Neutrosophic Discrete Facility Location Problems

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Abstract

Discrete facility location problems are classified as types of facility location problems, wherein decisions on choosing facilities in specific locations are made to serve the demand points of customers, thus minimizing the total cost. The covering- and median-based problems are the common classified types of discrete facility location problems, which both comprise different classes of discrete problems as reviewed in this research. However, the discrete facility location problems shown in deterministic and known information and data under uncertain, vague, and ambiguous environments have usually been solved using intuitionistic fuzzy approaches. Neutrosophic is recently applied to tackle the uncertainty and ambiguity of information and data. This paper considered solving the discrete facility location problems under the neutrosophic environment, wherein the information of the locations, distances, times, and costs is uncertain. The mathematical models for the main types of neutrosophic discrete facility location problems, which remain unclear till now despite previous related works, are formulated in this study. Numerical examples demonstrated testing of the neutrosophic discrete models and comparison with the optimization solutions obtained from the normal situations.

Keywords: Discrete Facility Problems; Neutrosophic Facility Problems; Neutrosophic Theory; Set Covering Facility Location Problems; Median Based Facility Location Problems

1. Introduction

Facility location problems (FLPs), which are also called location-allocation problems [8, 9, 31], are designed to choose the number of suggested facility locations to cover customer service. These customers are also called demand points or supply centers that must be served from specific facilities to minimize the total cost (distance/time). FLPs have been widely used for solving various types of allocation problems, such as transportation [4], healthcare [2, 20], and supply chain issues [6, 25, 26]. The efficiency of these types of allocating/assigning problems led to the design of different types of models, algorithms, and approaches [3, 15, 12, and 20]. Four classes of facility location problems are generally available: analytic allocation problems, network allocation problems, and continuous, and discrete types. The most common types of FLPs can be either continuous or discrete. In the continuous types, the facilities [39] can be chosen anywhere in a specific area; in the discrete types [11, 17, and 24], the facilities are in certain locations (Fig. 1).

Discrete facility location problems (DFLPs) are widely applied to solve the discrete type of facility problems and are classified into two types, namely covering- and median-based models. The DFLP is generally usually known in a deterministic environment, wherein the information or data are available and known. However, in many cases, this information or data of DFLPs is imprecise, vague, and uncertain [1, 19, 34]; thus, in this case, DFLPs need special models and algorithms for their solution. Vagueness and ambiguity in DFLPs are usually related to

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distance or time for delivery and cost parameters or even lack of information related to these types of problems [16, 18, and 21]. The FLPs were first considered in a fuzzy environment, and various studies have been established. [5, 6] demonstrated goal programming for multi and bi-objective FLPs under fuzzy situations. A novel fuzzy set theory based on factor rating and a simple additive weighting system is presented to solve the fuzziness of FLPs [37], while a fuzzy FLP with a value at risk is proposed based on the discretization of fuzzy variables to approximate the value at risk [36]. A fuzzy modeling method is presented based on a simulation method and a heuristic approach to solve the FLPs [22]. A two-stage fuzzy optimization model-based hybrid particle swarm approach is established to solve the three-level location-allocation problem [40]. A hybrid intelligent approach is produced based on integrating simple, random fuzzy simulation and genetic algorithm to solve the fuzzy facility location-allocation problems [23]. This paper is given an original insight into applying the neutrosophic models for the discrete facility problems, and the work of the author on this paper is unprecedented in the neutrosophic theory.

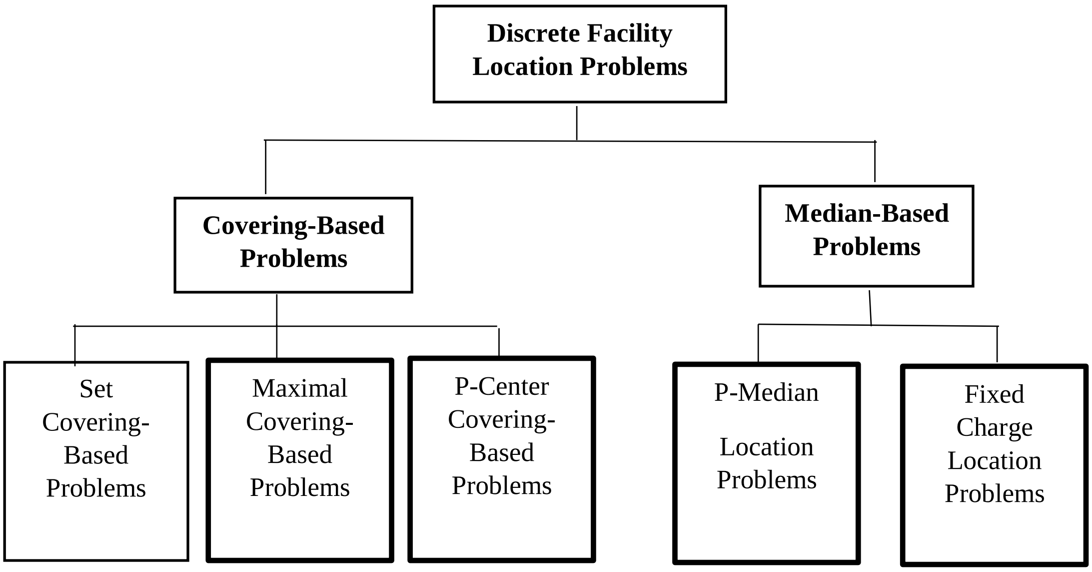

Figure 1: Diagram of the type of discrete facility location problems

This paper is organized as follows. Section 2 reviews the FLPs with their neutrosophic models. Section 3 shows the numerical examples. Section 4 concludes this paper.

2. Models of Discrete Facility Location Problems

FLPS are generally classified as integer linear programming problems (ILPPs). The classical ILPP is recalled as follows [25, 27]: $\frac{Min}{Max} Z = \sum_{j=1}^{n} c_j x_j$, (1)

S.t.

