Exact Solution for Neutrosophic System of Ordinary Differential Equations by Neutrosophic Thick Function Using Laplace Transform

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Abstract

In this work, the idea of neutrosophic thick function has been proposed for solving a neutrosophic system of ordinary differential equations of various orders by the Laplace transform. This proposed scheme method is effective and simple to reach an exact solution.

Keywords: Neutrosophic equations; Neutrosophic thick function; a system of ordinary differential equations; Laplace transforms; exact solution.

1. Introduction

Neutrosophic was proposed first by Smarandache. F, where he has defined neutrosophic precalculus and neutrosophic calculus [1], neutrosophic measure, neutrosophic integral and neutrosophic sets and systems [2], neutrosophic closed sets, and continuous functions in neutrosophic sets and systems by Salama. An *et al* [3], Neutrosophic crisp for Grayscale image by Salama. A. *et al* [4], cosine similarity measures of the bipolar neutrosophic set for diagnosis of bipolar diseases by Abdel-Basset. M *et al* [5], a novel group decision-making model based on neutrosophic sets by Abdel-Basset. M *et al* [6], the basic structure of some classes o neutrosophic crips nearly open sets by Salama. A [7], separation axioms in neutrosophic crips topological spaces by Al-Nafee. An *et al* [8], a study of multi-topological spaces by Al-Hamido. R [9].
In 2020 Malath. F studied the integration of neutrosophic thick function [10] and he studied some neutrosophic differential equations by using a neutrosophic thick function [11].

Definition 1. [10,11]
Let a neutrosophic thick function as follows:

$$f : R \to P(R)\,;\ f(x) = [\, f_1(x),\, f_2(x)\,]$$

Where $P(R)$ is the set of all subsets R.

Definition 2. Neutrosophic integration thick function [10,11]
Let $f(x) = [\, f_1(x), f_2(x)\,]$ be a neutrosophic thick function. The integration of thick function defined by

$$\int f = \left[\int f_1, \int f_2\right] \qquad \text{Or} \qquad \int f(x)\,dx = \left[\int f_1(x)\,dx + k_1, \int f_2(x)\,dx + k_2\right]$$

Definition 3. Laplace transform of a neutrosophic thick function.
Let $[\, f_1(x), f_2(x)\,]\,; x \in R$ is a neutrosophic thick function, we can define the Laplace transform of the thick function as:

Doi: https://doi.org/10.54216/IJNS.190116
Received: May 02, 2022 Accepted: September 09, 2022

$$L[\,f_1(x),\,f_2(x)\,]=[\,F_1(p),F_2(p)\,]=\int_0^{+\infty} e^{-px}[\,f_1(x),\,f_2(x)\,]dx=\int_0^{+\infty}\left[e^{-px}f_1(x),e^{-px}f_2(x)\,\right]dx$$

$$=\left[\int_0^{+\infty} e^{-px}f_1(x)\,dx,\int_0^{+\infty} e^{-px}f_2(x)\,dx\right]\qquad ;\ p\ \text{is a complex number}$$

2. Methodology of solving a neutrosophic system of ordinary differential equations by the Laplace transform:

Consider the following neutrosophic system of general ordinary differential equations of n^th^ order:

