



## Nonagonal Neutrosophic Number and its Application in Optimization Technique

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### Abstract

This article discusses Nonagonal Neutrosophic number and m-valued Nonagonal Neutrosophic number. The score function, the accuracy function, hamming distance, normalized hamming distance, Euclidean distance and normalized Euclidean distance of Nonagonal and m-polar Nonagonal Neutrosophic number are derived. Some de-neutrosophication method for Nonagonal Neutrosophic number and some properties of m-valued Nonagonal Neutrosophic number are proved. In this article the optimal path of an acyclic network is estimated using Neutrosophic  $\alpha$ -cut grade, Neutrosophic Euclidean grade technique and dynamic programming recursion method through Nonagonal Neutrosophic number. The score function and the removal area method are used to transform the Nonagonal Neutrosophic number to crisp number and the results obtained in both the methods are compared.

**Keywords:** Nonagonal Neutrosophic number; m- polar Nonagonal Neutrosophic number; Neutrosophic  $\alpha$  – cut grade; Euclidean grade, Removal area

### 1. Introduction

Smarandache [1] introduced the notion of Neutrosophic set, is a conception of classical set, fuzzy set and intuitionistic fuzzy set. Various kinds of generalized linear and non-linear triangular neutrosophical numbers are also discussed. Smarandache and Wang [2] exhibited a set of single-valued neutrosophics and its properties. Deli and Smarandache [3] proposed the concept of bipolar Neutrosophic set and its manipulation in problem of decision making. Peng and Wang [4] presented neutrosophical multivalued sets. Ye, J. [6] propounded trapezoidal Neutrosophic set and its properties. Mohamed and Smarandache [7] introduced the mathematical representation of PERT in neutrosophical environment. Ye, J.[8] presented three unique single-valued Neutrosophic soft set methods. Peng and Liu [9] initiated single valued Neutrosophic similarity measure. Poursmaeil [10] proposed a method to estimate the weightiness of decision creators. Zavadskas [12] extended the Neutrosophic Multi – Aspect Market value Calculation method. Abdel-Basset [13] introduced the

concept of additive approximation coherence for reciprocal arrays of triangular neutrosophical additives. Deli [14] proposed the concepts of cut sets of Single valued Neutrosophic numbers. This article contains six sections. Section 1, introduces the article. Section 2, designates the definition of Nonagonal Neutrosophic number and distance measures of Nonagonal Neutrosophic number and proves a theorem in distance measurement. The third section explains the concept of the m-polar Nonagonal Neutrosophical number and its properties. The fourth section explains the grade technique and the Euclidian grade method to estimate the optimal path of the neutrosophical network. Section five, details the dynamic programming recursion to find the optimal path of the Neutrosophic network. Section six, compares all methodologies and concludes the paper.

## 2. Discussion

Phase 1: Neutrosophic network is taken where the route distance is taken in terms of Nonagonal number

Phase 2: The length of all the feasible routes is calculated and identified the largest route.

Phase 3: The Neutrosophic  $\alpha$  – cut grade and the Neutrosophic Euclidean grade for all the Feasible routes are determined

Phase 4: The optimal path is the route which is having highest  $\alpha$  – cut grade and Euclidean grade in truth, indeterminacy and falsity function.

## 3. Practical Example

### 2. Nonagonal Neutrosophic number:

#### 2.1 Definition: score function

The score function is the mean positive of the neutrosophical Nonagonal components T, I, F.

#### 2.2 Definition: Verbose phrases for Nonagonal Neutrosophical number.

(Very strong, strongest, medium strong, strong, medium, very low, lowest, medium low, low)

#### 2.3 Definition: single valued Nonagonal Neutrosophic number:

A single valued Nonagonal Neutrosophic number ( $\tilde{N}_a$ ) is defined as

$$\tilde{N}_a = [(\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5, \varsigma_6, \varsigma_7, \varsigma_8, \varsigma_9): \alpha], [(\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \varpi_6, \varpi_7, \varpi_8, \varpi_9): \beta], \\ [(\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6, \varrho_7, \varrho_8, \varrho_9): \gamma]$$

Where  $\alpha, \beta, \gamma \in [0, 1]$

The truth membership function ( $T_s$ ):  $R \rightarrow [0, \alpha]$

The indeterminacy membership function ( $I_s$ ):  $R \rightarrow [\beta, 1]$   $\varrho_3$

The falsity membership function ( $F_s$ ):  $R \rightarrow [\gamma, 1]$

In  $T_s(y)$   $\Omega_1 \leq y \leq \Omega_2$

#### 2.4 Definition: Score function for Nonagonal Neutrosophic number

$$S_{\{Na\}} = \frac{1}{3} \left[ 2 + \frac{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5, \varsigma_6, \varsigma_7, \varsigma_8, \varsigma_9}{9} - \frac{\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \varpi_6, \varpi_7, \varpi_8, \varpi_9}{9} \right. \\ \left. - \frac{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6, \varrho_7, \varrho_8, \varrho_9}{9} \right]$$

$$A_{\{Na\}} \in [-1, 1]$$

For any Nonagonal single valued Neutrosophic number

$$B_{\{NaNeu\}} = [(\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5, \varsigma_6, \varsigma_7, \varsigma_8, \varsigma_9)], [(\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \varpi_6, \varpi_7, \varpi_8, \varpi_9)], \\ [(\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6, \varrho_7, \varrho_8, \varrho_9)]$$

This is concluded that

If  $B_{\{NaNeu\}} = [(1, 1, 1, 1, 1, 1, 1, 1, 1); (0, 0, 0, 0, 0, 0, 0, 0, 0); (0, 0, 0, 0, 0, 0, 0, 0, 0)]$

Then  $S_{\{Na\}} = 1$ ,  $S_{\{NaNeu\}} = 1$

If  $B_{\{NaNeu\}} = [(0, 0, 0, 0, 0, 0, 0, 0, 0); (1, 1, 1, 1, 1, 1, 1, 1, 1); (1, 1, 1, 1, 1, 1, 1, 1, 1)]$  then

$S_{\{Na\}} = 0$   $A_{\{NaNeu\}} = -1$

#### 2.5 Definition: Relationship between any two Nonagonal Neutrosophic numbers

Look at two Nonagonal neutrosophical fuzzy numbers.

$$B_{1NaNeu} = (T_{NaNeu}, I_{NaNeu}, F_{NaNeu})$$

$$B_{2NaNeu} = (T_{NaNeu}, I_{NaNeu}, F_{NaNeu})$$

$S_{B1NaNeu} < S_{B2NaNeu}$  then  $B_{1NaNeu} > B_{2NaNeu}$

$S_{B_1NaNeu} < S_{B_2NaNeu}$  then  $B_{1NaNeu} < B_{2NaNeu}$

$S_{B_1NaNeu} = S_{B_2NaNeu}$  then

1)  $A_{B_1NaNeu} > A_{B_2NaNeu}$ , then  $B_{1NaNeu} > B_{2NaNeu}$

2)  $A_{B_1NaNeu} < A_{B_2NaNeu}$ , then  $B_{1NaNeu} < B_{2NaNeu}$

3)  $A_{B_1NaNeu} = A_{B_2NaNeu}$ , then  $B_{1NaNeu} \sim B_{2NaNeu}$

**2.6 Theorem: The normalized hamming distance between two Nonagonal Neutrosophic number  $\tilde{W}_1$  &  $\tilde{W}_2$  complies with the conditions below.**

a)  $d(\tilde{W}_1, \tilde{W}_2) \geq 0$

b)  $d(\tilde{W}_1, \tilde{W}_2) = d(\tilde{W}_2, \tilde{W}_1)$

c)  $d(\tilde{W}_1, \tilde{W}_3) \leq d(\tilde{W}_1, \tilde{W}_2) + d(\tilde{W}_2, \tilde{W}_3)$

**Proof:**

Consider two Nonagonal Neutrosophic number

$$\tilde{W}_1 = (\eta_{11}, \eta_{22}, \eta_{33}, \eta_{44}, \eta_{55}, \eta_{66}, \eta_{77}, \eta_{88}, \eta_{99}); (T_{11}, I_{11}, F_{11})$$

$$\tilde{W}_2 = (\xi_{11}, \xi_{22}, \xi_{33}, \xi_{44}, \xi_{55}, \xi_{66}, \xi_{77}, \xi_{88}, \xi_{99}); (T_{22}, I_{22}, F_{22})$$

Normalized hamming distance between two Nonagonal Neutrosophic number is defined as follows

$$H_d(\tilde{W}_1, \tilde{W}_2) = \frac{1}{27} \{ |\eta_{11}(2 + T_{11} - I_{11} - F_{11}) - \xi_{11}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{22}(2 + T_{11} - I_{11} - F_{11}) - \xi_{22}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{33}(2 + T_{11} - I_{11} - F_{11}) - \xi_{33}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{44}(2 + T_{11} - I_{11} - F_{11}) - \xi_{44}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{55}(2 + T_{11} - I_{11} - F_{11}) - \xi_{55}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{66}(2 + T_{11} - I_{11} - F_{11}) - \xi_{66}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{77}(2 + T_{11} - I_{11} - F_{11}) - \xi_{77}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{88}(2 + T_{11} - I_{11} - F_{11}) - \xi_{88}(2 + T_{22} - I_{22} - F_{22})| + |\eta_{99}(2 + T_{11} - I_{11} - F_{11}) - \xi_{99}(2 + T_{22} - I_{22} - F_{22})| \}$$

a)  $d(\tilde{W}_1, \tilde{W}_2) > 0$  holds for any two  $(\tilde{W}_1, \tilde{W}_2)$

If  $(\tilde{W}_1, \cong \tilde{W}_2)$  then  $\eta_{11} = \xi_{11}, \eta_{22} = \xi_{22}, \eta_{33} = \xi_{33}, \eta_{44} = \xi_{44}, \eta_{55} = \xi_{55}, \eta_{66} = \xi_{66}, \eta_{77} = \xi_{77},$

