Application of MBJ - Neutrosophic Set on Filters of Incline Algebra

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Abstract

The concept of fuzzy set has been generalized to many kinds and one among them is neutrosophic set which is a developed from intuitionistic fuzzy set by adding a components called indeterminate function between truth and falsity membership function. This neutrosophic set also moved a step forward and shows a variation in indeterminate function alone as an interval valued indeterminate function and other functions remains same and this is named as MBJ - neutrosophic set. Here, this work on the MBJ - neutrosophic set merged with incline algbera and introduces the idea of MBJ - neutrosophic filter of an incline algebraic structure and is engrossing with several results. Also, the level, cartesian product, projection, characteristic function and chain conditions are also disputed.

Keywords: Incline algebra; Filter, MBJ - neutrosophic set; MBJ - neutrosophic filter; the level; projection and product on MBJ - neutrosophic filter.

AMS Classifications: 16Y60, 16Y99.

1 Introduction

This paper work on Incline algebra which is derived from the boolean algebra was originated by a Chinese Cybernetics expert Cao Zhiqiang.[?] This create a path to merge algebras of ordered pair structures which indicates the degree of intensity with two relations. Υ is an incline of the frame with associativity under addition, commutativity under addition, right and left distributivity under multiplication. Also, it is a semiring and a poset structure. This theory has been demonstrated from semiring and lattice theory, these are formed from the ideals in a ring or semigroup, as do the topologizing filters in a ring. Cao. Z. Q, Kim and Roush[?,5] used incline and incline matrix as a tool in different areas such as monograph , automata theory and so on. The concept of quotient incline was investigated by Ahn et al.[1] As a consequence several theories like probability, fuzzy sets, intuitionistic fuzzy sets, have been evolved in this way.

Lotfi A. Zadeh[14] proposed the idea of fuzzy set(FS) which is mathematically characterized by single components in the universe of discourse and a value interpreting the degree of truth funciton. Ranging the values truth grades from $[0, 1]$. This is a generalization of classical sets. Zadeh made the researches to focus out in this FS and many new types of fuzzy set has been evoked from this fuzzy set. At first, K.T. Atanassov[3] initiated the concept of intuitionistic fuzzy set(IFS) in this a new membership function(MF) has been brought and called as non - membership / falsity function(NMF) and this set has been developed to the great extend. Later on F. Smarandache[8] put forward an another membership function called indeterminate / neutrality function(IMF) in between truth and falsity membership function and named it as neutrosophic set(NS). This neutrosophic set has been sub - divided by showing a sight variation in indeterminate function interms of interval valued

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indeterminate function(IVIMF) and other two function remains the same and this set is called as MBJ - neutrosophic set by M. M. Talkalloo and Y. B. Jun.[10]

Introducing the fuzzy set(FS) in incline algebra and arises a new concept called fuzzy substructures of incline algbera was brought out by Jun[4] and the substructure such as fuzzy subincline and ideal, filter and so on. Later on, this has been extended to other FS namely (IFS)intuitionistic fuzzy set, interval valued and so on. As an inspiration this paper work on incline algebra interlinking with MBJ - NS and defines the content of MBJ - Neutrosophic filter(NF) with an example and discusses with an examples and interesting results like intersection and so on. Also, the level set on MBJ - neutrosophic filter is defined with an example. Further talks about the projection, cartesian product, characterisitc function and chain conditions of MBJ - neutrosophic filter of incline algebra.

The present paper is characterized into several section **Section 1** presents the introduction and motivational of the work. **Section 2** gives the fundamental definition to move forward. **Section 3** defines the layout of MBJ - NF on incline algebra with example thereby the concept of level is also discussed here. Further, **Section 4** introduces some more sub - divisions of MBJ - NF. Finally, **Section 5** ends with the conclusion of the work.

2 Preliminaries

The basic definitions of incline algebra and fuzzy sets are investigated here in this part.

Definition 2.1. [2,11] Incline algebra is an non - empty set $(\Upsilon, \oplus, \otimes)$ if $\forall\, \mathfrak{x}, \mathfrak{y} \in \Upsilon$,
(i) Commutative and associative under $\oplus$.
(ii) Associative under $\otimes$ and distributive (both left and right) under $\oplus$
(iii) $\mathfrak{x}_o \oplus \mathfrak{x}_o = \mathfrak{x}_o$ (idempotent)
(iv) $\mathfrak{x}_o \oplus (\mathfrak{x}_o \otimes \mathfrak{y}_o) = \mathfrak{x}_o$
(v) $\mathfrak{y}_o \oplus (\mathfrak{x}_o \otimes \mathfrak{y}_o) = \mathfrak{y}_o$.

Note:
In an incline algebra the partial order is defined as $\mathfrak{x}_o \leq \mathfrak{y}_o \Leftrightarrow \mathfrak{x}_o \oplus \mathfrak{y}_o = \mathfrak{y}_o$.
The notation $\curlywedge$ represents min, $\overline{\curlywedge}$ as rmin, $\curlyvee$ as max and the representation of $\geq$, $\leq$ in membership functions are $\succeq$ and $\preceq$ respectively whereas in terms they are treated as the same.

Definition 2.2. [2,11] $\mathfrak{H}$ is a not empty subset in incline Υ which is known as subincline of Υ if it is closed under addition and multiplication.

Definition 2.3. [2,13] A subincline $\mathfrak{H}$ of Υ is said to be filter of $\mathfrak{H}$ if it is an upper set, that is $\mathfrak{x}_o \in \mathfrak{H}, \mathfrak{y}_o \in \Upsilon$ & $\mathfrak{x}_o \leq \mathfrak{y}_o$ then $\mathfrak{y}_o \in \mathfrak{H}$.

Definition 2.4. [4] A structure $\mathcal{A} = \{\mathfrak{x}_o, \rho_{\mathcal{A}}(\mathfrak{x}_o) : \mathfrak{x}_o \in \Upsilon\}$ is said to be a fuzzy subincline of Υ if $\rho_{\mathcal{A}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) \curlywedge \rho_{\mathcal{A}}(\mathfrak{x}_o \otimes \mathfrak{y}_o) \succeq \zeta_{\mathcal{A}}(\mathfrak{x}_o) \curlywedge \rho_{\mathcal{A}}(\mathfrak{y}_o) \,\forall\, \mathfrak{x}_o, \mathfrak{y}_o \in \Upsilon$.

Definition 2.5. [14] A function ρ is a mapping from $\mathfrak{U}$ to $[0, 1]$ is referred as an fuzzy set(FS) in a univere $\mathfrak{U}$ where $\rho(\mathfrak{x})$ is the truth value of $\mathfrak{x}_o$, $\forall\, \mathfrak{x}_o \in \mathfrak{U}$.

Definition 2.6. [3] An Intuitionistic FS $\mathcal{B}$ in $\mathfrak{U}$ in the structure of $\{\mathfrak{x}_o, \rho_{\mathcal{B}}(\mathfrak{x}_o), \upsilon_{\mathcal{B}}(\mathfrak{x}_o)/\mathfrak{x}_o \in \mathfrak{U}\}$ where $\rho_{\mathcal{B}} : \mathfrak{U} \to [0, 1]$ as MF and $\upsilon_{\mathcal{B}} : \mathfrak{U} \to [0, 1]$ to be as NMF satisfying $0 \leq \rho_{\mathcal{B}}(\mathfrak{x}_o) + \upsilon_{\mathcal{B}}(\mathfrak{x}_o) \leq 1,\ \forall\, \mathfrak{x}_o \in \mathfrak{U}$.