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \ , (\mathrm{i} = 1,2, \dots, \mathrm{m}), \quad (2)$$

$$x_j \geq 0 \ , (\mathrm{j} = 1,2, \dots, \mathrm{n}) \ , x_j \in \{0, 1\}, \text{are integers} \quad (3)$$

The neutrosophic integer linear programming problem (NILPP) model has been formulated as follows [25]:

$$Min/Max\ Z = \sum_{j=1}^{n} c_j^{\sim n} x_j, \quad (4)$$

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S.t.

$$\sum_{j=1}^{n} a_{ij}^{\sim n} x_j \le b_i \ , (\mathrm{i} = 1,2, \dots, \mathrm{n}). \qquad (5)$$

$$x_j \ge 0, (\mathrm{j} = 1,2, \dots, \mathrm{m}), x_j \in \{0, 1\} \text{ are integers, and } c_j^{\sim n}, a_{ij}^{\sim n} \text{ are neutrosophic numbers.} \qquad (6)$$

Notably, the neutrosophic numbers $c_j^{\sim n}, a_{nd\,aij}^{\sim n}$ are single-valued neutrosophic numbers (SVNNs) denoted by $A = \ (a,\ \mathrm{b},\ c)$, where a, b, $c \in [0,\ 1]$, and $a + b + c \le 3$. Herein, the SVNN is presented as a triangular neutrosophic number of a neutrosophic set on the real number set R as demonstrated in [27].

To solve the NILPP, the same algorithms for solving the integer programming models can be applied but under a neutrosophic environment. These algorithms are as follows: neutrosophic branch and bound neutrosophic cutting plane, and neutrosophic branch and price. Since then, the cutting plane method has been known to solve all types of integer programming problems, particularly small FLPs. Thus, the neutrosophic cutting plane [27] algorithm, which was first suggested to convert the neutrosophic integer programming numbers (such as $\tilde{a} = \langle (a,\ b,\ c\ ),\ \mu\tilde{a},\ v\tilde{a},\ \lambda\tilde{a}\ \rangle$) into their crisp values based on the following score function, will be applied [15, 27].

$$\mathrm{S}(\tilde{a}) = 1/16\ [a + b + c] \times (2 + \mu\tilde{a} - v\tilde{a} - \lambda\tilde{a}\ ). \qquad (7)$$

The formula aims to create a decision set, which includes the highest degree of truth membership, the least degree of falsity, and indeterminacy memberships. The problem is then solved as a relaxed linear programming problem while ignoring the integrality. In this case, if the optimal solution is an integer, then the solution is completed; otherwise, a constraint should be generated and added to the problem and steps will be repeated until the problem is solved.

2.1. Covering-Based Facility Location Problems

Covering-based facility problems are designed to solve the issues of covering demand/service points within a coverage distance/time from available facilities. The covering-based FLPs include the following three types: set covering, maximal covering, and p-center covering problems [2].

2.1.1. Set Covering Facility Location Problems (SCFLPs)

The objective function of the SCFLP, which was first suggested by Toregas et al. (1971), aims to minimize the number of located facilities that cover all demand points. This function already leads to the minimization of the total cost of distance traveled between production and consumption centers. The mathematical model of the SCFLP is as follows [2]:

$$Min\ Z = \sum_{j \in J} f_j x_j, \qquad (8)$$

S.t.

$$\sum_{j \in J} a_{ij} x_j \ \ge 1, \mathrm{i} \in \mathrm{I}, (\mathrm{i} = 1,2, \dots, \mathrm{n}), \qquad (9)$$

$$x_j \in \{0,1\}, \mathrm{j} \in \mathrm{J}, (\mathrm{j} = 1,2, \dots, \mathrm{m}), \qquad (10)$$

where x_j is a binary decision variable, which indicates that the facility is located at chosen location j∈J and 0 otherwise. Sets I and J refer to the demand points and available facility locations, respectively. The parameter a_{ij} represents the travel distance/time from the consumption centers to the available production centers. The objective function (8) aims to minimize the allocation cost of the chosen facilities to cover the demand points. Constraint (9) ensures that all demand points are covered, while Constraint (10) represents integrality constraint. The model of the neutrosophic set covering location problems, in this case, can be suggested as follows:

$$Min\ Z = \sum_{j \in J} f_j^{\sim n} x_j, \qquad (11)$$

S.t.

$$\sum_{j \in J} a_{ij} x_j \ \ge 1, \mathrm{i} \in \mathrm{I}, (\mathrm{i} = 1,2, \dots, \mathrm{n}), \qquad (12)$$

$$x_j \in \{0,1\} \ \ \mathrm{j} \in \mathrm{J}, (\mathrm{j} = 1,2, \dots, \mathrm{m}), \qquad (13)$$

where $f_j^{\sim n}$ is a trapezoidal neutrosophic number.

2.1.2. Maximal Covering Location Facility Problems

The second type of the set covering FLPs is known as maximal covering facility location problems (MCFLPs), which aims to find the maximum number of facilities to be built to cover the demand points. The mathematical model of this type of covering-based problem is as follows [7]:

$$Max\ Z = \sum_{i \in I} g_i\, y_i, \qquad (14)$$

S.t.

$$\sum_{j \in N_i} x_{ij} \geq y_i\ , \forall\, i \in I, \qquad (15)$$

$$\sum_{j \in J} x_{ij} \leq p, \qquad (16)$$

$$x_j \in \{0,1\}, y_j \in \{0,1\}. \qquad (17)$$

In the above model, the objective function (14) aims to maximize the total covered demand. Constraint (15) refers to the number of facilities that must be established. Constraint (16) ensures that demand points are covered only by the chosen locations of facilities. Constraint (17) refers to integrality.

where I is the set of demand points and J is the set of candidate facility locations. Parameter g_i refers to the distance/time from demand points $i \in I$ to candidate facility location $j \in J$. Decision variables $y_j = 1$ if a facility is established at candidate facility location $j \in J$ and equal to 0 otherwise. Meanwhile, decision variable x_{ij} = 1 if demand point $i \in I$ is covered by facility $j \in J$ and equal to 0 otherwise.

The neutrosophic model for the maximal covering location problems is as follows:

$$Max\ \mathrm{Z} = \sum_{i \in I} g_i^{\sim n}\, y_i, \qquad (18)$$

S.t.

$$\sum_{j \in N_i} x_{ij} \geq y_i\ , \forall\, i \in I, \qquad (19)$$

$$\sum_{j \in J} x_{ij} \leq p, \qquad (20)$$

$$x_j \in \{0,1\}, y_j \in \{0,1\}, \qquad (21)$$

where $g_i^{\sim n}$ is a trapezoidal neutrosophic number.