$\eta_{88} = \xi_{88}, \eta_{99} = \xi_{99}$

$T_1 = T_2, I_1 = I_2, F_1 = F_2$

Then we get  $d(\tilde{W}_1, \tilde{W}_2) = 0$   $d(\tilde{W}_1, \tilde{W}_2) \geq 0$

b) Obviously  $d(\tilde{W}_1, \tilde{W}_2) = d(\tilde{W}_2, \tilde{W}_1)$

c) The normalized hamming distance between  $\tilde{W}_1$  and  $\tilde{W}_3$  is

$$d(\tilde{W}_1, \tilde{W}_3) = \frac{1}{27} \{ |\eta_{11}(2 + T_1 - I_1 - F_1) - \xi_{11}(2 + T_3 - I_3 - F_3)| + |\eta_{22}(2 + T_1 - I_1 - F_1) - \xi_{22}(2 + T_3 - I_3 - F_3)| + |\eta_{33}(2 + T_1 - I_1 - F_1) - \xi_{33}(2 + T_3 - I_3 - F_3)| + |\eta_{44}(2 + T_1 - I_1 - F_1) - \xi_{44}(2 + T_3 - I_3 - F_3)| + |\eta_{55}(2 + T_1 - I_1 - F_1) - \xi_{55}(2 + T_3 - I_3 - F_3)| + |\eta_{66}(2 + T_1 - I_1 - F_1) - \xi_{66}(2 + T_3 - I_3 - F_3)| + |\eta_{77}(2 + T_1 - I_1 - F_1) - \xi_{77}(2 + T_3 - I_3 - F_3)| + |\eta_{88}(2 + T_1 - I_1 - F_1) - \xi_{88}(2 + T_3 - I_3 - F_3)| + |\eta_{99}(2 + T_1 - I_1 - F_1) - \xi_{99}(2 + T_3 - I_3 - F_3)| \}$$

$$d(\tilde{W}_1, \tilde{W}_3) = \frac{1}{27} \{ |\eta_{11}(2 + T_1 - I_1 - F_1) + \xi_{11}(2 + T_2 - I_2 - F_2) - \xi_{11}(2 + T_2 - I_2 - F_2) - r_3(2 + T_3 - I_3 - F_3)| + |\eta_{22}(2 + T_1 - I_1 - F_1) + \xi_{22}(2 + T_2 - I_2 - F_2) - \xi_{22}(2 + T_2 - I_2 - F_2) - s_3(2 + T_3 - I_3 - F_3)| + |\eta_{33}(2 + T_1 - I_1 - F_1) + \xi_{33}(2 + T_2 - I_2 - F_2) - \xi_{33}(2 + T_2 - I_2 - F_2) - t_3(2 + T_3 - I_3 - F_3)| + |\eta_{44}(2 + T_1 - I_1 - F_1) + \xi_{44}(2 + T_2 - I_2 - F_2) - \xi_{44}(2 + T_2 - I_2 - F_2) - u_3(2 + T_3 - I_3 - F_3)| + |\eta_{55}(2 + T_1 - I_1 - F_1) + \xi_{55}(2 + T_2 - I_2 - F_2) - \xi_{55}(2 + T_2 - I_2 - F_2) - v_3(2 + T_3 - I_3 - F_3)| + |\eta_{66}(2 + T_1 - I_1 - F_1) + \xi_{66}(2 + T_2 - I_2 - F_2) - \xi_{66}(2 + T_2 - I_2 - F_2) - w_3(2 + T_3 - I_3 - F_3)| + |\eta_{77}(2 + T_1 - I_1 - F_1) + \xi_{77}(2 + T_2 - I_2 - F_2) - \xi_{77}(2 + T_2 - I_2 - F_2) - x_3(2 + T_3 - I_3 - F_3)| + |\eta_{88}(2 + T_1 - I_1 - F_1) + \xi_{88}(2 + T_2 - I_2 - F_2) - \xi_{88}(2 + T_2 - I_2 - F_2) - y_3(2 + T_3 - I_3 - F_3)| + |\eta_{99}(2 + T_1 - I_1 - F_1) + \xi_{99}(2 + T_2 - I_2 - F_2) - \xi_{99}(2 + T_2 - I_2 - F_2) - z_3(2 + T_3 - I_3 - F_3)| \}$$

$$\begin{aligned}
 d(\tilde{W}_1, \tilde{W}_3) \leq & \frac{1}{24} \{ |\eta_{11}(2 + T_1 - I_1 - F_1) - \xi_{11}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{22}(2 + T_1 - I_1 - F_1) - \xi_{22}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{33}(2 + T_1 - I_1 - F_1) - \xi_{33}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{44}(2 + T_1 - I_1 - F_1) - \xi_{44}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{55}(2 + T_1 - I_1 - F_1) - \xi_{55}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{66}(2 + T_1 - I_1 - F_1) - \xi_{66}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{77}(2 + T_1 - I_1 - F_1) - \xi_{77}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{88}(2 + T_1 - I_1 - F_1) - \xi_{88}(2 + T_2 - I_2 - F_2)| \\
 & + |\eta_{99}(2 + T_1 - I_1 - F_1) - \xi_{99}(2 + T_2 - I_2 - F_2)| \} \\
 & + \frac{1}{24} \{ |\eta_{11}(2 + T_2 - I_1 - F_1) - r_3(2 + T_3 - I_3 - F_3)| + |\eta_{22}(2 + T_1 - I_1 - F_1) - \\
 & s_3(2 + T_3 - I_3 - F_3)| + |\eta_{33}(2 + T_1 - I_1 - F_1) - t_3(2 + T_3 - I_3 - F_3)| + |\eta_{44}(2 + T_1 - I_1 - \\
 & F_1) - u_3(2 + T_3 - I_3 - F_3)| \\
 & |\eta_{55}(2 + T_1 - I_1 - F_1) - v_3(2 + T_3 - I_3 - F_3)| + |\eta_{66}(2 + T_1 - I_1 - F_1) - w_3(2 + T_3 - I_3 - F_3)| \\
 & + |\eta_{77}(2 + T_1 - I_1 - F_1) - x_3(2 + T_3 - I_3 - F_3)| \\
 & + |\eta_{88}(2 + T_1 - I_1 - F_1) - y_3(2 + T_3 - I_3 - F_3)| \\
 & + |\eta_{99}(2 + T_1 - I_1 - F_1) - z_3(2 + T_3 - I_3 - F_3)| \\
 & \leq d(\tilde{W}_1, \tilde{W}_2) + d(\tilde{W}_2, \tilde{W}_3)
 \end{aligned}$$

**2.7 Definition: Euclidean distance for Nonagonal Neutrosophic number:**

Consider two Nonagonal Neutrosophic number

$$\tilde{B}_1 = (\eta_{11}, \eta_{22}, \eta_{33}, \eta_{44}, \eta_{55}, \eta_{66}, \eta_{77}, \eta_{88}, \eta_{99}); (T_{11}, I_{11}, F_{11})$$

$$\tilde{B}_2 = (\xi_{11}, \xi_{22}, \xi_{33}, \xi_{44}, \xi_{55}, \xi_{66}, \xi_{77}, \xi_{88}, \xi_{99}); (T_{22}, I_{22}, F_{22})$$

The normalized Euclidean distance is defined by

$$\tilde{E}(\tilde{B}_1, \tilde{B}_2) = \sqrt{\sum_{i=1}^9 \frac{[(T_{B_1}(x_i) - T_{B_2}(x_i))^2 + (I_{B_1}(x_i) - I_{B_2}(x_i))^2 + (F_{B_1}(x_i) - F_{B_2}(x_i))^2]}{27}}$$

**3. m-polar Nonagonal Neutrosophic number**

**3.1 Definition:**

**m-polar Neutrosophic set is defined as**

$$\tilde{L} = \{ (\phi_L^1(x_\theta), \phi_L^2(x_\theta), \dots, \phi_L^m(x_\theta)), (\Psi_L^1(x_\theta), \Psi_L^2(x_\theta), \dots, \Psi_L^m(x_\theta)), \frac{(\eta_L^1(x_\theta), \eta_L^2(x_\theta), \dots, \eta_L^m(x_\theta))}{x_\theta} \in X \}$$

Where  $\phi_L^j: X \rightarrow [0,1]$ ,  $\Psi_L^j: X \rightarrow [0,1]$ ,  $\eta_L^j: X \rightarrow [0,1]$  (for all  $j=1,2,3\dots m$ )

Denote the  $j^{th}$  truth membership,  $j^{th}$  indeterminacy membership,  $j^{th}$  falsity membership respectively for each element  $x \in X$  to the set L

$$0 \leq \phi_L^1(x_\theta) + \Psi_L^1(x_\theta) + \eta_L^1(x_\theta) \leq 3 \text{ for all } j=1,2,3\dots m$$

**3.2 Definition: m-polar Nonagonal Neutrosophic set**

Let X be a universal set and an m-polar nanagonal Neutrosophic set (NNS)  $\tilde{L}$  over universe X is defined as

$$\tilde{L} = \{ (N\phi_L^1(x_\theta), N\phi_L^2(x_\theta), \dots, N\phi_L^m(x_\theta)), (N\Psi_L^1(x_\theta), N\Psi_L^2(x_\theta), \dots, N\Psi_L^m(x_\theta)), \\
 (N\eta_L^1(x_\theta), N\eta_L^2(x_\theta), \dots, N\eta_L^m(x_\theta)) / x_\theta \in X$$

$$N\phi_L^j(x_\theta) = [\phi_L^{j1}(x_\theta), \phi_L^{j2}(x_\theta), \phi_L^{j3}(x_\theta), \phi_L^{j4}(x_\theta), \phi_L^{j5}(x_\theta), \phi_L^{j6}(x_\theta), \phi_L^{j7}(x_\theta), \phi_L^{j8}(x_\theta), \phi_L^{j9}(x_\theta)]$$

$$N\Psi_L^j(x_\theta) = [\Psi_L^{j1}(x_\theta), \Psi_L^{j2}(x_\theta), \Psi_L^{j3}(x_\theta), \Psi_L^{j4}(x_\theta), \Psi_L^{j5}(x_\theta), \Psi_L^{j6}(x_\theta), \Psi_L^{j7}(x_\theta), \Psi_L^{j8}(x_\theta), \Psi_L^{j9}(x_\theta)]$$

$$N\eta_L^j(x_\theta) = [\eta_L^{j1}(x_\theta), \eta_L^{j2}(x_\theta), \eta_L^{j3}(x_\theta), \eta_L^{j4}(x_\theta), \eta_L^{j5}(x_\theta), \eta_L^{j6}(x_\theta), \eta_L^{j7}(x_\theta), \eta_L^{j8}(x_\theta), \eta_L^{j9}(x_\theta)]$$

For all  $j = 1,2,3,\dots m$

$$0 \leq \sup(N\phi_L^j(x_\theta)) + \sup(N\Psi_L^j(x_\theta)) + \sup(N\eta_L^j(x_\theta)) \leq 3 \text{ for all } j = 1,2,3\dots m$$

Consider two m-polar Nonagonal Neutrosophic set  $\tilde{L}_1$  &  $\tilde{L}_2$  over a entire set X. If  $L_1$  is a subset of  $L_2$

$$\tilde{L}_1 \subseteq \tilde{L}_2 \text{ if } \phi_{L_1}^j(x_\theta) \leq \phi_{L_2}^j(x_\theta), \Psi_{L_1}^j(x_\theta) \leq \Psi_{L_2}^j(x_\theta), \eta_{L_1}^j(x_\theta) \geq \eta_{L_2}^j(x_\theta)$$