$$\begin{cases}[\alpha_1,\alpha_2]u_1^{(n)}(x)+g_1\begin{pmatrix}[f_1(x),\,f_2(x)],[\alpha_3,\alpha_4]u_1,[\alpha_5,\alpha_6]u_2,...,[\alpha_7,\alpha_8]u_m,\,[\alpha_9,\alpha_{10}]u_1',...,[\alpha_{11},\alpha_{12}]u_m',\\ [\alpha_{13},\alpha_{14}]u_1^{(n-1)},...,[\alpha_{15},\alpha_{16}]u_m^{(n-1)}\end{pmatrix}=0\\ [\alpha_{17},\alpha_{18}]u_2^{(n)}(x)+g_2\begin{pmatrix}[f_3(x),\,f_4(x)],[\alpha_{19},\alpha_{20}]u_1,[\alpha_{21},\alpha_{22}]u_2,...,[\alpha_{23},\alpha_{24}]u_m,\,[\alpha_{25},\alpha_{26}]u_1',...,[\alpha_{27},\alpha_{28}]u_m',\\ [\alpha_{29},\alpha_{30}]u_1^{(n-1)},...,[\alpha_{31},\alpha_{32}]u_m^{(n-1)}\end{pmatrix}=0\\ \vdots\\ [\alpha_{33},\alpha_{34}]u_m^{(n)}(x)+g_m\begin{pmatrix}[f_5(x),\,f_6(x)],[\alpha_{35},\alpha_{36}]u_1,[\alpha_{37},\alpha_{38}]u_2,...,[\alpha_{39},\alpha_{40}]u_m,\,[\alpha_{41},\alpha_{42}]u_1',...,[\alpha_{43},\alpha_{44}]u_m',\\ [\alpha_{45},\alpha_{46}]u_1^{(n-1)},...,[\alpha_{47},\alpha_{48}]u_m^{(n-1)}\end{pmatrix}=0\end{cases}$$

Where $\alpha_i\in R$, $i=1,2,...,48$ and f_k ;$k=1,2,...,6$ are known analytical functions.

Step 1. Taking the Laplace to transform for each equation.

Step 2. Using the initial conditions.

Step 3. Taking the inverse Laplace transform.

3. Test problem

Example 1.

Consider the following neutrosophic system of ordinary differential equations

$$\begin{cases}u''(x)+[-3,5]v'(x)+[1,1]u(x)=[x^3-x,\,16x^2+2]\\ v''(x)+[1,-3]u'(x)+[0,1]v(x)=[3x^2+1,\,x^3]\end{cases}\tag{1}$$

With the initial conditions

$$u(0)=[0,0]\quad ,\quad u'(0)=[-1,0]\quad ,\quad v(0)=[-1,0]\quad ,\quad v'(0)=[0,0]$$

Solution.

By taking the Laplace to transform for each equation in (1), finds

$$\begin{cases}L(u''(x))+L(\,[-3,5]v'(x)\,)+L(\,[1,1]u(x)\,)=L([x^3-x,16x^2+2])\\ L(v''(x))+L(\,[1,-3]u'(x)\,)+L(\,[0,1]v(x)\,)=L([3x^2+1,\,x^3])\end{cases}$$

Then we have

$$\begin{cases}[p^2,p^2]U(p)-pu(0)-u'(0)+[-3,5](\,pV(p)-v(0))+[1,1]U(p)=[L(x^3-x),\,L(16x^2+2)]\\ [p^2,p^2]V(p)-pv(0)-v'(0)+[1,-3](\,pU(p)-u(0))+[0,1]V(p)=[L(3x^2+1),\,L(x^3)]\end{cases}\tag{2}$$

System (2) can be written as

$$\begin{cases}[p^2,p^2]U(p)-p\,[0,0]-[-1,0]+[-3,5](\,pV(p)-[-1,0])+[1,1]U(p)=\left[\dfrac{6}{p^4}-\dfrac{1}{p^2},\dfrac{32}{p^3}+\dfrac{2}{p}\right]\\ [p^2,p^2]V(p)-p\,[-1,0]-[0,0]+[1,-3](\,pU(p)-[0,0])+[0,1]V(p)=\left[\dfrac{6}{p^3}+\dfrac{1}{p},\dfrac{6}{p^4}\right]\end{cases}$$

yields

Doi: https://doi.org/10.54216/IJNS.190116
Received: May 02, 2022 Accepted: September 09, 2022

$$\begin{cases} \left[p^2+1, p^2+1\right]U(p)+\left[-3p, 5p\right]V(p)=\left[\dfrac{6}{p^4}-\dfrac{1}{p^2}+2, \dfrac{32}{p^3}+\dfrac{2}{p}\right] \\ \left[p, -3p\right]U(p)+\left[p^2, p^2+1\right]V(p)=\left[\dfrac{6}{p^3}+\dfrac{1}{p}-p, \dfrac{6}{p^4}\right] \end{cases} \tag{3}$$

The solution of system (3) is

$$\begin{cases} U(p)=\left[\dfrac{6}{p^4}-\dfrac{1}{p^2}, \dfrac{2}{p^3}\right] \\ V(p)=\left[\dfrac{2}{p^3}-\dfrac{1}{p}, \dfrac{6}{p^4}\right] \end{cases} \tag{4}$$

Then we take the inverse Laplace transform of (4), and then the exact solution of (1) with the initial conditions is

$$\begin{cases} u(x)=\left[x^3-x, x^2\right] \\ v(x)=\left[x^2-1, x^3\right] \end{cases}$$

Example 2.