Definition 2.7. [8] A structure $\mathcal{R} = \{\mathfrak{x}_o, \rho_{\mathcal{R}}(\mathfrak{x}_o), \zeta_{\mathcal{R}}(\mathfrak{x}_o), \upsilon_{\mathcal{R}}(\mathfrak{x}_o)/\mathfrak{x}_o \in \mathfrak{U}\}$ on the universe $\mathfrak{U}$ is called neutrosophic set where $\rho_{\mathcal{R}}, \zeta_{\mathcal{R}}, \upsilon_{\mathcal{R}} : \mathfrak{U} \to [0, 1]$ and $\rho_{\mathcal{R}}$ as TMF, $\zeta_{\mathcal{R}}$ represents IMF and $\upsilon_{\mathcal{R}}$ to be FMF.

Definition 2.8. [10] An MBJ - neutrosophic set $\mathcal{R}$ on $\mathfrak{U}$ is defined as $\mathcal{R} = \{\mathfrak{x}_o, \rho_{\mathcal{R}}(\mathfrak{x}_o), \overline{\zeta}_{\mathcal{R}}(\mathfrak{x}_o), \upsilon_{\mathcal{R}}(\mathfrak{x}_o)/\mathfrak{x}_o \in \mathfrak{U}\}$, $\rho_{\mathcal{R}}, , \upsilon_{\mathcal{R}} : \mathfrak{U} \to [0, 1], \overline{\zeta}_{\mathcal{R}} : \mathfrak{U} \to D[0, 1]$ such that $\rho_{\mathcal{R}}$ is TMF, $\overline{\zeta}_{\mathcal{R}}$ as IIMF and $\upsilon_{\mathcal{R}}$ to be FMF.

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3 MBJ - Neutrosophic Filter of Incline

This segment introduces the frame of MBJ - NF on incline algebra and related outcomes are disputed.

Definition 3.1. The collection $\mathfrak{M} = \{\mathfrak{x}_\circ, \rho_{\mathfrak{M}}(\mathfrak{x}_\circ), \zeta_{\mathfrak{M}}(\mathfrak{x}_\circ), \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ)/\mathfrak{x}_\circ \in \Upsilon\}$ be a neutrosophic subincline of a incline algebra Υ if
(i) $\rho_{\mathfrak{M}}(\mathfrak{x}_\circ \oplus \mathfrak{y}_\circ) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{x}_\circ \otimes \mathfrak{y}_\circ) \succeq \rho_{\mathfrak{M}}(\mathfrak{x}_\circ) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{y}_\circ)$
(ii) $\zeta_{\mathfrak{M}}(\mathfrak{x}_\circ \oplus \mathfrak{y}_\circ) \curlywedge \zeta_{\mathfrak{M}}(\mathfrak{x}_\circ \otimes \mathfrak{y}_\circ) \succeq \zeta_{\mathfrak{M}}(\mathfrak{x}_\circ) \curlywedge \zeta_{\mathfrak{M}}(\mathfrak{y}_\circ)$
(iii) $\upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ \oplus \mathfrak{y}_\circ) \curlyvee \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ \otimes \mathfrak{y}_\circ) \preceq \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ) \curlyvee \upsilon_{\mathfrak{M}}(\mathfrak{y}_\circ)$

Definition 3.2. The neutrosophic set $\mathfrak{M}$ of incline algbera is said to be MBJ - neutrosophic subincline of Υ if,
(i) $\rho_{\mathfrak{M}}(\mathfrak{x}_\circ \oplus \mathfrak{y}_\circ) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{x}_\circ \otimes \mathfrak{y}_\circ) \succeq \rho_{\mathfrak{M}}(\mathfrak{x}_\circ) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{y}_\circ)$,
(ii) $\overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_\circ \oplus \mathfrak{y}_\circ) \overline{\curlywedge}\, \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_\circ \otimes \mathfrak{y}_\circ) \succeq \zeta_{\mathfrak{M}}(\mathfrak{x}_\circ) \overline{\curlywedge}\, \zeta_{\mathfrak{M}}(\mathfrak{y}_\circ)$,
(iii) $\upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ \oplus \mathfrak{y}_\circ) \curlyvee \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ \otimes \mathfrak{y}_\circ) \preceq \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ) \curlyvee \upsilon_{\mathfrak{M}}(\mathfrak{y}_\circ)$,
for all $\mathfrak{x}_\circ, \mathfrak{y}_\circ \in \Upsilon$.

Definition 3.3. Let $\mathfrak{M}$ be a fuzzy subincline of Υ is known as fuzzy filter if $\rho_{\mathfrak{M}}(\mathfrak{y}_\circ) \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_\circ)$ when $\mathfrak{y}_\circ \leq \mathfrak{x}_\circ,\ \forall\, \mathfrak{x}_\circ, \mathfrak{y}_\circ\ \in \Upsilon$.

Definition 3.4. A neutrosophic subincline $\mathfrak{M} = \{\mathfrak{x}_\circ, \rho_{\mathfrak{M}}(\mathfrak{x}_\circ), \zeta_{\mathfrak{M}}(\mathfrak{x}_\circ), \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ)/\mathfrak{x}_\circ \in \Upsilon\}$ is referred as NF of Υ if,
$(i)\rho_{\mathfrak{M}}(\mathfrak{y}_\circ) \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_\circ)$
$(ii)\zeta_{\mathfrak{M}}(\mathfrak{y}_\circ) \preceq \zeta_{\mathfrak{M}}(\mathfrak{x}_\circ)$
$(iii)\upsilon_{\mathfrak{M}}(\mathfrak{y}_\circ) \succeq \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ)$ whenever $\mathfrak{y}_\circ \leq \mathfrak{x}_\circ,\ \forall\, \mathfrak{x}_\circ, \mathfrak{y}_\circ \in \Upsilon$.

Definition 3.5. A neutrosophic subincline $\mathfrak{M}$ of Υ is $\{\mathfrak{x}_\circ, \rho_{\mathfrak{M}}(\mathfrak{x}_\circ), \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_\circ), \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ)/\mathfrak{x}_\circ \in \Upsilon\}$ and is said to be MBJ - NF of Υ if,
$(i)\rho_{\mathfrak{M}}(\mathfrak{y}_\circ) \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_\circ)$
$(ii)\overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_\circ) \preceq \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_\circ)$
$(iii)\upsilon_{\mathfrak{M}}(\mathfrak{y}_\circ) \succeq \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ)$ whenever $\mathfrak{y}_\circ \leq \mathfrak{x}_\circ,\ \forall\, \mathfrak{x}_\circ, \mathfrak{y}_\circ \in \Upsilon$.

Example 3.6. An incline algbera $\Upsilon = \{\varepsilon^*, \mathfrak{a}^*, \mathfrak{b}^*, \mathfrak{c}^*\}$ with $\oplus$ and $\otimes$ defined by cayley's tables.