**3.3 Definition: Union of m-polar Nonagonal Neutrosophic set**

Consider two m-polar Nonagonal Neutrosophic set  $\tilde{L}_1$  &  $\tilde{L}_2$  over a entire set X. The union of  $\tilde{L}_1$  and  $\tilde{L}_2$  is  $\tilde{L}_1 \cup \tilde{L}_2 = \{N\phi_{\tilde{L}_1 \cup \tilde{L}_2}^j(x_\theta), N\Psi_{\tilde{L}_1 \cup \tilde{L}_2}^j(x_\theta), N\eta_{\tilde{L}_1 \cup \tilde{L}_2}^j(x_\theta) | x_\theta \in X\}$

$$N\phi_{\tilde{L}_1 \cup \tilde{L}_2}^j(x) = \sup \left( \left( \phi_{\tilde{L}_1}^{j1}(x_\theta), \phi_{\tilde{L}_1}^{j2}(x_\theta), \phi_{\tilde{L}_1}^{j3}(x_\theta), \phi_{\tilde{L}_1}^{j4}(x_\theta), \phi_{\tilde{L}_1}^{j5}(x_\theta), \phi_{\tilde{L}_1}^{j6}(x_\theta), \phi_{\tilde{L}_1}^{j7}(x_\theta), \phi_{\tilde{L}_1}^{j8}(x_\theta), \phi_{\tilde{L}_1}^{j9}(x_\theta) \right), \left( \phi_{\tilde{L}_2}^{j1}(x_\theta), \phi_{\tilde{L}_2}^{j2}(x_\theta), \phi_{\tilde{L}_2}^{j3}(x_\theta), \phi_{\tilde{L}_2}^{j4}(x_\theta), \phi_{\tilde{L}_2}^{j5}(x_\theta), \phi_{\tilde{L}_2}^{j6}(x_\theta), \phi_{\tilde{L}_2}^{j7}(x_\theta), \phi_{\tilde{L}_2}^{j8}(x_\theta), \phi_{\tilde{L}_2}^{j9}(x_\theta) \right) \right)$$

**3.4 Definition: Intersection of m-polar Nonagonal Neutrosophic set**

Let  $\tilde{L}_1$  and  $\tilde{L}_2$  be two m-polar nanogonal Neutrosophic set over a set universe X then the intersection of  $\tilde{L}_1$  and  $\tilde{L}_2$  defined as  $\tilde{L}_1 \cap \tilde{L}_2 = \{N\phi_{\tilde{L}_1 \cap \tilde{L}_2}^j(x_\theta), N\Psi_{\tilde{L}_1 \cap \tilde{L}_2}^j(x_\theta), N\eta_{\tilde{L}_1 \cap \tilde{L}_2}^j(x_\theta) | x_\theta \in X\}$

$$N\phi_{\tilde{L}_1 \cap \tilde{L}_2}^j(x_\theta) = \inf \left( \left( \phi_{\tilde{L}_1}^{j1}(x_\theta), \phi_{\tilde{L}_1}^{j2}(x_\theta), \phi_{\tilde{L}_1}^{j3}(x_\theta), \phi_{\tilde{L}_1}^{j4}(x_\theta), \phi_{\tilde{L}_1}^{j5}(x_\theta), \phi_{\tilde{L}_1}^{j6}(x_\theta), \phi_{\tilde{L}_1}^{j7}(x_\theta), \phi_{\tilde{L}_1}^{j8}(x_\theta), \phi_{\tilde{L}_1}^{j9}(x_\theta) \right), \left( \phi_{\tilde{L}_2}^{j1}(x_\theta), \phi_{\tilde{L}_2}^{j2}(x_\theta), \phi_{\tilde{L}_2}^{j3}(x_\theta), \phi_{\tilde{L}_2}^{j4}(x_\theta), \phi_{\tilde{L}_2}^{j5}(x_\theta), \phi_{\tilde{L}_2}^{j6}(x_\theta), \phi_{\tilde{L}_2}^{j7}(x_\theta), \phi_{\tilde{L}_2}^{j8}(x_\theta), \phi_{\tilde{L}_2}^{j9}(x_\theta) \right) \right)$$

**3.5 Properties of m-polar Nonagonal Neutrosophic set**

**Property 1: Idempotent laws**

- i.  $\tilde{L}_{11} \cap \tilde{L}_{11} = \tilde{L}_{11}$
- ii.  $\tilde{L}_{11} \cup \tilde{L}_{11} = \tilde{L}_{11}$

Proof:  $\tilde{L}_{11} \cap \tilde{L}_{11} = \{N\phi_{\tilde{L}_{11} \cap \tilde{L}_{11}}^j(x_\theta), N\Psi_{\tilde{L}_{11} \cap \tilde{L}_{11}}^j(x_\theta), N\eta_{\tilde{L}_{11} \cap \tilde{L}_{11}}^j(x_\theta) | x_\theta \in X\}$

$$= \inf \left( \left( \phi_{\tilde{L}_{11}}^{j1}(x_\theta), \phi_{\tilde{L}_{11}}^{j2}(x_\theta), \phi_{\tilde{L}_{11}}^{j3}(x_\theta), \phi_{\tilde{L}_{11}}^{j4}(x_\theta), \phi_{\tilde{L}_{11}}^{j5}(x_\theta), \phi_{\tilde{L}_{11}}^{j6}(x_\theta), \phi_{\tilde{L}_{11}}^{j7}(x_\theta), \phi_{\tilde{L}_{11}}^{j8}(x_\theta), \phi_{\tilde{L}_{11}}^{j9}(x_\theta) \right), \left( \phi_{\tilde{L}_{11}}^{j1}(x_\theta), \phi_{\tilde{L}_{11}}^{j2}(x_\theta), \phi_{\tilde{L}_{11}}^{j3}(x_\theta), \phi_{\tilde{L}_{11}}^{j4}(x_\theta), \phi_{\tilde{L}_{11}}^{j5}(x_\theta), \phi_{\tilde{L}_{11}}^{j6}(x_\theta), \phi_{\tilde{L}_{11}}^{j7}(x_\theta), \phi_{\tilde{L}_{11}}^{j8}(x_\theta), \phi_{\tilde{L}_{11}}^{j9}(x_\theta) \right) \right)$$

$$= \phi_{\tilde{L}_{11}}^{j1}(x_\theta), \phi_{\tilde{L}_{11}}^{j2}(x_\theta), \phi_{\tilde{L}_{11}}^{j3}(x_\theta), \phi_{\tilde{L}_{11}}^{j4}(x_\theta), \phi_{\tilde{L}_{11}}^{j5}(x_\theta), \phi_{\tilde{L}_{11}}^{j6}(x_\theta), \phi_{\tilde{L}_{11}}^{j7}(x_\theta), \phi_{\tilde{L}_{11}}^{j8}(x_\theta), \phi_{\tilde{L}_{11}}^{j9}(x_\theta)$$

$$\tilde{L}_{11} \cap \tilde{L}_{11} = \tilde{L}_{11}$$

Hence proved. Similarly, we can prove  $\tilde{L}_{11} \cup \tilde{L}_{11} = \tilde{L}_{11}$

**Property 2: Identity laws**  $\tilde{L} \cap U = \tilde{L} = U \cup \tilde{L}$

**Property 3: Domination laws:**

- i.  $\tilde{L} \cap \tilde{\phi} = \tilde{\phi} = \tilde{\phi} \cap \tilde{L}$
- ii.  $\tilde{L} \cup \tilde{U} = \tilde{U} = \tilde{U} \cup \tilde{L}$
- iii.  $\tilde{L} \cup \tilde{\phi} = \tilde{L} = \tilde{\phi} \cup \tilde{L}$

**Property 4: Complement laws** i.  $\tilde{\phi}^c = \tilde{U}$  ii.  $\tilde{U}^c = \tilde{\phi}$

**Property 5: Double complement law**  $(L^c)^c = L$

**Property 6: Commutative laws**

- i.  $\tilde{L}_{11} \cup \tilde{L}_{22} = \tilde{L}_{22} \cup \tilde{L}_{11}$
- ii.  $\tilde{L}_{11} \cap \tilde{L}_{22} = \tilde{L}_{22} \cap \tilde{L}_{11}$

Proof :

$$L_{11} \cup L_{22} = \{ \sup \{ \left[ \phi_{\tilde{L}_1}^{j1}(x_\theta), \phi_{\tilde{L}_1}^{j2}(x_\theta), \phi_{\tilde{L}_1}^{j3}(x_\theta), \phi_{\tilde{L}_1}^{j4}(x_\theta), \phi_{\tilde{L}_1}^{j5}(x_\theta), \phi_{\tilde{L}_1}^{j6}(x_\theta), \phi_{\tilde{L}_1}^{j7}(x_\theta), \phi_{\tilde{L}_1}^{j8}(x_\theta), \phi_{\tilde{L}_1}^{j9}(x_\theta) \right], \left[ \phi_{\tilde{L}_2}^{j1}(x_\theta), \phi_{\tilde{L}_2}^{j2}(x_\theta), \phi_{\tilde{L}_2}^{j3}(x_\theta), \phi_{\tilde{L}_2}^{j4}(x_\theta), \phi_{\tilde{L}_2}^{j5}(x_\theta), \phi_{\tilde{L}_2}^{j6}(x_\theta), \phi_{\tilde{L}_2}^{j7}(x_\theta), \phi_{\tilde{L}_2}^{j8}(x_\theta), \phi_{\tilde{L}_2}^{j9}(x_\theta) \right] \}, \sup \{ \left[ \Psi_{\tilde{L}_1}^{j1}(x_\theta), \Psi_{\tilde{L}_1}^{j2}(x_\theta), \Psi_{\tilde{L}_1}^{j3}(x_\theta), \Psi_{\tilde{L}_1}^{j4}(x_\theta), \Psi_{\tilde{L}_1}^{j5}(x_\theta), \Psi_{\tilde{L}_1}^{j6}(x_\theta), \Psi_{\tilde{L}_1}^{j7}(x_\theta), \Psi_{\tilde{L}_1}^{j8}(x_\theta), \Psi_{\tilde{L}_1}^{j9}(x_\theta) \right], \left[ \Psi_{\tilde{L}_2}^{j1}(x_\theta), \Psi_{\tilde{L}_2}^{j2}(x_\theta), \Psi_{\tilde{L}_2}^{j3}(x_\theta), \Psi_{\tilde{L}_2}^{j4}(x_\theta), \Psi_{\tilde{L}_2}^{j5}(x_\theta), \Psi_{\tilde{L}_2}^{j6}(x_\theta), \Psi_{\tilde{L}_2}^{j7}(x_\theta), \Psi_{\tilde{L}_2}^{j8}(x_\theta), \Psi_{\tilde{L}_2}^{j9}(x_\theta) \right] \}, \inf \{ \left[ \eta_{\tilde{L}_1}^{j1}(x_\theta), \eta_{\tilde{L}_1}^{j2}(x_\theta), \eta_{\tilde{L}_1}^{j3}(x_\theta), \eta_{\tilde{L}_1}^{j4}(x_\theta), \eta_{\tilde{L}_1}^{j5}(x_\theta), \eta_{\tilde{L}_1}^{j6}(x_\theta), \eta_{\tilde{L}_1}^{j7}(x_\theta), \eta_{\tilde{L}_1}^{j8}(x_\theta), \eta_{\tilde{L}_1}^{j9}(x_\theta) \right], \left[ \eta_{\tilde{L}_2}^{j1}(x_\theta), \eta_{\tilde{L}_2}^{j2}(x_\theta), \eta_{\tilde{L}_2}^{j3}(x_\theta), \eta_{\tilde{L}_2}^{j4}(x_\theta), \eta_{\tilde{L}_2}^{j5}(x_\theta), \eta_{\tilde{L}_2}^{j6}(x_\theta), \eta_{\tilde{L}_2}^{j7}(x_\theta), \eta_{\tilde{L}_2}^{j8}(x_\theta), \eta_{\tilde{L}_2}^{j9}(x_\theta) \right] \} \}$$

