Neutrosophic Nano M Open Sets

A. Vadivel[1,*], C. John Sundar[2], K. Saraswathi[3], S. Tamilselvan[4]
[1]PG and Research Department of Mathematics, Government Arts College (Autonomous), Karur - 639 005, India.
[1,2]Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.
[3]Department of Mathematics, Thiruvalluvar Government Arts College, Rasipuram, Namakkal - 637 401, India.
[4]Engineering Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.
Emails: avmaths@gmail.com[1], johnphdau@hotmail.com[2], srivanji2015@gmail.com[3], tamil_au@yahoo.com[4]

Abstract

In this paper, we introduce the concepts of neutrosophic nano M-open sets and some stronger and weaker forms of neutrosophic nano open sets in neutrosophic nano topological spaces. Further, we dealt with the concepts of neutrosophic nano M-interior and M-closure operators. Moreover, we define the product related neutrosophic nano topological spaces and proved some theorems related to this.

Keywords: neutrosophic nano open; neutrosophic nano θ-open; neutrosophic nano θ-semi open; neutrosophic nano δ-pre open and neutrosophic nano M open
AMS (2000) subject classification: 03E72, 54A05, 54A40.

1 Introduction

Zadeh[29] was the first to present fuzzy set between intervals in mathematics. Later, Chang[4] was the first to present the idea of fuzzy topology. Atanassov[2,3] introduced a further generalisation of this fuzzy set in 1986 under the name intuitionistic fuzzy sets. In addition to this, intuitionistic fuzzy sets are widely explored in the topological framework proposed by Coker.[5] In 1995, Smarandache[17,18] introduced neutrosophic logic. It is a logic in which each assertion is determined to have a certain amount of truth, uncertainty, and falsity. Neutosophic topological spaces were first introduced in 2012 by Salama et al.[16] The neutrosophic set has numerous real-time applications in the many areas by several authors.[1,6,10,13,23,25] Rough set theory was first introduced by Pawlak[14] as a substitute mathematical tool for describing and handling vagueness and uncertainty in reasoning and decision-making. Rough set concepts are frequently expressed in a very generic manner using approximations, which are topological procedures for interior and closure. Rough set theory was extended in 2013 by Lellis Thivagar,[8] who also presented a new topology called nano topology and nano topological spaces.

Additionally,[11] investigated nano δ open sets in nano topological space. Vadivel et al.[21,22,26–28] presented the concepts of neutrosophic δ closure derived from neutrosophic regular closed sets and open sets in a neutrosophic topological space. El-Maghrabi and Al-Juhani[?] in 2011 introduced M-open sets in topological spaces and Padma et al.[12] introduced it in nano topological spaces and studied some of their properties. Recently, a novel idea of neutrosophic nano topology was investigated by Lellis Thivagar et al.[9] Recently, Thangammal et al.[19,20] introduced Z-open sets in fuzzy nano topological spaces and Vadivel et al.[24] investigated δ open

https://doi.org/10.54216/IJNS.190110
Received: March 02, 2022 Accepted: September 15, 2022

sets and their weaker sets in neutrosophic nano topological spaces. Kalaiyarasan et al.[7] introduced normal spaces associated with M-open sets in fuzzy nano topological spaces and their applications using fuzzy score function.

In section 2 of this paper, certain foundational ideas needed for our work are simply recalled. In section 3, we introduce the concept of neutrosophic nano M (resp. θ)-interior, neutrosophic nano M (resp. θ)-closure operators. It also established $N_seu\mathcal{N}\theta o$, $N_seu\mathcal{N}\theta So$, $N_seu\mathcal{N}\delta\mathcal{P}o$, $N_seu\mathcal{N}eo$ and $N_seu\mathcal{N}Mo$ sets and discuss about some of their properties. In section 4, we dealt with the concepts of neutrosophic nano interior and closure operators in various nano open and closed sets. Finally, in section 5, is to define the product related neutrosophic nano M open sets via neutrosophic nano topological spaces.

2 Preliminaries

Definition 2.1. [15] Let U be a universe of discourse with a generic element in U denoted by s, the neutrosophic set (briefly, $N_seu\ s$)is an object having the form $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : x \in U\}$, where $\mu_S, \sigma_S, \nu_S : U \to [0,1]$ denote the degree of membership, indeterminacy and non-membership functions respectively of each element $s \in U$ to the set S and $0 \leq \mu_S(s) + \sigma_S(s) + \nu_S(s) \leq 3$ for each $s \in U$.

Definition 2.2. [9] Let U be a non-empty set and Re be an equivalence relation on U.

1. Let S be a $N_seut\ s$ in U with μ_S, σ_S and ν_S. The neutrosophic nano lower (resp. upper) approximations and neutrosophic nano boundary of S in the approximation (U, Re) denoted by $\underline{N_seu\mathcal{N}}(S), \overline{N_seu\mathcal{N}}(S)$ & $B_{N_seu\mathcal{N}}(S)$ are

   (i) $\underline{N_seu\mathcal{N}}(S) = \left\{\langle s, \mu_{\underline{Re}(J)}(s), \sigma_{\underline{Re}(J)}(s), \nu_{\underline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (ii) $\overline{N_seu\mathcal{N}}(S) = \left\{\langle s, \mu_{\overline{Re}(J)}(s), \sigma_{\overline{Re}(J)}(s), \nu_{\overline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (iii) $B_{N_seu\mathcal{N}}(S) = \overline{N_seu\mathcal{N}}(S) - \underline{N_seu\mathcal{N}}(S)$

   where $\mu_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \mu_J(t), \sigma_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \sigma_J(t), \nu_{\underline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \nu_J(t)$.

   $\mu_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \mu_J(t), \sigma_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \sigma_J(t), \nu_{\overline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \nu_J(t)$.

   The collection $\tau_N(S) = \left\{0_N, 1_N, \underline{N_seu\mathcal{N}}(S), \overline{N_seu\mathcal{N}}(S), B_{N_seu\mathcal{N}}(S)\right\}$ forms a topology called as neutrosophic nano topology and $(U, \tau_N(S))$ as neutrosophic nano topological space (briefly, $N_seu\mathcal{N}ts$). The elements of $\tau_N(S)$ are called neutrosophic nano open (briefly, $N_seu\mathcal{N}o$) sets. Elements of $[\tau_N(S)]^c$ are called neutrosophic nano closed (briefly, $N_seu\mathcal{N}c$) sets.

2. Let S be an intuitionistic set (briefly, $Int\ s$) in U with μ_S and ν_S. The intuitionistic nano lower (resp. upper) approximations and intuitionistic nano boundary of S in the approximation (U, Re) denoted by $\underline{Int\mathcal{N}}(S), \overline{Int\mathcal{N}}(S)$ & $B_{Int\mathcal{N}}(S)$ are

   (i) $\underline{Int\mathcal{N}}(S) = \left\{\langle s, \mu_{\underline{Re}(J)}(s), \nu_{\underline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (ii) $\overline{Int\mathcal{N}}(S) = \left\{\langle s, \mu_{\overline{Re}(J)}(s), \nu_{\overline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (iii) $B_{Int\mathcal{N}}(S) = \overline{Int\mathcal{N}}(S) - \underline{Int\mathcal{N}}(S)$

   where $\mu_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \mu_J(t), \nu_{\underline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \nu_J(t)$.

   $\mu_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \mu_J(t), \nu_{\overline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \nu_J(t)$.

   The collection $\tau_I(S) = \left\{0_I, 1_I, \underline{Int\mathcal{N}}(S), \overline{Int\mathcal{N}}(S), B_{Int\mathcal{N}}(S)\right\}$ forms a topology called as an intuitionistic nano topology and $(U, \tau_I(S))$ as an intuitionistic nano topological space (briefly, $Int\mathcal{N}ts$). The elements of $\tau_I(S)$ are called intuitionistic nano open (briefly, $Int\mathcal{N}o$) sets. Elements of $[\tau_I(S)]^c$ are called intuitionistic nano closed (briefly, $N_seu\mathcal{N}c$) sets.

3. Let S be a fuzzy set (briefly, $\mathcal{F}\ s$) in U with μ_S. The fuzzy nano lower (resp. upper) approximations and fuzzy nano boundary of S in the approximation (U, Re) denoted by $\underline{\mathcal{FN}}(S), \overline{\mathcal{FN}}(S)$ & $B_{\mathcal{FN}}(S)$ are

Received: March 02, 2022 Accepted: September 15, 2022

(i) $\underline{\mathcal{FN}}(S) = \{\langle s, \mu_{\underline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\}$

(ii) $\overline{\mathcal{FN}}(S) = \left\{\langle s, \mu_{\overline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

(iii) $B_{\mathcal{FN}}(S) = \overline{\mathcal{FN}}(S) - \underline{\mathcal{FN}}(S)$

where $\mu_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \mu_J(t)$. $\mu_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \mu_J(t)$.

The collection $\tau_{\mathcal{F}}(S) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{FN}}(S), \overline{\mathcal{FN}}(S), B_{\mathcal{FN}}(S)\}$ forms a topology called as fuzzy nano topology and $(U, \tau_{\mathcal{F}}(S))$ as fuzzy nano topological space (briefly, $\mathcal{FN}ts$). The elements of $\tau_{\mathcal{F}}(S)$ are called fuzzy nano open (briefly, $\mathcal{FN}o$) sets. Elements of $[\tau_{\mathcal{F}}(S)]^c$ are called fuzzy nano closed (briefly, $N_seu\mathcal{N}c$) sets.

Remark 2.3. Thus from the definitions of intuitionistic nano and fuzzy nano topologies we can assure that throughout this paper all the properties and examples also holds good when it is possible for neutrosophic nano topology.

Definition 2.4. [9] Let U be a nonempty set and the neutrosophic subsets (briefly, $N_seu\ subs$'s) S and T in the form $S = \{\langle s : \mu_S(s), \sigma_S(s), \nu_S(s)\rangle, s \in U\}, T = \{\langle s : \mu_T(s), \sigma_T(s), \nu_T(s)\rangle, s \in U\}$. Then the statements are hold:

(i) $0_N = \{\langle s, 0, 0, 1\rangle : s \in U\}$.

(ii) $1_N = \{\langle s, 1, 1, 0\rangle : s \in U\}$.

(iii) $S \subseteq T$ iff $\mu_S(s) \leq \mu_T(s), \sigma_S(s) \leq \sigma_T(s), \nu_S(s) \geq \nu_T(s)\ \forall\ s \in U\}$.

(iv) $S = T$ iff $S \subseteq T$ and $T \subseteq S$

(v) $S^c = \{\langle s, \nu_S(s), 1 - \sigma_S(s), \mu_S(s)\rangle : s \in U\}$

(vi) $S \cap T = \{s, \mu_S(s) \wedge \mu_T(s), \sigma_S(s) \wedge \sigma_T(s), \nu_S(s) \vee \nu_T(s)\ \forall\ s \in U\}$.

(vii) $S \cup T = \{s, \mu_S(s) \vee \mu_T(s), \sigma_S(s) \vee \sigma_T(s), \nu_S(s) \wedge \nu_T(s)\ \forall\ s \in U\}$.