$\oplus$	ε^*	$\mathfrak{a}^*$	$\mathfrak{b}^*$	$\mathfrak{c}^*$
ε^*	ε^*	$\mathfrak{a}^*$	$\mathfrak{b}^*$	$\mathfrak{c}^*$
$\mathfrak{a}^*$	ε^*	ε^*	ε^*	ε^*
$\mathfrak{b}^*$	$\mathfrak{b}^*$	$\mathfrak{a}^*$	$\mathfrak{b}^*$	$\mathfrak{a}^*$
$\mathfrak{c}^*$	$\mathfrak{c}^*$	$\mathfrak{a}^*$	$\mathfrak{a}^*$	$\mathfrak{b}^*$

$\otimes$	ε^*	$\mathfrak{a}^*$	$\mathfrak{b}^*$	$\mathfrak{c}^*$
ε^*	ε^*	ε^*	ε^*	ε^*
$\mathfrak{a}^*$	ε^*	$\mathfrak{a}^*$	$\mathfrak{b}^*$	$\mathfrak{c}^*$
$\mathfrak{b}^*$	ε^*	$\mathfrak{b}^*$	$\mathfrak{b}^*$	ε^*
$\mathfrak{c}^*$	ε^*	$\mathfrak{c}^*$	ε^*	$\mathfrak{c}^*$

Table 1

Consider $\mathfrak{M}$ be a MBJ - N subset on Υ and the following membership functions

$$\rho_{\mathfrak{M}} = \begin{cases} 0.7: & \mathfrak{x}_\circ = \varepsilon^*, \mathfrak{a}^* \\ 0.4: & \mathfrak{y}_\circ = \mathfrak{b}^*, \mathfrak{c}^* \end{cases}, \zeta_{\mathfrak{M}} = \begin{cases} [0.61, 0.62]: & \mathfrak{x}_\circ = \varepsilon^*, \mathfrak{a}^* \\ [0.51, 0.52]: & \mathfrak{y}_\circ = \mathfrak{b}^*, \mathfrak{c}^* \end{cases}, \upsilon_{\mathfrak{M}} = \begin{cases} 0.1: & \mathfrak{x}_\circ = \varepsilon^*, \mathfrak{a}^* \\ 0.3: & \mathfrak{y}_\circ = \mathfrak{b}^*, \mathfrak{c}^* \end{cases}$$

Thus, $\mathfrak{M}$ is an MBJ - NF of Υ.

Lemma 3.7. *Let $\mathfrak{M}$ be an MBJ - NF of Υ if and only if $\rho_{\mathfrak{M}}$ is a fuzzy filter, $\upsilon'_{\mathfrak{M}}$ is a anti - fuzzy filter and $\zeta^L_{\mathfrak{M}}, \zeta^U_{\mathfrak{M}}$ are fuzzy filter of Υ.*

Proof: *Considering $\mathfrak{M}$ as an MBJ - NF of Υ. Clearly, $\rho_{\mathfrak{M}}, \zeta^U_{\mathfrak{M}}, \zeta^L_{\mathfrak{M}}$ are fuzzy filter of Υ , then to prove $\upsilon'_{\mathfrak{M}}$ is also a fuzzy filter of Υ. Now,*

$$\begin{aligned} \upsilon'_{\mathfrak{M}}(\mathfrak{y}_\circ) &= (1 - \upsilon_{\mathfrak{M}}(\mathfrak{y}_\circ)) \\ &\succeq 1 - (\upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ) \\ &= \upsilon'_{\mathfrak{M}}(\mathfrak{x}_\circ). \end{aligned}$$

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Conversely, let $\rho_{\mathfrak{M}}, \zeta^U_{\mathfrak{M}}, \zeta^L_{\mathfrak{M}}$ *and* $\upsilon^{'}_{\mathfrak{M}}$ *are filter of* Υ, *then*

$$\begin{aligned} 1 - \upsilon_{\mathfrak{M}}(\mathfrak{y}_o) &= \upsilon^{'}_{\mathfrak{M}}(\mathfrak{y}_o) \\ &\succeq (\upsilon^{'}_{\mathfrak{M}}(\mathfrak{x}_o)) \\ &= 1 - (\upsilon_{\mathfrak{M}}(\mathfrak{x}_o) \\ \Rightarrow \upsilon_{\mathfrak{M}}(\mathfrak{y}_o) &\succeq \upsilon_{\mathfrak{M}}(\mathfrak{x}_o). \end{aligned}$$

Thus, $\mathfrak{M}$ *is an MBJ - NF of* Υ.

Theorem 3.8. *Let* $\mathfrak{M}_1, \mathfrak{M}_2$ *be two MBJ - NF of* Υ, *then* $\mathfrak{M}_1 \bigcap \mathfrak{M}_2$ *is also MBJ - NF of* Υ.

Proof: *Consider* $\mathfrak{M}_1$ *and* $\mathfrak{M}_2$ *be two MBJ - NF's of* Υ, *then*

$$\begin{aligned} \rho_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{y}_o) &= (\rho_{\mathfrak{M}_1}(\mathfrak{y}_o) \curlywedge \rho_{\mathfrak{M}_2}(\mathfrak{y}_o)) \\ &\preceq (\rho_{\mathfrak{M}_1}(\mathfrak{x}_o) \curlywedge \rho_{\mathfrak{M}_2}(\mathfrak{x}_o)) \\ &= \rho_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{x}_o) \end{aligned}$$

Thus, $\rho_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{y}_o) \preceq \rho_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{x}_o)$
Similarly,

$$\begin{aligned} \overline{\zeta}_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{y}_o) &\preceq \overline{\zeta}_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{x}_o) \\ \upsilon_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{x}_o) &\succeq \upsilon_{\mathfrak{M}_1 \bigcap \mathfrak{M}_2}(\mathfrak{x}_o) \end{aligned}$$

$\Rightarrow \mathfrak{M}_1 \bigcap \mathfrak{M}_2$ *is also an MBJ - NF of* Υ.

Theorem 3.9. *An MBJ - NF* $\mathfrak{M}$ *of* Υ, *then* $\rho_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \rho_{\mathfrak{M}}(\mathfrak{x}_o) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{y}_o)$; $\overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_o) \,\overline{\curlywedge}\, \overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_o)$ *and* $\upsilon_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \upsilon_{\mathfrak{M}}(\mathfrak{x}_o) \curlyvee \upsilon_{\mathfrak{M}}(\mathfrak{y}_o)\ \forall\, \mathfrak{x}_o, \mathfrak{y}_o \in \Upsilon$.

Proof: *Clearly,* $\forall\, \mathfrak{x}_o, \mathfrak{y}_o \in \Upsilon$.
$\rho_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) \succeq \rho_{\mathfrak{M}}(\mathfrak{x}_o) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{y}_o)$. *Note*

$$\begin{aligned} \mathfrak{x}_o \oplus (\mathfrak{x}_o \oplus \mathfrak{y}_o) &= \mathfrak{y}_o \oplus (\mathfrak{x}_o \oplus \mathfrak{y}_o) \\ &= (\mathfrak{x}_o \oplus \mathfrak{y}_o) \oplus \mathfrak{y}_o \\ &= \mathfrak{x}_o \oplus (\mathfrak{y}_o \oplus \mathfrak{y}_o) \\ &= \mathfrak{x}_o \oplus \mathfrak{y}_o. \end{aligned}$$

such that $\mathfrak{x}_o \leq \mathfrak{x}_o + \mathfrak{y}_o$, $\mathfrak{y}_o \leq \mathfrak{x}_o + \mathfrak{y}_o$.
Since $\rho_{\mathfrak{M}}$ *is upper set, implies* $\rho_{\mathfrak{M}}(\mathfrak{y}_o) \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_o)$
then $\rho_{\mathfrak{M}}(\mathfrak{x}_o) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{y}_o) \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o)$.
Thus, $\rho_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \rho_{\mathfrak{M}}(\mathfrak{x}_o) \curlywedge \rho_{\mathfrak{M}}(\mathfrak{y}_o))$.
Analogously, $\overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_o) \,\overline{\curlywedge}\, \overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_o), \upsilon_{\mathfrak{M}}(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \upsilon_{\mathfrak{M}}(\mathfrak{x}_o) \curlyvee \upsilon_{\mathfrak{M}}(\mathfrak{y}_o))$.

