On Neutrosophic Generalized Alpha Generalized Separation Axioms

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Abstract

The paper provided a new notion of neutrosophic separation axioms as neutrosophic gαg-R_i-space & neutrosophic gαg-T_j-space (note that the indexes i & j are natural numbers of the spaces R & T are from 0 to 1 & from 0 to 2 alternately).

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1. Introduction

F. Smarandache [1,2] furnished the impression of a "neutrosophic set". A. Alblowi et al. [3] offered the evidence of neutrosophic topological space (or artlessly NTS). I. Arokiarani et al. [4] combined the interpretation of neutrosophic α-open subsets of neutrosophic topological spaces. Q. H. Imran et al. [5] proposed neutrosophic semi-open sets in neutrosophic topological spaces. R. Dhavaseelan et al. [6,7] offered the notion of generalized neutrosophic closed sets & neutrosophic α^m-continuity. Md. Hanif PAGE et al. [8] gave the idea of neutrosophic generalized homeomorphism. Q. H. Imran et al. [9] presented the concepts of neutrosophic generalized αg-closed sets & neutrosophic generalized αg-continuous functions. The purpose is to initiate a newfangled idea of neutrosophic separation axioms such as neutrosophic gαg-R_i-space, $i = 0,1$ & neutrosophic gαg-T_j-space, $j = 0,1,2$ & affirm some of their primary characteristics.

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2. Preliminaries

During this paper, $(\mathcal{X},\xi)$ (or artlessly $\mathcal{X}$) constantly recouple to NTS. The sequel of a neutrosophic open set (N-OS) is named the neutrosophic closed set (N-CS) in $(\mathcal{X},\xi)$. For an NS $\mathcal{M}$ in an NTS $(\mathcal{X},\xi)$, $Ncl(\mathcal{M})$, $Nint(\mathcal{M})$ & $\mathcal{M}^c = 1_N - \mathcal{M}$ signifies the neutrosophic closure of $\mathcal{M}$, the neutrosophic interior of $\mathcal{M}$ & the neutrosophic sequel of $\mathcal{M}$ alternately.

Definition 2.1:
A NS $\mathcal{M}$ of a NTS $(\mathcal{X},\xi)$ is uttered:
(i) a neutrosophic α-closed set (artlessly N^{α}-CS) if $Ncl(Nint(Ncl(\mathcal{M}))) \sqsubseteq \mathcal{M}$. The supplement of a N^{α}-CS in $\mathcal{X}$ is a neutrosophic α- open set (artlessly N^{α}-OS) in $\mathcal{X}$. The neutrosophic α-closure of a NS $\mathcal{M}$ of a NTS $(\mathcal{X},\xi)$ is the junction ∀ N^{α}-CSs that restrain $\mathcal{M}$ & is signalized by $N^{\alpha}cl(\mathcal{M})$. [4]
(ii) a neutrosophic g-closed set (artlessly N^{g}-CS) if $Ncl(\mathcal{M}) \sqsubseteq \mathcal{U}$ if $\mathcal{M} \sqsubseteq \mathcal{U}$ & $\mathcal{U}$ is a N-OS in $\mathcal{X}$. The sequel of a N^{g}-CS in $\mathcal{X}$ is a N^{g}-OS in $\mathcal{X}$. [10]
(iii) a neutrosophic αg-closed set (artlessly $\mathrm{N}^{\alpha \mathrm{g}}$-CS) if $N^{\alpha}cl(\mathcal{M}) \sqsubseteq \mathcal{U}$ if $\mathcal{M} \sqsubseteq \mathcal{U}$ & $\mathcal{U}$ is a N^{α}-OS in $\mathcal{X}$. The supplement of a $\mathrm{N}^{\alpha \mathrm{g}}$-CS in $\mathcal{X}$ is a $\mathrm{N}^{\alpha \mathrm{g}}$-OS in $\mathcal{X}$. [11]
(iv) a neutrosophic indiscriminate αg-closed set (artlessly $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS) if $Ncl(\mathcal{M}) \sqsubseteq \mathcal{U}$ if $\mathcal{M} \sqsubseteq \mathcal{U}$ & $\mathcal{U}$ is a $\mathrm{N}^{\alpha \mathrm{g}}$-OS in $\mathcal{X}$. The collection of all $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CSs of a NTS $(\mathcal{X},\xi)$ is designated by $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-$\mathrm{C}(\mathcal{X})$. The sequel of a $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS in $\mathcal{X}$ is a $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OS in $\mathcal{X}$. The collection of all $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OSs of a NTS $(\mathcal{X},\xi)$ is popularized as $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-$\mathrm{O}(\mathcal{X})$. [9]