Definition 2.5. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. Let S be a $N_seu\ subs$ of U. Then neutrosophic nano

(i) interior of S[9] (briefly, $N_seu\mathcal{N}int(S)$) is described as $N_seu\mathcal{N}int(S) = \bigcup\{C : C \subseteq S\ \&\ C$ is a $N_seut\ \mathcal{N}o\}$.

(ii) closure of S[9] (briefly, $N_seu\mathcal{N}cl(S)$) is described as $N_seu\mathcal{N}cl(S) = \bigcap\{C : S \subseteq C\ \&\ C$ is a $N_seut\ \mathcal{N}c\}$.

(iii) regular open[12] (briefly, $N_seu\mathcal{N}ro$) set if $S = N_seu\mathcal{N}int(N_seu\mathcal{N}cl(S))$.

(iv) regular closed[12] (briefly, $N_seu\mathcal{N}rc$) set if $S = N_seu\mathcal{N}cl(N_seu\mathcal{N}int(S))$.

Definition 2.6. [24] Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$ with respect to F where F is a $N_seu\ subs$ of U. Let S be a $N_seu\ subs$ of U. Then a neutrosophic nano

(i) δ interior of S (briefly, $N_seu\mathcal{N}\delta int(S)$) is defined by $N_seu\mathcal{N}\delta int(S) = \bigcup\{C : C \subseteq S\ \&\ C$ is a $N_seu\mathcal{N}ro$ set in $U\}$.

(ii) δ closure of S (briefly, $N_seu\mathcal{N}\delta cl(S)$) is defined by $N_seu\mathcal{N}\delta cl(S) = \bigcap\{C : S \subseteq C\ \&\ C$ is a $N_seu\mathcal{N}rc$ set in $U\}$.

(iii) δ-open (briefly, $N_seu\mathcal{N}\delta o$) set if $S = N_seu\mathcal{N}\delta int(S)$.

(iv) δ-pre open (briefly, $N_seu\mathcal{N}\delta\mathcal{P}o$) set if $S \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))$.

(v) e-open (briefly, $N_seu\mathcal{N}eo$) set if $S \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta\ int(S)) \cup N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))$.

The complement of a $N_seu\mathcal{N}\delta o$ (resp. $N_seu\mathcal{N}\delta\mathcal{P}o\ \&\ N_seu\mathcal{N}eo$) set is called a neutrosophic nano δ (resp. neutrosophic nano δ-pre & neutrosophic nano e) closed (briefly, $N_seu\mathcal{N}\delta c$ (resp. $N_seu\mathcal{N}\delta\mathcal{P}c\ \&\ N_seu\mathcal{N}ec$)) in U.

https://doi.org/10.54216/IJNS.190110

Received: March 02, 2022 Accepted: September 15, 2022

3 More on neutrosophic nano open sets via nano θ-open sets

Definition 3.1. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. Let S be a $N_seu\ subs$ of U. Then a neutrosophic nano

(i) θ interior of S (briefly, $N_seu\mathcal{N}\theta int(S)$) is defined by $N_seu\mathcal{N}\theta int(S) = \bigcup\{N_seu\mathcal{N}int(C) : C \subseteq S$ & C is a $N_seu\mathcal{N}c$ set in $U\}$.

(ii) θ closure of S (briefly, $N_seu\mathcal{N}\theta cl(S)$) is defined by $N_seu\mathcal{N}\theta cl(S) = \bigcap\{N_seu\mathcal{N}cl(C) : S \subseteq C$ & C is a $N_seu\mathcal{N}o$ set in $U\}$.

Definition 3.2. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. Then a $N_seu\ subs\ S$ in U is said to be a neutrosophic nano

(i) θ open (briefly, $N_seu\mathcal{N}\theta o$) set if $S = N_seu\mathcal{N}\theta int(S)$.

(ii) θ semi open (briefly, $N_seu\mathcal{N}\theta\mathcal{S}o$) set if $S \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S))$.

(iii) θ pre open (briefly, $N_seu\mathcal{N}\theta\mathcal{P}o$) set if $S \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S))$.

(iv) M-open (briefly, $N_seu\mathcal{N}Mo$) set if $S \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)) \cup N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))$.

The complement of a $N_seu\mathcal{N}\theta o$ (resp. $N_seu\mathcal{N}\theta\mathcal{S}o$, $N_seu\mathcal{N}\theta\mathcal{P}o$ & $N_seu\mathcal{N}Mo$) set is called a neutrosophic nano θ (resp. neutrosophic nano θ semi, neutrosophic nano θ pre & neutrosophic nano M) closed (briefly, $N_seu\mathcal{N}\theta c$ (resp. $N_seu\mathcal{N}\theta\mathcal{S}c$, $N_seu\mathcal{N}\theta\mathcal{P}c$ & $N_seu\mathcal{N}Mc$)) in U.

Proposition 3.3. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. Then the statements are hold for $N_seu\mathcal{N}ts$. Every

(i) $N_seu\mathcal{N}\theta o$ set (resp. $N_seu\mathcal{N}\theta c$ set) is a $N_seu\mathcal{N}o$ set (resp. $N_seu\mathcal{N}c$ set).

(ii) $N_seu\mathcal{N}\theta o$ set (resp. $N_seu\mathcal{N}\theta c$ set) is a $N_seu\mathcal{N}\theta\mathcal{S}o$ set (resp. $N_seu\mathcal{N}\theta\mathcal{S}c$ set).

(iii) $N_seu\mathcal{N}o$ set (resp. $N_seu\mathcal{N}c$ set) is a $N_seu\mathcal{N}\delta\mathcal{P}o$ set (resp. $N_seu\mathcal{N}\delta\mathcal{P}c$ set).

(iv) $N_seu\mathcal{N}\theta\mathcal{S}o$ set (resp. $N_seu\mathcal{N}\theta\mathcal{S}c$ set) is a $N_seu\mathcal{N}Mo$ set (resp. $N_seu\mathcal{N}Mc$ set).

(v) $N_seu\mathcal{N}\delta\mathcal{P}o$ set (resp. $N_seu\mathcal{N}\delta\mathcal{P}c$ set) is a $N_seu\mathcal{N}Mo$ set (resp. $N_seu\mathcal{N}Mc$ set).

(vi) $N_seu\mathcal{N}\delta\mathcal{P}o$ set (resp. $N_seu\mathcal{N}\delta\mathcal{P}c$ set) is a $N_seu\mathcal{N}eo$ set (resp. $N_seu\mathcal{N}ec$ set).

(vii) $N_seu\mathcal{N}Mo$ set (resp. $N_seu\mathcal{N}Mc$ set) is a $N_seu\mathcal{N}eo$ set (resp. $N_seu\mathcal{N}ec$ set).

But not converse.

Proof. (i) Let S_o is a $N_seu\mathcal{N}\theta o$, then $S_o = N_seu\mathcal{N}\theta int(S_o) \subseteq N_seu\mathcal{N}int(S_o)$. Therefore S_o is a $N_seu\mathcal{N}o$.

(ii) Let S_o is a $N_seu\mathcal{N}\theta o$, then $S_o = N_seu\mathcal{N}\theta int(S_o) \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S_o))$. Therefore S_o is a $N_seu\mathcal{N}\theta\mathcal{S}o$.

(iii) Let S_o is a $N_seu\mathcal{N}o$, then $S_o = N_seu\mathcal{N}int(S_o) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o))$. Therefore S_o is a $N_seu\mathcal{N}\delta\mathcal{P}o$.

(iv) Let S_o is a $N_seu\mathcal{N}\theta\mathcal{S}o$, then $S_o \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S_o)) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S_o))$. Therefore S_o is a $N_seu\mathcal{N}Mo$.

(v) Let S_o is a $N_seu\mathcal{N}\delta\mathcal{P}os$, then $S_o \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S_o))$. Therefore S_o is a $N_seu\mathcal{N}Mo$.

(vi) Let S_o is a $N_seu\mathcal{N}\delta\mathcal{P}os$, then $S_o \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta int(S_o))$. Therefore S_o is a $N_seu\mathcal{N}eo$.

(vi) Let S_o is a $N_seu\mathcal{N}Mo$, then $S_o \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S_o)) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S_o)) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}\ \delta int(S_o))$. Therefore S_o is a $N_seu\mathcal{N}eo$.

Proof of the closed sets are also in a similar way. □

https://doi.org/10.54216/IJNS.190110

Received: March 02, 2022 Accepted: September 15, 2022

[arrows] $N_seu\mathcal{N}\theta o$ [d] [r] $N_seu\mathcal{N}\theta\mathcal{S}o$ [r] $N_seu\mathcal{N}Mo$ [d]
$N_seu\mathcal{N}o$ [r] $N_seu\mathcal{N}\delta\mathcal{P}o$ [r] [ur] $N_seu\mathcal{N}eo$

Figure 1: $N_seu\mathcal{N}Mos$'s in $N_seu\mathcal{N}ts$.

Example 3.4. Assume $U = \{s_1, s_2, s_3, s_4\}$ and $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$.

Let $S = \left\{\left\langle\frac{s_1}{0.2,0.5,0.8}\right\rangle, \left\langle\frac{s_2}{0.3,0.5,0.7}\right\rangle, \left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle, \left\langle\frac{s_4}{0.1,0.5,0.9}\right\rangle\right\}$ be a $Neu\ sub$ of U.

$$\underline{N_seu\mathcal{N}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.1, 0.5, 0.9}\right\rangle, \left\langle\frac{s_2}{0.3, 0.5, 0.7}\right\rangle, \left\langle\frac{s_3}{0.4, 0.5, 0.6}\right\rangle\right\},$$
$$\overline{N_seu\mathcal{N}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.2, 0.5, 0.8}\right\rangle, \left\langle\frac{s_2}{0.3, 0.5, 0.7}\right\rangle, \left\langle\frac{s_3}{0.4, 0.5, 0.6}\right\rangle\right\},$$
$$B_{N_seu\mathcal{N}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.2, 0.5, 0.8}\right\rangle, \left\langle\frac{s_2}{0.3, 0.5, 0.7}\right\rangle, \left\langle\frac{s_3}{0.4, 0.5, 0.6}\right\rangle\right\}.$$

Thus $\tau_N(S) = \{0_N, 1_N, \underline{N_seu\mathcal{N}}(S), \overline{N_seu\mathcal{N}}(S) = B_{N_seu\mathcal{N}}(S)\}$.