2.1.3. P-Center Facility Location Problems

The third type of set covering problems is the p-center facility location problem (P-CFLP) type, wherein the objective is to minimize the maximum travel distance/time between (or time) demand points and facilities to ensure that all demand points are covered by the chosen facilities. The mathematical model for this type of problem is as follows [35]:

$$\text{Min L} \qquad (22)$$

S.t.

$$\sum_{j \in J} y_{ij} = 1 \quad ,\mathrm{i} \in \mathrm{I} \qquad (23)$$

$$\sum_{j \in J} x_j = p \qquad (24)$$

$$\sum_{j \in J} d_{ij} y_{ij} \leq L\ , \mathrm{i} \in \mathrm{I} \qquad (25)$$

$$y_{ij} \leq x_j,\ \forall\ i \in I, j \in \mathrm{J} \qquad (26)$$

$$y_{ij} \in \{0,1\}, x_j \in \{0,1\} \forall\ i \in I, j \in \mathrm{J} \qquad (27)$$

$$\mathrm{L} \geq 0 \qquad (28)$$

where L refers to the maximum distance/time between demand points and the nearest facility, I is the set of demand points, and J is the set of candidate facility locations. Parameter d_{ij} refers to the distance/time from the demand points $i \in I$ to candidate facility location $j \in J$. Decision variables $y_{ij} = 1$ if demand point $i \in I$ is assigned to facility location $j \in J$ and is equal to 0 otherwise; meanwhile, decision variables x_j = 1 if facility location $j \in J$ is chosen to be built and is equal to 0 otherwise.

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The objective function (22) for the P-CFLP model is to minimize the maximum distance/time between demand points and the nearest facility. Constraint (23) ensures that each demand point is only covered by one facility. Constraint (24) refers to the total number of needed facilities, while Constraint (25) aims to allocate the maximum demand weighted distance/time. Constraints (26) ensure that demand points are only assigned to chosen facilities and finally Constraints (27, 28) are the domain constraints.

The neutrosophic model for the PCFLAP is as follows [25, 27]:

$$\text{Min } L \qquad (29)$$

S.t.

$$\sum_{j\in J} y_{ij} = 1 \quad ,\text{i} \in \text{I} \qquad (30)$$

$$\sum_{j\in J} x_j = P \qquad (31)$$

$$\sum_{j\in J} \tilde{d_{ij}}^{n} x_{ij} \leq \tilde{L} \ , \quad \text{i} \in \text{I} \qquad (32)$$

$$y_{ij} \leq x_j, \ \forall \ i \in I\,, j \in \text{J} \qquad (33)$$

$$y_{ij} \in \{0,1\}, \ x_j \in \{0,1\} \forall \ i \in I\,, j \in \text{J} \qquad (34)$$

$$\text{L} \geq 0 \qquad (35)$$

where $\tilde{d_{ij}}^{n}$ is a trapezoidal neutrosophic number.

2.2. Median-Based Facility Location Problems

Median-based facility location-allocation problems are the second type of DFLPs, which consider detecting facilities at median locations to minimize the distance/time cost between demand points and the facilities assigned to them. This type of problem comprises the following two types: p-median and fixed-charge median location problems [2, 15, 39].

2.2.1. p-Median Facility Location Problems

The first class of median-based FLPs is the p-median FLPs based on the chosen specific number of facilities to cover the demand requirements. The optimization model for this type of problem is represented as shown below [2]:

$$Min \sum_{i\in I} \sum_{j\in J} d_{ij} y_{ij}\,, \qquad (36)$$

S.t.

$$\sum_{j\in J} y_{ij} = 1\,.\ \text{i} \in \text{I}, \qquad (37)$$

$$\sum_{j\in J} x_j = p, \qquad (38)$$

$$y_{ij} \leq x_j, \quad i \in I\,, j \in \text{J}، \qquad (39)$$

$$y_{ij} \in \{0,1\}, x_j \in \{0,1\} \forall \ i \in I\,, j \in \text{J}. \qquad (40)$$

In the above model, the objective function (36) aims to minimize the total travel distance/time. Constraint (37) ensures that each point of demand is only assigned to a single facility location. Constraint (38) determines the number of required facilities. Constraint (39) limits the assignments of demand points only to the chosen facilities. Constraint (40) is domain constraint.

Set I is the set of demand points, J is the set of candidate facility locations, and p refers to the number of required facility locations. Parameter d_{ij} refers to the distance/time from demand points $i \in I$ to candidate facility location $j \in J$. Decision variables $y_{ij} = 1$ if demand point $i \in I$ is assigned to median facility location

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$j \in J$ and equal to 0 otherwise; meanwhile, decision variable $x_j = 1$ if median facility location $j \in J$ is chosen to be built and equal to 0 otherwise.
The neutrosophic model for the p-median location problems is then presented as follows:

$$Min\ Z = \sum_{i \in I} \sum_{j \in J} \tilde{d_{ij}}^{n} y_{ij}\ , \qquad (41)$$

S.t.

$$\sum_{j \in J} y_{ij} = 1\ , \qquad i \in I, \qquad (42)$$

$$\sum_{j \in J} x_j = P, \qquad (43)$$

$$y_{ij} \leq x_j, \quad i \in I\ , j \in J, \qquad (44)$$

$$y_{ij} \in \{0,1\}, \forall\ i \in I\ , j \in J، \qquad (45)$$

$$x_j \in \{0,1\}\ , \forall\ i \in I\ , j \in J, \qquad (46)$$

Where $\tilde{d_{ij}}^{n}$ is a trapezoidal neutrosophic number.