where  $j = 1, 2, 3, \dots, m$

$$L_{22} \cup L_{11} = \{ \sup \{ \left[ \phi_{\tilde{L}_2}^{j1}(x_\theta), \phi_{\tilde{L}_2}^{j2}(x_\theta), \phi_{\tilde{L}_2}^{j3}(x_\theta), \phi_{\tilde{L}_2}^{j4}(x_\theta), \phi_{\tilde{L}_2}^{j5}(x_\theta), \phi_{\tilde{L}_2}^{j6}(x_\theta), \phi_{\tilde{L}_2}^{j7}(x_\theta), \phi_{\tilde{L}_2}^{j8}(x_\theta), \phi_{\tilde{L}_2}^{j9}(x_\theta) \right], \left[ \phi_{\tilde{L}_1}^{j1}(x_\theta), \phi_{\tilde{L}_1}^{j2}(x_\theta), \phi_{\tilde{L}_1}^{j3}(x_\theta), \phi_{\tilde{L}_1}^{j4}(x_\theta), \phi_{\tilde{L}_1}^{j5}(x_\theta), \phi_{\tilde{L}_1}^{j6}(x_\theta), \phi_{\tilde{L}_1}^{j7}(x_\theta), \phi_{\tilde{L}_1}^{j8}(x_\theta), \phi_{\tilde{L}_1}^{j9}(x_\theta) \right] \}, \sup \{ \left[ \Psi_{\tilde{L}_2}^{j1}(x_\theta), \Psi_{\tilde{L}_2}^{j2}(x_\theta), \Psi_{\tilde{L}_2}^{j3}(x_\theta), \Psi_{\tilde{L}_2}^{j4}(x_\theta), \Psi_{\tilde{L}_2}^{j5}(x_\theta), \Psi_{\tilde{L}_2}^{j6}(x_\theta), \Psi_{\tilde{L}_2}^{j7}(x_\theta), \Psi_{\tilde{L}_2}^{j8}(x_\theta), \Psi_{\tilde{L}_2}^{j9}(x_\theta) \right], \left[ \Psi_{\tilde{L}_1}^{j1}(x_\theta), \Psi_{\tilde{L}_1}^{j2}(x_\theta), \Psi_{\tilde{L}_1}^{j3}(x_\theta), \Psi_{\tilde{L}_1}^{j4}(x_\theta), \Psi_{\tilde{L}_1}^{j5}(x_\theta), \Psi_{\tilde{L}_1}^{j6}(x_\theta), \Psi_{\tilde{L}_1}^{j7}(x_\theta), \Psi_{\tilde{L}_1}^{j8}(x_\theta), \Psi_{\tilde{L}_1}^{j9}(x_\theta) \right] \}, \inf \{ \left[ \eta_{\tilde{L}_2}^{j1}(x_\theta), \eta_{\tilde{L}_2}^{j2}(x_\theta), \eta_{\tilde{L}_2}^{j3}(x_\theta), \eta_{\tilde{L}_2}^{j4}(x_\theta), \eta_{\tilde{L}_2}^{j5}(x_\theta), \eta_{\tilde{L}_2}^{j6}(x_\theta), \eta_{\tilde{L}_2}^{j7}(x_\theta), \eta_{\tilde{L}_2}^{j8}(x_\theta), \eta_{\tilde{L}_2}^{j9}(x_\theta) \right], \left[ \eta_{\tilde{L}_1}^{j1}(x_\theta), \eta_{\tilde{L}_1}^{j2}(x_\theta), \eta_{\tilde{L}_1}^{j3}(x_\theta), \eta_{\tilde{L}_1}^{j4}(x_\theta), \eta_{\tilde{L}_1}^{j5}(x_\theta), \eta_{\tilde{L}_1}^{j6}(x_\theta), \eta_{\tilde{L}_1}^{j7}(x_\theta), \eta_{\tilde{L}_1}^{j8}(x_\theta), \eta_{\tilde{L}_1}^{j9}(x_\theta) \right] \} \}$$

$$[\eta_{\tilde{L}_1}^{j_1}(x_\theta), \eta_{\tilde{L}_1}^{j_2}(x_\theta), \eta_{\tilde{L}_1}^{j_3}(x_\theta), \eta_{\tilde{L}_1}^{j_4}(x_\theta), \eta_{\tilde{L}_1}^{j_5}(x_\theta), \eta_{\tilde{L}_1}^{j_6}(x_\theta), \eta_{\tilde{L}_1}^{j_7}(x_\theta), \eta_{\tilde{L}_1}^{j_8}(x_\theta), \eta_{\tilde{L}_1}^{j_9}(x_\theta)] \}} \\ \text{where } j = 1, 2, 3, \dots, m \\ \therefore \tilde{L}_{11} \cup \tilde{L}_{22} = \tilde{L}_{22} \cup \tilde{L}_{11}$$

The proof is similar for intersection also.

**Property 7: Associative laws**

- i.  $\tilde{L}_{11} \cup (\tilde{L}_{22} \cup \tilde{L}_{33}) = (\tilde{L}_{11} \cup \tilde{L}_{22}) \cup \tilde{L}_{33}$
- ii.  $\tilde{L}_{11} \cap (\tilde{L}_{22} \cap \tilde{L}_{33}) = (\tilde{L}_{11} \cap \tilde{L}_{22}) \cap \tilde{L}_{33}$

**Proof:**

$$\tilde{L}_{11} \cup (\tilde{L}_{22} \cup \tilde{L}_{33}) \\ = \{ \sup [\phi_{\tilde{L}_{11}}^j(x_\theta), \sup [\phi_{\tilde{L}_{22}}^j(x_\theta), \phi_{\tilde{L}_{33}}^j(x_\theta)]] , \sup [\Psi_{\tilde{L}_{11}}^j(x_\theta), \sup [\Psi_{\tilde{L}_{22}}^j(x_\theta), \Psi_{\tilde{L}_{33}}^j(x_\theta)]]], \\ \inf [\eta_{\tilde{L}_{11}}^j(x_\theta), \inf [\eta_{\tilde{L}_{22}}^j(x_\theta), \eta_{\tilde{L}_{33}}^j(x_\theta)]] \} \\ = \{ \sup [\phi_{\tilde{L}_{11}}^j, \phi_{\tilde{L}_{22}}^j, \phi_{\tilde{L}_{33}}^j], \sup [\Psi_{\tilde{L}_{11}}^j, \Psi_{\tilde{L}_{22}}^j, \Psi_{\tilde{L}_{33}}^j], \inf [\eta_{\tilde{L}_{11}}^j, \eta_{\tilde{L}_{22}}^j, \eta_{\tilde{L}_{33}}^j] \} \\ = \{ \sup [\sup \{ \phi_{\tilde{L}_{11}}^j, \phi_{\tilde{L}_{22}}^j \}, \phi_{\tilde{L}_{33}}^j], \sup [\sup \{ \Psi_{\tilde{L}_{11}}^j, \Psi_{\tilde{L}_{22}}^j \}, \Psi_{\tilde{L}_{33}}^j], \inf [\inf \{ \eta_{\tilde{L}_{11}}^j, \eta_{\tilde{L}_{22}}^j \}, \eta_{\tilde{L}_{33}}^j] \} \\ \forall i = 1, 2, 3, \dots, m \\ (\tilde{L}_{11} \cup \tilde{L}_{22}) \cup \tilde{L}_{33} \quad \therefore \tilde{L}_{11} \cup (\tilde{L}_{22} \cup \tilde{L}_{33}) = (\tilde{L}_{11} \cup \tilde{L}_{22}) \cup \tilde{L}_{33}$$

The proof is similar for intersection also.

**Property 8: Distributive laws:**

- i.  $\tilde{L}_{11} \cup (\tilde{L}_{22} \cap \tilde{L}_{33}) = (\tilde{L}_{11} \cup \tilde{L}_{22}) \cap (\tilde{L}_{11} \cup \tilde{L}_{33})$
- ii.  $\tilde{L}_{11} \cap (\tilde{L}_{22} \cup \tilde{L}_{33}) = (\tilde{L}_{11} \cap \tilde{L}_{22}) \cup (\tilde{L}_{11} \cap \tilde{L}_{33})$

**Proof:**

$$\tilde{L}_{11} \cap (\tilde{L}_{22} \cup \tilde{L}_{33}) \\ = \sup \{ \phi_{\tilde{L}_{11}}^j(x_\theta), \inf \{ \phi_{\tilde{L}_{22}}^j(x_\theta), \phi_{\tilde{L}_{33}}^j(x_\theta) \} \}, \sup \{ \Psi_{\tilde{L}_{11}}^j(x_\theta), \inf \{ \Psi_{\tilde{L}_{22}}^j(x_\theta), \Psi_{\tilde{L}_{33}}^j(x_\theta) \} \}, \\ \inf \{ \eta_{\tilde{L}_{11}}^j(x_\theta), \sup \{ \eta_{\tilde{L}_{22}}^j(x_\theta), \eta_{\tilde{L}_{33}}^j(x_\theta) \} \} \\ = \inf \{ \sup \{ \phi_{\tilde{L}_{11}}^j(x_\theta), \phi_{\tilde{L}_{22}}^j(x_\theta) \}, \sup \{ \phi_{\tilde{L}_{11}}^j(x_\theta), \phi_{\tilde{L}_{33}}^j(x_\theta) \} \}, \\ \inf \{ \sup \{ \Psi_{\tilde{L}_{11}}^j(x_\theta), \Psi_{\tilde{L}_{22}}^j(x_\theta) \}, \sup \{ \Psi_{\tilde{L}_{11}}^j(x_\theta), \Psi_{\tilde{L}_{33}}^j(x_\theta) \} \}, \\ \sup \{ \inf \{ \eta_{\tilde{L}_{11}}^j(x_\theta), \eta_{\tilde{L}_{22}}^j(x_\theta) \}, \inf \{ \eta_{\tilde{L}_{11}}^j(x_\theta), \eta_{\tilde{L}_{33}}^j(x_\theta) \} \} \\ \forall j = 1, 2, 3 \dots, m \\ = (\tilde{L}_{11} \cup \tilde{L}_{22}) \cap (\tilde{L}_{11} \cup \tilde{L}_{33})$$

Similarly, we can prove  $\tilde{L}_{11} \cap (\tilde{L}_{22} \cup \tilde{L}_{33}) = (\tilde{L}_{11} \cap \tilde{L}_{22}) \cup (\tilde{L}_{11} \cap \tilde{L}_{33})$

**3.6 Theorem: Demorgan's law**

$$i. (\tilde{L}_1 \cup \tilde{L}_2)^C = \tilde{L}_1^C \cap \tilde{L}_2^C \quad ii. (\tilde{L}_1 \cap \tilde{L}_2)^C = \tilde{L}_1^C \cup \tilde{L}_2^C$$