Definition 3.10. MBJ - NS $\mathfrak{M}$ in Υ then it's level subset is defined as $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}} = \{\rho_{\mathfrak{M}}(\mathfrak{x}_o) \succeq \mathfrak{p},\ \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_o) \succeq \overline{\mathfrak{q}},\ \upsilon_{\mathfrak{M}}(\mathfrak{x}_o) \preceq \mathfrak{r} / \mathfrak{x}_o \in \Upsilon\}$, where $\mathfrak{p}, \mathfrak{r} \in [0,1]$ and $\overline{\mathfrak{q}} \in D[0,1]$.

Theorem 3.11. *MBJ - neutrosophic subincline* $\mathfrak{M}$ *of* Υ *is an MBJ - NF if and only if the MBJ - neutrosophic level subset* $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *are filters of* Υ.

Proof: *Let* $\mathfrak{x}_o \in \mathfrak{M}_{\mathfrak{p}}$ *and* $\mathfrak{y}_o \leq \mathfrak{x}_o$ *then* $\rho_{\mathfrak{M}}(\mathfrak{y}_o) \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_o) \preceq \mathfrak{p}$. *Thus,* $\mathfrak{y}_o \in \mathfrak{M}_{\mathfrak{p}}$. *Similarly, carried out for the other two membership function. Conversely, assume that the MBJ - neutrosophic level sets* $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *of* Υ, *then* $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *is MBJ - NF's of* Υ.
Now assume in a contradiction that if $\mathfrak{M}$ *is not an MBJ - NF* $\ni$: $\mathfrak{x}_1, \mathfrak{y}_1 \in \Upsilon$ *s.t* $\mathfrak{y}_1 \leq \mathfrak{x}_1$ *and* $\rho_{\mathfrak{M}}(\mathfrak{y}_1) \succ \rho_{\mathfrak{M}}(\mathfrak{x}_1)$.
Let $u = \frac{1}{2}\{\rho_{\mathfrak{M}}(\mathfrak{x}_1) \oplus \rho_{\mathfrak{M}}(\mathfrak{y}_1))\}$, *then* $\rho_{\mathfrak{M}}(\mathfrak{x}_1) \succ u \succ \rho_{\mathfrak{M}}(\mathfrak{y}_1)$ *and so* $\mathfrak{x}_1 \in \rho_{\mathfrak{M}}(u)$.
Thus, $\mathfrak{y}_1 \in \rho_{\mathfrak{M}}(u) \preceq u$ *arrives at a contradiction. Similarly, in the same manner for the other two MF's can be verified.*
Thus, $\mathfrak{M}$ *is an MBJ - NF of* Υ.

Theorem 3.12. *Letting* $\mathfrak{H}$ *as an filter of* Υ *and* $\mathfrak{M}$ *is defined by*
$\rho_{\mathfrak{M}}(\mathfrak{x}_o) = \begin{cases} \mathfrak{p}_1 & if \quad \mathfrak{x}_o \in \mathfrak{H} \\ \mathfrak{p}_2 & if \quad \mathfrak{x}_o \notin \mathfrak{H} \end{cases}$; $\overline{\zeta}_{\mathfrak{M}}(x) = \begin{cases} \overline{\mathfrak{q}}_1 & if \quad \mathfrak{x}_o \in \mathfrak{H} \\ \overline{\mathfrak{q}}_2 & if \quad \mathfrak{x}_o \notin \mathfrak{H} \end{cases}$ *and* $\upsilon_{\mathfrak{M}}(\mathfrak{x}_o) = \begin{cases} \mathfrak{r}_1 & if \quad \mathfrak{x}_o \in \mathfrak{H} \\ \mathfrak{r}_2 & if \quad \mathfrak{x}_o \notin \mathfrak{H} \end{cases}$

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where $\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{r}_1, \mathfrak{r}_2 \in [0,1]$ *and* $\overline{\mathfrak{q}}_1, \overline{\mathfrak{q}}_2 \in D[0,1]$ *also* $\mathfrak{p}_1 > \mathfrak{p}_2; \overline{\mathfrak{q}}_1 > \overline{\mathfrak{q}}_2$; $\mathfrak{r}_1 > \mathfrak{r}_2$. *Then* $\mathfrak{M}$ *is a MBJ - NF of* Υ *and* $\mathfrak{M}_{\mathfrak{p}_1,\overline{\mathfrak{q}}_1,\mathfrak{r}_1} = \mathfrak{H}$.

Proof: *Consider* $\mathfrak{x}_\circ \leq \mathfrak{y}_\circ$ *and if* $\mathfrak{x}_\circ \in \mathfrak{H}$ *and* $\mathfrak{y}_\circ \in \Upsilon$ *then* $\mathfrak{y}_\circ \in \mathfrak{H}$.
Since $\mathfrak{H}$ *is a filter of* Υ, *i.e.,* $\rho_{\mathfrak{M}}(\mathfrak{x}_\circ) = \rho_{\mathfrak{M}}(\mathfrak{y}_\circ) = \mathfrak{p}_1$
Suppose if $\mathfrak{y}_\circ \notin \mathfrak{H}$, *then* $\rho_{\mathfrak{M}}(\mathfrak{y}_\circ) = \mathfrak{p}_2 \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_\circ)$. *Thus* $\rho_{\mathfrak{M}}$ *is an MBJ - NF of* Υ. *Hence* $\overline{\zeta}_{\mathfrak{M}}$ & $\upsilon_{\mathfrak{M}}$ *are also MBJ -NF's of* Υ *and also* $\mathfrak{M}_{\mathfrak{p}_1,\mathfrak{q}_1,\mathfrak{r}_1} = \mathfrak{H}$.

Corollary 3.13. *Any filter of* Υ *can be realized as the level filter for some MBJ - NF of* Υ.

Theorem 3.14. $\mathfrak{H}$ *is an non - empty subset of* Υ *and* $\mathfrak{M}$ *is an characteristic function of* $\mathfrak{H}$. *Then* $\mathfrak{M}$ *is an MBJ - NF of* Υ *if and only if* $\mathfrak{H}$ *is an filter of* Υ.