Then

1. $\left\{\left\langle\frac{s_1,s_4}{0.1,0.5,0.9}\right\rangle, \left\langle\frac{s_2}{0.3,0.5,0.7}\right\rangle, \left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$ is a $N_seu\mathcal{N}o$ set but not $N_seu\mathcal{N}\theta o$ set.
2. $\left\{\left\langle\frac{s_1,s_4}{0.1,0.5,0.9}\right\rangle, \left\langle\frac{s_2}{0.1,0.5,0.9}\right\rangle, \left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$ is a $N_seu\mathcal{N}\delta\mathcal{P}o$ (resp. $N_seu\mathcal{N}Mo$) set but not $N_seu\mathcal{N}o$ (resp. $N_seu\mathcal{N}\theta\mathcal{S}o$) set.
3. $\left\{\left\langle\frac{s_1,s_4}{0.8,0.5,0.2}\right\rangle, \left\langle\frac{s_2}{0.7,0.5,0.3}\right\rangle, \left\langle\frac{s_3}{0.6,0.5,0.4}\right\rangle\right\}$ is a $N_seu\mathcal{N}\theta\mathcal{S}o$ (resp. $N_seu\mathcal{N}Mo$) set but not $N_seu\mathcal{N}\theta o$ (resp. $N_seu\mathcal{N}\delta\mathcal{P}o$) set.
4. $\left\{\left\langle\frac{s_1,s_4}{0.4,0.5,0.6}\right\rangle, \left\langle\frac{s_2}{0.6,0.5,0.4}\right\rangle, \left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$ is a $N_seu\mathcal{N}eo$ set but not $N_seu\mathcal{N}\delta\mathcal{P}o$ (resp. $N_seu\mathcal{N}Mo$) set.

Proposition 3.5. If S_o and T_o are two $N_seu\mathcal{N}Mo$ (resp. $N_seu\mathcal{N}\theta o$, $N_seu\mathcal{N}\theta\mathcal{S}o$ & $N_seu\mathcal{N}\delta\mathcal{P}o$) sets, then $S_o \cup T_o$ is $N_seu\mathcal{N}Mo$ (resp. $N_seu\mathcal{N}\theta o$, $N_seu\mathcal{N}\theta\mathcal{S}o$ & $N_seu\mathcal{N}\delta\mathcal{P}o$) set.

Proof. If S_o and T_o are two $N_seu\mathcal{N}Mo$ sets. Then by definition $S_o = N_seu\mathcal{N}Mint(S_o)$ and $T_o = N_seu\mathcal{N}Mint(T_o)$. Now $S_o \cup T_o = N_seu\mathcal{N}Mint(S_o) \cup N_seu\mathcal{N}Mint(T_o) \subseteq N_seu\mathcal{N}Mint(S_o \cup T_o)$. Since, $N_seu\mathcal{N}Mint(S_o \cup T_o) \subseteq S_o \cup T_o$. So $S_o \cup T_o = N_seu\mathcal{N}Mint(S_o \cup T_o)$. Hence, $S_o \cup T_o$ is $N_seu\mathcal{N}Mo$ set.

The proof in the other cases is comparable. □

Proposition 3.6. Arbitrary union of $N_seu\mathcal{N}Mo$ (resp. $N_seu\mathcal{N}\theta o$, $N_seu\mathcal{N}\theta\mathcal{S}o$ & $N_seu\mathcal{N}\delta\mathcal{P}o$) sets is a $N_seu\mathcal{N}Mo$ (resp. $N_seu\mathcal{N}\theta o$, $N_seu\mathcal{N}\theta\mathcal{S}o$ & $N_seu\mathcal{N}\delta\mathcal{P}o$) set.

Proof. Let $\{S_k\}$ be a collection of $N_seu\mathcal{N}Mo$ sets of a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then by definition $S_k = N_seu\mathcal{N}Mint(S_k)$ for each k. Now $\bigcup S_k = \bigcup N_seu\mathcal{N}Mint(S_k) \subseteq N_seu\mathcal{N}Mint(\bigcup S_k)$. Since, $N_seu\mathcal{N}M$ $int(\bigcup S_k) \subseteq \bigcup S_k$. So $\bigcup S_k = N_seu\mathcal{N}Mint(\bigcup S_k)$. Hence, $\bigcup S_k$ is $N_seu\mathcal{N}Mo$ set.

The proof in the other cases is comparable. □

Remark 3.7. Intersection of any two $N_seu\mathcal{N}Mo$ (resp. $N_seu\mathcal{N}\theta\mathcal{S}o$ & $N_seu\mathcal{N}\delta\mathcal{P}o$) sets need not be $N_seu\mathcal{N}Mo$ (resp. $N_seu\mathcal{N}\theta\mathcal{S}o$ & $N_seu\mathcal{N}\delta\mathcal{P}o$) set as shown below.

Received: March 02, 2022 Accepted: September 15, 2022

Example 3.8. In Example 3.4, let $A = \left\{\left\langle \frac{s_1,s_4}{0.3,0.5,0.7} \right\rangle, \left\langle \frac{s_2}{0.5,0.5,0.5} \right\rangle, \left\langle \frac{s_3}{0.8,0.5,0.2} \right\rangle\right\}$ and

$B = \left\{\left\langle \frac{s_1,s_4}{0.8,0.5,0.2} \right\rangle, \left\langle \frac{s_2}{0.8,0.5,0.2} \right\rangle, \left\langle \frac{s_3}{0.5,0.5,0.5} \right\rangle\right\}$ are $N_s eu\mathcal{N}Mo$ sets but

$A \cap B = \left\{\left\langle \frac{s_1,s_4}{0.3,0.5,0.7} \right\rangle, \left\langle \frac{s_2}{0.5,0.5,0.5} \right\rangle, \left\langle \frac{s_3}{0.5,0.5,0.5} \right\rangle\right\}$ is not $N_s eu\mathcal{N}Mo$ set.

4 More on neutrosophic nano interior and closure operators

Definition 4.1. A set S is said to be a

(i) neutrosophic nano M (resp. neutrosophic nano θ-semi & neutrosophic nano δ-pre) interior of S (briefly, $N_s eu\mathcal{N}Mint(S)$ (resp. $N_s eu\mathcal{N}\theta\mathcal{S}int(S)$ & $N_s eu\mathcal{N}\delta\mathcal{P}int(S)$)) is the union of all $N_s eu\mathcal{N}Mo$ (resp. $N_s eu\mathcal{N}\theta\mathcal{S}o$ & $N_s eu\mathcal{N}\delta\mathcal{P}o$) set contained in S.

(ii) neutrosophic nano M (resp. neutrosophic nano θ-semi & neutrosophic nano δ-pre) closure of S (briefly, $N_s eu\mathcal{N}Mcl(S)$ (resp. $N_s eu\mathcal{N}\theta\mathcal{S}cl(S)$ & $N_s eu\mathcal{N}\delta\mathcal{P}cl(S)$)) is the intersection of all $N_s eu\mathcal{N}Mc$ (resp. $N_s eu\mathcal{N}\theta\mathcal{S}c$ & $N_s eu\mathcal{N}\delta\mathcal{P}c$) set containing S.

Theorem 4.2. Let $(U, \tau_N(F))$ be a $N_s eu\mathcal{N}ts$. Let S be a $N_s eu\ subs$ of U. Then

(i) $1_N - N_s eu\mathcal{N}Mint(S) = N_s eu\mathcal{N}Mcl(1_N - S)$ (or) $(N_s eu\mathcal{N}Mint(S))^c = N_s eu\mathcal{N}Mcl(S^c)$.

(ii) $1_N - N_s eu\mathcal{N}Mcl(S) = N_s eu\mathcal{N}Mint(1_N - S)$ (or) $(N_s eu\mathcal{N}Mcl(S))^c = N_s eu\mathcal{N}Mint(S^c)$.

Proof. (i) By Definition 3.1, $N_s eu\mathcal{N}Mint(S) = \bigcup\{G : G \text{ is a } N_s eu\mathcal{N}Mo \text{ in } U \ \& \ G \subseteq S\}$. Taking complement on both sides, $(N_s eu\mathcal{N}Mint(S))^c = (\bigcup\{G : G \text{ is a } N_s eu\mathcal{N}Mo \text{ set in } U \ \& \ G \subseteq S\})^c = \bigcap\{G^c : G^c \text{ is a } N_s eu\mathcal{N}Mc \text{ set in } U \ \& \ S^c \subseteq G^c\}$. Replacing G^c by K, we get $(N_s eu\mathcal{N}Mint(S))^c = \bigcap\{K : K \text{ is a } N_s eu\mathcal{N}Mc \text{ set in } U \ \& \ S^c \subseteq K\}$. By Definition 3.1, $(N_s eu\mathcal{N}Mint(S))^c = N_s eu\mathcal{N}Mcl(S^c)$. This proves (i).

(ii) By using (i), $(N_s eu\mathcal{N}Mint(S^c))^c = N_s eu\mathcal{N}Mcl((S^c)^c) = N_s eu\mathcal{N}Mcl(S)$. Taking complement on both sides, we get $N_s eu\mathcal{N}M\ int(S^c) = (N_s eu\mathcal{N}Mcl(S))^c$. Hence proved (ii). □

Remark 4.3. Taking complements on either side of (i) and (ii) of Theorem 4.2, we get $N_s eu\mathcal{N}Mint(S) = 1_N - N_s eu\mathcal{N}Mcl(1_N - S)$ and $N_s eu\mathcal{N}Mcl(S) = 1_N - N_s eu\mathcal{N}Mint(1_N - S)$.

Example 4.4. In Example 3.4, let $S = \left\{\left\langle \frac{s_1,s_4}{(0.2,0.5,0.8)} \right\rangle, \left\langle \frac{s_2}{(0.3,0.5,0.7)} \right\rangle, \left\langle \frac{s_3}{(0.5,0.5,0.5)} \right\rangle\right\}$

1. $N_s eu\mathcal{N}Mint(S) = 1_N - N_s eu\mathcal{N}Mcl(1_N - S)$,

$$N_s eu\mathcal{N}Mint(S) = \left\{\left\langle \frac{s_1, s_4}{(0.2, 0.5, 0.8)} \right\rangle, \left\langle \frac{s_2}{(0.3, 0.5, 0.7)} \right\rangle, \left\langle \frac{s_3}{(0.4, 0.5, 0.6)} \right\rangle\right\}$$

$$\begin{aligned}
&1_N - N_s eu\mathcal{N}Mcl(1_N - S) \\
&\quad = 1_N - N_s eu\mathcal{N}Mcl\left(\left\{\left\langle \frac{s_1, s_4}{(0.8, 0.5, 0.2)} \right\rangle, \left\langle \frac{s_2}{(0.3, 0.5, 0.3)} \right\rangle, \left\langle \frac{s_3}{(0.5, 0.5, 0.4)} \right\rangle\right\}\right) \\
&\quad = 1_N - \left\{\left\langle \frac{s_1, s_4}{(0.8, 0.5, 0.2)} \right\rangle, \left\langle \frac{s_2}{(0.7, 0.5, 0.3)} \right\rangle, \left\langle \frac{s_3}{(0.6, 0.5, 0.4)} \right\rangle\right\} \\
&\quad = \left\{\left\langle \frac{s_1, s_4}{(0.2, 0.5, 0.8)} \right\rangle, \left\langle \frac{s_2}{(0.3, 0.5, 0.7)} \right\rangle, \left\langle \frac{s_3}{(0.4, 0.5, 0.6)} \right\rangle\right\}.
\end{aligned}$$

https://doi.org/10.54216/IJNS.190110
Received: March 02, 2022 Accepted: September 15, 2022