2.2.2. Fixed-Charge Facility Location Problems

This type of median FLP deals with related p-median problems [35, 38]. However, the difference lies in the similarity of the cost of locating any facilities; in this case, this difference will not affect the choice of the location. Choosing locations in this type of FLP is based on the travel distance (time) between median and non-median locations, which also indicates that the cost of chosen locations is the same. The demand is not explicitly considered; thus, the demand in all locations is also assumed to be equal. Similarly, the capacities are also disregarded; each median can be infinite or have the capacity to cover demand locations. Assume the presence of i facilities required to be located in points i with a fixed cost of locating a facility c_i and maximum production, which refers to the maximum production that this facility can produce. A unit cost transportation of c_{ij} aims to meet the demand Dj between facilities and demand locations. The decision variable $y_i = 1$ if location i is chosen as a facility and 0 otherwise. By contrast, the decision variable $x_{ij} = 1$ refers to the quantity transported from i facility to j demand location and 0 otherwise. The general optimization model for the fixed-charge location problems is as follows [34, 38]:

$$Min\ Z = \sum_{i \in I} f_j y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}, \qquad (47)$$

S.t.

$$\sum_{i \in I} y_i = p\ , i \in I, \qquad (48)$$

$$\sum_{j \in J} x_{ij}\ \leq C_i y_i,\ \forall\ i \in I, \qquad (49)$$

$$y_i \in \{0,1\}, \qquad \forall\ i \in I\ , j \in J, \qquad (50)$$

$$x_{ij} \geq\ 0, \forall\ i \in I, j \in J, \qquad (51)$$

In this model, the objective function (47) minimizes the total cost, which includes the fixed cost of facility opening in addition to the quantity transported. Constraint (48) refers to the number of required facilities, while Constraint (49) limits capacities to open facilities. Constraints (50) and (51) are integrality constraints.

The neutrosophic model of the fixed charge of FLPs becomes as follows:

$$Min\ Z = \sum_{i \in I} \tilde{f_j}^{n} y_j + \sum_{i \in I} \sum_{j \in J} \tilde{c_{ij}}^{n} x_{ij}, \qquad (52)$$

S.t.

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$$\sum_{i\in I} y_i = p \ , \mathrm{i} \in \mathrm{I}, \tag{53}$$

$$\sum_{j\in J} x_{ij} \ \le C_i y_i, \ \forall \mathrm{i} \in \mathrm{I}, \tag{54}$$

$$y_i \in \{0,1\}, \qquad \forall \mathrm{i} \in \mathrm{I}, \mathrm{j} \in \mathrm{J}, \tag{55}$$

$$x_{ij} \ge \ 0, \forall \mathrm{i} \in \mathrm{I}, \mathrm{j} \in \mathrm{J}, \tag{56}$$

where $f_j^{\sim n}, c_{ij}^{\sim n}$ is a trapezoidal neutrosophic number.

3. Numerical Experiments

Example 1: Neutrosophic set covering location problems

Assume that locating the number of facilities to cover the demand of Z areas is necessary. Times between the centers of demand areas are given in the time matrix a_{ij} as shown below:

$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & 4 & 12 & 6 & 15 & 10 & 8 \\ 2 & 8 & 0 & 15 & 60 & 7 & 2 & 3 \\ 3 & 20 & 13 & 0 & 8 & 6 & 5 & 9 \\ 4 & 9 & 11 & 8 & 0 & 9 & 10 & 3 \\ 5 & 50 & 8 & 4 & 10 & 0 & 2 & 27 \\ 6 & 30 & 5 & 7 & 9 & 3 & 0 & 27 \\ 7 & 8 & 5 & 9 & 7 & 25 & 27 & 0 \end{array}$$

Assume also that the costs of locations of facilities are as follows: 100, 80,120, 110, 90, 90, and 110. Moreover, the travel speed is assumed to be 60 mph and the response to each demand requirement should be within 8 min. The objective is to locate the minimum number of facilities to cover the demand of all areas to minimize the total cost.

Sol: Distance parameter matrix, which takes the value 1 if the time is less or equal to 8 or 0; otherwise, changes are made as shown below.

$$a_{ij} = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 5 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 6 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 7 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}$$

The integer programming model for the set covering FLPs is as follows:

$Min\ Z = 100x_1 + 80x_2 + 120x_3 + 110x_4 + 90x_5 + 90x_6 + 110x_7$

S.t.

$x_1 + x_2 + x_4 + x_7 \ge 1$

$x_1 + x_2 + x_5 + x_6 + x_7 \ge 1$

$x_3 + x_4 + x_5 + x_6 \ge 1$

$x_3 + x_4 + x_7 \ge 1$

$x_2 + x_3 + x_5 + x_6 \ge 1$

$x_2 + x_3 + x_5 + x_6 \ge 1$

$x_1 + x_2 + x_4 + x_7 \ge 1$

$x_j \in (0,1), j = 1,2, \dots, 7$ and x_j integer

Solving the above model using MatLab, the solution representing the minimum total cost is as follows:

$$x_1 = x_3 = x_5 = x_6 = x_7 = 0, x_2 = x_4 = 1, Z = 190$$

The neutrosophic integer programming model for the above example is as follows:

$Min\ Z = \widetilde{100}x_1 + \widetilde{80}x_2 + \widetilde{120}x_3 + \widetilde{110}x_4 + \widetilde{90}x_5 + \widetilde{90}x_6 + \widetilde{110}x_7$
S.t.
$x_1 + x_2 + x_4 + x_7 \geq 1$
$x_1 + x_2 + x_5 + x_6 + x_7 \geq 1$
$x_3 + x_4 + x_5 + x_6 \geq 1$
$x_3 + x_4 + x_7 \geq 1$
$x_2 + x_3 + x_5 + x_6 \geq 1$
$x_2 + x_3 + x_5 + x_6 \geq 1$
$x_1 + x_2 + x_4 + x_7 \geq 1$
$x_j \in (0,1), j = 1,2,\dots,7, x_j$ Binary integer

The above constraints can be written as shown below to simplify the constraints.

$$\sum_{j=1}^{7} a_{ij}x_j \geq 1\ , i = 1,2,\dots,7$$

The cost coefficients in the objective function are neutrosophic numbers, which are created as triangular neutrosophic numbers as explained in [27]. Thus, as explained in the neutrosophic cutting plane algorithm, the neutrosophic numbers are first investigated as follows:

$\widetilde{100} = \langle(50,100,150), 0.9, 0.6, 0.5\rangle$
$\widetilde{80}\ = \langle(60,80,100), 0.8, 0.6, 0.5\rangle$
$\widetilde{120} = \langle(100,120,140), 1,0.5,0\rangle$
$\widetilde{110} = \langle(100,110,120), 0.7,0.5,0.3\rangle$
$\widetilde{90} = \langle(70,90,110), 0.6,0.4,0.2\rangle$
$\widetilde{90} = \langle(70,90,110), 0.8,0.6,0.3\rangle$
$\widetilde{110} = \langle(100,110,120), 0.9,0.7,0.6\rangle$

The neutrosophic model is then converted into the crisp model as shown below using the score function (7).