Theorem 3.15. *Consider* ξ *to be an totally ordered set and let* $\{\mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}/\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r} \in \xi\}$ *be the family of filters of* Υ *such that* $\forall\, \mathfrak{p}, \mathfrak{p}_\circ, \overline{\mathfrak{q}}, \overline{\mathfrak{q}}_\circ, \mathfrak{r}, \mathfrak{r}_\circ \in \xi$, $\mathfrak{p}_\circ > \mathfrak{p}, \overline{\mathfrak{q}}_\circ > \overline{\mathfrak{q}}$; $\mathfrak{r}_\circ > \mathfrak{r}$ *if and only if* $\mathfrak{H}_{\mathfrak{p}_\circ,\overline{\mathfrak{q}}_\circ,\mathfrak{r}_\circ} \subset \mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$. *Then* $\bigcup_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}\in\xi} \mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *and* $\bigcap_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}\in\xi} \mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *are filters of* $\mathfrak{H}$, *where* ξ *is a non - empty subset of [0,1].*

Theorem 3.16. *Let* $\{\mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}/\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r} \in \xi\}$ *be a collections of filter of* Υ *s.t* $\Upsilon = \bigcup_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}\in\xi} \mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *and* $\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r},\mathfrak{p}_\circ,\overline{\mathfrak{q}}_\circ,\mathfrak{r}_\circ \in \xi$ *iff* $\mathfrak{H}_{\mathfrak{p}_\circ,\overline{\mathfrak{q}}_\circ,\mathfrak{r}_\circ} \subset \mathfrak{H}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *then* $\mathfrak{M} = \{\rho_{\mathfrak{M}}, \overline{\zeta}_{\mathfrak{M}}, \upsilon_{\mathfrak{M}}\}$ *in* Υ *defines* $\rho_{\mathfrak{M}}(\mathfrak{x}_\circ) = sup\{\mathfrak{p}/\mathfrak{x}_\circ \in \mathfrak{H}_{\mathfrak{p}}\}, \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_\circ) = rsup\{\overline{\mathfrak{q}}/\mathfrak{x}_\circ \in \mathfrak{H}_{\overline{\mathfrak{q}}}\}; \upsilon_{\mathfrak{M}}(\mathfrak{x}_\circ) = inf\{\mathfrak{r}/\mathfrak{x}_\circ \in \mathfrak{H}_{\mathfrak{r}}\}\ \forall\, \mathfrak{x}_\circ\ \in \Upsilon$ *is an MBJ - NF of* Υ.

Proof: *Let us consider the following cases:*
(i) $\mathfrak{p}_\circ = \{\mathfrak{p} \in \xi/\mathfrak{p} < \mathfrak{p}_\circ\}$
$(ii)\mathfrak{p}_\circ \neq \{\mathfrak{p} \in \xi/\mathfrak{p} < \mathfrak{p}_\circ\}$, *to prove* $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ *is a filter of* Υ.
$(i)^{st}$ *implies that* $\mathfrak{y} \in \rho_{\mathfrak{M}_{\mathfrak{p}_\circ}} \Leftrightarrow \mathfrak{y}_\circ \in \mathfrak{H}_{\mathfrak{p}_\circ}\ \forall\, \mathfrak{p} < \mathfrak{p}_\circ$
$\Leftrightarrow \mathfrak{y}_\circ \in \bigcap_{\mathfrak{p}<\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}}$, *i.e.,* $\rho_{\mathfrak{M}_{\mathfrak{p}}} = \bigcap_{\mathfrak{p}<\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}_\circ}$ *which is a filter of* Υ.
(ii) *Next to claim* $\rho_{\mathfrak{M}_{\mathfrak{p}_\circ}} = \bigcup_{\mathfrak{p}\geq\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}}$ *and if* $\mathfrak{y}_\circ \in \bigcup_{\mathfrak{p}\geq\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}}$ *then* $\mathfrak{y}_\circ \in \mathfrak{M}_{\mathfrak{p}}$ *for* $\mathfrak{p} \geq \mathfrak{p}_\circ$, *then* $\rho_{\mathfrak{M}}(\mathfrak{y}_\circ) \preceq \mathfrak{p} \preceq \mathfrak{p}_\circ$ *so that* $\mathfrak{y}_\circ \in \rho_{\mathfrak{M}_{\mathfrak{p}_\circ}}$. *Thus* $\bigcup_{\mathfrak{p}\geq\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}} \subseteq \rho_{\mathfrak{M}_{\mathfrak{p}_\circ}}$.
Now take $\mathfrak{y}_\circ \notin \bigcup_{\mathfrak{p}\geq\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}}$, *then* $\mathfrak{y}_\circ \notin \mathfrak{H}_{\mathfrak{p}}$ *for* $\mathfrak{p} \geq \mathfrak{p}_\circ$. *Since* $\mathfrak{p}_\circ \neq sup\{\mathfrak{p} \in \chi/\mathfrak{p} < \mathfrak{p}_\circ\}$, $\ni$: $\epsilon > 0$ *such that* $(\mathfrak{p}_\circ - \epsilon, \mathfrak{p}_\circ) \bigcap \xi = \varphi$.
Hence $\mathfrak{y}_\circ \notin \mathfrak{H}_{\mathfrak{p}}$ *for* $\mathfrak{p} > \mathfrak{p}_\circ - \epsilon$ *that is if* $\mathfrak{y}_\circ \in \mathfrak{H}_{\mathfrak{p}}$ *then* $\mathfrak{p} \geq \mathfrak{p}_\circ - \epsilon$ *thus* $\rho_{\mathfrak{M}}(\mathfrak{y}_\circ) \succeq \mathfrak{p}_\circ - \epsilon \succ \mathfrak{p}_\circ$ *and* $\mathfrak{y}_\circ \notin \rho_{\mathfrak{M}_{\mathfrak{p}_\circ}}$.
Thus $\rho_{\mathfrak{M}_{\mathfrak{p}_\circ}} \subseteq \bigcup_{\mathfrak{p}\geq\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}}$. $\rho_{\mathfrak{M}_{\mathfrak{p}_\circ}} = \bigcup_{\mathfrak{p}\geq\mathfrak{p}_\circ} \mathfrak{H}_{\mathfrak{p}}$ *is an filter of* Υ.
In the same way, for $\overline{\zeta}_{\mathfrak{M}_{\overline{\mathfrak{q}}}}$ *and* $\upsilon_{\mathfrak{M}_{\mathfrak{r}}}$ *and hence* $\mathfrak{M}$ *is an MBJ - NF of* Υ.

4 Further Results on MBJ - Neutrosophic Filter

This part deals with other sub - divisions such as product, projection, charactersitic function and chain condition of an MBJ - neutrosophic filter of an incline algebra.