2. $N_seu\mathcal{N}Mcl(S) = 1_N - N_seu\mathcal{N}Mint(1_N - S)$,

$$N_seu\mathcal{N}Mcl(S) = \left\{\left\langle\frac{s_1, s_4}{(0.8, 0.5, 0.2)}\right\rangle, \left\langle\frac{s_2}{(0.7, 0.5, 0.3)}\right\rangle, \left\langle\frac{s_3}{(0.6, 0.5, 0.4)}\right\rangle\right\}$$

$$\begin{aligned} &1_N - N_seu\mathcal{N}Mint(1_N - S) \\ &= 1_N - N_seu\mathcal{N}Mint\left(\left\{\left\langle\frac{s_1, s_4}{(0.8, 0.5, 0.2)}\right\rangle, \left\langle\frac{s_2}{(0.3, 0.5, 0.7)}\right\rangle, \left\langle\frac{s_3}{(0.5, 0.5, 0.5)}\right\rangle\right\}\right) \\ &= 1_N - \left\{\left\langle\frac{s_1, s_4}{(0.2, 0.5, 0.8)}\right\rangle, \left\langle\frac{s_2}{(0.3, 0.5, 0.7)}\right\rangle, \left\langle\frac{s_3}{(0.4, 0.5, 0.6)}\right\rangle\right\} \\ &= \left\{\left\langle\frac{s_1, s_4}{(0.8, 0.5, 0.2)}\right\rangle, \left\langle\frac{s_2}{(0.7, 0.5, 0.3)}\right\rangle, \left\langle\frac{s_3}{(0.6, 0.5, 0.4)}\right\rangle\right\}. \end{aligned}$$

Remark 4.5. By Definition of $N_seu\mathcal{N}Mcl(S)$, it is clear that for any neutosophic set S,

$N_seu\mathcal{N}cl(N_seu\mathcal{N}Mcl(S)) = N_seu\mathcal{N}Mcl(S)$ and we have the following equality:

$$\begin{aligned} N_seu\mathcal{N}Mint(S) &= 1_N - N_seu\mathcal{N}Mcl(1_N - S) \\ &= 1_N - \bigcap\{F : 1_N - S \subseteq F, F = N_seu\mathcal{N}cl(N_seu\mathcal{N}int(F))\} \\ &= \bigcup\{1_N - F : 1_N - F \subseteq S, 1_N - F = 1_N - N_seu\mathcal{N}cl(N_seu\mathcal{N}int(F))\} \\ &= \bigcup\{G : G \subseteq S, G = N_seu\mathcal{N}int(N_seu\mathcal{N}cl(G))\} \end{aligned}$$

That is, $N_seu\mathcal{N}Mint(S)$ is the union of all $N_seu\mathcal{N}Mo$ subsets of S. Since any $N_seu\mathcal{N}Mo$ set is the complement of a $N_seu\mathcal{N}Mc$ set, G is a $N_seu\mathcal{N}Mo$ set if and only if $G = N_seu\mathcal{N}Mint(G)$.

Theorem 4.6. If F is a $N_seu\ subs$ of U, then let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. The following claims are true if S_o and T_o are $N_seu\ subs$'s of U.

(i) $N_seu\mathcal{N}Mint(S_o) \subseteq S_o$.

(ii) S_o is $N_seu\mathcal{N}Mo$ iff $N_seu\mathcal{N}Mint(S_o) = S_o$.

(iii) $N_seu\mathcal{N}Mint(N_seu\mathcal{N}Mint(S_o)) = N_seu\mathcal{N}Mint(S_o)$.

(iv) $S_o \subseteq T_o \Rightarrow N_seu\mathcal{N}Mint(S_o) \subseteq N_seu\mathcal{N}Mint(T_o)$.

(v) $N_seu\mathcal{N}Mint(S_o \cap T_o) = N_seu\mathcal{N}Mint(S_o) \cap N_seu\mathcal{N}Mint(T_o)$.

(vi) $N_seu\mathcal{N}Mint(S_o \cup T_o) \supseteq N_seu\mathcal{N}Mint(S_o) \cup N_seu\mathcal{N}Mint(T_o)$.

(vii) $N_seu\mathcal{N}Mint(0_N) = 0_N$ & $N_seu\mathcal{N}Mint(1_N) = 1_N$.

Proof. (i) Follows from definition, $N_seu\mathcal{N}Mint(S_o)$.

(ii) S_o is $N_seu\mathcal{N}Mo$ iff $1_N - S_o$ is $N_seu\mathcal{N}Mc$, iff $N_seu\mathcal{N}Mcl(1_N - S_o) = 1_N - S_o$, iff $1_N - N_seu\mathcal{N}Mcl(1_N - S_o) = S_o$ iff $N_seu\mathcal{N}Mint(S_o) = S_o$, by Remark 4.3.

(iii) By using (ii), $N_seu\mathcal{N}Mint(N_seu\mathcal{N}Mint(S_o)) = N_seu\mathcal{N}Mint(S_o)$. This proves (iii).

(iv) $S_o \subseteq T_o \Rightarrow 1_N - T_o \subseteq 1_N - S_o$. Therefore, $N_seu\mathcal{N}Mcl(1_N - T_o) \subseteq N_seu\mathcal{N}Mcl(1_N - S_o)$. (i.e.) $1_N - N_seu\mathcal{N}Mcl(1_N - S_o) \subseteq 1_N - N_seu\mathcal{N}Mcl(1_N - T_o)$. (i.e.) $N_seu\mathcal{N}Mint(S_o) \subseteq N_seu\mathcal{N}Mint(T_o)$.

(v) Since $S_o \cap T_o \subseteq S_o$ and $S_o \cap T_o \subseteq T_o$, by using (iv), $N_seu\mathcal{N}Mint(S_o \cap T_o) \subseteq N_seu\mathcal{N}Mint(S_o)$ and $N_seu\mathcal{N}Mint(S_o \cap T_o) \subseteq N_seu\mathcal{N}Mint(T_o)$. This implies that $N_seu\mathcal{N}Mint(S_o \cap T_o) \subseteq N_seu\mathcal{N}Mint(S_o) \cap N_seu\mathcal{N}Mint(T_o)$. Now $N_seu\mathcal{N}Mint(S_o) \subseteq S_o$ and $N_seu\mathcal{N}Mint(T_o) \subseteq T_o$, we get, $N_seu\mathcal{N}Mint(S_o) \cap N_seu\mathcal{N}Mint(T_o) \subseteq S_o \cap T_o$.

Received: March 02, 2022 Accepted: September 15, 2022

$\Rightarrow N_s eu\mathcal{N}Mint(N_s eu\mathcal{N}Mint(S_o) \cap N_s eu\mathcal{N}Mint(T_o)) \subseteq N_s eu\mathcal{N}Mint(S_o \cap T_o)$, which implies $N_s eu\mathcal{N}Mint(N_s eu\mathcal{N}Min$ $N_s eu\mathcal{N}Mint(N_s eu\mathcal{N}M\ int(T_o)) \subseteq N_s eu\mathcal{N}Mint(S_o \cap T_o)$.

$\Rightarrow N_s eu\mathcal{N}Mint(S_o) \cap N_s eu\mathcal{N}Mint(T_o) \subseteq N_s eu\mathcal{N}Mint(S_o \cap T_o)$. Hence, $N_s eu\mathcal{N}Mint(S_o) \cap N_s eu\mathcal{N}Mint(T_o) = N_s eu\mathcal{N}Mint(S_o \cap T_o)$.

(vi) Since $S_o \subseteq S_o \cup T_o$ and $T_o \subseteq S_o \cup T_o$, by using (iv), $N_s eu\mathcal{N}Mint(S_o) \subseteq N_s eu\mathcal{N}Mint(S_o \cup T_o)$ and $N_s eu\mathcal{N}Mint(T_o) \subseteq N_s eu\mathcal{N}Mint(S_o \cup T_o)$. This implies that, $N_s eu\mathcal{N}Mint(S_o) \cup N_s eu\mathcal{N}Mint(T_o) \subseteq N_s eu\mathcal{N}Mint(S_o \cup T_o)$.

(vii) Since 0_N and 1_N are $N_s eu\mathcal{N}Mo$, $N_s eu\mathcal{N}Mint(0_N) = 0_N$ and $N_s eu\mathcal{N}Mint(1_N) = 1_N$. □

Remark 4.7. The following example shows that the equality need not be hold in Theorem 4.6 (vi).

Example 4.8. In Example 3.4, the sets $A = \left\{\left\langle \frac{s_1,s_4}{0.1,0.5,0.9} \right\rangle, \left\langle \frac{s_2}{0.5,0.5,0.5} \right\rangle, \left\langle \frac{s_3}{0.4,0.5,0.6} \right\rangle\right\}$ and

$B = \left\{\left\langle \frac{s_1,s_4}{0.2,0.5,0.8} \right\rangle, \left\langle \frac{s_2}{0.3,0.5,0.7} \right\rangle, \left\langle \frac{s_3}{0.8,0.5,0.2} \right\rangle\right\}$, then

$A \cup B = \left\{\left\langle \frac{s_1,s_4}{0.2,0.5,0.8} \right\rangle, \left\langle \frac{s_2}{0.5,0.5,0.5} \right\rangle, \left\langle \frac{s_3}{0.8,0.5,0.2} \right\rangle\right\}$.

$N_s eu\mathcal{N}Mint(A) = \left\{\left\langle \frac{s_1,s_4}{0.1,0.5,0.9} \right\rangle, \left\langle \frac{s_2}{0.3,0.5,0.7} \right\rangle, \left\langle \frac{s_3}{0.4,0.5,0.6} \right\rangle\right\}$,

$N_s eu\mathcal{N}Mint(B) = \left\{\left\langle \frac{s_1,s_4}{0.2,0.5,0.8} \right\rangle, \left\langle \frac{s_2}{0.3,0.5,0.7} \right\rangle, \left\langle \frac{s_3}{0.8,0.5,0.2} \right\rangle\right\}$ and

$N_s eu\mathcal{N}Mint(A \cup B) = \left\{\left\langle \frac{s_1,s_4}{0.2,0.5,0.8} \right\rangle, \left\langle \frac{s_2}{0.5,0.5,0.5} \right\rangle, \left\langle \frac{s_3}{0.8,0.5,0.2} \right\rangle\right\}$.

Thus $N_s eu\mathcal{N}Mint(A \cup B) \not\subseteq N_s eu\mathcal{N}Mint(A) \cup N_s eu\mathcal{N}Mint(B)$.

Theorem 4.9. Let $(U, \tau_N(F))$ be a $N_s eu\mathcal{N}ts$. Let S_o and T_o be $N_s eu\ subs$'s of U, then the following statements hold.

(i) $S_o \subseteq N_s eu\mathcal{N}Mcl(S_o)$.

(ii) S_o is $N_s eu\mathcal{N}Mc$ iff $N_s eu\mathcal{N}Mcl(S_o) = S_o$.

(iii) $N_s eu\mathcal{N}Mcl(N_s eu\mathcal{N}Mcl(S_o)) = N_s eu\mathcal{N}Mcl(S_o)$.

(iv) $S_o \subseteq T_o \Rightarrow N_s eu\mathcal{N}Mcl(S_o) \subseteq N_s eu\mathcal{N}Mcl(T_o)$.