Consider the following neutrosophic system of ordinary differential equations

$$\begin{cases} u^{(3)}(x)+[2,1]v''(x)+[3,5]u'(x)=[0, 7\cosh(x)] \\ v^{(3)}(x)+[0,-2]u''(x)+[-1,0]u'(x)+[0,3]v'(x)+[0,-2]u(x)+[2,0]v(x)=[\sin(x)+\cos(x), 0] \end{cases} \tag{5}$$

With the initial conditions

$u(0)=[0,0]$, $u'(0)=[1,1]$, $u''(0)=[0,0]$

$v(0)=[1,1]$, $v'(0)=[0,0]$, $v''(0)=[-1,1]$

Solution.

By taking the Laplace to transform for each equation in (5), finds

$$\begin{cases} L\left(u^{(3)}(x)\right)+L\left([2,1]v''(x)\right)+L\left([3,5]u'(x)\right)=L\left([0, 7\cosh(x)]\right) \\ L\left(v^{(3)}(x)\right)+L\left([0,-2]u''(x)\right)+L\left([-1,0]u'(x)\right)+L\left([0,3]v'(x)\right)+L\left([0,-2]u(x)\right)+L\left([2,0]v(x)\right)=L\left([\sin(x)+\cos(x), 0]\right) \end{cases}$$

Then we have

$$\begin{cases} \left[p^3, p^3\right]U(p)-p^2u(0)-pu'(0)-u''(0)+[2,1]\left(p^2V(p)-pv(0)-v'(0)\right)+[3,5]\left(pU(p)-u(0)\right) \\ \qquad =\left[L(0), L(7\cosh(x))\right] \\ \left[p^3, p^3\right]V(p)-p^2v(0)-pv'(0)-v''(0)+[0,-2]\left(p^2U(p)-pu(0)-u'(0)\right)+[-1,0]\left(pU(p)-u(0)\right) \\ \qquad +[0,3]\left(pV(p)-v(0)\right)+[0,-2]U(p)+[2,0]V(p)=\left[L(\sin(x)+\cos(x)), L(0)\right] \end{cases} \tag{6}$$

By using the initial conditions, then the system (6) can be written as

$$\begin{cases} \left[p^3+3p, p^3+5p\right]U(p)+\left[2p^2, p^2\right]V(p)=\left[3p, 2p+\dfrac{7p}{p^2-1}\right] \\ \left[-p, -2p^2-2\right]U(p)+\left[p^3+2, p^3+3p\right]V(p)=\left[\dfrac{p+1}{p^2+1}-1+p^2, p^2+2\right] \end{cases} \tag{7}$$

The solution of system (7) is

$$\begin{cases} U(p)=\left[\dfrac{1}{p^2+1}, \dfrac{1}{p^2-1}\right] \\ V(p)=\left[\dfrac{p}{p^2+1}, \dfrac{p}{p^2-1}\right] \end{cases} \tag{8}$$

Then we take the inverse Laplace transform of (8), and then the exact solution of (5) with the initial conditions is

$$\begin{cases} u(x)=\left[\sin(x), \sinh(x)\right] \\ v(x)=\left[\cos(x), \cosh(x)\right] \end{cases}$$

Doi: https://doi.org/10.54216/IJNS.190116
Received: May 02, 2022 Accepted: September 09, 2022

Example 3.