Definition 4.1. Let $\mathfrak{M}_1, \mathfrak{M}_2$ be two MBJ - neutrosophic sets in inclines Υ_1, Υ_2 and $\mathfrak{M}_1 \in \Upsilon_1$, $\mathfrak{M}_2 \in \Upsilon_2$. Then the cartesian product of $\mathfrak{M}_1$ and $\mathfrak{M}_2$ of $\Upsilon_1 \times \Upsilon_2$, is defined as $(\mathfrak{M}_1 \times \mathfrak{M}_2)(\mathfrak{x}_\circ, \mathfrak{y}_\circ) = \{(\mathfrak{x}_\circ, \mathfrak{y}_\circ), \rho_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{x}_\circ, \mathfrak{y}_\circ), \overline{\zeta}_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{x}_\circ, \mathfrak{y}_\circ); \upsilon_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{x}_\circ, \mathfrak{y}_\circ)/\mathfrak{x}_\circ, \mathfrak{y}_\circ \in \Upsilon_1 \times \Upsilon_2\}$, where $\rho_{\mathfrak{M}_1\times\mathfrak{M}_2} = \rho_{\mathfrak{M}_1}(\mathfrak{x}_\circ) \curlywedge \rho_{\mathfrak{M}_2}(\mathfrak{y}_\circ); \overline{\zeta}_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{x}_\circ, \mathfrak{y}_\circ) = \overline{\zeta}_{\mathfrak{M}_1}(\mathfrak{x}_\circ) \mathbin{\overline{\curlywedge}} \overline{\zeta}_{\mathfrak{M}_2}(\mathfrak{y}_\circ)$ and $\upsilon_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{x}_\circ, \mathfrak{y}_\circ) = \upsilon_{\mathfrak{M}_1}(\mathfrak{x}_\circ) \curlyvee \upsilon_{\mathfrak{M}_2}(\mathfrak{y}_\circ)\ \forall\, (\mathfrak{x}_\circ, \mathfrak{y}_\circ) \in \Upsilon_1 \times \Upsilon_2$.

Lemma 4.2. *If* $\mathfrak{M}_1 \in \Upsilon_1$ *and* $\mathfrak{M}_2 \in \Upsilon_2$ *satisfy upper set property then so is* $\mathfrak{M}_1 \times \mathfrak{M}_2 \in \Upsilon_1 \times \Upsilon_2$.

Theorem 4.3. *If* $\mathfrak{M}_k$ *is a MBJ - NF's of* Υ_k, *then* $\mathfrak{M}_1 \times \mathfrak{M}_2$ *is a MBJ - NF of* $\Upsilon_1 \times \Upsilon_2$.

Proof: *By using lemma(4.2), it is concluded that when* $\mathfrak{y}_\circ \leq \mathfrak{x}_\circ$
$\Longrightarrow\ \rho_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{y}_\circ) \preceq \rho_{\mathfrak{M}_1\times\mathfrak{M}_2}(\mathfrak{x}_\circ)$.
The proof is similar for the other functions.
$\therefore \mathfrak{M}_1 \times \mathfrak{M}_2$ *is an MBJ - NF of* $\Upsilon_1 \times \Upsilon_2$.

Definition 4.4. $\mathfrak{M}_1$ is a MBJ - NS in incline Υ_1, the projection of $\mathfrak{M}_1$ on Υ_1 is the MBJ - neutrosophic subset and its represented as $proj(\mathfrak{M}_1) \in \Upsilon_1$ is defined by $proj_1(\rho_{\mathfrak{M}_1})(\mathfrak{x}_\circ) = sup\{\rho_{\mathfrak{M}_1}(\mathfrak{x}_\circ, \mathfrak{y}_\circ)/\mathfrak{y}_\circ \in \Upsilon_2\}, proj_1(\overline{\zeta}_{\mathfrak{M}_1})(\mathfrak{x}_\circ) = rsup\{\overline{\zeta}_{\mathfrak{M}_1}(\mathfrak{x}_\circ, \mathfrak{y}_\circ)/\mathfrak{y}_\circ \in \Upsilon_2\}$ and $proj_1(\upsilon_{\mathfrak{M}_1})(\mathfrak{x}_\circ) = inf\{\upsilon_{\mathfrak{M}_1}(\mathfrak{x}_\circ, \mathfrak{y}_\circ)/\mathfrak{y}_\circ \in \Upsilon_2\}\ \forall\, \mathfrak{x}_\circ \in \Upsilon_1$ respectively, $proj_2(\rho_{\mathfrak{M}_2})(\mathfrak{y}_\circ) = sup\{\rho_{\mathfrak{M}_2}(\mathfrak{x}_\circ, \mathfrak{y}_\circ)/\mathfrak{x}_\circ \in \Upsilon_1\}$ and similarly for $proj_2(\overline{\zeta}_{\mathfrak{M}_2})(\mathfrak{y}_\circ), proj_2(\upsilon_{\mathfrak{M}_2})(\mathfrak{y}_\circ)$ $\forall\, \mathfrak{y}_\circ \in \Upsilon_2$.

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Theorem 4.5. *Let Υ_2 be an idempotent incline and $\mathfrak{M}_1$ in $\Upsilon_1 \times \Upsilon_2$ is a MBJ - NF, then the projection $proj_k(\mathfrak{M}_1)$ is also an MBJ - NF of $\Upsilon_k, k = 1, 2, ...$ respectively.*

Proof: *For $\mathfrak{x}_o, \mathfrak{y}_o \in \Upsilon_1$ and $\mathfrak{M}_1 \in \Upsilon_1 \times \Upsilon_2$ which is upper set and $\mathfrak{y}_o \leq \mathfrak{x}_o$ in Υ_1 then*

$$\begin{aligned} proj_1(\rho_{\mathfrak{M}_1}(\mathfrak{y}_o)) &= sup\{\rho_{\mathfrak{M}_1}(\mathfrak{y}_o, \mathfrak{r})/\mathfrak{r} \in \Upsilon_2\} \\ &\leq sup\{\rho_{\mathfrak{M}_1}(\mathfrak{x}_o, \mathfrak{r})/\mathfrak{r} \in \Upsilon_2\} \\ &= proj_1(\rho_{\mathfrak{M}_1})(\mathfrak{x}_o) \end{aligned}$$

Thus, $proj_1(\rho_{\mathfrak{M}_1}))$ is a MBJ - NF of Υ_1.
Analogously, for the other functions and hence, $proj_k(\rho_{\mathfrak{M}_1})$ is a MBJ - NF of $\Upsilon_k (k = 1, 2)$.

Definition 4.6. Let $\mathfrak{M}$ be a MBJ - NF of incline algbera $\Upsilon, \varepsilon : \Upsilon \to \Upsilon$ and define a mapping $\mathfrak{M}^\varepsilon : \mathfrak{M} \to$ closed interval of zero, one by $\mathfrak{M}^\varepsilon(\mathfrak{x}_o) = \{\rho^\varepsilon_{\mathfrak{M}}(\mathfrak{x}_o), \overline{\zeta}^\varepsilon_{\mathfrak{M}}(\mathfrak{x}_o), \upsilon^\varepsilon_{\mathfrak{M}}(\mathfrak{x}_o)\} = \{\rho_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o)), \overline{\zeta}_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o)), \upsilon_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o))/\mathfrak{x}_o \in \mathfrak{M}\}$.

Theorem 4.7. *If $\mathfrak{M}$ is a MBJ - NF of Υ and ϵ is an endomorphism of Υ, then $\mathfrak{M}^\varepsilon$ is an MBJ - NF of Υ.*

Proof: *For $\mathfrak{x}_o, \mathfrak{y}_o \in \Upsilon$ and $\mathfrak{M} = (\rho_{\mathfrak{M}}, \overline{\zeta}_{\mathfrak{M}}, \Upsilon_{\mathfrak{M}})$. Let*

$$\begin{aligned} \mathfrak{y}_o \leq \mathfrak{x}_o \;\&\; \mathfrak{x}_o \oplus \mathfrak{y}_o = \mathfrak{y}_o, \varepsilon(\mathfrak{y}_o) &= \varepsilon(\mathfrak{x}_o \oplus \mathfrak{y}_o) = \varepsilon(\mathfrak{x}_o) \oplus \varepsilon(\mathfrak{y}_o) \\ Thus, \varepsilon(\mathfrak{x}_o) &\leq \varepsilon(\mathfrak{y}_o) \\ \rho^\varepsilon_{\mathfrak{M}}(\mathfrak{y}_o) &= \rho_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) \preceq \rho_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o)) \\ &\preceq \rho^\varepsilon_{\mathfrak{M}}(\mathfrak{x}_o). \end{aligned}$$

In the same manner for the other two components and thus, $\mathfrak{M}^\varepsilon$ is an MBJ - NF of Υ.