(v) $N_s eu\mathcal{N}Mcl(S_o \cap T_o) \subseteq N_s eu\mathcal{N}Mcl(S_o) \cap N_s eu\mathcal{N}Mcl(T_o)$.

(vi) $N_s eu\mathcal{N}Mcl(S_o \cup T_o) = N_s eu\mathcal{N}Mcl(S_o) \cup N_s eu\mathcal{N}Mcl(T_o)$.

(vii) $N_s eu\mathcal{N}Mcl(0_N) = 0_N$ & $N_s eu\mathcal{N}Mcl(1_N) = 1_N$.

Proof. (i) Follows from definition, $N_s eu\mathcal{N}Mcl(S_o)$.

(ii) Let S_o be $N_s eu\mathcal{N}Mc$ set in U. By using Definition 3.2, S_o^c is a $N_s eu\mathcal{N}Mo$ set in U. By Theorem 4.6 and Proposition 4.2, $N_s eu\mathcal{N}Mint(S_o^c) = S_o^c \Leftrightarrow (N_s eu\mathcal{N}Mcl(S_o))^c = S_o^c \Leftrightarrow N_s eu\mathcal{N}Mcl(S_o) = S_o$. This proved (ii).

(iii) By using (ii), $N_s eu\mathcal{N}Mcl(N_s eu\mathcal{N}Mcl(S_o)) = N_s eu\mathcal{N}Mcl(S_o)$. This proves (iii).

(iv) $S_o \subseteq T_o$, $T_o^c \subseteq S_o^c$. By using Theorem 4.6 (iv), $N_s eu\mathcal{N}Mint(T_o^c) \subseteq N_s eu\mathcal{N}Min(S_o^c)$. Taking complement on both sides, $(N_s eu\mathcal{N}Mint(T_o^c))^c \supseteq (N_s eu\mathcal{N}Mint(S_o^c))^c$. By proposition 4.2 (ii), $N_s eu\mathcal{N}M$ $cl(S_o) \subseteq N_s eu\mathcal{N}Mcl(T_o)$. This proves (iv).

Received: March 02, 2022 Accepted: September 15, 2022

(v) Since $S_o \cap T_o \subseteq S_o$ and $S_o \cap T_o \subseteq T_o$, by using (iv), $N_s eu\mathcal{N}Mcl(S_o \cap T_o) \subseteq N_s eu\mathcal{N}Mcl(S_o)$ and $N_s eu\mathcal{N}Mcl(S_o \cap T_o) \subseteq N_s eu\mathcal{N}Mcl(T_o)$. This implies that $N_s eu\mathcal{N}Mcl(S_o \cap T_o) \subseteq N_s eu\mathcal{N}Mcl(S_o) \cap N_s eu\mathcal{N}Mcl(T_o)$. This proves (v).

(vi) Since $S_o \subseteq S_o \cup T_o$ and $T_o \subseteq S_o \cup T_o$, by using (iv), $N_s eu\mathcal{N}Mcl(S_o) \subseteq N_s eu\mathcal{N}Mcl(S_o \cup T_o)$ and $N_s eu\mathcal{N}Mcl(T_o) \subseteq N_s eu\mathcal{N}Mcl(S_o \cup T_o)$. This implies that, $N_s eu\mathcal{N}Mcl(S_o) \cup N_s eu\mathcal{N}Mcl(T_o) \subseteq N_s eu\mathcal{N}Mcl(S_o \cup T_o)$.

Now $S_o \subseteq N_s eu\mathcal{N}Mcl(S_o)$ and $T_o \subseteq N_s eu\mathcal{N}Mcl(T_o)$, we get, $S_o \cup T_o \subseteq N_s eu\mathcal{N}Mcl(S_o) \cup N_s eu\mathcal{N}Mcl(T_o)$.

$\Rightarrow N_s eu\mathcal{N}Mcl(S_o \cup T_o) \subseteq N_s eu\mathcal{N}Mcl(N_s eu\mathcal{N}Mcl(S_o) \cup N_s eu\mathcal{N}Mcl(T_o))$, which implies $N_s eu\mathcal{N}M cl(S_o \cup T_o) \subseteq N_s eu\mathcal{N}Mcl(N_s eu\mathcal{N}Mcl(S_o)) \cup N_s eu\mathcal{N}Mcl(N_s eu\mathcal{N}Mcl(T_o))$.

$\Rightarrow N_s eu\mathcal{N}Mcl(S_o \cup T_o) \subseteq N_s eu\mathcal{N}Mcl(S_o) \cup N_s eu\mathcal{N}Mcl(T_o)$. Hence, $N_s eu\mathcal{N}Mcl(S_o) \cup N_s eu\mathcal{N}M cl(T_o) = N_s eu\mathcal{N}Mcl(S_o \cup T_o)$.

(vii) Since 0_N and 1_N are $N_s eu\mathcal{N}Mc$, $N_s eu\mathcal{N}Mcl(0_N) = 0_N$ and $N_s eu\mathcal{N}Mcl(1_N) = 1_N$. □

Remark 4.10. The following example shows that the equality need not be hold in Theorem 4.9 (v).

Example 4.11. In Example 3.4, the sets $A = \left\{\left\langle\frac{s_1,s_4}{0.1,0.5,0.9}\right\rangle,\left\langle\frac{s_2}{0.5,0.5,0.5}\right\rangle,\left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$ and

$B = \left\{\left\langle\frac{s_1,s_4}{0.2,0.5,0.8}\right\rangle,\left\langle\frac{s_2}{0.3,0.5,0.7}\right\rangle,\left\langle\frac{s_3}{0.8,0.5,0.2}\right\rangle\right\}$, then

$A \cap B = \left\{\left\langle\frac{s_1,s_4}{0.1,0.5,0.9}\right\rangle,\left\langle\frac{s_2}{0.3,0.5,0.7}\right\rangle,\left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$.

$N_s eu\mathcal{N}Mcl(A) = \left\{\left\langle\frac{s_1,s_4}{0.1,0.5,0.9}\right\rangle,\left\langle\frac{s_2}{0.5,0.5,0.5}\right\rangle,\left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$,

$N_s eu\mathcal{N}Mcl(B) = \left\{\left\langle\frac{s_1,s_4}{0.8,0.5,0.2}\right\rangle,\left\langle\frac{s_2}{0.7,0.5,0.3}\right\rangle,\left\langle\frac{s_3}{0.8,0.5,0.2}\right\rangle\right\}$ and

$N_s eu\mathcal{N}Mcl(A \cap B) = \left\{\left\langle\frac{s_1,s_4}{0.1,0.5,0.9}\right\rangle,\left\langle\frac{s_2}{0.3,0.5,0.7}\right\rangle,\left\langle\frac{s_3}{0.4,0.5,0.6}\right\rangle\right\}$.

Thus $N_s eu\mathcal{N}Mcl(A \cup B) \not\supseteq N_s eu\mathcal{N}Mcl(A) \cap N_s eu\mathcal{N}Mcl(B)$.

The operators $N_s eu\mathcal{N}\theta\mathcal{S}int(.)$ (resp. $N_s eu\mathcal{N}\delta\mathcal{P}int(.)$) and their respective closure operators are also satisfies the Theorems 4.2, 4.6 and 4.9.

Proposition 4.12. Let $(U, \tau_N(F))$ be a $N_s eu\mathcal{N}ts$. Then for any $N_s eu\ subs\ S$ of U,

1. $N_s eu\mathcal{N}\theta int(S) \subseteq N_s eu\mathcal{N}\theta\mathcal{S}int(S) \subseteq N_s eu\mathcal{N}Mint(S) \subseteq N_s eu\mathcal{N}eint(S) \subseteq S$.
2. $N_s eu\mathcal{N}\theta int(S) \subseteq N_s eu\mathcal{N}int(S) \subseteq N_s eu\mathcal{N}\delta\mathcal{P}int(S) \subseteq N_s eu\mathcal{N}Mint(S) \subseteq N_s eu\mathcal{N}eint(S) \subseteq S$.
3. $N_s eu\mathcal{N}\theta cl(S) \supseteq N_s eu\mathcal{N}\theta\mathcal{S}cl(S) \supseteq N_s eu\mathcal{N}Mcl(S) \supseteq N_s eu\mathcal{N}ecl(S) \supseteq S$.
4. $N_s eu\mathcal{N}\theta cl(S) \supseteq N_s eu\mathcal{N}cl(S) \supseteq N_s eu\mathcal{N}\delta\mathcal{P}cl(S) \supseteq N_s eu\mathcal{N}Mcl(S) \supseteq N_s eu\mathcal{N}ecl(S) \supseteq S$.

Lemma 4.13. The following hold for a subset S in a $N_s eu\mathcal{N}ts$ $(U, \tau_N(F))$.

1. $N_s eu\mathcal{N}\delta\mathcal{P}cl(S) = S \cup N_s eu\mathcal{N}cl(N_s eu\mathcal{N}\delta int(S))$ and $N_s eu\mathcal{N}\delta\mathcal{P}int(S) = S \cap N_s eu\mathcal{N}int(N_s eu\mathcal{N} \delta cl(S))$,
2. $N_s eu\mathcal{N}\delta\mathcal{P}cl(N_s eu\mathcal{N}\delta\mathcal{P}int(S)) = N_s eu\mathcal{N}\delta\mathcal{P}int(S) \cup N_s eu\mathcal{N}cl(N_s eu\mathcal{N}\delta int(S))$ and $N_s eu\mathcal{N}\delta\mathcal{P}int(N_s eu\mathcal{N}\delta\mathcal{P}cl(S)) = N_s eu\mathcal{N}\delta\mathcal{P}cl(S) \cap N_s eu\mathcal{N}int(N_s eu\mathcal{N}\delta cl(S))$.

Theorem 4.14. Let $(U, \tau_N(F))$ be a $N_s eu\mathcal{N}ts$ and $S \subseteq A$. Then S is a $N_s eu\mathcal{N}Mo$ iff $S = N_s eu\mathcal{N}\theta\mathcal{S} int(S) \cup N_s eu\mathcal{N}\delta\mathcal{P}int(S)$.

https://doi.org/10.54216/IJNS.190110
Received: March 02, 2022 Accepted: September 15, 2022

Proof. Let S be a $N_seu\mathcal{N}Mo$ set. Then $S \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)) \cup N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))$ and hence by Theorem 4.2 and Lemma 4.13, $N_seu\mathcal{N}\theta\mathcal{S}int(S) \cup N_seu\mathcal{N}\delta\mathcal{P}int(S) = (A \cap N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S))) \cup (A \cap N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))) = S \cap (N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)) \cup N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))) = S$. The Converse follows from Theorem 4.2 & Lemma 4.13. □

Theorem 4.15. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$ and $S \subseteq A$. Then S is a $N_seu\mathcal{N}Mc$ iff $S = N_seu\mathcal{N}\theta\mathcal{S}cl(S) \cap N_seu\mathcal{N}\delta\mathcal{P}cl(S)$.