Proof:

$$\tilde{L}_1^C \cap \tilde{L}_2^C = \left[ \left( \eta_{\tilde{L}_1}^{j_1}(x_\theta), \eta_{\tilde{L}_1}^{j_2}(x_\theta), \eta_{\tilde{L}_1}^{j_3}(x_\theta), \eta_{\tilde{L}_1}^{j_4}(x_\theta), \eta_{\tilde{L}_1}^{j_5}(x_\theta), \eta_{\tilde{L}_1}^{j_6}(x_\theta), \eta_{\tilde{L}_1}^{j_7}(x_\theta), \eta_{\tilde{L}_1}^{j_8}(x_\theta), \eta_{\tilde{L}_1}^{j_9}(x_\theta) \right), 1 - \right. \\ \left. \left( \Psi_{\tilde{L}_1}^{j_1}(x_\theta), \Psi_{\tilde{L}_1}^{j_2}(x_\theta), \Psi_{\tilde{L}_1}^{j_3}(x_\theta), \Psi_{\tilde{L}_1}^{j_4}(x_\theta), \Psi_{\tilde{L}_1}^{j_5}(x_\theta), \Psi_{\tilde{L}_1}^{j_6}(x_\theta), \Psi_{\tilde{L}_1}^{j_7}(x_\theta), \Psi_{\tilde{L}_1}^{j_8}(x_\theta), \Psi_{\tilde{L}_1}^{j_9}(x_\theta) \right) \right) \\ \left( \phi_{\tilde{L}_1}^{j_1}(x_\theta), \phi_{\tilde{L}_1}^{j_2}(x_\theta), \phi_{\tilde{L}_1}^{j_3}(x_\theta), \phi_{\tilde{L}_1}^{j_4}(x_\theta), \phi_{\tilde{L}_1}^{j_5}(x_\theta), \phi_{\tilde{L}_1}^{j_6}(x_\theta), \phi_{\tilde{L}_1}^{j_7}(x_\theta), \phi_{\tilde{L}_1}^{j_8}(x_\theta), \phi_{\tilde{L}_1}^{j_9}(x_\theta) \right) \cap \\ \left( \eta_{\tilde{L}_2}^{j_1}(x_\theta), \eta_{\tilde{L}_2}^{j_2}(x_\theta), \eta_{\tilde{L}_2}^{j_3}(x_\theta), \eta_{\tilde{L}_2}^{j_4}(x_\theta), \eta_{\tilde{L}_2}^{j_5}(x_\theta), \eta_{\tilde{L}_2}^{j_6}(x_\theta), \eta_{\tilde{L}_2}^{j_7}(x_\theta), \eta_{\tilde{L}_2}^{j_8}(x_\theta), \eta_{\tilde{L}_2}^{j_9}(x_\theta) \right), 1 \\ - \left( \Psi_{\tilde{L}_2}^{j_1}(x_\theta), \Psi_{\tilde{L}_2}^{j_2}(x_\theta), \Psi_{\tilde{L}_2}^{j_3}(x_\theta), \Psi_{\tilde{L}_2}^{j_4}(x_\theta), \Psi_{\tilde{L}_2}^{j_5}(x_\theta), \Psi_{\tilde{L}_2}^{j_6}(x_\theta), \Psi_{\tilde{L}_2}^{j_7}(x_\theta), \Psi_{\tilde{L}_2}^{j_8}(x_\theta), \Psi_{\tilde{L}_2}^{j_9}(x_\theta) \right), \\ \left( \phi_{\tilde{L}_2}^{j_1}(x_\theta), \phi_{\tilde{L}_2}^{j_2}(x_\theta), \phi_{\tilde{L}_2}^{j_3}(x_\theta), \phi_{\tilde{L}_2}^{j_4}(x_\theta), \phi_{\tilde{L}_2}^{j_5}(x_\theta), \phi_{\tilde{L}_2}^{j_6}(x_\theta), \phi_{\tilde{L}_2}^{j_7}(x_\theta), \phi_{\tilde{L}_2}^{j_8}(x_\theta), \phi_{\tilde{L}_2}^{j_9}(x_\theta) \right) \} \\ \forall j = 1, 2, 3 \dots, m \\ = \left( \eta_{\tilde{L}_1}^{j_1}(x_\theta), \eta_{\tilde{L}_1}^{j_2}(x_\theta), \eta_{\tilde{L}_1}^{j_3}(x_\theta), \eta_{\tilde{L}_1}^{j_4}(x_\theta), \eta_{\tilde{L}_1}^{j_5}(x_\theta), \eta_{\tilde{L}_1}^{j_6}(x_\theta), \eta_{\tilde{L}_1}^{j_7}(x_\theta), \eta_{\tilde{L}_1}^{j_8}(x_\theta), \eta_{\tilde{L}_1}^{j_9}(x_\theta) \right) \\ \cap \left( \eta_{\tilde{L}_2}^{j_1}(x_\theta), \eta_{\tilde{L}_2}^{j_2}(x_\theta), \eta_{\tilde{L}_2}^{j_3}(x_\theta), \eta_{\tilde{L}_2}^{j_4}(x_\theta), \eta_{\tilde{L}_2}^{j_5}(x_\theta), \eta_{\tilde{L}_2}^{j_6}(x_\theta), \eta_{\tilde{L}_2}^{j_7}(x_\theta), \eta_{\tilde{L}_2}^{j_8}(x_\theta), \eta_{\tilde{L}_2}^{j_9}(x_\theta) \right), \left( 1 \right. \\ \left. - \left( \Psi_{\tilde{L}_1}^{j_1}(x_\theta), \Psi_{\tilde{L}_1}^{j_2}(x_\theta), \Psi_{\tilde{L}_1}^{j_3}(x_\theta), \Psi_{\tilde{L}_1}^{j_4}(x_\theta), \Psi_{\tilde{L}_1}^{j_5}(x_\theta), \Psi_{\tilde{L}_1}^{j_6}(x_\theta), \Psi_{\tilde{L}_1}^{j_7}(x_\theta), \Psi_{\tilde{L}_1}^{j_8}(x_\theta), \Psi_{\tilde{L}_1}^{j_9}(x_\theta) \right) \right) \\ \cap \left( 1 - \left( \phi_{\tilde{L}_2}^{j_1}(x_\theta), \phi_{\tilde{L}_2}^{j_2}(x_\theta), \phi_{\tilde{L}_2}^{j_3}(x_\theta), \phi_{\tilde{L}_2}^{j_4}(x_\theta), \phi_{\tilde{L}_2}^{j_5}(x_\theta), \phi_{\tilde{L}_2}^{j_6}(x_\theta), \phi_{\tilde{L}_2}^{j_7}(x_\theta), \phi_{\tilde{L}_2}^{j_8}(x_\theta), \phi_{\tilde{L}_2}^{j_9}(x_\theta) \right) \right),$$

$$\begin{aligned}
 & \left( \phi_{\tilde{L}_1}^{j_1}(x_\theta), \phi_{\tilde{L}_1}^{j_2}(x_\theta), \phi_{\tilde{L}_1}^{j_3}(x_\theta), \phi_{\tilde{L}_1}^{j_4}(x_\theta), \phi_{\tilde{L}_1}^{j_5}(x_\theta), \phi_{\tilde{L}_1}^{j_6}(x_\theta), \phi_{\tilde{L}_1}^{j_7}(x_\theta), \phi_{\tilde{L}_1}^{j_8}(x_\theta), \phi_{\tilde{L}_1}^{j_9}(x_\theta) \right) \\
 & \cap \left( \phi_{\tilde{L}_2}^{j_1}(x_\theta), \phi_{\tilde{L}_2}^{j_2}(x_\theta), \phi_{\tilde{L}_2}^{j_3}(x_\theta), \phi_{\tilde{L}_2}^{j_4}(x_\theta), \phi_{\tilde{L}_2}^{j_5}(x_\theta), \phi_{\tilde{L}_2}^{j_6}(x_\theta), \phi_{\tilde{L}_2}^{j_7}(x_\theta), \phi_{\tilde{L}_2}^{j_8}(x_\theta), \phi_{\tilde{L}_2}^{j_9}(x_\theta) \right) \\
 & \quad \forall j = 1, 2, 3 \dots m \\
 & = \left[ \left( \eta_{\tilde{L}_1}^{j_1}(x_\theta), \eta_{\tilde{L}_1}^{j_2}(x_\theta), \eta_{\tilde{L}_1}^{j_3}(x_\theta), \eta_{\tilde{L}_1}^{j_4}(x_\theta), \eta_{\tilde{L}_1}^{j_5}(x_\theta), \eta_{\tilde{L}_1}^{j_6}(x_\theta), \eta_{\tilde{L}_1}^{j_7}(x_\theta), \eta_{\tilde{L}_1}^{j_8}(x_\theta), \eta_{\tilde{L}_1}^{j_9}(x_\theta) \right) \right. \\
 & \cup \left( \eta_{\tilde{L}_2}^{j_1}(x_\theta), \eta_{\tilde{L}_2}^{j_2}(x_\theta), \eta_{\tilde{L}_2}^{j_3}(x_\theta), \eta_{\tilde{L}_2}^{j_4}(x_\theta), \eta_{\tilde{L}_2}^{j_5}(x_\theta), \eta_{\tilde{L}_2}^{j_6}(x_\theta), \eta_{\tilde{L}_2}^{j_7}(x_\theta), \eta_{\tilde{L}_2}^{j_8}(x_\theta), \eta_{\tilde{L}_2}^{j_9}(x_\theta) \right) \left. \right]^c, \left[ \left( 1 \right. \right. \\
 & \left. \left. - \left( \Psi_{\tilde{L}_1}^{j_1}(x_\theta), \Psi_{\tilde{L}_1}^{j_2}(x_\theta), \Psi_{\tilde{L}_1}^{j_3}(x_\theta), \Psi_{\tilde{L}_1}^{j_4}(x_\theta), \Psi_{\tilde{L}_1}^{j_5}(x_\theta), \Psi_{\tilde{L}_1}^{j_6}(x_\theta), \Psi_{\tilde{L}_1}^{j_7}(x_\theta), \Psi_{\tilde{L}_1}^{j_8}(x_\theta), \Psi_{\tilde{L}_1}^{j_9}(x_\theta) \right) \right) \right. \\
 & \left. \cup \left( 1 - \left( \Psi_{\tilde{L}_2}^{j_1}(x_\theta), \Psi_{\tilde{L}_2}^{j_2}(x_\theta), \Psi_{\tilde{L}_2}^{j_3}(x_\theta), \Psi_{\tilde{L}_2}^{j_4}(x_\theta), \Psi_{\tilde{L}_2}^{j_5}(x_\theta), \Psi_{\tilde{L}_2}^{j_6}(x_\theta), \Psi_{\tilde{L}_2}^{j_7}(x_\theta), \Psi_{\tilde{L}_2}^{j_8}(x_\theta), \Psi_{\tilde{L}_2}^{j_9}(x_\theta) \right) \right) \right] \left. \right]^c \\
 & \left[ \left( \phi_{\tilde{L}_1}^{j_1}(x_\theta), \phi_{\tilde{L}_1}^{j_2}(x_\theta), \phi_{\tilde{L}_1}^{j_3}(x_\theta), \phi_{\tilde{L}_1}^{j_4}(x_\theta), \phi_{\tilde{L}_1}^{j_5}(x_\theta), \phi_{\tilde{L}_1}^{j_6}(x_\theta), \phi_{\tilde{L}_1}^{j_7}(x_\theta), \phi_{\tilde{L}_1}^{j_8}(x_\theta), \phi_{\tilde{L}_1}^{j_9}(x_\theta) \right) \cup \right. \\
 & \left. \left( \phi_{\tilde{L}_2}^{j_1}(x_\theta), \phi_{\tilde{L}_2}^{j_2}(x_\theta), \phi_{\tilde{L}_2}^{j_3}(x_\theta), \phi_{\tilde{L}_2}^{j_4}(x_\theta), \phi_{\tilde{L}_2}^{j_5}(x_\theta), \phi_{\tilde{L}_2}^{j_6}(x_\theta), \phi_{\tilde{L}_2}^{j_7}(x_\theta), \phi_{\tilde{L}_2}^{j_8}(x_\theta), \phi_{\tilde{L}_2}^{j_9}(x_\theta) \right) \right] \\
 & = (\tilde{L}_1 \cup \tilde{L}_2)^c
 \end{aligned}$$