Example 2.2:
Let $\mathcal{X} = \{\mathscr{k}, \ell\}$ & $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$, so NSs $\mathcal{A} = \langle x, (0.6{,}0.7), (0.1{,}0.1), (0.4{,}0.2)\rangle$ & $\mathcal{B} = \langle x, (0.1{,}0.2), (0.1{,}0.1), (0.8{,}0.8)\rangle$, so that $(\mathcal{X},\xi)$ is a NTS. However, the NS $\mathcal{C} = \langle x, (0.2{,}0.2), (0.1{,}0.1), (0.6{,}0.7)\rangle$ is a N^{α}-CS, N^{g}-CS, $\mathrm{N}^{\alpha \mathrm{g}}$-CS & $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS.

Remark 2.3 [9]:
In a NTS $(\mathcal{X},\xi)$, so the subsequent declaration engages, & the contrary of any confession is not true:
(i) Every N-OS (resp. N-CS) is a $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OS (resp. $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS).
(ii) Every $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OS (resp. $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS) is a N^{g}-OS (resp. N^{g}-CS).
(iii) Every $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OS (resp. $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS) is a $\mathrm{N}^{\alpha \mathrm{g}}$-OS (resp. $\mathrm{N}^{\alpha \mathrm{g}}$-CS).

Definition 2.4 [9]:
The crossroads ∀ $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CSs in a NTS $(\mathcal{X},\xi)$ comprise $\mathcal{M}$ is denominated neutrosophic gαg-closure of $\mathcal{M}$ & is designated by $N^{g\alpha g}cl(\mathcal{M})$, $N^{g\alpha g}cl(\mathcal{M}) = \sqcap\{\mathcal{N}: \mathcal{M} \sqsubseteq \mathcal{N}, \mathcal{N}$ is a $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-CS$\}$.

3. Neutrosophic $g\alpha g$ -R_i-Spaces, $i = 0, 1$

Definition 3.1:
The intersection of all $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OSs of a NTS $(\mathcal{X},\xi)$ comprises $\mathcal{A}$ is denominated by the neutrosophic gαg-kernel of $\mathcal{A}$ ($N^{g\alpha g}ker(\mathcal{A})$), signifying $N^{g\alpha g}ker(\mathcal{A}) = \sqcap\{\mathcal{M}: \mathcal{M} \in \mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-$\mathrm{O}(\mathcal{X})$ & $\mathcal{A} \sqsubseteq \mathcal{M}\}$.

Definition 3.2:
Let $\mathscr{k}_{\lambda}$ be a neutrosophic point (NP) of a NTS $(\mathcal{X},\xi)$. The $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-kernel of $\mathscr{k}_{\lambda}$, designated by $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$ is defined to be the NS $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) = \sqcap\{\mathcal{M}: \mathcal{M} \in \mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-$\mathrm{O}(\mathcal{X})$ & $\mathscr{k}_{\lambda} \in \mathcal{M}\}$.

Definition 3.3:
In a NTS $(\mathcal{X},\xi)$, a NS $\mathcal{A}$ is denominated weakly ultra $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-separated from $\mathcal{B}$ if ∃ a $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-OS $\mathcal{M}$ s.t. $\mathcal{M} \sqcap \mathcal{B} = 0_N$ or $\mathcal{A} \sqcap N^{g\alpha g}cl(\mathcal{B}) = 0_N$.
By definition (3.3), got: For any two distinct NPs $\mathscr{k}_{\lambda}$ & ℓ_{μ} of a NTS $(\mathcal{X},\xi)$
(i) $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) = \{\ell_{\mu} : \{\ell_{\mu}\}$ is not weakly ultra $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-separated from $\{\mathscr{k}_{\lambda}\}\}$.
(ii) $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) = \{\ell_{\mu} : \{\mathscr{k}_{\lambda}\}$ is not weakly ultra $\mathrm{N}^{\mathrm{g}\alpha \mathrm{g}}$-separated from $\{\ell_{\mu}\}\}$.