$Min\ Z = 33.8x_1 + 27x_2 + 59.4\ x_3 + 18.6\ x_4 + 16.9x_5 + 15.2x_6 + 12.4x_7$
S.t.

$$\sum_{i=1}^{7} a_{ij}x_j \geq 1, i = 1,2,\dots,7$$

$x_j \in (0,1), j = 1,2,\dots,7$

Solving the above model involves the optimal solution $x_1 = x_2 = x_3 = x_4 = x_5 = 0$

while $x_6 = x_7 = 1$. Thus, areas 6 and 7 are chosen to locate the two facilities with a total minimum cost of 27.6. Comparison results of the two solutions of the normal and neutrosophic models reveal that the total cost is improved from the normal model (190) while that of the neutrosophic model is 27.6, and the chosen facilities are 6 and 7.

Example 2: Neutrosophic maximal covering facility location problems

Assume 10 demand centers and the number of 8 suggested facility locations with a radius of 1.5 (Figure 2) and coordinates as shown in Table 1.

Table 1: Coordinates of Demand Centers and Facilities

ID	X	Y	Demand	Facilities	X	Y
١	4.5	١٣	٢٣	A	2.5	12.5
٢	١١	١١	٢٥	B	6.5	13.5
٣	٩	١٠	٥٤	C	٥	8.5
٤	١٠	8.5	٧٤	D	٢	4.5
٥	١٠	7.5	١٣	E	٥	٥

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٦	٨	٥	٣٣	F	٩.٥	9.5
٧	٨	٤	٤٧	G	٨.٥	٦
٨	٣	٩	٩٢	H	١٠	10.5
٩	٣	٦	١٧			
١٠	١	٤	٣٩			

The objective is to determine which facilities can cover each demand center to maximize the covering demand.

Solution:

$Max\ z = 23y_1 + 25y_2 + 54y_3 + 74y_4 + 13y_5 + \ +33y_6 + 47y_7 + 92y_8 + 17y_9 + 39y_{10}$

S.t.

$0 \geq y_1, X_H \geq y_2,\ X_H + X_F \geq y_3, X_F \geq y_4$

$0 \geq y_5, X_G \geq y_6\ , 0 \geq y_7, 0 \geq y_8, 0 \geq y_9, X_D \geq y_{10}$

$X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H \leq p$

$x_j\ \in \{0,1\}, y_i\ \in \{0,1\},\ \ \mathrm{j = A\,, B\,, C\,, D\,, E\,, F\,, G\,, H\,;}\ i = (1,2, \ldots ,10)$

The optimal solution for the normal model (Figure 3) if $p = 1$ is $y_3 = y_4 = 1, X_F = 1$

$y_1 = y_2 = y_5 = y_6 = y_7 = y_8 = y_9 = y_{10} = 0$

$X_A = X_B = X_C = X_D = X_E = X_G = X_H = 0\ , Z = 128$

If $p = 2, then$ the solution will be Z = 167

$y_3 = y_4 = y_{10}\ = X_F = X_D = 1\ ,$

$y_1 = y_2 = y_5 = y_6 = y_7 = y_8 = y_9 = 0, X_A = X_B = X_C = X_E = X_G = X_H = 0.$

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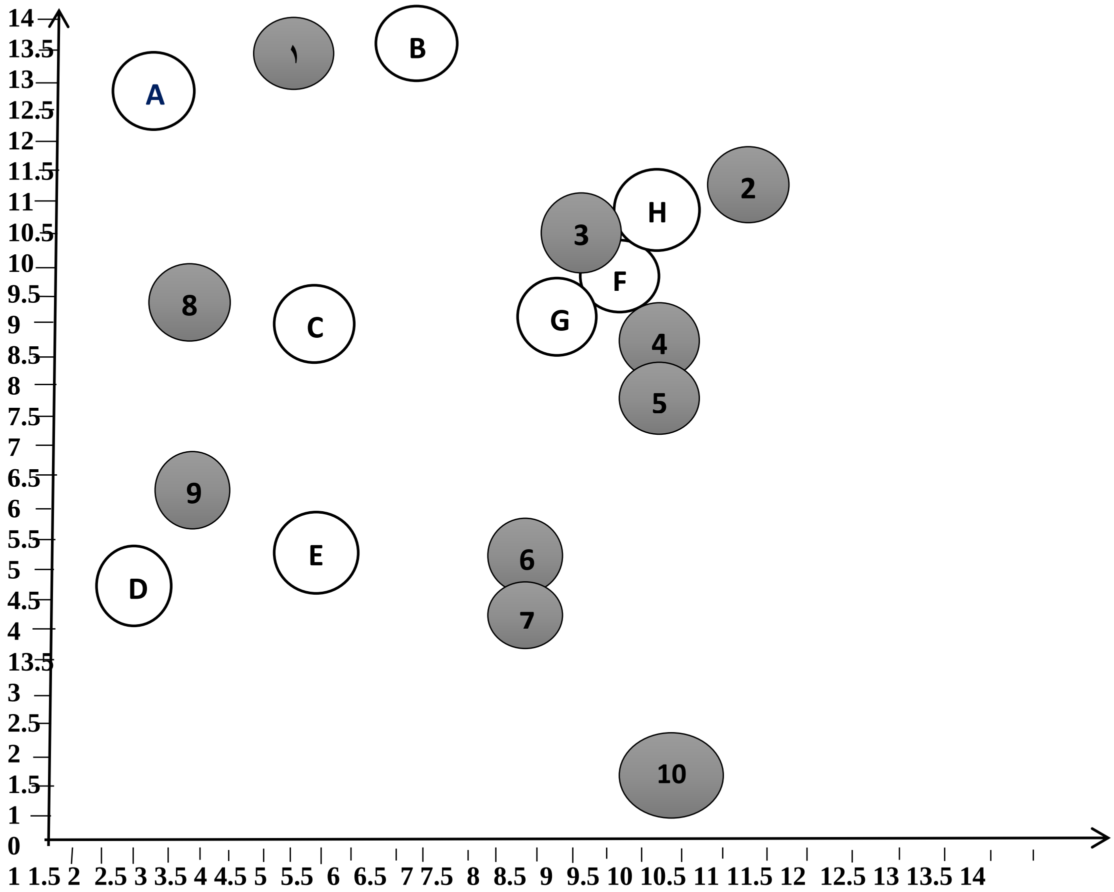