Similarly, we can prove

$$\tilde{L}_1^c \cup \tilde{L}_2^c = (\tilde{L}_1 \cap \tilde{L}_2)^c$$

**3.7 Definition: Distance Measure for m-polar Nonagonal Neutrosophic set**

Let  $\tilde{L}_1$  and  $\tilde{L}_2$  be two m-polar Nonagonal Neutrosophic set

$$\tilde{L}_1 = \{ \phi_{\tilde{L}_1}^{j_1}(x_\theta), \phi_{\tilde{L}_1}^{j_2}(x_\theta), \phi_{\tilde{L}_1}^{j_3}(x_\theta), \phi_{\tilde{L}_1}^{j_4}(x_\theta), \phi_{\tilde{L}_1}^{j_5}(x_\theta), \phi_{\tilde{L}_1}^{j_6}(x_\theta), \phi_{\tilde{L}_1}^{j_7}(x_\theta), \phi_{\tilde{L}_1}^{j_8}(x_\theta), \phi_{\tilde{L}_1}^{j_9}(x_\theta) | x_\theta \in X \}$$

$$\tilde{L}_2 = \{ \phi_{\tilde{L}_2}^{j_1}(x_\theta), \phi_{\tilde{L}_2}^{j_2}(x_\theta), \phi_{\tilde{L}_2}^{j_3}(x_\theta), \phi_{\tilde{L}_2}^{j_4}(x_\theta), \phi_{\tilde{L}_2}^{j_5}(x_\theta), \phi_{\tilde{L}_2}^{j_6}(x_\theta), \phi_{\tilde{L}_2}^{j_7}(x_\theta), \phi_{\tilde{L}_2}^{j_8}(x_\theta), \phi_{\tilde{L}_2}^{j_9}(x_\theta) | x_\theta \in X \}$$

a) The Hamming distance is defined as

$$\begin{aligned}
 Ha(\tilde{L}_1, \tilde{L}_2) &= \frac{1}{3m} \sum_{j=1}^m \sum_{i=1}^n \{ | \tilde{\phi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\phi}_{\tilde{L}_2}^j(x_{i_\theta}) | + | \tilde{\Psi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\Psi}_{\tilde{L}_2}^j(x_{i_\theta}) | \\
 & \quad + | \tilde{\eta}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\eta}_{\tilde{L}_2}^j(x_{i_\theta}) | \}
 \end{aligned}$$

b) Normalized Hamming distance is given by

$$\begin{aligned}
 NHa(\tilde{L}_1, \tilde{L}_2) &= \frac{1}{3n} \sum_{j=1}^m \sum_{i=1}^n \{ | \tilde{\phi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\phi}_{\tilde{L}_2}^j(x_{i_\theta}) | + | \tilde{\Psi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\Psi}_{\tilde{L}_2}^j(x_{i_\theta}) | \\
 & \quad + | \tilde{\eta}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\eta}_{\tilde{L}_2}^j(x_{i_\theta}) | \}
 \end{aligned}$$

c) Euclidean distance is given by

$$\begin{aligned}
 Eu(\tilde{L}_1, \tilde{L}_2) &= \left\{ \frac{1}{3m} \sum_{j=1}^m \sum_{i=1}^n \left\{ | \tilde{\phi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\phi}_{\tilde{L}_2}^j(x_{i_\theta}) |^2 + | \tilde{\Psi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\Psi}_{\tilde{L}_2}^j(x_{i_\theta}) |^2 \right. \right. \\
 & \quad \left. \left. + | \tilde{\eta}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\eta}_{\tilde{L}_2}^j(x_{i_\theta}) |^2 \right\} \right\}^{1/2}
 \end{aligned}$$

d) Normalized Euclidean distance

$$\begin{aligned}
 NEU(\tilde{L}_1, \tilde{L}_2) &= \left\{ \frac{1}{3mn} \sum_{j=1}^m \sum_{i=1}^n \left\{ | \tilde{\phi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\phi}_{\tilde{L}_2}^j(x_{i_\theta}) |^2 + | \tilde{\Psi}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\Psi}_{\tilde{L}_2}^j(x_{i_\theta}) |^2 \right. \right. \\
 & \quad \left. \left. + | \tilde{\eta}_{\tilde{L}_1}^j(x_{i_\theta}) - \tilde{\eta}_{\tilde{L}_2}^j(x_{i_\theta}) |^2 \right\} \right\}^{1/2}
 \end{aligned}$$

Where

$$\begin{aligned}
 \tilde{\phi}_{\tilde{L}_1}^j(x_{i_\theta}) &= \frac{\tilde{\phi}_{\tilde{L}_1}^{j_1}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_2}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_3}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_4}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_5}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_6}(x_{i_\theta}) + \\
 & \quad \tilde{\phi}_{\tilde{L}_1}^{j_7}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_8}(x_{i_\theta}) + \tilde{\phi}_{\tilde{L}_1}^{j_9}(x_{i_\theta})}{9}
 \end{aligned}$$

$$\tilde{\phi}_{L_2}^j(x_{i\theta}) = \frac{\tilde{\phi}_{L_2}^{j1}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j2}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j3}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j4}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j5}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j6}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j7}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j8}(x_{i\theta}) + \tilde{\phi}_{L_2}^{j9}(x_{i\theta})}{9}$$

$\forall i = 1, 2, \dots, n ; j = 1, 2, \dots, m$

Where

$$\tilde{\Psi}_{L_1}^j(x_{i\theta}) = \frac{\tilde{\Psi}_{L_1}^{j1}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j2}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j3}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j4}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j5}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j6}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j7}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j8}(x_{i\theta}) + \tilde{\Psi}_{L_1}^{j9}(x_{i\theta})}{9}$$

$$\tilde{\Psi}_{L_2}^j(x_{i\theta}) = \frac{\tilde{\Psi}_{L_2}^{j1}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j2}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j3}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j4}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j5}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j6}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j7}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j8}(x_{i\theta}) + \tilde{\Psi}_{L_2}^{j9}(x_{i\theta})}{9}$$

$\forall i = 1, 2, \dots, n ; j = 1, 2, \dots, m$

$v_{11} + \varsigma_{11} + \varepsilon_{11} + \sigma_{11} + \tau_{11} + \varrho_{11} + \vartheta_{11} + \varpi_{11} + \iota_{11}$

**4. Optimal path using new technique for the Neutrosophic Network**

**4.1 Definition: Neutrosophic  $\alpha$  – cut grade for indeterminacy membership function**

$$\rho(\tilde{L}_1) = \int_0^1 [\alpha(\varsigma_{22} - v_{22}) + v_{22} + \alpha(\varepsilon_{22} - \varsigma_{22}) + \varsigma_{22} + \alpha(\sigma_{22} - \varepsilon_{22}) + \varepsilon_{22} + \alpha(\tau_{22} - \sigma_{22}) + \sigma_{22} + \tau_{22} - \alpha(\tau_{22} - \varrho_{22}) + \vartheta_{22} - \alpha(\vartheta_{22} - \varrho_{22}) + \vartheta_{22} - \alpha(\vartheta_{22} - \varpi_{22})] \alpha \, d\alpha$$

$$= \left[ [v_{22} + \varsigma_{22} + \varepsilon_{22} + \sigma_{22} + \tau_{22} + 2\vartheta_{22} + \varpi_{22}] \left(\frac{\alpha^2}{2}\right) + \left(\frac{\alpha^3}{3}\right) [-v_{22} + 2\varrho_{22} - \vartheta_{22}] \right]_0^1$$

$$= \left\{ \frac{1}{2} [v_{22} + \varsigma_{22} + \varepsilon_{22} + \sigma_{22} + \tau_{22} + 2\vartheta_{22} + \varpi_{22}] + \frac{1}{3} [-v_{22} + 2\varrho_{22} - \vartheta_{22}] \right\}$$

$$= \frac{3v_{22} + 3\varsigma_{22} + 3\varepsilon_{22} + 3\sigma_{22} + 3\tau_{22} + 6\vartheta_{22} + 3\varpi_{22} - 2v_{22} + 4\varrho_{22} - 2\vartheta_{22}}{6}$$

$$= \frac{v_{22} + 3\varsigma_{22} + 3\varepsilon_{22} + 3\sigma_{22} + 3\tau_{22} + 4\varrho_{22} + 4\vartheta_{22} + 3\varpi_{22}}{6}$$

$$\rho(\tilde{L}_1) = \frac{1}{6} [v_{22} + 3\varsigma_{22} + 3\varepsilon_{22} + 3\sigma_{22} + 3\tau_{22} + 4\varrho_{22} + 4\vartheta_{22} + 3\varpi_{22}]$$

**4.2 Definition: Neutrosophic  $\alpha$  – cut grade for truth membership function**

$$\rho(\tilde{L}_1) = \int_0^1 [v_{11} + \alpha(\varsigma_{11} - v_{11}) + \varsigma_{11} + \alpha(\varepsilon_{11} - \varsigma_{11}) + \varepsilon_{11} + \alpha(\sigma_{11} - \varepsilon_{11}) + \sigma_{11} + \tau_{11} - \alpha(\tau_{11} - \varrho_{11}) + \vartheta_{11} - \alpha(\vartheta_{11} - \varrho_{11}) + \vartheta_{11} - \alpha(\vartheta_{11} - \varpi_{11}) + \varpi_{11} - \alpha(\varpi_{11} - \iota_{11})] \alpha \, d\alpha$$

$$= \left[ [v_{11} + \varsigma_{11} + \varepsilon_{11} + \sigma_{11} + \tau_{11} + \varrho_{11} + \vartheta_{11} + \varpi_{11}] \left(\frac{\alpha^2}{2}\right) + \left(\frac{\alpha^3}{3}\right) [\iota_{11}] \right]_0^1$$

$$= \left\{ \frac{1}{2} [v_{11} + \varsigma_{11} + \varepsilon_{11} + \sigma_{11} + \tau_{11} + \varrho_{11} + \vartheta_{11} + \varpi_{11}] + \frac{1}{3} [\iota_{11}] \right\}$$

$$= \frac{3v_{11} + 3\varsigma_{11} + 3\varepsilon_{11} + 3\sigma_{11} + 3\tau_{11} + 3\varrho_{11} + 3\vartheta_{11} + 3\varpi_{11} + 2\iota_{11}}{6}$$

$$\rho(\tilde{L}_1) = \frac{3v_{11} + 3\varsigma_{11} + 3\varepsilon_{11} + 3\sigma_{11} + 3\tau_{11} + 3\varrho_{11} + 3\vartheta_{11} + 3\varpi_{11} + 2\iota_{11}}{6}$$