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Lemma 3.4:
Let $(\mathcal{X},\xi)$ be a NTS, then $\ell_{\mu} \in N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$ iff $\mathscr{k}_{\lambda} \in N^{g\alpha g}cl(\{\ell_{\mu}\})$ for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$.
Proof:
Suppose that $\ell_{\mu} \notin N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$. Then $\exists$ $\mathrm{N}^{g\alpha g}$-OS $\mathcal{U}$ comprise $\mathscr{k}_{\lambda}$ s.t. $\ell_{\mu} \notin \mathcal{U}$. Therefore, we have $\mathscr{k}_{\lambda} \notin N^{g\alpha g}cl(\{\ell_{\mu}\})$. The same way can prove the contrariwise part. ▪

Definition 3.5:
A NTS $(\mathcal{X},\xi)$ is denominated neutrosophic gαg-R_0-space ($\mathrm{N}^{g\alpha g}$-R_0-space, for short) if for any $\mathrm{N}^{g\alpha g}$-OS $\mathcal{U}$ & $\mathscr{k}_{\lambda} \in \mathcal{U}$, then $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$.

Definition 3.6:
A NTS $(\mathcal{X},\xi)$ is denominated neutrosophic gαg-R_1-space ($\mathrm{N}^{g\alpha g}$-R_1-space, for short) if for any two distinct NPs $\mathscr{k}_{\lambda}$ & ℓ_{μ} of $\mathcal{X}$ with $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}cl(\{\ell_{\mu}\})$, there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_{\mu}\}) \sqsubseteq \mathcal{V}$.

Theorem 3.7:
Let $(\mathcal{X},\xi)$ be a NTS. Then $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space iff $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) = N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$, for any $\mathscr{k}_{\lambda} \in \mathcal{X}$.
Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-R_0-space. If $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$, for any $\mathscr{k}_{\lambda} \in \mathcal{X}$, then there exists another NP $\ell_{\mu} \neq \mathscr{k}_{\lambda}$ s.t. $\ell_{\mu} \in N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\})$ & $\ell_{\mu} \notin N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$ This recouple has there existed a $\mathrm{N}^{g\alpha g}$-OS $\mathcal{U}_{\mathscr{k}_{\lambda}}$, $\ell_{\mu} \notin \mathcal{U}_{\mathscr{k}_{\lambda}}$ implies $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \not\sqsubseteq \mathcal{U}_{\mathscr{k}_{\lambda}}$ This ambivalence. Consequently $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) = N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$.
Contrariwise, let $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) = N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$, for any $\mathrm{N}^{g\alpha g}$-OS $\mathcal{U}, \mathscr{k}_{\lambda} \in \mathcal{U}$, then $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) = N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$ [by def. (3.1)]. So by def. (3.5), $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space. ▪

Theorem 3.8:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-R_0-space iff for any $\mathrm{N}^{g\alpha g}$-CS $\mathcal{M}$ & $\mathscr{k}_{\lambda} \in \mathcal{M}$, then $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{M}$.
Proof:
Let for any $\mathcal{M}$ $\mathrm{N}^{g\alpha g}$-CS & $\mathscr{k}_{\lambda} \in \mathcal{M}$, then $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{M}$ & $\mathcal{U}$ be a $\mathrm{N}^{g\alpha g}$-OS, $\mathscr{k}_{\lambda} \in \mathcal{U}$ then, for any $\ell_{\mu} \notin \mathcal{U}$ implies $\ell_{\mu} \in \mathcal{U}^{c}$ is a $\mathrm{N}^{g\alpha g}$-CS implies $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqsubseteq \mathcal{U}^{c}$[by hypothesis]. Therefore $\mathscr{k}_{\lambda} \notin N^{g\alpha g}ker(\{\ell_{\mu}\})$ implies $\ell_{\mu} \notin N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\})$ [by lemma (3.4)]. So $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space.
Contrariwise, let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-R_0-space & $\mathcal{M}$ be a $\mathrm{N}^{g\alpha g}$-CS & $\mathscr{k}_{\lambda} \in \mathcal{M}$. Then for any $\ell_{\mu} \notin \mathcal{M}$ implies $\ell_{\mu} \in \mathcal{M}^{c}$ is a $\mathrm{N}^{g\alpha g}$-OS, then $N^{g\alpha g}cl(\{\ell_{\mu}\}) \sqsubseteq \mathcal{M}^{c}$ [since $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space], so $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) = N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\})$. Consequently $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{M}$. ▪

Corollary 3.9:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-R_0-space iff for any $\mathcal{U}$ $\mathrm{N}^{g\alpha g}$-OS & $\mathscr{k}_{\lambda} \in \mathcal{U}$, then $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})) \sqsubseteq \mathcal{U}$.
Proof:
Obviously. ▪