Figure 2: Coordinates of facilities and demand centers

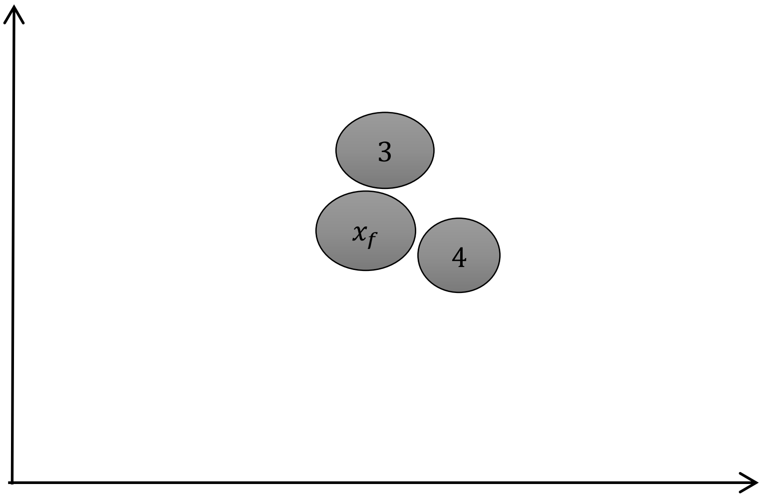

Figure 3: optimal facilities and demand centers locations

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The neutrosophic maximal covering location model, in this case, will be as follows:

$Max\ z = \widetilde{23}y_1 + \widetilde{25}y_2 + \widetilde{54}y_3 + \widetilde{74}y_4 + \widetilde{13}y_5 + \widetilde{33}y_6 + \widetilde{47}y_7 + \widetilde{92}y_8 + \widetilde{17}y_9 + \widetilde{39}y_{10}$

S.t

$0 \geq y_1, X_H \geq y_2,\ X_H + X_F \geq y_3, X_F \geq y_4$

$0 \geq y_5, X_G \geq y_6\ , 0 \geq y_7, 0 \geq y_8, 0 \geq y_9, X_D \geq y_{10}$

$X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H \leq p$

$x_j\ \in \{0,1\}, y_i\ \in \{0,1\},\ \ j = A, B, C, D, E, F, G, H\ ; i = (1,2, \dots, 10)$

The cost coefficients in the objective function are neutrosophic numbers; thus, as explained in the neutrosophic cutting plane algorithm, the neutrosophic numbers are first investigated as follows:

$\widetilde{23} = \langle (18,23,28), 0.9, 0.6, 0.5 \rangle$

$\widetilde{25}\ = \langle (20,25,30), 0.8, 0.6, 0.5 \rangle$

$\widetilde{54} = \langle (48,54, 60), 1,0.5,0 \rangle$

$\widetilde{74} = \langle (68,74,80), 0.7,0.5,0.3 \rangle$

$\widetilde{13} = \langle (9,13,20), 0.6,0.4,0.2 \rangle$

$\widetilde{33} = \langle (28,33,40), 0.8,0.6,0.3 \rangle$

$\widetilde{47} = \langle (40,47,55), 0.9,0.7,0.6 \rangle$

$\widetilde{92} = \langle (75,92,100), 0.6,0.4,0.2 \rangle$

$\widetilde{17} = \langle (10,17,25), 0.8,0.6,0.3 \rangle$

$\widetilde{39} = \langle (25,39,50), 0.9,0.7,0.6 \rangle$

The neutrosophic model is then converted into the crisp model using the score function(7), and the objective function is presented as shown below:

$Max\ Z = 7y_1 + 8y_2 + 25.3y_3 + 29y_4 + 5.3y_5 + 12y_6 + 14y_7 + 33.4y_8 + 6y_9 + 11.4y_{10}$

The constraints are the same as in the neutrosophic model. When the crisp model is solved, then the optimal solution if $p = 1$ is:

$Z = 54, y_3 = y_4 = 1\ , X_F = 1$

$y_1 = y_2 = y_5 = y_6 = y_7 = y_8 = y_9 = y_{10} = 0$

$X_A = X_B = X_C = X_D = X_E = X_G = X_H = 0$

When $p = 2$, the solution will be $\mathrm{Z} = 66.3$

$y_3 = y_4 = y_6 = 1\ , X_F = X_G = 1$

$y_1 = y_2 = y_5 = y_7 = y_8 = y_9 = y_{10} = 0$

$X_A = X_B = X_C = X_D = X_E = X_H = 0$

Example 3: Neutrosophic p-center facility location problems

Assume the number of 1–4 demand centers and **5-7** as suggested facilities as in T with an acceptance travel distance of 20 min and distance information as in Table (2).

Table 2: Distances between Demand Centers and Facilities

F D	5	6	7
١	20	25	60
٢	30	15	30
٣	30	15	30
٤	30	15	10

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The objective is to assign demand centers to facilities to minimize the maximum traveling time.

Sol:

The mathematical model for this p-center example when p = 2 is as follows:

Min. L
S.t.
$x_5 + x_6 + x_7 = 2$
$y_{15} + y_{16} + y_{17} = 1$
$y_{25} + y_{26} + y_{27} = 1$
$y_{35} + y_{36} + y_{37} = 1$
$y_{45} + y_{46} + y_{47} = 1$
$y_{15} \le x_5, y_{25} \le x_5, y_{35} \le x_5, y_{45} \le x_5$
$20\, y_{15} + 25y_{16} + 60y_{17} \le \mathrm{L}$
$30y_{25} + 15y_{26} + 30y_{27} \le \mathrm{L}$
$30y_{35} + 15y_{36} + 30y_{37} \le \mathrm{L}$
$30y_{45} + 15y_{46} + 10y_{47} \le \mathrm{L}$
$x_5, x_6, x_7 \ \in \{0,1\},\ \ y_{ij} \ \in \{0,1\}, \forall\, i \ \in I, j \ \in J$

The optimal solution when p = 2 is L = 20 min, $x_5 = x_6 = 1$, $x_7 = 0$
$y_{15} = y_{26} = y_{36} = 1$, while all other $y_{ij} = 0$.

Thus, facility 5 has been assigned to demand center 1 while facility 6 has been assigned to demand centers 2, 3, and 4.

The above example is then considered to be a neutrosophic p-center location problem; in this case, the neutrosophic model is as follows:

Min. L

S.t.