Proof. From Theorem 4.14, it follows. □

Theorem 4.16. Let S be a subset of $(U, \tau_N(F))$. Then:

1. $N_seu\mathcal{N}Mcl(S) = N_seu\mathcal{N}\theta\mathcal{S}cl(S) \cap N_seu\mathcal{N}\delta\mathcal{P}cl(S)$,
2. $N_seu\mathcal{N}Mint(S) = N_seu\mathcal{N}\theta\mathcal{S}int(S) \cup N_seu\mathcal{N}\delta\mathcal{P}int(S)$.

Proof. (i) It is easy to see that $N_seu\mathcal{N}Mcl(S) \subseteq N_seu\mathcal{N}\theta\mathcal{S}cl(S) \cap N_seu\mathcal{N}\delta\mathcal{P}cl(S)$. Also, $N_seu\mathcal{N}\theta\mathcal{S}cl(S) \cap N_seu\mathcal{N}\delta\mathcal{P}cl(S) = (S \cup N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S)) \cap (S \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta int(S)) = S \cup (N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S)) \cap N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta int(S)))$. Since $N_seu\mathcal{N}Mcl(S)$ is $N_seu\mathcal{N}Mc$, then $N_seu\mathcal{N}Mcl(S) \supseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(N_seu\mathcal{N}Mcl(S))) \cap N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta int(N_seu\mathcal{N}Mcl(S))) \supseteq N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S)) \cap N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta int(S))$. Thus $S \cup (N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S)) \cap N_seu\mathcal{N}cl(N_seu\mathcal{N}\delta int(S))) \subseteq S \cup N_seu\mathcal{N}Mcl(S) = N_seu\mathcal{N}Mcl(S)$ and hence, $N_seu\mathcal{N}\theta\mathcal{S}cl(S) \cap N_seu\mathcal{N}\delta\mathcal{P}cl(S) \subseteq N_seu\mathcal{N}Mcl(S)$. So, $N_seu\mathcal{N}Mcl(S) = N_seu\mathcal{N}\theta\mathcal{S}cl(S) \cap N_seu\mathcal{N}\delta\mathcal{P}cl(S)$.

(ii) It follows from (i). □

Lemma 4.17. Let S be a subset of a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then

(1) $N_seu\mathcal{N}\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S)) = N_seu\mathcal{N}\delta\mathcal{P}cl(S) \cap N_seu\mathcal{N}int(N_seu\mathcal{N}cl(S))$ and $N_seu\mathcal{N}\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S)) = N_seu\mathcal{N}\delta\mathcal{P}int(S) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}int(S))$.

(2) $N_seu\mathcal{N}\theta Scl(N_seu\mathcal{N}\theta int(S)) = N_seu\mathcal{N}Scl(N_seu\mathcal{N}\theta int(S)) = N_seu\mathcal{N}int(N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)))$.

(3) $N_seu\mathcal{N}\theta Scl(N_seu\mathcal{N}\theta int(S)) = N_seu\mathcal{N}Scl(N_seu\mathcal{N}\theta int(S)) = N_seu\mathcal{N}int(N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)))$.

Proposition 4.18. Let S be a $N_seu\ subs$ of a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then:

1. $N_seu\mathcal{N}Mcl(S) = S \cup N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$,
2. $N_seu\mathcal{N}Mint(S) = S \cap N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$.

Proof. (i) By Lemma 4.17, $S \cup N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S)) = S \cup (N_seu\mathcal{N}\delta\mathcal{P}cl(S) \cap N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S))) = (S \cup N_seu\mathcal{N}\delta\mathcal{P}cl(S)) \cap (S \cup N_seu\mathcal{N}int(N_seu\mathcal{N}\theta cl(S))) = N_seu\mathcal{N}\delta\mathcal{P}cl(S) \cap N_seu\mathcal{N}\theta\mathcal{S}cl(S) = N_seu\mathcal{N}Mcl(S)$.

(ii) It follows from (i). □

Theorem 4.19. Let S be subset in a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then S is an $N_seu\mathcal{N}\mathcal{M}o$ set iff $S \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$.

Proof. Let S be an $N_seu\mathcal{N}\mathcal{M}o$ set. Then by Theorem 4.6, $S = N_seu\mathcal{N}\mathcal{M}int(S)$ and by Proposition 4.18, $S = S \cap N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$ and hence, $S \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$.

Conversely, let $S \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$. Then by Proposition 4.18, $S \subseteq S \cap N_seu\mathcal{N}\theta\,\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S)) = N_seu\mathcal{N}\mathcal{M}int(S)$. So, $S \subseteq N_seu\mathcal{N}\mathcal{M}int(S)$. Then $S = N_seu\mathcal{N}\mathcal{M}int(S)$ and hence, S is $N_seu\mathcal{N}\mathcal{M}o$. □

Received: March 02, 2022 Accepted: September 15, 2022

Theorem 4.20. Let S be subset in a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then $S \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$ iff $N_seu\mathcal{N}\theta\mathcal{P}cl(S) = N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\ \delta\mathcal{P}int(S))$.

Proof. Let $S \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$. Then $N_seu\mathcal{N}\theta\mathcal{P}cl(S) \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$ and hence, $N_seu\mathcal{N}\theta\mathcal{P}cl(S) = N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$.

Conversely, let $N_seu\mathcal{N}\theta\mathcal{P}cl(S) = N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$. Then $N_seu\mathcal{N}\theta\mathcal{P}cl(S) \subseteq N_seu\mathcal{N}$ $\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$ and hence, $S \subseteq N_seu\mathcal{N}\theta\mathcal{P}cl(N_seu\mathcal{N}\delta\mathcal{P}int(S))$. □

Theorem 4.21. Let S be subset in a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then S is an $N_seu\mathcal{N}\mathcal{M}c$ set iff $N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S)) \subseteq S$.

Proof. Let S be an $N_seu\mathcal{N}\mathcal{M}c$ set. Then by Theorem 4.6, $S = N_seu\mathcal{N}\mathcal{M}cl(S)$ and by Proposition 4.18, $S = S \cup N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$ and hence, $S \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$.

Conversely, let $S \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$. Then by Proposition 4.18, $S \supseteq S \cup N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S)) = N_seu\mathcal{N}\mathcal{M}cl(S)$. so, $S \supseteq N_seu\mathcal{N}\mathcal{M}cl(S)$. Then $S = N_seu\mathcal{N}\mathcal{M}cl(S)$ and hence, S is $N_seu\mathcal{N}\mathcal{M}c$. □

Theorem 4.22. Let S be subset in a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$. Then $N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S)) \subseteq S$ iff $N_seu\mathcal{N}\theta\mathcal{P}int(S) = N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$

Proof. (1) ⇒ (2). Let S be an $N_seu\mathcal{N}\mathcal{M}c$ set. Then by Theorem 4.6, $S = N_seu\mathcal{N}\mathcal{M}cl(S)$ and by Proposition 4.18, $S = S \cup N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$ and hence, $S \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$.

(2) ⇒ (1). Let $S \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$. Then by Proposition 4.18, $S \supseteq S \cup N_seu\mathcal{N}\theta\mathcal{P}int$ $(N_seu\mathcal{N}\delta\mathcal{P}cl(S)) = N_seu\mathcal{N}\mathcal{M}cl(S)$. So, $S \supseteq N_seu\mathcal{N}\mathcal{M}cl(S)$. Then $S = N_seu\mathcal{N}\mathcal{M}cl(S)$ and hence, S is $N_seu\mathcal{N}\mathcal{M}c$.

(2) ⇒ (3). Let $S \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$. Then $N_seu\mathcal{N}\theta\mathcal{P}int(S) \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}$ $\delta\mathcal{P}cl(S))$ and hence, $N_seu\mathcal{N}\theta\mathcal{P}int(S) = N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$.

(3) ⇒ (2). Let $N_seu\mathcal{N}\theta\mathcal{P}int(S) = N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$. Then $N_seu\mathcal{N}\theta\mathcal{P}int(S) \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl($ and hence, $S \supseteq N_seu\mathcal{N}\theta\mathcal{P}int(N_seu\mathcal{N}\delta\mathcal{P}cl(S))$. □

Proposition 4.23. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. Then the closure of a $N_seu\mathcal{N}Mo$ of A is $N_seu\mathcal{N}\delta\mathcal{P}o$.

Proof. Let $S \in N_seu\mathcal{N}MO(A)$. Then $N_seu\mathcal{N}cl(S) \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)) \cup N_seu\mathcal{N}int$ $(N_seu\mathcal{N}\delta cl(S))) \subseteq N_seu\mathcal{N}cl(N_seu\mathcal{N}\theta int(S)) \cup N_seu\mathcal{N}cl(N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S))) = N_seu\mathcal{N}cl($ $N_seu\mathcal{N}int(N_seu\mathcal{N}\delta cl(S)))$.

Therefore, $N_seu\mathcal{N}cl(S)$ is $N_seu\mathcal{N}\delta\mathcal{P}o$. □

Proposition 4.24. Let S be a $N_seu\mathcal{N}Mo$ subset of a $N_seu\mathcal{N}ts$ $(U, \tau_N(F))$ and $N_seu\mathcal{N}\theta int(S) = \varphi$. Then S is $N_seu\mathcal{N}\delta\mathcal{P}o$.

Proof. Obvious. □

https://doi.org/10.54216/IJNS.190110
Received: March 02, 2022 Accepted: September 15, 2022

5 Product related neutrosophic nano topological spaces

Definition 5.1. Let $(U_1, \tau_N(F_1))$ & $(U_2, \tau_N(F_2))$ be a $N_seu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $N_seu\ subs$'s of U_1 and U_2. Let $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : s \in U_1\}$ and $T = \{\langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle : t \in U_2\}$ be $N_seu\ subs$'s of U_1 and U_2 respectively. Then $S \times T$ is a $N_seu\ subs$ of $U_1 \times U_2$ is defined by

(P_1) $(S \times T)(s,t) = \langle (s,t), \min(\mu_S(s), \mu_T(t)), \min(\sigma_S(s), \sigma_T(t)), \max(\nu_S(s), \nu_T(t))\rangle.$

(P_2) $(S \times T)(s,t) = \langle (s,t), \min(\mu_S(s), \mu_T(t)), \max(\sigma_S(s), \sigma_T(t)), \max(\nu_S(s), \nu_T(t))\rangle.$

$({P_1}^c)$ $((S \times T)(s,t))^c = \langle (s,t), \max(\mu_S(s), \mu_T(t)), \max(\sigma_S(s), \sigma_T(t)), \min(\nu_S(s), \nu_T(t))\rangle.$

$({P_2}^c)$ $((S \times T)(s,t))^c = \langle (s,t), \max(\mu_S(s), \mu_T(t)), \min(\sigma_S(s), \sigma_T(t)), \min(\nu_S(s), \nu_T(t))\rangle.$

Lemma 5.2. Let $(U_1, \tau_N(F_1))$ & $(U_2, \tau_N(F_2))$ be a $N_seu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $N_seu\ subs$'s of U_1 and U_2. If S and T be $N_seu\ subs$'s of U_1 and U_2, then

1. $(S \times 1_N) \cap (1_N \times T) = S \times T$,
2. $(S \times 1_N) \cup (1_N \times T) = (S^c \times T^c)^c$,
3. $(S \times T)^c = (S^c \times 1_N) \cup (1_N \times T^c)$.