Theorem 3.10:
Every $\mathrm{N}^{g\alpha g}$-R_1-space is a $\mathrm{N}^{g\alpha g}$-R_0-space.
Proof:
Assume $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-R_1-space & let $\mathcal{U}$ be a $\mathrm{N}^{g\alpha g}$-OS, $\mathscr{k}_{\lambda} \in \mathcal{U}$, then for any $\ell_{\mu} \notin \mathcal{U}$ then $\ell_{\mu} \in \mathcal{U}^{c}$ is a $\mathrm{N}^{g\alpha g}$-CS & $N^{g\alpha g}cl(\{\ell_{\mu}\}) \sqsubseteq \mathcal{U}^{c}$ got $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}cl(\{\ell_{\mu}\})$. Wherefore by def.(3.6), $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space. ▪

Theorem 3.11:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-R_1-space iff for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}ker(\{\ell_{\mu}\})$, then there exist $\mathrm{N}^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$.

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Proof:

Let $(\mathcal{X},\xi)$ be a $N^{g\alpha g}$-R_1-space. Then for any $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$. Such any $N^{g\alpha g}$-R_1-space is a $N^{g\alpha g}$-R_0-space [by Thm. (3.10)], & by Thm. (3.7), $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, then there exist $N^{g\alpha g}$-OSs $\mathcal{U}_1$,$\mathcal{U}_2$ s.t. $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{U}_1$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2$ & $\mathcal{U}_1 \sqcap \mathcal{U}_2 = 0_N$ [since $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_1-space], then $\mathcal{U}_1^c$ & $\mathcal{U}_2^c$ are $N^{g\alpha g}$-CSs s.t. $\mathcal{U}_1^c \sqcup \mathcal{U}_2^c = 1_N$. Put $\mathcal{M}_1 = \mathcal{U}_1^c$ & $\mathcal{M}_2 = \mathcal{U}_2^c$. Consequently $\mathscr{k}_\lambda \in \mathcal{U}_1 \sqsubseteq \mathcal{M}_2$ & $\ell_\mu \in \mathcal{U}_2 \sqsubseteq \mathcal{M}_1$ so $N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{U}_1 \sqsubseteq \mathcal{M}_2$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2 \sqsubseteq \mathcal{M}_1$.

Contrariwise, let for any $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$, there exist $N^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_\mu\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$, then $\mathcal{M}_1^c$ & $\mathcal{M}_2^c$ are $N^{g\alpha g}$-OSs s.t. $\mathcal{M}_1^c \sqcap \mathcal{M}_2^c = 0_N$. Put $\mathcal{M}_1^c = \mathcal{U}_2$ & $\mathcal{M}_2^c = \mathcal{U}_1$. Consequently, $N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{U}_1$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2$ & $\mathcal{U}_1 \sqcap \mathcal{U}_2 = 0_N$, so that $\mathscr{k}_\lambda \in \mathcal{U}_1$ & $\ell_\mu \in \mathcal{U}_2$ implies $\mathscr{k}_\lambda \notin N^{g\alpha g}cl(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}cl(\{\mathscr{k}_\lambda\})$, then $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{U}_1$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2$. Consequently, $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_1-space. ▪

Corollary 3.12:

A NTS $(\mathcal{X},\xi)$ is $N^{g\alpha g}$-R_1-space iff for any $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$ there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$.

Proof:

Let $(\mathcal{X},\xi)$ be a $N^{g\alpha g}$-R_1-space & let $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, then there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{V}$. Likewise, $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_0-space [by theorem (3.10)] implies that any $\mathscr{k}_\lambda \in \mathcal{X}$, then $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) = N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})$ [by Thm. (3.7)], but $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) = N^{g\alpha g}cl(N^{g\alpha g}cl(\{\mathscr{k}_\lambda\})) = N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}))$.

Consequently $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$.

Contrariwise, let $\forall \mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$ there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$. Since $\{\mathscr{k}_\lambda\} \sqsubseteq N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})$, then $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \sqsubseteq N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_\lambda\}))$ for any $\mathscr{k}_\lambda \in \mathcal{X}$. So we get $N^{g\alpha g}cl(\{\mathscr{k}_\lambda\}) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{V}$. Consequently, $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_1-space. ▪

4. Neutrosophic $g\alpha g$-T_j-Spaces, $j = 0, 1, 2$

Definition 4.1:

Let $(\mathcal{X},\xi)$ be a NTS, $\mathcal{X}$ is denominated:

(i) neutrosophic $g\alpha g$-T_0-space ($N^{g\alpha g}$-T_0-space, for short) iff any couple of distinct NPs in $\mathcal{X}$, $\exists N^{g\alpha g}$-OS in $\mathcal{X}$ comprises one & not the other.

