *is quasi* $\mathcal{N}g^{\#}-$ *open mapping if and only if for each* $\mathcal{NS}$ $\mathcal{B}_{\mathcal{N}}$ *of* $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *and for each* $\mathcal{N}g^{\#}CS$ $\mathcal{C}_{\mathcal{N}}$ *of* $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ *containing* $f_{\mathcal{N}}^{-1}(\mathcal{B}_{\mathcal{N}})$ *there is a* $\mathcal{NCS}$ $\mathcal{A}_{\mathcal{N}}$ *of* $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *such that* $\mathcal{B}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}$ *and* $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{B}_{\mathcal{N}}$.

Proof. Assume that $f_{\mathcal{N}}$ is a quasi $\mathcal{N}g^{\#}-$ open mapping. Let $\mathcal{B}_{\mathcal{N}}$ be a Neutrosophic set in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ and $\mathcal{C}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $f_{\mathcal{N}}^{-1}(\mathcal{B}_{\mathcal{N}}) \subseteq \mathcal{C}_{\mathcal{N}}$. Then $\mathcal{A}_{\mathcal{N}} = (f_{\mathcal{N}}^{-1}(\mathcal{C}_{\mathcal{N}}{}^{c}))^{c}$ is $\mathcal{NCS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ such that $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{C}_{\mathcal{N}}$.
Conversely, Assume that $\mathcal{G}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $f_{\mathcal{N}}^{-1}(f_{\mathcal{N}}(\mathcal{G}_{\mathcal{N}})^{c}) \subseteq \mathcal{G}_{\mathcal{N}}{}^{c}$ and $\mathcal{G}_{\mathcal{N}}{}^{c}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. By hypothesis, there is a $\mathcal{NCS}$ $\mathcal{A}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ such that $(f_{\mathcal{N}}(\mathcal{G}_{\mathcal{N}}))^{c} \subseteq \mathcal{A}_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{G}_{\mathcal{N}}{}^{c}$. Therefore, $\mathcal{G}_{\mathcal{N}} \subseteq (f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))^{c}$. Hence $\mathcal{A}_{\mathcal{N}}{}^{c} \subseteq f_{\mathcal{N}}(\mathcal{G}_{\mathcal{N}}) \subseteq f_{\mathcal{N}}((f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))^{c}) \subseteq \mathcal{A}_{\mathcal{N}}{}^{c}$ which implies that $f_{\mathcal{N}}(\mathcal{G}_{\mathcal{N}}) = \mathcal{A}_{\mathcal{N}}{}^{c}$. Since $\mathcal{A}_{\mathcal{N}}{}^{c}$ is a $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$, $f_{\mathcal{N}}(\mathcal{G}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping. □

Theorem 3.12. *A Neutrosophic mapping* $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *is quasi* $\mathcal{N}g^{\#}-$ *open mapping if and only if* $f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$ *for every neutrosophic set* $\mathcal{A}_{\mathcal{N}}$ *in* $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Proof. Suppose that $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping. For any $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$,
$f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$. Therefore, by Theorem 3.11, there exists a $\mathcal{NCSB}_{\mathcal{N}}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ such that $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{B}_{\mathcal{N}}$ and $f_{\mathcal{N}}^{-1}(\mathcal{B}_{\mathcal{N}}) \subseteq \mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$. Therefore, we obtain $f_{\mathcal{N}}^{-1}(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}^{-1}(\mathcal{B}_{\mathcal{N}}) \subseteq \mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$.
Conversely, let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ and $\mathcal{B}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ containing $f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})$. Put $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) = \mathcal{W}_{\mathcal{N}}$, then we have $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{W}_{\mathcal{N}}$ and $\mathcal{W}_{\mathcal{N}}$ is $\mathcal{NCS}$ and $f_{\mathcal{N}}^{-1}(\mathcal{W}_{\mathcal{N}}) \subseteq \mathcal{N}g^{\#}cl(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{B}_{\mathcal{N}}$. Then, by Theorem 3.11, $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping. □