Proof. Let $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : s \in U_1\}$ and $T = \{\langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle : t \in U_2\}$.

(i) Since $S \times 1_N = \langle s, \min(\mu_S(s), 1_N), \min(\sigma_S(s), 1_N), \max(\nu_S(s), 0_N)\rangle = \langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle = S$ and similarly $1_N \times T = \langle t, \min(1_N, \mu_T(t)), \min(1_N, \sigma_T(t)), \max(0_N, \nu_T(t))\rangle = \langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle = T$, we have $(S \times 1_N) \cap (1_N \times T) = S(s) \cap T(t) = \langle (s,t), \mu_S(s) \wedge \mu_T(t), \sigma_S(s) \wedge \sigma_T(t), \nu_S(s) \vee \nu_T(t)\rangle = S \times T$.

(ii) Similarly to (i).

(iii) Obvious by putting S, T instead of S^c, T^c in (ii). □

Definition 5.3. let $(U_1, \tau_N(F_1))$ & $(U_2, \tau_N(F_2))$ be a $N_seu\mathcal{N}ts$'s. The $N_seu\mathcal{N}$ product topological space [$N_seu\mathcal{N}Pts$ for short] of $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ is the cartesian product $U_1 \times U_2$ of neutrosophic sets U_1 and U_2 together with the $N_seu\mathcal{N}$ topology $\tau_N(\xi)$ of $U_1 \times U_2$ which is generated by the family { $P_1^{-1}(S_i), P_2^{-1}(T_j) : S_i \in \tau_N(F_1)$, $T_j \in \tau_N(F_2)$ and P_1, P_2 are projections of $U_1 \times U_2$ onto U_1 & U_2 respectively } (i.e. the family $\{P_1^{-1}(S_i), P_2^{-1}(T_j) : S_i \in \tau_N(F_1), T_j \in \tau_N(F_2)\}$ is a subbase for $N_seu\mathcal{N}$ topology $\tau_N(\xi)$ of $U_1 \times U_2$).

Remark 5.4. In the Definition 5.3, since $P_1^{-1}(S_i) = S_i \times 1_N$ and $P_2^{-1}(T_j) = 1_N \times T_j$ and $S_i \times 1_N \cap 1_N \times T_j = S_i \times T_j$, the family $\chi = \{S_i \times T_j : S_i \in \tau_N(F_1), T_j \in \tau_N(F_2)\}$ forms a base for $N_seu\mathcal{N}Pts\ \tau_N(\xi)$ of $U_1 \times U_2$.

Lemma 5.5. Let $(U, \tau_N(F))$ be a $N_seu\mathcal{N}ts$. If S_1, S_2, T_1 and T_2 be $N_seu\ subs$'s of U, then $S_1 \subseteq T_1$, $S_2 \subseteq T_2 \Rightarrow S_1 \times S_2 \subseteq T_1 \times T_2$.

Proof. Let $S_1 = \langle x, \mu_{S_1}(x), \sigma_{S_1}(x), \nu_{S_1}(x)\rangle$, $S_2 = \langle x, \mu_{S_2}(x), \sigma_{S_2}(x), \nu_{S_2}(x)\rangle$, $T_1 = \langle x, \mu_{T_1}(x), \sigma_{T_1}(x), \nu_{T_1}(x)\rangle$ and $T_2 = \langle x, \mu_{T_2}(x), \sigma_{T_2}(x), \nu_{T_2}(x)\rangle$ be $N_seu\ s$'s. Since $S_1 \subseteq T_1 \Rightarrow \mu_{S_1} \leq \mu_{T_1}, \sigma_{S_1} \leq \sigma_{T_1}, \nu_{S_1} \leq \nu_{T_1}$ and also $S_2 \subseteq T_2 \Rightarrow \mu_{S_2} \leq \mu_{T_2}, \sigma_{S_2} \leq \sigma_{T_2}, \nu_{S_2} \leq \nu_{T_2}$, we have $\min(\mu_{S_1}, \mu_{S_2}) \leq \min(\mu_{T_1}, \mu_{T_2})$, $\min(\sigma_{S_1}, \sigma_{S_2}) \leq \min(\sigma_{T_1}, \sigma_{T_2})$ and $\max(\nu_{S_1}, \nu_{S_2}) \geq \max(\nu_{T_1}, \nu_{T_2})$. Hence the result. □

Lemma 5.6. Let $(U_1, \tau_N(F_1))$ & $(U_2, \tau_N(F_2))$ be a $N_seu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $N_seu\ subs$'s of U_1 and U_2 such that U_1 is neutrosophic product relative to U_2. Let S & T be $N_seu\mathcal{N}Mcs$'s of U_1 and U_2 respectively. Then $S \times T$ is the $N_seu\mathcal{N}Mcs$ in the $N_seu\mathcal{N}Pts$ of $U_1 \times U_2$.

https://doi.org/10.54216/IJNS.190110
Received: March 02, 2022 Accepted: September 15, 2022

Proof. Let $S = \langle x, \mu_S(x), \sigma_S(x), \nu_S(x)\rangle, T = \langle y, \mu_T(y), \sigma_T(y), \nu_T(y)\rangle$. From Lemma 5.2, $((S\times T)(x,y))^c = (S^c \times 1_N) \cup (1_N \times T^c)(x,y)$. Since $S^c \times 1_N$ and $1_N \times T^c$ are $N_s eu\mathcal{N}Mos$'s in U_1 and U_2 respectively. Hence $S^c \times 1_N \cup 1_N \times T^c$ is $N_s eu\mathcal{N}Mos$ of $U_1 \times U_2$. Hence $(S \times T)^c$ is a $N_s eu\mathcal{N}Mos$ of $U_1 \times U_2$ and consequently $S \times T$ is the $N_s eu\mathcal{N}cs$ of $U_1 \times U_2$. □

Theorem 5.7. If S & T are neutrosophic sets of $N_s eu\mathcal{N}ts$'s $(U, \tau_N(F_1))$ and $(V, \tau_N(F_2))$ respectively, then

(i) $N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T) \supseteq N_s eu\mathcal{N}Mcl(S \times T)$,

(ii) $N_s eu\mathcal{N}Mint(S) \times N_s eu\mathcal{N}Mint(T) \subseteq N_s eu\mathcal{N}Mint(S \times T)$.

Proof. (i) Since $S \subseteq N_s eu\mathcal{N}Mcl(S)$ and $T \subseteq N_s eu\mathcal{N}Mcl(T)$, hence $S \times T \subseteq N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T)$. This implies that $N_s eu\mathcal{N}Mcl(S \times T) \subseteq N_s eu\mathcal{N}Mcl(N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T))$ and from Lemma 5.6, $N_s eu\mathcal{N}Mcl(S \times T) \subseteq N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T)$.

(ii) Follows from (i) and the fact that $N_s eu\mathcal{N}Mint(S^c) = (N_s eu\mathcal{N}Mcl(S))^c$. □

Definition 5.8. Let $(U, \tau_N(F_1))$ and $(V, \tau_N(F_2))$ be $N_s eu\mathcal{N}ts$'s and $S \in \tau_N(F_1)$, $T \in \tau_N(F_2)$. We say that $(U, \tau_N(F_1))$ is $N_s eu\mathcal{N}$ product related to $(V, \tau_N(F_2))$ if for any neutrosophic sets P of U and Q of V, whenever $S^c \not\supseteq P$ and $T^c \not\supseteq Q \Rightarrow S^c \times 1_N \cup 1_N \times T^c \supseteq P \times Q$, there exist $S_1 \in \tau_N(F_1)$, $T_1 \in \tau_N(F_2)$ such that $S_1^c \supseteq P$ or $P(T_1) \supseteq Q$ and $P(S_1) \times 1_N \cup 1_N \times P(T_1) = P(S) \times 1_N \cup 1_N \times P(T)$.

Lemma 5.9. For NS's S_i's and T_j's of $N_s eu\mathcal{N}ts$'s U and V respectively, we have

(i) $\cap\{S_i, T_j\} = \min(\cap S_i, \cap T_j)$; $\cup\{S_i, T_j\} = \max(\cup S_i, \cup T_j)$.

(ii) $\cap\{S_i, 1_N\} = (\cap S_i) \times 1_N$; $\cup\{S_i, 1_N\} = (\cup S_i) \times 1_N$.

(iii) $\cap\{1_N \times T_j\} = 1_N \times (\cap T_j)$; $\cup\{1_N \times T_j\} = 1_N \times (\cup T_j)$.

Proof. Obvious. □

Theorem 5.10. Let $(U, \tau_N(F_1))$ and $(V, \tau_N(F_2))$ be $N_s eu\mathcal{N}ts$'s such that U is neutrosophic product related to V. Then for neutrosophic sets S of U & T of V, we have

(i) $N_s eu\mathcal{N}Mcl(S \times T) = N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T)$,

(ii) $N_s eu\mathcal{N}Mint(S \times T) = N_s eu\mathcal{N}Mint(S) \times N_s eu\mathcal{N}Mint(T)$.

Proof. (i) Since $N_s eu\mathcal{N}Mcl(S \times T) \subseteq N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T)$ (By Theorem 5.7), it is sufficient to show that $N_s eu\mathcal{N}Mcl(S \times T) \supseteq N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T)$. Let $S_i \in \tau_N(F_1)$ and $T_j \in \tau_N(F_2)$. Then $N_s eu\mathcal{N}Mcl(S \times T) = \langle (s,t), \cap(\{S_i \times T_j\})^c : (\{S_i \times T_j\})^c \supseteq S \times T, \cup\{S_i \times T_j\} : \{S_i \times T_j\} \subseteq S \times T\rangle = \langle (s,t), \cap((S_i)^c \times 1_N \cup 1_N \times (T_j)^c) : (S_i)^c \times 1_N \cup 1_N \times (T_j)^c \supseteq S \times T, \cup(S_i \times 1_N \cap 1_N \times T_j) : S_i \times 1_N \cap 1_N \times T_j \subseteq S \times T\rangle = \langle (s,t), \cap((S_i)^c \times 1_N \cup 1_N \times (T_j)^c) : (S_i)^c \supseteq S$ or $(T_j)^c \supseteq T, \cup(S_i \times 1_N \cap 1_N \times T_j) : S_i \subseteq S$ and $T_j \subseteq T\rangle = \langle (s,t), \min(\cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (S_i)^c \supseteq S\}, \cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (T_j)^c \supseteq T\}), \max(\cup\{S_i \times 1_N \cap 1_N \times T_j : S_i \subseteq S\}, \cup\{S_i \times 1_N \cap 1_N \times T_j : T_j \subseteq T\})\rangle$. Since $\langle (s,t), \cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (S_i)^c \supseteq S\}, \cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (T_j)^c \supseteq T\}\rangle \supseteq \langle (s,t), \cap\{(S_i)^c \times 1_N : (S_i)^c \supseteq S\}, \cap\{1_N \times (T_j)^c : (T_j)^c \supseteq T\}\rangle = \langle (s,t), \cap\{(S_i)^c : (S_i)^c \supseteq S\} \times 1_N, 1_N \times \cap\{(T_j)^c : (T_j)^c \supseteq T\}\rangle = \langle (s,t), N_s eu\mathcal{N}Mcl(S) \times 1_N, 1_N \times N_s eu\mathcal{N}Mcl(T)\rangle$ and $\langle (s,t), \cup\{S_i \times 1_N \cap 1_N \times T_j : S_i \subseteq S\}, \cup\{S_i \times 1_N \cap 1_N \times T_j : T_j \subseteq T\}\rangle \subseteq \langle (s,t), \cup\{S_i \times 1_N : S_i \subseteq S\}, \cup\{1_N \times T_j : T_j \subseteq T\}\rangle = \langle (s,t), \cup\{S_i : S_i \subseteq S\} \times 1_N, 1_N \times \cup\{T_j : T_j \subseteq T\}\rangle = \langle (s,t), N_s eu\mathcal{N}Mint(S) \times 1_N, 1_N \times N_s eu\mathcal{N}Mint(T)\rangle$, we have $N_s eu\mathcal{N}Mcl(S \times T) \supseteq \langle (s,t), \min(N_s eu\mathcal{N}Mcl(S) \times 1_N, 1_N \times N_s eu\mathcal{N}Mcl(T)), \max(N_s eu\mathcal{N}Mint(S) \times 1_N, 1_N \times N_s eu\mathcal{N}Mint(T))\rangle = \langle (s,t), \min(N_s eu\mathcal{N}Mcl(S), N_s eu\mathcal{N}Mcl(T)), \max(N_s eu\mathcal{N}Mint(S), N_s eu\mathcal{N}Mint(T))\rangle = N_s eu\mathcal{N}Mcl(S) \times N_s eu\mathcal{N}Mcl(T)$.