(ii) neutrosophic $g\alpha g$-T_1-space ($N^{g\alpha g}$-T_1-space, for short) iff for any couple of distinct NPs $\mathscr{k}_\lambda$ & ℓ_μ of $\mathcal{X}$, there exist $N^{g\alpha g}$-OSs $\mathcal{M},\mathcal{N}$ comprise $\mathscr{k}_\lambda$ & ℓ_μ alternately s.t. $\ell_\mu \notin \mathcal{M}$ & $\mathscr{k}_\lambda \notin \mathcal{N}$.

(iii) neutrosophic $g\alpha g$-T_2-space ($N^{g\alpha g}$-T_2-space, for short) iff for any couple of distinct NPs $\mathscr{k}_\lambda$ & ℓ_μ of $\mathcal{X}$, there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{M},\mathcal{N}$ in $\mathcal{X}$ s.t. $\mathscr{k}_\lambda \in \mathcal{M}$ & $\ell_\mu \in \mathcal{N}$.

Remark 4.2:

Every $N^{g\alpha g}$-T_k-space is a $N^{g\alpha g}$-T_{k-1}-space, $k = 1,2$.

Proof:

Obviously. ▪

Theorem 4.3:

A NTS $(\mathcal{X},\xi)$ is $N^{g\alpha g}$-T_0-space iff either $\ell_\mu \notin N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})$ or $\mathscr{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$, for any $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$.

Proof:

Let $(\mathcal{X},\xi)$ be a $N^{g\alpha g}$-T_0-space then for any $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, there exists a $N^{g\alpha g}$-OS $\mathcal{M}$ s.t. $\mathscr{k}_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ or $\mathscr{k}_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Consequently either $\mathscr{k}_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ implies $\ell_\mu \notin N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})$ or $\mathscr{k}_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$ implies $\mathscr{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$.

Contrariwise, let either $\ell_\mu \notin N^{g\alpha g}ker(\{\mathscr{k}_\lambda\})$ or $\mathscr{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$, for any $\mathscr{k}_\lambda \neq \ell_\mu \in \mathcal{X}$. Then there exists a $N^{g\alpha g}$-OS $\mathcal{M}$ s.t. $\mathscr{k}_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ or $\mathscr{k}_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Consequently, $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-T_0-space. ▪

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Theorem 4.4:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-T_0-space iff either $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$ or $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}$ for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$.

Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-T_0-space then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, there exists a $\mathrm{N}^{g\alpha g}$-OS $\mathcal{M}$ s.t. $\mathcal{k}_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ or $\mathcal{k}_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Now if $\mathcal{k}_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ implies $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$. Or if $\mathcal{k}_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$ implies $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}$.
Contrariwise, let either $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ be weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$ or $N^{g\alpha g}ker(\{\ell_\mu\})$ be weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}$. Then there exists a $\mathrm{N}^{g\alpha g}$-OS $\mathcal{M}$ s.t. $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqsubseteq \mathcal{M}$ & $\ell_\mu \notin \mathcal{M}$ or $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}$, $\mathcal{k}_\lambda \notin \mathcal{M}$ implies $\mathcal{k}_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ or $\mathcal{k}_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-T_0-space. ▪

Theorem 4.5:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-T_1-space iff for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$ & $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}$.

Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-T_1-space, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, there exist $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\mathcal{k}_\lambda \notin \mathcal{V}$, $\ell_\mu \in \mathcal{V}$. Implies $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$ & $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}$.
Contrariwise, let $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ be weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$ & $N^{g\alpha g}ker(\{\ell_\mu\})$ be weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}$. Then there exist $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqsubseteq \mathcal{U}, \ell_\mu \notin \mathcal{U}$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{V}, \mathcal{k}_\lambda \notin \mathcal{V}$ implies $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\mathcal{k}_\lambda \notin \mathcal{V}$, $\ell_\mu \in \mathcal{V}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-T_1-space. ▪

Theorem 4.6:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-T_1-space iff for any $\mathcal{k}_\lambda \in \mathcal{X}$, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) = \{\mathcal{k}_\lambda\}$.

Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-T_1-space & let $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \neq \{\mathcal{k}_\lambda\}$. Then $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ contains other NPs distinct from $\mathcal{k}_\lambda$ say ℓ_μ. So $\ell_\mu \in N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ implies $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is not weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$. Wherefore by Thm. (4.5), $(\mathcal{X},\xi)$ is not a $\mathrm{N}^{g\alpha g}$-T_1-space, this is ambivalence. Consequently, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) = \{\mathcal{k}_\lambda\}$.
Contrariwise, let $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) = \{\mathcal{k}_\lambda\}$, for any $\mathcal{k}_\lambda \in \mathcal{X}$ & let $(\mathcal{X},\xi)$ be not a $\mathrm{N}^{g\alpha g}$-T_1-space. Then by Thm. (4.5), $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is not weakly ultra $\mathrm{N}^{g\alpha g}$-separated from $\{\ell_\mu\}$ for some $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, this recouple has that for every $\mathrm{N}^{g\alpha g}$-OS $\mathcal{M}$ contains $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ then $\ell_\mu \in \mathcal{M}$ implies $\ell_\mu \in \sqcap\{\mathcal{M} \in \mathrm{N}^{g\alpha g}\text{-}O(\mathcal{X})\text{: } \mathcal{k}_\lambda \in \mathcal{M}\}$ implies $\ell_\mu \in N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$, this is ambivalence. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-T_1-space. ▪

Theorem 4.7:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-T_1-space iff for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, $\ell_\mu \notin N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ & $\mathcal{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$.

Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-T_1-space then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, $\exists\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\ell_\mu \in \mathcal{V}$, $\mathcal{k}_\lambda \notin \mathcal{V}$. Implies $\ell_\mu \notin N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ & $\mathcal{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$.
Contrariwise, let $\ell_\mu \notin N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ & $\mathcal{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$, for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$. Then $\exists\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\ell_\mu \in \mathcal{V}$, $\mathcal{k}_\lambda \notin \mathcal{V}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-T_1-space. ▪

Theorem 4.8:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-T_1-space iff for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ implies $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$.

Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-T_1-space. Then $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) = \{\mathcal{k}_\lambda\}$ & $N^{g\alpha g}ker(\{\ell_\mu\}) = \{\ell_\mu\}$ [by Thm. (4.6)]. Consequently, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$.

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Contrariwise, let for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ implies $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqcap N^{g\alpha g}ker(\{\ell_{\mu}\}) = 0_N$ & let $(\mathcal{X}, \xi)$ be not $\mathrm{N}^{g\alpha g}$-T_1-space, then for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ implies $\ell_{\mu} \in N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$ or $\mathscr{k}_{\lambda} \in N^{g\alpha g}ker(\{\ell_{\mu}\})$ [by Thm. (4.7)], then $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqcap N^{g\alpha g}ker(\{\ell_{\mu}\}) \neq 0_N$. This is ambivalence. Consequently, $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_1-space. ▪

Proposition 4.9:
A NTS $(\mathcal{X}, \xi)$ is $\mathrm{N}^{g\alpha g}$-T_1-space iff $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_0-space & $\mathrm{N}^{g\alpha g}$-R_0-space.

Proof:
Let $(\mathcal{X}, \xi)$ be a $\mathrm{N}^{g\alpha g}$-T_1-space & let $\mathscr{k}_{\lambda} \in \mathcal{U}$ be a $\mathrm{N}^{g\alpha g}$-OS, then for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$, $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqcap N^{g\alpha g}ker(\{\ell_{\mu}\}) = 0_N$ [by Thm. (4.8)] implies $\mathscr{k}_{\lambda} \notin N^{g\alpha g}ker(\{\ell_{\mu}\})$ & $\ell_{\mu} \notin N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})$, this recouple tos $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) = \{\mathscr{k}_{\lambda}\}$, wherefore $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$. Consequently, $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space.
Contrariwise, let $(\mathcal{X}, \xi)$ be a $\mathrm{N}^{g\alpha g}$-T_0-space & $\mathrm{N}^{g\alpha g}$-R_0-space, then for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ $\exists \mathrm{N}^{g\alpha g}$-OS $\mathcal{U}$ s.t. $\mathscr{k}_{\lambda} \in \mathcal{U}$, $\ell_{\mu} \notin \mathcal{U}$ or $\mathscr{k}_{\lambda} \notin \mathcal{U}$, $\ell_{\mu} \in \mathcal{U}$. Say $\mathscr{k}_{\lambda} \in \mathcal{U}$, $\ell_{\mu} \notin \mathcal{U}$ since $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space, then $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{U}$, this recouple tos $\exists \mathrm{N}^{g\alpha g}$-OS $\mathcal{V}$ s.t. $\ell_{\mu} \in \mathcal{V}$, $\mathscr{k}_{\lambda} \notin \mathcal{V}$. Consequently, $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_1-space. ▪

Theorem 4.10:
A NTS $(\mathcal{X}, \xi)$ is $\mathrm{N}^{g\alpha g}$-T_2-space iff
(i) $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_0-space & $\mathrm{N}^{g\alpha g}$-R_1-space.
(ii) $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_1-space & $\mathrm{N}^{g\alpha g}$-R_1-space.