Theorem 3.13. *Let* $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *and* $g_{\mathcal{N}} : (\mathcal{Y}, \zeta_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ *be two Neutrosophic mappings and let* $(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ *is quasi* $\mathcal{N}g^{\#}-$ *open mapping. If* $g_{\mathcal{N}}$ *is* $\mathcal{N}-$ *continuous and one to one function, then* $f_{\mathcal{N}}$ *is quasi* $\mathcal{N}g^{\#}-$ *open mapping.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, then $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$, since $(g_{\mathcal{N}} \circ f_{\mathcal{N}})$ is quasi $\mathcal{N}g^{\#}-$ open. Since $g_{\mathcal{N}}$ is $\mathcal{N}-$ continuous and one to one function, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}^{-1}(g_{\mathcal{N}} \circ f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. This shows that $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping. □

Theorem 3.14. *Let* $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ *and* $g_{\mathcal{N}} : (\mathcal{Y}_{\mathcal{N}}, \zeta_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ *be any two Neutrosophic mappings. Then*

1. $(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ *is Neutrosophic open mapping if* $g_{\mathcal{N}}$ *is quasi* $\mathcal{N}g^{\#}-$ *open mapping and* $f_{\mathcal{N}}$ *is* $\mathcal{N}g^{\#}-$ *open mapping.*

2. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is strongly $\mathcal{N}g^{\#}-$ open mapping if $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping and $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping.*
3. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is quasi $\mathcal{N}g^{\#}-$ open mapping if $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping and $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ open mapping.*

Proof. :
(1) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NOS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{NOS}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is Neutrosophic open mapping.
(2) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ open mapping.
(3) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ open mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{NOS}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping. ☐

4 Quasi $\mathcal{N}g^{\#}-$ closed Mappings

Definition 4.1. A Neutrosophic mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be quasi $\mathcal{N}g^{\#}-$ closed mapping (quasi- $\mathcal{N}g^{\#}CM$ for short) if $f_{\mathcal{N}}(\mathcal{V}_{\mathcal{N}})$ is a Neutrosophic closed set in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{N}g^{\#}CS$ $\mathcal{V}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Example 4.2. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.4, 0.6)\rangle, \langle m, (0.5, 0.4, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.2, 0.4, 0.7)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\} = (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = l$ and $f_{\mathcal{N}}(m) = m$. Here $= \mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\} = \mathcal{NCS}(\mathcal{Y}_{\mathcal{N}})$. Therefore $f_{\mathcal{N}}$ is quasi- $\mathcal{N}g^{\#}CM$.

Theorem 4.3. *Every quasi- $\mathcal{N}g^{\#}CM$ is $\mathcal{NOM}$.*

Proof. Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}CM$. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}CM$, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is $\mathcal{NOM}$. ☐

Remark 4.4. Every $\mathcal{NOM}$ is need not be a quasi- $\mathcal{N}g^{\#}CM$.

Example 4.5. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.2, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.3, 0.3, 0.6)\rangle, \langle m, (0.4, 0.5, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.4, 0.5, 0.6)\rangle, \langle m, (0.3, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = m$ and $f_{\mathcal{N}}(m) = l$. Here $\mathcal{NCS}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}, \mathcal{NCS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$. Hence $f_{\mathcal{N}}$ is $\mathcal{NOM}$. Now $\mathcal{M}_{\mathcal{N}1}^c$ and $\mathcal{M}_{\mathcal{N}2}^c$ both are $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but $f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}1}^c)$ is not $\mathcal{NCS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore, $f_{\mathcal{N}}$ is not quasi- $\mathcal{N}g^{\#}CM$.

Theorem 4.6. *Every quasi- $\mathcal{N}g^{\#}CM$ is $\mathcal{N}g^{\#}CM$.*

Proof. Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi- $\mathcal{N}g^{\#}CM$. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}CM$, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Note that every $\mathcal{NCS}$ is $\mathcal{N}g^{\#}CS$. Therefore, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CM$. ☐

Remark 4.7. Every $\mathcal{N}g^{\#}CM$ is need not be a quasi- $\mathcal{N}g^{\#}CM$ can be proved by following example.