(ii) follows from (i). □

Theorem 5.11. Let $(U, \tau_N(F))$ be a $N_s eu\mathcal{N}ts$. Then for a $N_s eu\ subs$ S and T of U we have,

https://doi.org/10.54216/IJNS.190110

Received: March 02, 2022 Accepted: September 15, 2022

(i) $N_seu\mathcal{N}Mcl(S) \supseteq S \cup N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S))$,

(ii) $N_seu\mathcal{N}Mint(S) \subseteq S \cap N_seu\mathcal{N}Mint(N_seu\mathcal{N}Mcl(S))$,

(iii) $N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(S)) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}cl(S))$,

(iv) $N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(S)) \supseteq N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S)))$.

Proof. By Theorem 4.9 (i),

$$S \subseteq N_seu\mathcal{N}Mcl(S). \tag{1}$$

Again using Theorem 4.6 (i), $N_seu\mathcal{N}Mint(S) \subseteq S$. Then

$$N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S)) \subseteq N_seu\mathcal{N}Mcl(S). \tag{2}$$

By (1) & (2) we have, $S \cup N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S)) \subseteq N_seu\mathcal{N}Mcl(S)$. This proves (i).

By Theorem 4.6 (i),

$$N_seu\mathcal{N}Mint(S) \subseteq S. \tag{3}$$

Again using Theorem 4.9 (i), $S \subseteq N_seu\mathcal{N}Mcl(S)$. Then

$$N_seu\mathcal{N}Mint(S) \subseteq N_seu\mathcal{N}Mint(N_seu\mathcal{N}Mcl(S)). \tag{4}$$

From (3)& (4), we have $N_seu\mathcal{N}Mint(S) \subseteq S \cap N_seu\mathcal{N}Mint(N_seu\mathcal{N}Mcl(S))$. This proves(ii).

By Proposition 4.12, $N_seu\mathcal{N}Mcl(S) \subseteq N_seu\mathcal{N}cl(S)$. We get $N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(S)) \subseteq N_seu\mathcal{N}int(N_seu\mathcal{N}cl(S))$. Hence (iii).

By (i), $N_seu\mathcal{N}Mcl(S) \supseteq S \cup N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S))$. We have $N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(S) \supseteq N_seu\mathcal{N}int(S \cup N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S)))$. Since $N_seu\mathcal{N}int(S \cup T) \supseteq N_seu\mathcal{N}int(S) \cup N_seu\mathcal{N}int(T)$, $N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(S) \supseteq N_seu\mathcal{N}int(S)) \cup N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S))) \supseteq N_seu\mathcal{N}int(N_seu\mathcal{N}Mcl(N_seu\mathcal{N}Mint(S)))$. Hence (iv). □

The operators $N_seu\mathcal{N}\theta\mathcal{S}int(.)$ (resp. $N_seu\mathcal{N}\delta\mathcal{P}int(.)$) and their respective closure is satisfy Lemma 5.6 and Theorems 5.7, 5.10 and 5.11.

6 Conclusions

In this work, sets named $N_seu\mathcal{N}\theta o$, $N_seu\mathcal{N}\theta\mathcal{S}o$, $N_seu\mathcal{N}\delta\mathcal{P}o$ and $N_seu\mathcal{N}Mo$ sets were presented, and some of their properties were addressed. Modeling spatial regions with ambiguous boundaries and under ambiguity is necessary in GIS. As a result, this neutrosophic nano topological space can be expanded to a neutrosophic spatial region.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] R. K. Al-Hamido, *On neutrosophic crisp supra bi-topological spaces*, International Journal of Neutrosophic Sciences, **4** (1) (2020), 36-46.

[2] K. Atanassov and S. Stoeva, *Intuitionistic fuzzy sets*, In Proceedings of the Polish Symposium on Interval and Fuzzy Mathematics, Poznan, Poland, (1983), 23-26.

[3] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.

[4] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182-190.

[5] D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy sets and systems, **88** (1997), 81-89.

[6] S. K. Das and S. A. Edalatpanah, *A new ranking function of triangular neutrosophic number and its application in inter programming*, International Journal of Neutrosophic Sciences, **4** (2) (2020), 82-92.

[7] V. Kalaiyarasan, S. Tamilselvan, A. Vadivel and C. John Sundar, *Normal spaces associated with fuzzy nano M-open sets and its application*, Journal of Mathematics and Computer Science, **29** (2) (2023), 156-166.

[8] M. Lellis Thivagar and C. Richard, *On nano forms of weekly open sets*, International journal of mathematics and statistics invention, **1** (1) (2013), 31-37.

[9] M. Lellis Thivagar, S. Jafari, V. Sutha Devi and V. Antonysamy, *A novel approach to nano topology via neutrosophic sets*, Neutrosophic Sets and Systems, **20** (2018), 86-94.

[10] M. Mullai, K. Sangeetha, R. Surya, G. Madhan Kumar, R. Jayabalan and S. Broumi, *A single valued neutrosophic inventory model with neutrosophic random variable*, International Journal of Neutrosophic Sciences, **1** (2) (2020), 52-63.

[11] V. Pankajam and K. Kavitha, *δ-open sets and δ-nano continuity in δ-nano topological spaces*, International Journal of Innovative Science and Research Technology, **2** (12) (2017), 110-118.

[12] M. Parimala, R. Jeevitha, S. Jafari, F. Samarandache and M. Karthika, *Neutrosophic nano $\alpha\psi$-closed sets in neutrosophic nano topological spaces*, Journal of advance Research in Dynamical and Control Systems, **10** (2018), 523-531.

[13] M. Parimala, K. Karthika, F. Smarandache and S. Broumi, *On αw-closed sets and its connectedness in terms of neutrosophic topological spaces*, International Journal of Neutrosophic Sciences, **2** (2020), 82-88.

[14] Z. Pawlak, *Rough sets*, International Journal of Computer and Information Sciences, 11 (1982), 341-356.

[15] A. A. Salama and S. A. Alblowi, *Generalized neutrosophic set and generalized neutrosophic topological spaces*, Journal of Computer Sci. Engineering, **2** (7) (2012), 129-132.

[16] A. A. Salama and S. A. Alblowi, *Neutrosophic Set and Neutrosophic Topological Spaces*, IOSR Journal of Mathematics, **3** (4) (2012), 31-35.

[17] F. Smarandache, *A Unifying field in logics: neutrosophic logic. neutrosophy, neutrosophic set, neutrosophic probability*, (Rehoboth: American Research Press) (1999).

[18] F. Smarandache, *Neutrosophy and neutrosophic logic*, 1st Int. Conf. on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics, University of New Mexico, Gallup NM, 87301, USA (2002).

[19] R. Thangammal, M. Saraswathi, A. Vadivel and C. John Sundar, *Fuzzy nano Z -open sets in fuzzy nano topological spaces*, Journal of Linear and Topological Algebra, **11** (01) (2022), 27-38.

[20] R. Thangammal, M. Saraswathi, A. Vadivel, Samad Noeiaghdam, C. John Sundar, V. Govindan and Aiyared Iampan, *Fuzzy Nano Z-locally Closed Sets, Extremally Disconnected Spaces, Normal Spaces, and Their Application*, Advances in Fuzzy Systems, **2022** (2002), 3364170.

[21] A. Vadivel and C. John Sundar, *Neutrosophic δ-Open Maps and Neutrosophic δ-Closed Maps*, International Journal of Neutrosophic Science (IJNS), **13** (2) (2021), 66-74.

[22] A. Vadivel and C. John Sundar, *New Operators Using Neutrosophic δ-Open Set*, Journal of Neutrosophic and Fuzzy Systems, **1** (2) (2021), 61-70.

[23] A. Vadivel and C. John Sundar, *Application of Neutrosophic Sets Based on Mobile Network Using Neutrosophic Functions* , Emerging Trends in Industry 4.0 (ETI 4.0), (2021), 1-8.

[24] A. Vadivel, C. John Sundar, K. Kirubadevi and S. Tamilselvan, More on Neutrosophic Nano Open Sets, International Journal of Neutrosophic Science, **18** (4) (2022), 204-222.

[25] A. Vadivel, N. Moogambigai, S. Tamilselvan and P. Thangaraja, *Application of Neutrosophic Sets Based on Neutrosophic Score Function in Material Selection*, 2022 First International Conference on Electrical, Electronics, Information and Communication Technologies (ICEEICT), 2022, 1-5.

[26] A. Vadivel, M. Seenivasan and C. John Sundar, *An Introduction to δ-open sets in a Neutrosophic Topological Spaces*, Journal of Physics: Conference Series, 1724 (2021), 012011.

[27] A. Vadivel, P. Thangaraja and C. John Sundar, *Neutrosophic e-continuous maps and neutrosophic e-irresolute maps*, Turkish Journal of Computer and Mathematics Education, **12** (1S) (2021), 369-375.

[28] A. Vadivel, P. Thangaraja and C. John Sundar, *Neutrosophic e-Open Maps, Neutrosophic e-Closed Maps and Neutrosophic e-Homeomorphisms in Neutrosophic Topological Spaces*, AIP Conference Proceedings, **2364** (2021), 020016.

[29] L. A. Zadeh, *Fuzzy sets*, Information and control, **8** (1965), 338-353.

https://doi.org/10.54216/IJNS.190110
Received: March 02, 2022 Accepted: September 15, 2022