Proof:
(i) Let $(\mathcal{X}, \xi)$ be a $\mathrm{N}^{g\alpha g}$-T_2-space, then it is a $\mathrm{N}^{g\alpha g}$-T_0-space. Wherefore $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_2-space, then for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$, there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U}, \mathcal{V}$ s.t. $\mathscr{k}_{\lambda} \in \mathcal{U}$ & $\ell_{\mu} \in \mathcal{V}$ implies $\mathscr{k}_{\lambda} \notin N^{g\alpha g}cl(\{\ell_{\mu}\})$ & $\ell_{\mu} \notin N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\})$, therefore $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) = \{\mathscr{k}_{\lambda}\} \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_{\mu}\}) = \{\ell_{\mu}\} \sqsubseteq \mathcal{V}$. Consequently, $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-R_1-space.
Contrariwise, let $(\mathcal{X}, \xi)$ be a $\mathrm{N}^{g\alpha g}$-T_0-space & $\mathrm{N}^{g\alpha g}$-R_1-space, then for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$, there exists a $\mathrm{N}^{g\alpha g}$-OS $\mathcal{U}$ s.t. $\mathscr{k}_{\lambda} \in \mathcal{U}$, $\ell_{\mu} \notin \mathcal{U}$ or $\ell_{\mu} \in \mathcal{U}$, $\mathscr{k}_{\lambda} \notin \mathcal{U}$, got $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}cl(\{\ell_{\mu}\})$, since $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-R_1-space [by hypothesis], then there exists disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{M}, \mathcal{N}$ s.t. $\mathscr{k}_{\lambda} \in \mathcal{M}$ & $\ell_{\mu} \in \mathcal{N}$. Consequently, $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_2-space.
(ii) Similarly to (i), $\mathrm{N}^{g\alpha g}$-T_2-space is a $\mathrm{N}^{g\alpha g}$-T_1-space & $\mathrm{N}^{g\alpha g}$-R_1-space.
Contrariwise, let $(\mathcal{X}, \xi)$ be a $\mathrm{N}^{g\alpha g}$-T_1-space & $\mathrm{N}^{g\alpha g}$-R_1-space, then for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$, $\exists \mathrm{N}^{g\alpha g}$-OSs $\mathcal{U}, \mathcal{V}$ s.t. $\mathscr{k}_{\lambda} \in \mathcal{U}$, $\ell_{\mu} \notin \mathcal{U}$ & $\ell_{\mu} \in \mathcal{V}$, $\mathscr{k}_{\lambda} \notin \mathcal{V}$ implies $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}cl(\{\ell_{\mu}\})$, since $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-R_1-space, then there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{M}, \mathcal{N}$ s.t. $\mathscr{k}_{\lambda} \in \mathcal{M}$ & $\ell_{\mu} \in \mathcal{N}$. Consequently, $(\mathcal{X}, \xi)$ is a $\mathrm{N}^{g\alpha g}$-T_2-space. ▪

Corollary 4.11:
A $\mathrm{N}^{g\alpha g}$-T_0-space is $\mathrm{N}^{g\alpha g}$-T_2-space iff for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}ker(\{\ell_{\mu}\})$, then $\exists \mathrm{N}^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$.

Proof:
By Thm. (3.11) & Thm. (4.10). ▪

Corollary 4.12:
A $\mathrm{N}^{g\alpha g}$-T_1-space is $\mathrm{N}^{g\alpha g}$-T_2-space iff one of the following satisfies:
(i) for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}cl(\{\ell_{\mu}\})$, then there exist $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U}, \mathcal{V}$ s.t. $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_{\mu}\})) \sqsubseteq \mathcal{V}$.
(ii) for any $\mathscr{k}_{\lambda} \neq \ell_{\mu} \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \neq N^{g\alpha g}ker(\{\ell_{\mu}\})$, then there exist $\mathrm{N}^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\mathscr{k}_{\lambda}\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_{\mu}\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$.

Proof:
(i) By corollary (3.12) & Thm. (4.10).
(ii) By Thm. (3.11) & Thm. (4.10). ▪

Theorem 4.13:
A $\mathrm{N}^{g\alpha g}$-R_1-space is $\mathrm{N}^{g\alpha g}$-T_2-space iff one of the following satisfies:

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(i) for any $ꝁ_\lambda \in \mathcal{X}$, $N^{g\alpha g}ker(\{ꝁ_\lambda\}) = \{ꝁ_\lambda\}$.
(ii) for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{ꝁ_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$ implies $N^{g\alpha g}ker(\{ꝁ_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$.
(iii) for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$, either $ꝁ_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ or $\ell_\mu \notin N^{g\alpha g}ker(\{ꝁ_\lambda\})$.
(iv) for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$ then $ꝁ_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{ꝁ_\lambda\})$.