**4.3 Definition: Neutrosophic  $\alpha$  – cut grade for falsity membership function**

$$\rho(\tilde{L}_1) = \left[ [v_{33} + \varsigma_{33} + \varepsilon_{33} + \sigma_{33} + \tau_{33} + \varrho_{11} + \vartheta_{33} + \varpi_{33}] \left(\frac{\alpha^2}{2}\right) + \left(\frac{\alpha^3}{3}\right) [-v_{33} + \iota_{33}] \right]_0^1$$

$$= \frac{1}{2} [v_{33} + \varsigma_{33} + \varepsilon_{33} + \sigma_{33} + \tau_{33} + \varrho_{11} + \vartheta_{33} + \varpi_{33}] + \frac{1}{3} [-v_{33} + \iota_{33}]$$

$$= \frac{3v_{33} + 3\zeta_{33} + 3\varepsilon_{33} + 3\sigma_{33} + 3\tau_{33} + 3\rho_{11} + 3\vartheta_{33} + 3\omega_{33} - 2v_{33} + 2l_{33}}{6}$$

$$\rho(\tilde{L}_1) = \frac{v_{33} + 3\zeta_{33} + 3\varepsilon_{33} + 3\sigma_{33} + 3\tau_{33} + 3\rho_{11} + 3\vartheta_{33} + 3\omega_{33} + 2l_{33}}{6}$$

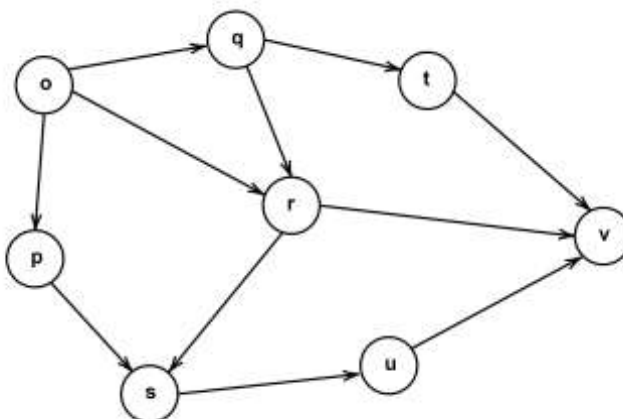


Figure:1 Nonagonal Neutrosophic Network

Table 1: Edge Weight - Nonagonal Neutrosophic Network

Edge	Edge weight
<i>o</i> – <i>q</i>	(0.45,0.46,0.51,0.52,0.53,0.62,0.65,0.71,0.75)(0.47,0.49,0.52,0.56,0.54,0.64,0.67,0.78,0.79)(0.48,0.49,0.53,0.57,0.58,0.63,0.68,0.79,0.80)
<i>o</i> – <i>p</i>	(0.46,0.48,0.51,0.52,0.53,0.55,0.62,0.64,0.72)(0.52,0.56,0.57,0.62,0.63,0.72,0.78,0.80,0.82)(0.61,0.63,0.67,0.72,0.74,0.76,0.82,0.85,0.87)
<i>o</i> – <i>r</i>	(0.52,0.54,0.56,0.58,0.59,0.60,0.62,0.64,0.68)(0.54,0.56,0.58,0.59,0.62,0.64,0.67,0.69,0.72)(0.78,0.79,0.82,0.84,0.85,0.87,0.88,0.89,0.90)
<i>p</i> – <i>s</i>	(0.32,0.34,0.36,0.45,0.47,0.49,0.51,0.53,0.55)(0.34,0.35,0.37,0.39,0.40,0.42,0.43,0.45,0.48)(0.42,0.44,0.46,0.47,0.52,0.54,0.56,0.57,0.59)
<i>q</i> – <i>t</i>	(0.42,0.45,0.48,0.52,0.54,0.56,0.62,0.64,0.66)(0.52,0.53,0.55,0.57,0.58,0.59,0.60,0.62,0.70)(0.32,0.34,0.36,0.42,0.44,0.46,0.52,0.55,0.56)
<i>q</i> – <i>r</i>	(0.45,0.47,0.52,0.56,0.59,0.63,0.67,0.70,0.73)(0.42,0.45,0.46,0.52,0.53,0.55,0.57,0.59,0.62)(0.42,0.45,0.47,0.52,0.54,0.56,0.58,0.60,0.64)
<i>s</i> – <i>u</i>	(0.52,0.54,0.56,0.62,0.64,0.66,0.68,0.70,0.72)(0.62,0.63,0.65,0.68,0.72,0.74,0.76,0.78,0.82)(0.53,0.55,0.62,0.64,0.66,0.68,0.72,0.82,0.84)
<i>r</i> – <i>v</i>	(0.53,0.55,0.58,0.60,0.64,0.66,0.68,0.72,0.73)(0.52,0.54,0.56,0.57,0.62,0.64,0.66,0.68,0.70)(0.32,0.34,0.36,0.38,0.40,0.42,0.43,0.45,0.52)
<i>r</i> – <i>s</i>	(0.5,0.52,0.55,0.58,0.59,0.62,0.65,0.66,0.72)(0.42,0.44,0.46,0.48,0.52,0.53,0.55,0.62,0.65)(0.32,0.34,0.42,0.44,0.52,0.55,0.57,0.59,0.62)
<i>u</i> – <i>v</i>	(0.42,0.44,0.46,0.48,0.52,0.53,0.55,0.56,0.62)(0.42,0.44,0.46,0.52,0.53,0.55,0.56,0.62,0.72)(0.72,0.73,0.75,0.80,0.82,0.84,0.86,0.92,0.94)
<i>t</i> – <i>v</i>	(0.4,0.5,0.6,0.72,0.74,0.76,0.80,0.82,0.84)(0.54,0.56,0.58,0.62,0.63,0.65,0.67,0.68,0.72)(0.32,0.34,0.35,0.42,0.46,0.47,0.52,0.53,0.55)

Table 2: Distance in terms of Nonagonal

Feasible routes	Distance in terms of Nonagonal
<i>o</i> – <i>q</i> – <i>t</i>	(1.27,1.41,1.59,1.76,1.81,1.942.07,2.172.25)
– <i>v</i>	(1.53.1.58,1.65,1.75,1.75,1.88,1.942.08,2.21)
	(1.12,1.17,1.24,1.41,1.48,1.56,1.72,1.87,1.91)

$o - r - v$	(1.05,1.09,1.14,1.18,1.23,1.26,1.3,1.36,1.41) (1.06,1.1,1.14,1.16,1.24,1.28,1.33,1.37,1.42,11.1) (1.1,1.13,1.18,1.22,1.2,1.29,1.31,1.34,1.42)
$o - p - s$ $- u - v$	(1.72,1.81,1.89,2.07,2.16,2.23,2.36,2.43,2.61) (1.9,1.98,2.0,2.21,2.28,2.43,2.53,2.65,2.84) (2.28,2.35,2.5,2.63,2.74,2.82,2.96,3.16,3.24)
$o - r - s$ $- u - v$	(1.96,2.04,2.13,2.26,2.34,2.41,2.5,2.56,2.74) (2,2.07,2.15,2.27,2.39,2.46,2.54,2.71,2.91) (2.3,2.41,2.61,2.72,2.85,2.94,3.03,3.22,3.3)
$o - q - r$ $- v$	(1.5,1.55,1.65,1.7,1.76,1.88,1.95,2.07,2.16) (1.53,1.59,1.66,1.72,1.78,1.92,2.2,1.15,2.21) (1.58,1.62,1.71,1.79,1.83,1.92,1.99,2.13,2.22)
$o - q - r$ $- s - u$ $- v$	(2.41,2.5,2.64,2.78,2.87,3.03,3.1,3.27,3.49) (2.47,2.56,2.67,2.83,2.93,3.1,3.21,3.49,3.7) (2.83,2.9,3.14,3.29,3.43,3.57,3.71,4.01,4.1)

**4.4 Definition: Euclidean grade technique for Nonagonal Neutrosophic number**

Let  $D_i = \{(d_{11}, d_{22}, d_{33}, d_{44}, d_{55}, d_{66}, d_{77}, d_{88}, d_{99}), (d'_{11}, d'_{22}, d'_{33}, d'_{44}, d'_{55}, d'_{66}, d'_{77}, d'_{88}, d'_{99}), (d''_{11}, d''_{22}, d''_{33}, d''_{44}, d''_{55}, d''_{66}, d''_{77}, d''_{88}, d''_{99})\}$  be the longest route

Consider the  $j^{th}$  Neutrosophic path distance is

$$L_i = \{(l_{11}, l_{22}, l_{33}, l_{44}, l_{55}, l_{66}, l_{77}, l_{88}, l_{99}), (l'_{11}, l'_{22}, l'_{33}, l'_{44}, l'_{55}, l'_{66}, l'_{77}, l'_{88}, l'_{99}), (l''_{11}, l''_{22}, l''_{33}, l''_{44}, l''_{55}, l''_{66}, l''_{77}, l''_{88}, l''_{99})\}$$

a)The Euclidean grade formula for Nonagonal Neutrosophic number for truth membership function

$$\varepsilon(L_i) = \left\{ \frac{(d_{11}-l_{11})^2 + (d_{22}-l_{22})^2 + (d_{33}-l_{33})^2 + (d_{44}-l_{44})^2 + (d_{55}-l_{55})^2 + (d_{66}-l_{66})^2 + (d_{77}-l_{77})^2 + (d_{88}-l_{88})^2 + (d_{99}-l_{99})^2}{27} \right\}^{\frac{1}{2}}$$

b)The Euclidean grade formula for Nonagonal Neutrosophic number for indeterminacy membership function

$$\varepsilon(L_i) = \left\{ \frac{(d'_{11}-l'_{11})^2 + (d'_{22}-l'_{22})^2 + (d'_{33}-l'_{33})^2 + (d'_{44}-l'_{44})^2 + (d'_{55}-l'_{55})^2 + (d'_{66}-l'_{66})^2 + (d'_{77}-l'_{77})^2 + (d'_{88}-l'_{88})^2 + (d'_{99}-l'_{99})^2}{27} \right\}^{\frac{1}{2}}$$

c)The Euclidean grade formula for Nonagonal Neutrosophic number for falsity membership function.

$$\varepsilon(L_i) = \left\{ \frac{(d''_{11}-l''_{11})^2 + (d''_{22}-l''_{22})^2 + (d''_{33}-l''_{33})^2 + (d''_{44}-l''_{44})^2 + (d''_{55}-l''_{55})^2 + (d''_{66}-l''_{66})^2 + (d''_{77}-l''_{77})^2 + (d''_{88}-l''_{88})^2 + (d''_{99}-l''_{99})^2}{27} \right\}^{\frac{1}{2}}$$

Table 3: Neutrosophic  $\alpha - cut$  grade and Neutrosophic Euclidean grade

Feasible Route	Neutrosophic $\alpha - cut$ grade	Neutrosophic Euclidean grade
$o - q - t - v$	(8.135,7.2066,6.048)	(3.099,3.449,5.605)
$o - r - v$	(5.51,4.92,5.02)	(3.54,3.57,3.60)
$o - p - s - u$ $- v$	(9.635,9.208,11.04)	(6.26,6.76,7.97)
$o - r - s - u$ $- v$	(10.47,9.46,11.38)	(6.75,6.95,8.21)
$o - q - r - v$	(8.11,7.32,7.50)	(5.24,5.36, .41)
$o - q - r - s$ $- u - v$	(13.07,11.86,13.86)	(8.45,8.73,10.03)

From the above table it is identified that the route  $o - q - r - s - u - v$  is having highest  $\alpha - cut$  grade and Euclidean grade in truth, indeterminacy and falsity function. Therefore  $o - q - r - s - u - v$  is the optimal path.