Consider the following neutrosophic system of ordinary differential equations

$$\begin{cases} [1,0]u^{(4)}(x)+[0,1]u^{(3)}(x)+[2,0]v''(x)+[0,3]w''(x)+[-3,0]w'(x)+[-3,0]u(x)+[0,-5]v(x)+[0,1]w(x) \\ \qquad =\left[-6e^{2x}, 8\cosh(2x)-5\cosh(x)+28e^{3x}+x^2-5x+6\right] \\ [1,1]v^{(4)}(x)+[-3,0]u^{(3)}(x)+[2,0]v'(x)+[0,3]w'(x)+[0,-2]u(x)+[1,0]w(x) \\ \qquad =\left[e^{2x}+3, -2\cosh(2x)+\cosh(x)+9e^{3x}+6x\right] \\ [0,1]w^{(4)}(x)+[1,0]w^{(3)}(x)+[-2,7]w''(x)+[-5,-2]v'(x)+[1,1]u(x) \\ \qquad =\left[-4e^{x},\ \sinh(2x)-2\sinh(x)+144e^{3x}+12\right] \end{cases} \tag{9}$$

With the initial conditions

$u(0)=[1,0]$, $u'(0)=[1,2]$, $u''(0)=[1,0]$, $u^{(3)}(0)=[1,8]$

$v(0)=[0,1]$, $v'(0)=[1,1]$, $v''(0)=[1,1]$, $v^{(3)}(0)=[1,0]$

$w(0)=[4,1]$, $w'(0)=[2,3]$, $w''(0)=[4,11]$, $w^{(3)}(0)=[8,27]$

Solution.

By taking the Laplace to transform for each equation in (9) and the same technique as Examples 1 & 2, yields

$$\begin{cases} \left[p^4-3, p^3\right]U(p)+\left[2p^2,5\right]V(p)+\left[-3p,3p^2+1\right]W(p) \\ \qquad =\left[\frac{-6}{p-2}+p^3+p^2+p-9, \frac{8p}{p^2-4}-\frac{5p}{p^2-1}+\frac{28}{p-3}+\frac{2}{p^3}-\frac{5}{p^2}+\frac{6}{p}+5p+9\right] \\ \left[-3p^3,-2\right]U(p)+\left[p^4+2p, p^4\right]V(p)+[1,3p]W(p) \\ \qquad =\left[\frac{1}{p-2}+\frac{3}{p}-2p^2-2p-2, \frac{p}{p^2-1}+\frac{6}{p^2}-\frac{4}{p^2-4}+\frac{9}{p-3}+p^3+p^2+p+3\right] \\ [1,1]U(p)+[-5p,-2p]V(p)+\left[p^3-2p^2, p^4+7p^2\right]W(p) \\ \qquad =\left[\frac{-4}{p-1}+4p^2-6p, \frac{-2}{p^2-1}+\frac{12}{p}+\frac{2}{p^2-4}+\frac{144}{p-3}+p^3+3p^2+18p+46\right] \end{cases} \tag{10}$$

The solution of system (10) is

$$\begin{cases} U(p)=\left[\frac{1}{p-1}, \frac{2}{p^2-4}\right] \\ V(p)=\left[\frac{1}{p-1}-\frac{1}{p}, \frac{1}{p^2}+\frac{p}{p^2-1}\right] \\ W(p)=\left[\frac{1}{p-2}+\frac{3}{p}, \frac{2}{p^3}+\frac{1}{p-3}\right] \end{cases} \tag{11}$$

Then we take the inverse Laplace transform of (11), and then the exact solution of (9) with the initial conditions is

$$\begin{cases} u(x)=\left[e^{x},\ \sinh(2x)\right] \\ v(x)=\left[e^{x}-1,\ x+\cosh(x)\right] \\ w(x)=\left[e^{2x}+3, x^2+e^{3x}\right] \end{cases}$$

4. Conclusion

In this work, the Laplace transform successfully deals with a new type of neutrosophic equations which are neutrosophic systems of ordinary differential equations by thick function. The obtained results show that the Laplace transform is simple, good, and powerful for obtaining the exact solution to these systems.

Doi: https://doi.org/10.54216/IJNS.190116
Received: May 02, 2022 Accepted: September 09, 2022

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Doi: https://doi.org/10.54216/IJNS.190116
Received: May 02, 2022 Accepted: September 09, 2022