Definition 4.8. An MBJ - NF $\mathfrak{M} = (\rho_{\mathfrak{M}}, \overline{\zeta}_{\mathfrak{M}}, \upsilon_{\mathfrak{M}})$ of Υ is called as a MBJ - N characteristic if $\rho_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o)) = \rho_{\mathfrak{M}}(\mathfrak{x}_o), \overline{\zeta}_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o)) = \overline{\zeta}_{\mathfrak{M}}(\mathfrak{x}_o), \upsilon_{\mathfrak{M}}(\varepsilon(\mathfrak{x}_o)) = \upsilon_{\mathfrak{M}}(\mathfrak{x}_o) \;\forall\; \mathfrak{x}_o \in \Upsilon \; and \; \varepsilon \in Aut(\Upsilon)$, where $Aut(\Upsilon)$ is the set of all automorphism of Υ.

Theorem 4.9. *Let $\mathfrak{M} \in \Upsilon$ be an MBJ - N characteristic filter of Υ, then each level ideal of $\mathfrak{M}$ is a characteristic filter of Υ.*

Proof: *$\mathfrak{M}$ is an MBJ - neutrosophic characteristic filter of an incline Υ, $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}, where$ $\mathfrak{p}, \overline{\mathfrak{q}}, \mathfrak{r} \in Im(\mathfrak{M})$ is a filter of incline Υ.*
To show that $\varepsilon(\rho_{\mathfrak{M}_\mathfrak{p}}) = \rho_{\mathfrak{M}_\mathfrak{p}}, \varepsilon(\overline{\zeta}_{\mathfrak{M}_{\overline{\mathfrak{q}}}}) = \overline{\zeta}_{\mathfrak{M}_{\overline{\mathfrak{q}}}}, \varepsilon(\upsilon_{\mathfrak{M}_\mathfrak{r}}) = \upsilon_{\mathfrak{M}_\mathfrak{r}},\; \mathfrak{p} \in Im(\rho_{\mathfrak{M}}),\; \overline{\mathfrak{q}} \in Im(\overline{\zeta}_{\mathfrak{M}}),\; \mathfrak{r} \in Im(\upsilon_{\mathfrak{M}})$.
Let $\varepsilon \in Aut(\Upsilon)$ and $\mathfrak{y}_o \in \mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ since $\mathfrak{M}$ is a MBJ - neutrosophic characteristic, $\rho_{\mathfrak{M}}(\epsilon(\mathfrak{y}_o)) = \rho_{\mathfrak{M}}(\mathfrak{y}_o) \preceq \mathfrak{p}$.
It follows that $\varepsilon(\mathfrak{y}_o) \in \rho_{\mathfrak{M}_\mathfrak{p}}$ and hence $\varepsilon(\rho_{\mathfrak{M}_\mathfrak{p}}) \subseteq \rho_{\mathfrak{M}_\mathfrak{p}}$. Similarly, for $\overline{\zeta}_{\mathfrak{M}_{\overline{\mathfrak{q}}}}, \upsilon_{\mathfrak{M}_\mathfrak{r}}$.
Conversely, let $\mathfrak{x}_o \in \rho_{\mathfrak{M}_\mathfrak{p}}$ & $\mathfrak{y} \in \Upsilon$ s.t
$\varepsilon(\mathfrak{y}_o) = \mathfrak{x}_o$ then $\rho_{\mathfrak{M}}(\mathfrak{y}_o) = \rho_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) = \rho_{\mathfrak{M}}(\mathfrak{x}_o) \preceq \mathfrak{p},\; \mathfrak{y}_o \in \rho_{\mathfrak{M}_\mathfrak{p}}$
$\implies \mathfrak{x}_o = \varepsilon(\mathfrak{y}_o) \in \varepsilon(\rho_{\mathfrak{M}_\mathfrak{p}})$ so that $\rho_{\mathfrak{M}_\mathfrak{p}} \subseteq \varepsilon(\rho_{\mathfrak{M}_\mathfrak{p}})$.
Similarly, for $\overline{\zeta}_{\mathfrak{M}}, \upsilon_{\mathfrak{M}}$. Thus, $\mathfrak{M}_{\mathfrak{p},\overline{\mathfrak{q}},\mathfrak{r}}$ is a characteristic filter of Υ.

Lemma 4.10. *Let $\mathfrak{M}$ be an MBJ - NF of Υ and $\mathfrak{x}_o \in \Upsilon$ then $\mathfrak{M}(\mathfrak{y}_o) = (\rho_{\mathfrak{M}}(\mathfrak{y}_o), \overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_o), \upsilon_{\mathfrak{M}}(\mathfrak{y}_o)) = (\mathfrak{p}, \overline{\mathfrak{q}}, \mathfrak{r}) \Leftrightarrow \mathfrak{y} \in \rho_{\mathfrak{M}_\mathfrak{p}}\;,\; \mathfrak{y}_o \notin \rho_{\mathfrak{M}_{\mathfrak{p}_1}}, \mathfrak{y}_o \in \overline{\zeta}_{\mathfrak{M}_{\overline{\mathfrak{q}}}}, \mathfrak{y}_o \notin \overline{\zeta}_{\mathfrak{M}_{\overline{\mathfrak{q}}_1}}\;;\; \mathfrak{y}_o \in \upsilon_{\mathfrak{M}_\mathfrak{r}}, \mathfrak{y}_o \notin \upsilon_{\mathfrak{M}_{\mathfrak{r}_1}} \;\forall\; p_1 \geq p, \overline{\mathfrak{q}}_1 \geq \overline{\mathfrak{q}} \; and \; \mathfrak{r}_1 \geq \mathfrak{r}$.*

Theorem 4.11. *Let $\mathfrak{M}$ be an MBJ - neutrosophic filter of Υ and if each level filter of $\mathfrak{M}$ is characteristic, then $\mathfrak{M}$ is an MBJ - N characteristic filter of Υ.*