**5. Technique to find Neutrosophic optimal path using dynamic programming recursion**

- 1) The Neutrosophic network is taken where the route distance is taken in terms of Nonagonal Neutrosophic number.
- 2) The route distances are De-Neutrosophicted by the score function of the Nonagonal Neutrosophic number and also by the removal area method.
- 3) The dynamic programming recursion formula is given by

$$R^*(j) = \max[D_{ij} + R^*(j)] | i, j \in E$$

For the end node n  $R^*(n) = 0$

**5.1 Optimal path using Dynamic programming recursion through score function**

The deneutrosophication done by the score function of the Nonagonal Neutrosophic number

Table 4: De-Neutrosophic value using Score function

Edge	De-Neutrosophic value
<i>o</i> – <i>p</i>	0.451481
<i>o</i> – <i>p</i>	0.382963
<i>o</i> – <i>r</i>	0.374074
<i>p</i> – <i>s</i>	0.511852
<i>q</i> – <i>t</i>	0.505926
<i>q</i> – <i>r</i>	0.512222
<i>s</i> – <i>u</i>	0.414074
<i>r</i> – <i>v</i>	0.54
<i>r</i> – <i>s</i>	0.531481
<i>u</i> – <i>v</i>	0.384444
<i>t</i> – <i>v</i>	0.53963

Let  $\tau_1^*(v) = 0$

$$\begin{aligned} \tau_1^*(u) &= \rho^*(u, v) + \tau_1^*(v) = 0.384444 + 0 = 0.384444 \\ \tau_1^*(t) &= \rho^*(t, v) + \tau_1^*(v) = 0.53963 + 0 = 0.53963 \\ \tau_1^*(s) &= \rho^*(s, u) + \tau_1^*(u) = 0.414074 + 0.384444 = 0.798518 \\ \tau_1^*(r) &= \max\{\rho^*(r, v) + \tau_1^*(v), \rho^*(r, s) + \tau_1^*(s)\} \\ &= \max\{0.54 + 0, 0.531481 + 0.79851\} \\ &= \max\{0.54, 1.329991\} = 1.329991 \\ \tau_1^*(q) &= \max\{\rho^*(q, t) + \tau_1^*(t), \rho^*(q, r) + \tau_1^*(r)\} \\ &= \max\{0.5050926 + 0.53963, 0.512222 + 1.329991\} \\ &= \max\{1.045556, 1.842213\} = 1.842213 \\ \tau_1^*(p) &= \max\{\rho^*(p, s) + \tau_1^*(s), \rho^*(p, r) + \tau_1^*(r) + \rho^*(p, q) + \tau_1^*(q)\} \\ &= \max\{0.41481 + 1.842213, 0.374074 + 1.329991, 0.382963 + 1.31037\} \\ &= \max\{2.293694, 1.704065, 1.693333\} = 2.293694 \\ \tau_1^*(o) &= \rho^*(o, q) + \tau_1^*(q) = \rho^*(o, q) + \rho^*(q, r) + \tau_1^*(r) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \tau_1^*(s) = \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \tau_1^*(u) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) + \tau_1^*(v) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) \end{aligned}$$

Therefore *o – q – r – s – u – v* is the optimal path

**5.2 Optimal path using dynamic programming recursion through removal area method**

The De-Neutrosophic value of Nonagonal Neutrosophic number using removal area method is given below

$$\begin{aligned} \tilde{A} = & [(a_1 + 2b_1 + 2c_1 + d_1 + f_1 + 2g_1 + 2h_1 + i_1 + d_2 + 2e_2 + f_2 + d_3 + 2e_3 + f_3)\delta \\ & + (d_1 + 2e_1 + f_1 + a_2 + 2b_2 + 2c_2 + d_2 + f_2 + 2g_2 + 2h_2 + i_2 + a_3 + 2b_3 \\ & + 2c_3 + d_3 + f_3 + 2g_3 + 2h_3 + i_3)(1 - \delta)]/28 \end{aligned}$$

Table:5 De-Neutrosophic value using Removal Area method

Edge	De-Neutrosophic value		
	$\delta$	$\delta$	$\delta$
<i>o</i>	0.704821	1.104643	1.374429
<i>o</i>	0.773929	1.193286	1.511643
<i>o</i>	0.805893	1.217643	1.542286
<i>p</i>	0.530179	0.815214	1.016571
<i>q</i>	0.627857	0.984143	1.210571
<i>q</i>	0.646786	1.007	1.239143
<i>s</i>	0.799643	1.236429	1.542071
<i>r</i>	0.661964	1.039143	1.2535
<i>r</i>	0.628214	0.986786	1.204
<i>u</i>	0.718929	1.093286	1.398786
<i>t</i>	0.707679	1.113071	1.343143

The optimal path using dynamic programming recursion is given below

Case:1 Let  $\delta = 0.5$

Let  $\tau_1^*(v) = 0$

$$\begin{aligned} \tau_1^*(u) &= \rho^*(u, v) + \tau_1^*(v) = 0.718929 \\ \tau_1^*(t) &= \rho^*(t, v) + \tau_1^*(v) = 0.707679 \\ \tau_1^*(s) &= \rho^*(s, u) + \tau_1^*(u) = 0.799643 + 0.718929 = 1.518572 \\ \tau_1^*(r) &= \max\{\rho^*(r, v) + \tau_1^*(v), \rho^*(r, s) + \tau_1^*(s)\} \\ &= \max\{0.661964 + 0, 0.628214 + 1.518572\} = 2.146786 \\ \tau_1^*(q) &= \max\{\rho^*(q, t) + \tau_1^*(t), \rho^*(q, r) + \tau_1^*(r)\} \\ &= \max\{0.62785 + 0.707679, 0.646786 + 2.146786\} = 2.793572 \\ \tau_1^*(p) &= \rho^*(p, s) + \tau_1^*(s) = 0.530179 + 1.518572 = 2.048751 \\ \tau_1^*(o) &= \max\{\rho^*(o, q) + \tau_1^*(q), \rho^*(o, r) + \tau_1^*(r), \rho^*(o, p) + \tau_1^*(p)\} \\ &= \max\{0.704821 + 2.793572, 0.805893 + 2.146786, 0.773929 + 2.048751\} \\ &= \max\{3.498393, 2.952679, 2.82268\} = 3.498393 \\ \tau_1^*(o) &= \rho^*(o, q) + \tau_1^*(q) = \rho^*(o, q) + \rho^*(q, r) + \tau_1^*(r) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \tau_1^*(s) = \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \tau_1^*(u) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) + \tau_1^*(v) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) \end{aligned}$$

Therefore  $o - q - r - s - u - v$  is the optimal path

Case: 2 Let  $\delta = 0.6$

Let  $\tau_1^*(v) = 0$

$$\begin{aligned} \tau_1^*(u) &= \rho^*(u, v) + \tau_1^*(v) = 1.093286 \\ \tau_1^*(t) &= \rho^*(t, v) + \tau_1^*(v) = 1.113071 \\ \tau_1^*(s) &= \rho^*(s, u) + \tau_1^*(u) = 2.329715 \\ \tau_1^*(r) &= \max\{\rho^*(r, v) + \tau_1^*(v), \rho^*(r, s) + \tau_1^*(s)\} \\ &= \max\{1.039143, 3.316501\} = 3.316501 \\ \tau_1^*(q) &= \max\{\rho^*(q, t) + \tau_1^*(t), \rho^*(q, r) + \tau_1^*(r)\} \\ &= \max\{2.097214, 4.323501\} = 4.323501 \\ \tau_1^*(p) &= \rho^*(p, s) + \tau_1^*(s) = 3.144929 \\ \tau_1^*(o) &= \max\{\rho^*(o, q) + \tau_1^*(q), \rho^*(o, r) + \tau_1^*(r), \rho^*(o, p) + \tau_1^*(p)\} \\ &= \max\{5.428144, 4.534144, 4.338215\} = 5.428144 \\ \tau_1^*(o) &= \rho^*(o, q) + \tau_1^*(q) = \rho^*(o, q) + \rho^*(q, r) + \tau_1^*(r) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \tau_1^*(s) = \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \tau_1^*(u) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) + \tau_1^*(v) \\ &= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) \end{aligned}$$

Therefore  $o - q - r - s - u - v$  is the optimal path

Case: 3 Let  $\delta = 0.4$

Let  $\tau_1^*(v) = 0$ ,  $\tau_1^*(u) = \rho^*(u, v) + \tau_1^*(v) = 1.398786$

$\tau_1^*(t) = \rho^*(t, v) + \tau_1^*(v) = 1.343143$

$\tau_1^*(s) = \rho^*(s, u) + \tau_1^*(u) = 2.940857$

$\tau_1^*(r) = \max\{\rho^*(r, v) + \tau_1^*(v), \rho^*(r, s) + \tau_1^*(s)\}$   
 $= \max\{1.25, 4.144857\} = 4.144857$

$\tau_1^*(q) = \max\{\rho^*(q, t) + \tau_1^*(t), \rho^*(q, r) + \tau_1^*(r)\}$   
 $= \max\{2.553714, 5.384\} = 5.384$

$\tau_1^*(p) = \rho^*(p, s) + \tau_1^*(s) = 3.346286$

$\tau_1^*(o) = \max\{\rho^*(o, q) + \tau_1^*(q), \rho^*(o, r) + \tau_1^*(r), \rho^*(o, p) + \tau_1^*(p)\}$   
 $= \max\{6.758429, 5.687143, 4.857929\} = 6.758429$

$\tau_1^*(o) = \rho^*(o, q) + \tau_1^*(q) = \rho^*(o, q) + \rho^*(q, r) + \tau_1^*(r)$   
 $= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \tau_1^*(s) = \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \tau_1^*(u)$   
 $= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v) + \tau_1^*(v)$   
 $= \rho^*(o, q) + \rho^*(q, r) + \rho^*(r, s) + \rho^*(s, u) + \rho^*(u, v)$

Therefore  $o - q - r - s - u - v$  is the optimal path

## 6. Conclusion

The optimal path of the Nonagonal Neutrosophic network is obtained using Neutrosophic  $\alpha$  – cut grade technique and Neutrosophic Euclidean grade technique. In both the methods  $o - q - r - s - u - v$  is the optimal path. Also, the optimal path is obtained using dynamic programming recursion through score function and removal area method. In removal area method three cases are analyzed. In all the three cases,  $o - q - r - s - u - v$  is the optimal path of the network. It is concluded that the optimal path of the given network is same for using all the above said methods. In the network, if there is indeterminacy then the Neutrosophic number will be very useful. In future, this may be extended for m-valued Nonagonal Neutrosophic number.

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