Proof:

(i) Assume $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_2-space. So $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_1-space & N$^{g\alpha g}$-R_1-space [by Thm. (4.10)]. Wherefore by Thm. (4.6), $N^{g\alpha g}ker(\{ꝁ_\lambda\}) = \{ꝁ_\lambda\}$ for any $ꝁ_\lambda \in \mathcal{X}$.

Contrariwise, let for any $ꝁ_\lambda \in \mathcal{X}$, $N^{g\alpha g}ker(\{ꝁ_\lambda\}) = \{ꝁ_\lambda\}$, then by Thm. (4.6), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_1-space. Likewise, $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-R_1-space by hypothesis. Wherefore by Thm. (4.10), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_2-space.

(ii) Let $(\mathcal{X},\xi)$ be a N$^{g\alpha g}$-T_2-space. Then $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_1-space [by remark (4.2)]. Wherefore by Thm. (4.8), $N^{g\alpha g}ker(\{ꝁ_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$ for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$.

Contrariwise, assume that for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{ꝁ_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$ implies $N^{g\alpha g}ker(\{ꝁ_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$. So by Thm. (4.8), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_1-space, likewise $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-R_1-space by hypothesis. Wherefore by Thm. (4.10), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_2-space.

(iii) Let $(\mathcal{X},\xi)$ be a N$^{g\alpha g}$-T_2-space. Then $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_0-space [by remark (4.2)]. Wherefore by Thm. (4.3), either $ꝁ_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ or $\ell_\mu \notin N^{g\alpha g}ker(\{ꝁ_\lambda\})$ for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$.

Contrariwise, assume that for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$, either $ꝁ_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ or $\ell_\mu \notin N^{g\alpha g}ker(\{ꝁ_\lambda\})$ for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$. So by Thm. (4.3), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_0-space, likewise $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-R_1-space by hypothesis. Consequently, $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_2-space [by Thm. (4.10)].

(iv) Let $(\mathcal{X},\xi)$ be a N$^{g\alpha g}$-T_2-space. Then $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_1-space & N$^{g\alpha g}$-R_1-space [by Thm. (4.10)]. Wherefore by Thm. (4.7), $ꝁ_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{ꝁ_\lambda\})$.

Contrariwise, let for any $ꝁ_\lambda \neq \ell_\mu \in \mathcal{X}$ then $ꝁ_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{ꝁ_\lambda\})$. Then by Thm. (4.7), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_1-space. Likewise, $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-R_1-space by hypothesis. Wherefore by Thm. (4.10), $(\mathcal{X},\xi)$ is a N$^{g\alpha g}$-T_2-space. ▪

Remark 4.14:

Any N$^{g\alpha g}$-separation axiom is defined as the conjunction of two weaker neutrosophic axioms: N$^{g\alpha g}$-T_k-space = N$^{g\alpha g}$-R_{k-1}-space & N$^{g\alpha g}$-T_{k-1}-space = N$^{g\alpha g}$-R_{k-1}-space & N$^{g\alpha g}$-T_0-space, $k = 1,2$.

Remark 4.15:

The relevance between N$^{g\alpha g}$-separation axioms can be illustrated as a matrix. Therefore, a_{ij} refers to this relation. As the following matrix impersonation shows:

&	N$^{g\alpha g}$-T_0	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-R_0	N$^{g\alpha g}$-R_1
N$^{g\alpha g}$-T_0	N$^{g\alpha g}$-T_0	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_2
N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_2
N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2
N$^{g\alpha g}$-R_0	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_1	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-R_0	N$^{g\alpha g}$-R_1
N$^{g\alpha g}$-R_1	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-T_2	N$^{g\alpha g}$-R_1	N$^{g\alpha g}$-R_1

Matrix Representation
Figure 4.1: The relation between N$^{g\alpha g}$-separation axioms

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5. Conclusions

We have provided some new concepts of neutrosophic separation axioms, such as neutrosophic $g\alpha$g-R_i-space, $i = 0{,}1$ & neutrosophic $g\alpha$g-T_j-space, $= 0{,}1{,}2$. Furthermore, likewise proved some of their related attributes.

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