$x_5 + x_6 + x_7 = 2$

$y_{15} + y_{16} + y_{17} = 1$

$y_{25} + y_{26} + y_{27} = 1$

$y_{35} + y_{36} + y_{37} = 1$

$y_{45} + y_{46} + y_{47} = 1$

$y_{15} \le x_5, y_{25} \le x_5, y_{35} \le x_5, y_{45} \le x_5$

$\widetilde{20}\, y_{15} + \widetilde{25}y_{16} + \widetilde{60}y_{17} \le \mathrm{L}$

$\widetilde{30}y_{25} + \widetilde{15}y_{26} + \widetilde{30}y_{27} \le \mathrm{L}$

$\widetilde{30}y_{35} + \widetilde{15}y_{36} + \widetilde{30}y_{37} \le \mathrm{L}$

$\widetilde{30}y_{45} + \widetilde{15}y_{46} + \widetilde{10}y_{47} \le \mathrm{L}$
$x_5 + x_6 + x_7 \ \in \{0,1\},\ \ y_{ij} \ \in \{0,1\}, \forall\, i \ \in \boldsymbol{I}, \boldsymbol{j} \ \in \boldsymbol{J}$

The neutrosophic numbers are considered and converted into crisp numbers as follows:
$\widetilde{20} = \langle (15,20,35), 0.8, 0.6, 0.5 \rangle$
$\widetilde{25} = \langle (18,25,30), 1,0.5,0 \rangle$
$\widetilde{60} = \langle (50,60,80), 0.7,0.5,0.3 \rangle$
$\widetilde{30} = \langle (20,30,45), 0.6,0.4,0.2 \rangle$
$\widetilde{15} = \langle (10,15,25), 0.8,0.6,0.3 \rangle$
$\widetilde{30} = \langle (20,30,45), 0.9,0.7,0.6 \rangle$
$\widetilde{30} = \langle (18,30,40), 0.6,0.4,0.2 \rangle$
$\widetilde{15} = \langle (10,15,25), 0.8,0.6,0.3 \rangle$
$\widetilde{30} = \langle (20,30,50), 0.9,0.7,0.6 \rangle$

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$\widetilde{30} = \langle (18,30,40), 0.6,0.4,0.2 \rangle$
$\widetilde{15} = \langle (10,15,25), 0.8,0.6,0.3 \rangle$
$\widetilde{10} = \langle (5,10,20), 0.9,0.7,0.6 \rangle$

The neutrosophic model can be converted into the crisp model as shown below.

Min. L
S.t.
$x_5 + x_6 + x_7 = 2$
$y_{15} + y_{16} + y_{17} = 1$
$y_{25} + y_{26} + y_{27} = 1$
$y_{35} + y_{36} + y_{37} = 1$
$y_{45} + y_{46} + y_{47} = 1$
$y_{15} \le x_5, y_{25} \le x_5, y_{35} \le x_5, y_{45} \le x_5$
$7.4\, y_{15} + 11.4 y_{16} + 22.6 y_{17} \le \mathrm{L}$
$11.9 y_{25} + 5.9 y_{26} + 9.5 y_{27} \le \mathrm{L}$
$11 y_{35} + 5.9 y_{36} + 10 y_{37} \le \mathrm{L}$
$11 y_{45} + 5.9 y_{46} + 3.5 y_{47} \le \mathrm{L}$
$x_5, x_6, x_7 \in \{0,1\},\ \ y_{ij} \in \{0,1\}, \forall\, i \in I, j \in J$
The optimal solution when p = 2 is L = 15 min, $x_5 = x_6 = 1$, $x_7 = 0$
$y_{15} = y_{26} = y_{36} = 1$, while all other $y_{ij} = 0$.

Example 4: Neutrosophic p-median facility location problems

Assume six demand centers, and the distance matrix is shown below:

$$d_{ij} = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 20 & 18 & 14 & 16 & 12 \\ 2 & 20 & 0 & 22 & 18 & 30 & 26 \\ 3 & 18 & 22 & 0 & 32 & 20 & 22 \\ 4 & 14 & 18 & 32 & 0 & 20 & 22 \\ 5 & 16 & 30 & 20 & 20 & 0 & 30 \\ 6 & 12 & 26 & 22 & 22 & 30 & 0 \end{array}.$$

The objective is to assign demand centers to facilities to minimize the demand weighted total travel distances.

Sol: The mathematical model for the above p-median problem example when p = 2 is as follows:

$$Min\ Z = \sum_{i=1}^{6} \sum_{j=1}^{6} d_{ij}\, y_{ij}$$

S.t.

$$\sum_{j=1}^{6} x_j = 2, \qquad \mathrm{j} \in \mathrm{J}$$

$$\sum_{j=1}^{6} y_{ij} = 1, \qquad \mathrm{i} \in \mathrm{I}$$

$y_{ij} \le x_j,\quad i \in I, j \in J$
$x_j \in \{0,1\}, \forall\, j \in J;\ \ y_{ij} \in \{0,1\}, \forall\, i \in I, j \in J$

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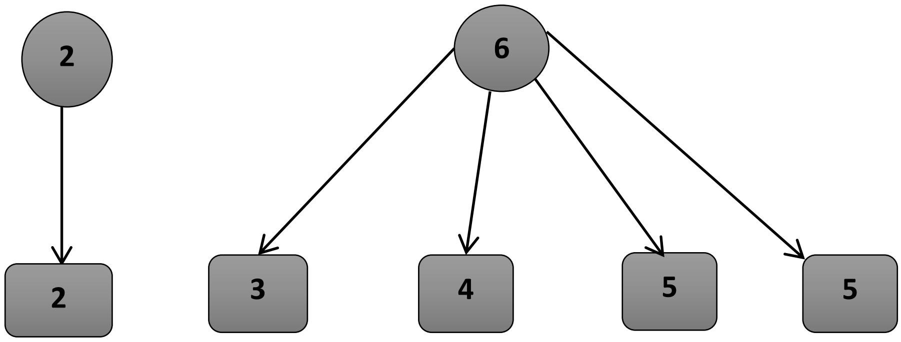

Figure 4: Optimal solution to p-median facility location problems

The optimal solution when p = 2 is Z = 60, $x_1 = 1, x_2 = 1$, which means that locations 1 and 2 are chosen as median facility locations;

$y_{13} = y_{14} = y_{15} = y_{16} = y_{22} = 1$, which indicates that demand locations 3, 4, 5, and 6 are assigned to facility 1 while demand location 2 is assigned to facility 2 (Figure 4).