Example 4.8. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.4, 0.3, 0.6)\rangle, \langle m, (0.5, 0.5, 0.6)\rangle\}$
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.5, 0.5, 0.6)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$ are

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Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}$ and $\zeta_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_\mathcal{N}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_\mathcal{N}$ and $\mathcal{Y}_\mathcal{N}$ respectively. Define a mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ by $f_\mathcal{N}(l) = m$ and $f_\mathcal{N}(m) = l$. Here $\mathcal{NCS}(\mathcal{Y}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_\mathcal{N}\}$, $\mathcal{NCS}(\mathcal{X}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_\mathcal{N}\}$, $\mathcal{N}g^\#\mathcal{CS}(\mathcal{X}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_\mathcal{N}\}$, $\mathcal{N}g^\#\mathcal{CS}(\mathcal{Y}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_\mathcal{N}\}$. Hence $f_\mathcal{N}$ is $\mathcal{N}g^\#CM$. Now $\mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c$ both are $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ but $f_\mathcal{N}(\mathcal{M}_{\mathcal{N}1}^c)$ is not $\mathcal{NCS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Therefore $f_\mathcal{N}$ is not quasi- $\mathcal{N}g^\#CM$.

Theorem 4.9. *A Neutrosophic mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ be a quasi $\mathcal{N}g^\#-$ closed if and only if for every $\mathcal{NS}$ $\mathcal{A}_\mathcal{N}$ of $(\mathcal{X}_\mathcal{N}, \tau_N)$, $\mathcal{N}cl(f_\mathcal{N}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))$.*

Proof. Assume that $f_\mathcal{N}$ is quasi-$\mathcal{N}g^\#CM$ and $\mathcal{A}_\mathcal{N}$ is any $\mathcal{NS}$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Then $\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N})$ is a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Therefore, $f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))$ is a $\mathcal{N}g^\#CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Since $f_\mathcal{N}(\mathcal{A}_\mathcal{N}) \subseteq f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))$ which implies that $\mathcal{N}cl(f_\mathcal{N}(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}cl(f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))) = f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))$. This implies $\mathcal{N}cl(f_\mathcal{N}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))$.
Conversely, Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Then $\mathcal{A}_\mathcal{N} = \mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N})$. Therefore, $f_\mathcal{N}(\mathcal{A}_\mathcal{N}) = f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N}))$. By hypothesis, $\mathcal{N}cl(f_\mathcal{N}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}(\mathcal{N}g^\# cl(\mathcal{A}_\mathcal{N})) = f_\mathcal{N}(\mathcal{A}_\mathcal{N})$. Hence $\mathcal{N}cl(f_\mathcal{N}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}(\mathcal{A}_\mathcal{N})$. But $f_\mathcal{N}(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{N}cl(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$. This implies $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is a $\mathcal{NCS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Therefore, $f_\mathcal{N}$ is quasi-$\mathcal{N}g^\#CM$. ☐

Lemma 4.10. *A Neutrosophic mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ be a quasi $\mathcal{N}g^\#-$ closed then for every $\mathcal{NS}$ $\mathcal{A}_\mathcal{N}$ of $(\mathcal{Y}_\mathcal{N}, \zeta_N)$, $f_\mathcal{N}^{-1}(\mathcal{N}int(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$.*

Proof. Let $\mathcal{A}_\mathcal{N}$ be any neutrosophic set in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Then $\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$ is a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ and $f_\mathcal{N}$ is quasi- $\mathcal{N}g^\#CM$. Hence $f_\mathcal{N}(\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))) \subseteq \mathcal{N}int(f_\mathcal{N}(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))) \subseteq \mathcal{N}int(\mathcal{A}_\mathcal{N})$. Which implies $f_\mathcal{N}(\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))) \subseteq \mathcal{N}int(\mathcal{A}_\mathcal{N})$. Therefore, $f_\mathcal{N}^{-1}(\mathcal{N}int(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$. ☐

Theorem 4.11. *Let $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ and $g_\mathcal{N} : (\mathcal{Y}_\mathcal{N}, \zeta_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ be any two Neutrosophic mappings. Then*