Proof: *$\mathfrak{M} \in \Upsilon$ be an MBJ - neutrosophic filter and let $\mathfrak{y}_o \in \Upsilon, \varepsilon \in Aut(\Upsilon)$ and $\rho_{\mathfrak{M}}(\mathfrak{y}_o) = \mathfrak{p}, \overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_o) = \overline{\mathfrak{q}}$ and $\upsilon_{\mathfrak{M}}(\mathfrak{y}_o) = \mathfrak{r}$. By lemma 4.10, $\mathfrak{y}_o \in \rho_{\mathfrak{M}_\mathfrak{p}}\;,\; \mathfrak{y}_o \notin \rho_{\mathfrak{M}_\mathfrak{p}} \;\forall\; \mathfrak{p}_1 \geq \mathfrak{p}$. It follow $\varepsilon(\rho_{\mathfrak{M}_\mathfrak{p}}) = \rho_{\mathfrak{M}_\mathfrak{p}}$.*
Thus $\varepsilon(\mathfrak{y}_o) \in \varepsilon(\rho_{\mathfrak{M}_\mathfrak{p}}) = \rho_{\mathfrak{M}_\mathfrak{p}}$ and so $\rho_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) \preceq \mathfrak{p}$. Let $\rho_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) = \mathfrak{p}_1$ and assume that $\mathfrak{p}_1 \geq \mathfrak{p}$, then $\varepsilon(\mathfrak{y}_0) \in \rho_{\mathfrak{M}_{\mathfrak{p}_1}} = \varepsilon(\rho_{\mathfrak{M}_{\mathfrak{p}_1}})$. Since ε is one - one $\implies$ $\mathfrak{y}_o \in \rho_{\mathfrak{M}_{\mathfrak{p}_1}}$ which is a contradiction. Thus, $\rho_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) = \mathfrak{p} = \rho_{\mathfrak{M}}(\mathfrak{y}_o)$ similarly , $\overline{\zeta}_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) = \overline{\mathfrak{q}} = \overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_o)$ and $\upsilon_{\mathfrak{M}}(\varepsilon(\mathfrak{y}_o)) = \mathfrak{r} = \upsilon_{\mathfrak{M}}(\mathfrak{y}_o)$. Thus, $\mathfrak{M}$ is an MBJ - neutrosophic characteristic filter of Υ.

Definition 4.12. [4] An ascending chain condition in an incline algebra Υ with respect to filter if an incline algbera Υ contains no infinite proper ascending chain of filters
$\mathfrak{H}_1 \subsetneq \mathfrak{H}_2 \subsetneq \mathfrak{H}_3 \subsetneq$ .... Analogously, for the descending chain.

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Theorem 4.13. *Presume that an incline algebra Υ satisifies the descending chain condition with respect to filter and let $\mathfrak{M}$ be an MBJ - NF then there is no finite ascending sequence of elements of $Im(\mathfrak{M})$.*

Proof: *Suppose $|Im(\chi_{\mathfrak{M}})| = \infty$ and $\{\mathfrak{p}_n\}$ is an strictly increasing sequence of elements of $Im(\xi_{\mathfrak{M}})$ then $0 \leq \mathfrak{p}_1 < \mathfrak{p}_2 .... < 1$.*
Now define $\xi_{\mathfrak{M}_{\mathfrak{p}}} = \{\mathfrak{y} \in \Upsilon / \xi_{\mathfrak{M}}(\mathfrak{y}) \preceq \mathfrak{p}_k\},\ k = 2, 3, ...$ then $\xi_{\mathfrak{M}_k}$ is an filter of Υ.
Let $\mathfrak{y}_o \in \xi_{\mathfrak{M}_k}$, $\xi_k(\mathfrak{y}_o) \preceq \mathfrak{p}_k < \mathfrak{p}_{k-1}$ implies $\mathfrak{y}_o \in \xi_{\mathfrak{M}_{k-1}}$.
Since $\mathfrak{p}_{k-1} \in Im(\mathfrak{M})$, there exists $\mathfrak{y}_{o k-1} \in \Upsilon$ such that $\xi_{\mathfrak{M}}(\mathfrak{y}_{o k-1}) = \mathfrak{p}_{k-1}$.
Hence $\xi_{\mathfrak{M}_k} \subseteq \xi_{\mathfrak{M}_{k-1}}$. Thus, $\xi_{\mathfrak{M}_k} \subsetneq \xi_{\mathfrak{M}_{k-1}}$ and so on.
Will get as, $\xi_{\mathfrak{M}_1} \supsetneq \xi_{\mathfrak{M}_2} \supsetneq \xi_{\mathfrak{M}_3} \supsetneq .....$ of filter of Υ which goes on.
This contradicts and the same manner is carried out for other two components.

Theorem 4.14. *If $|Im(\mathfrak{M})| < \infty$ for every MBJ - neutrosophic filter $\mathfrak{M}$ of Υ then Υ satisfies the descending chain condition with respect to filter.*

Proof: *Assume in a contradiction, that Υ does not satisfies the descending chain condition with respect to filter, then existing an strict infinite descending chain $\mathfrak{H}_0 \supsetneq \mathfrak{H}_1 \supsetneq \mathfrak{H}_2 \supsetneq .....$ of filters of incline Υ.*
Now define $\mathfrak{M} \in \Upsilon$ by

$$\rho_{\mathfrak{M}}(\mathfrak{y}_o) = \begin{cases} \frac{s}{s \oplus 1} : & if\ \mathfrak{y}_o \in \frac{\mathfrak{H}_s}{\mathfrak{H}_{s\oplus 1}},\ s = 0, 1... \\ 1 : & if\ \mathfrak{y}_o \in \cap_{s=0}^{\infty} \mathfrak{H}_s \end{cases}$$

$$\overline{\zeta}_{\mathfrak{M}}(\mathfrak{y}_o) = \begin{cases} \frac{\overline{s}}{\overline{s \oplus 1}} : & if\ \mathfrak{y}_o \in \frac{\mathfrak{H}_s}{\mathfrak{H}_{s\oplus 1}},\ s = 0, 1... \\ \overline{1} : & if\ \mathfrak{y}_o \in \cap_{s=0}^{\infty} \mathfrak{H}_s \end{cases}$$

$$\upsilon_{\mathfrak{M}}(\mathfrak{y}_o) = \begin{cases} \frac{s}{s \oplus 1} : & if\ \mathfrak{y}_o \in \frac{\mathfrak{H}_s}{\mathfrak{H}_{s\oplus 1}},\ s = 0, 1... \\ 1 : & if\ \mathfrak{y}_o \in \cap_{s=0}^{\infty} \mathfrak{H}_s \end{cases}$$

where $\mathfrak{H}_0$ stands for Υ
And if $\mathfrak{y}_o \in \frac{\mathfrak{H}_s}{\mathfrak{H}_{s\oplus 1}}, \mathfrak{x}_o \leq \mathfrak{y}_o$ then $\mathfrak{x}_o \in \frac{\mathfrak{H}_s}{\mathfrak{H}_{s\oplus 1}}$. Hence $\Phi_{\mathfrak{M}}(\mathfrak{y}_o) = \frac{r}{r+1} \preceq \Phi_{\mathfrak{M}}(\mathfrak{x}_o)$
If $\mathfrak{y}_o \in \cap_{s=0}^{\infty} \mathfrak{H}_s$, $\mathfrak{x}_o \leq \mathfrak{y}_o$ then $\rho_{\mathfrak{M}}(\mathfrak{y}_o) = 1 \preceq \rho_{\mathfrak{M}}(\mathfrak{x}_o)$, $\mathfrak{M}$ is an MBJ - NF of Υ having different values in infinite number, arrives in a contradicts.

5 Conclusion

Correlating an incline algebra and an MBJ - neutrosophic set in this paper and the structure called MBJ - neutrosophic filter of incline algebra is defined with an example and examines related results. Further moved to MBJ - neutrosophic filter and various divisions such as level set, product, projection, characteristic function and finally chain condition are also studied. One can carry this work to any other algebras.

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