Next, the neutrosophic case is considered, and the model will be as follows:

$$Min\ Z = \sum_{i=1}^{6}\sum_{j=1}^{6} d_{\widetilde{ij}}^{\,n} y_{ij}$$

S.t.

$$\sum_{j=1}^{6} x_j = 2, \qquad j \in J$$

$$\sum_{j=1}^{6} y_{ij} = 1, \qquad i \in I$$

$$y_{ij} \le x_j, \quad i \in I, j \in J$$

$$x_j \in \{0,1\}, \forall j \in J; \quad y_{ij} \in \{0,1\}, \forall i \in I, j \in J$$

The neutrosophic case applied for the above model for the distances $d_{\widetilde{ij}}^{\,n}$.

The optimal solution when p = 2 is Z = 21.5, $x_1 = 1, x_2 = 1$, which means that locations 1 and 3 are chosen as median facility locations;

$y_{13} = y_{14} = y_{15} = y_{16} = y_{22} = 1$, which indicates that demand locations 3, 4, 5, and 6 are assigned to facility 1 while demand location 2 is assigned to facility 2. Thus, the difference between the solution of the normal and neutrosophic models is the total cost, which is less in the neutrosophic case. This condition is normal because the application of Equation (4) will reduce the distances between demand locations, which is normally the total cost obtained from the solution (Figure 5).

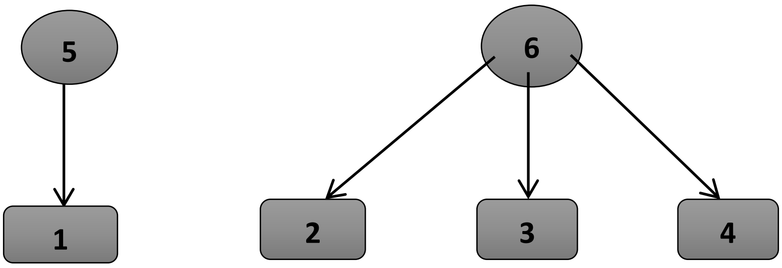

Figure 5: Optimal solution to neutrosophic p-median facility location problems

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Example 5: Neutrosophic fixed-charge facility location problems

Assume six demand centers and four available facility centers with capacity and fixed cost as provided in Table (3).

Table 3:

Facilities	**Demand Centers**						**Fixed Cost**	**Capacity**
	D1	D2	D3	D4	D5	D6		
F1	25	180	230	300	40	120	5000	250
F2	180	25	40	110	180	40	7500	200
F3	230	40	25	40	180	80	9000	350
F4	300	110	40	25	180	120	10000	450
Demand	100	90	125	150	80	75		

Information on Demand Centers and Facilities

The objective is to minimize the total cost, including the fixed cost of facility opening and the quantity transported cost.

Solution:

The objective is to minimize the total cost of fixed and transportation costs, and the mathematical model will be the same as below:

Min. Z = $5000y_1 + 7500y_2 + 9000y_3 + 10000y_3 + 25x_{11} + 180x_{12} + \cdots + 180x_{46}$

S.t.

$x_{11} + x_{21} + x_{31} + x_{41} = 100$

$x_{12} + x_{22} + x_{32} + x_{42} = 90$

$x_{13} + x_{23} + x_{33} + x_{43} = 125$

$x_{14} + x_{24} + x_{34} + x_{44} = 150$

$x_{15} + x_{25} + x_{35} + x_{45} = 80$

$x_{16} + x_{26} + x_{36} + x_{46} = 75$

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} \leq 250y_1$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} \leq 200y_2$

$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} \leq 350y_3$

$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} \leq 450y_4$

$x_{ij} \geq 0, \forall\, i = 1,2,3,4;\ \boldsymbol{j} = 1,2,3,4,5,6$

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$y_i \in \{0,1\}, \forall\, i \in I, j \in J$

The optimal solution for this example is Z = 36175, and Open Facilities are F1 and F3. Costs of transported are coefficients of the following: $x_{11}, x_{15}, x_{32}, x_{33}, x_{36}, x_{44}$

Next, the example is solved for the neutrosophic model, as demonstrated below.

Min. Z = $\widetilde{5000}y_1 + \widetilde{7500}y_2 + \widetilde{9000}y_3 + \widetilde{10000}y_4 + \widetilde{25}x_{11} + \widetilde{180}x_{12} + \cdots + \widetilde{180}x_{46}$

S.t.

$x_{11} + x_{21} + x_{31} + x_{41} = 100$

$x_{12} + x_{22} + x_{32} + x_{42} = 90$

$x_{13} + x_{23} + x_{33} + x_{43} = 125$

$x_{14} + x_{24} + x_{34} + x_{44} = 150$

$x_{15} + x_{25} + x_{35} + x_{45} = 80$

$x_{16} + x_{26} + x_{36} + x_{46} = 75$

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} \leq 250y_1$

$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} \leq 200y_2$

$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} \leq 350y_3$

$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} \leq 450y_4$

$x_{ij} \geq 0, \forall\, i = 1,2,3,4; \boldsymbol{j} = 1,2,3,4,5,6$

$y_i \in \{0,1\}, \forall\, i \in I, j \in J$

The optimal solution of the neutrosophic case of the example is Z = 24285, Open Facilities are F1 and F4, and transport costs are as follows: $x_{11} = 25, x_{15} = 40, x_{22} = 25, x_{26} = 40, x_{33} = 25, x_{34} = 25.$

4. Conclusion

DFLPs is the common type of FLPs, wherein the decision of choosing the number of facilities to cover the service demand is required. This type of FLP aims to find the specific number (minimum or maximum) of locations of facilities to be established to serve customers as represented by demand centers. In some cases, information and data on facilities, demand centers, and distance/time are deterministic, while the information and data related to facilities and demand points are vague and ambiguous in other cases. This paper designed neutrosophic models for the different discrete facility problems to solve uncertain and vague situations. Numerical examples are illustrated to test the neutrosophic nature of the discrete facility location models and compared them with the normal ones. In the future, additional neutrosophic discrete facility problems can be considered by including constraints to the models based on the problem requirement. Thus, various types of FLPs under uncertain (neutrosophic) situations, such as continuous, analytic, and network types, can also be considered for future work.

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