1. *$(g_\mathcal{N} \circ f_\mathcal{N}) : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ is Neutrosophic closed mapping if $g_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping and $f_\mathcal{N}$ is $\mathcal{N}g^\#-$ closed mapping.*

2. *$(g_\mathcal{N} \circ f_\mathcal{N}) : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ is strongly $\mathcal{N}g^\#-$ closed mapping if $g_\mathcal{N}$ is $\mathcal{N}g^\#-$ closed mapping and $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping.*

3. *$(g_\mathcal{N} \circ f_\mathcal{N}) : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ is quasi $\mathcal{N}g^\#-$ closed mapping if $g_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping and $f_\mathcal{N}$ is strongly $\mathcal{N}g^\#-$ closed mapping.*

Proof. : (1) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NCS}$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $f_\mathcal{N}$ is $\mathcal{N}g^\#-$ closed mapping, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^\#CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Since $g_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping, $(g_\mathcal{N} \circ f_\mathcal{N})(\mathcal{A}_\mathcal{N}) = g_\mathcal{N}(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ is $\mathcal{NCS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. Therefore $g_\mathcal{N} \circ f_\mathcal{N}$ is Neutrosophic closed mapping.
(2) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{NCS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Since $g_\mathcal{N}$ is $\mathcal{N}g^\#-$ closed mapping, $(g_\mathcal{N} \circ f_\mathcal{N})(\mathcal{A}_\mathcal{N}) = g_\mathcal{N}(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ is $\mathcal{N}g^\#CS$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. Therefore $g_\mathcal{N} \circ f_\mathcal{N}$ is strongly $\mathcal{N}g^\#-$ closed mapping.
(3) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $f_\mathcal{N}$ is strongly $\mathcal{N}g^\#-$ closed mapping, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^\#CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Since $g_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping, $(g_\mathcal{N} \circ f_\mathcal{N})(\mathcal{A}_\mathcal{N}) = g_\mathcal{N}(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ is $\mathcal{NCS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. Therefore $g_\mathcal{N} \circ f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed mapping. ☐

Theorem 4.12. *Let $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ and $g_\mathcal{N} : (\mathcal{Y}_\mathcal{N}, \zeta_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ be any two Neutrosophic mappings such that $(g_\mathcal{N} \circ f_\mathcal{N}) : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ is quasi $\mathcal{N}g^\#-$ closed mapping.*

1. *If $f_\mathcal{N}$ is $\mathcal{N}g^\#-$ irresolute and onto then $g_\mathcal{N}$ is Neutrosophic closed mapping.*

2. *If $g_\mathcal{N}$ is $\mathcal{N}g^\#-$ continuous and one to one then $f_\mathcal{N}$ is strongly $\mathcal{N}g^\#-$ closed mapping.*

Proof. : (1) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NCS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Then $\mathcal{A}_\mathcal{N}$ is $\mathcal{N}g^\#CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Since $f_\mathcal{N}$ is $\mathcal{N}g^\#-$ irresolute mapping, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $(g_\mathcal{N} \circ f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ closed and $f_\mathcal{N})$ is onto, $(g_\mathcal{N} \circ f_\mathcal{N})(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N})) = g_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{NCS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. This implies that $g_\mathcal{N}$ is Neutrosophic closed mapping.
(2) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $(g_\mathcal{N} \circ f_\mathcal{N})$ is quasi $\mathcal{N}g^\#-$ closed, $(g_\mathcal{N} \circ f_\mathcal{N})(\mathcal{A}_\mathcal{N}) = g_\mathcal{N}(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ is $\mathcal{NCS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. Since $g_\mathcal{N}$ is $\mathcal{N}g^\#-$ continuous and one to one mapping, $g_\mathcal{N}^{-1}(g_\mathcal{N} \circ f_\mathcal{N}(\mathcal{A}_\mathcal{N})) = f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^\#CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. This implies that $f_\mathcal{N}$ is strongly $\mathcal{N}g^\#-$ closed mapping. ☐

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