The Application of AH-Isometry in the Study of Neutrosophic Conic Sections

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Abstract:

One of the most important areas of analytic geometry involves the concept of conic sections. The objective of this paper is to introduce the concept of neutrosophic conic sections, so that each neutrosophic conic section represents two classic conic section in the general case. On the other hand, all special cases resulting from the expansion by moving to the neutrosophic systems will be discussed and handled.

Keywords: Neutrosophic circles; Neutrosophic ellipses; Neutrosophic parabolas; Neutrosophic hyperbolas

Introduction

Neutrosophic logic is a generalization of intuitionistic fuzzy logic by adding an indeterminacy I with property I=I^2 . Neutrosophic set concept has wide applications in different areas of science [1-3].

On the other hand, neutrosophic sets played an interesting role in pure mathematics such as topology and analysis, linear algebra, and algebraic structures [6-8,10-12].

Neutrosophic spaces theory began with Agboola et.al [4], where they studied neutrosophic vector spaces and their properties. Recently, many studies have been carried out on these spaces, where AH-subspaces and homomorphisms were presented. In [5,9], Hatip et. al studied neutrosophic modules (a generalized form of neutrosophic spaces) with their substructures such as homomorphisms and AH-submodules.

In [13-14] the concept of the neutrosophic plane with n neutrosophic dimensions is obtained. In addition, Euclidean geometric concepts are extended neutrosophicaly such as neutrosophic distance, neutrosophic midpoint, neutrosophic vectors, neutrosophic circles, and lines.

This work is considered the first study in neutrosophic geometry by defining the neutrosophic conic section concepts based on neutrosophic numbers and spaces.

Preliminaries

Definition 2.1 : Classical neutrosophic number has the form $a+bI$ where a,b are real or complex numbers and I is the literal indeterminacy such that $0 \cdot I=0$ and $I^2=I$ which results that
$I^n=I$for all positive integers n.
Definition 2.2: Let K be a field, the neutrosophic file generated by $\langle K \cup I\rangle$ which is denoted by $K(I)=\langle K \cup I\rangle$.
Definition 2.3:
(a) **(Two dimensional AH-isometry)** Let $M=R(I)^2=R(I)\times R(I), V=R^2\times R^2$
Be the neutrosophic plane with two N-dimensions and the Cartesian product of the classical Euclidean space R^2 with itself, we define the AH-isometry map as follows:
$f: M \rightarrow V; f(a+bI, c+dI)=((a, a+b),(c, c+d))$.

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(b) (**One dimensional AH-isometry**) We can define the one-dimensional isometry between R(I) and the space R×R as follows:
$g: R(I) \to R \times R; g(a+bI) = (a, a+b)$.
Remark: The one dimensional isometry is an isometry, i.e. an algebraic isomorphism between R(I) and $R \times R$. Also, it preserve distances on R(I).
Theorem 2.4: (Fundamental Theorem In neutrosophic Euclidean Geometry)
Let $f: M \to V; f(a+bI, c+dI) = ((a, a+b), (c, c+d))$ be the AH-isometry defined above, we have:
(a) f preserves addition operation between vectors.
(b) f preserves distances between points.
(c) f is a bijection one-to-one between M and V.
(d) Multiplying a neutrosophic vector by a neutrosophic real number is preserved up to isometry, i.e. The direct image of a neutrosophic vector multiplied by a neutrosophic real number is exactly equal to its AH-isometric image multiplied by the one dimensional isometric image of the corresponding neutrosophic real number.

Main Discussion

Theorem3.1:

Let $R^2(I)$ be the neutrosophic plane with two dimensions, let $a = a_0 + a_1 I$, $b = b_0 + b_1 I$, $x = x_0 + x_1 I$, $y = y_0 + y_1 I$, $p = p_0 + p_1 I$, then if p is invertible, the neutrosophic parabola

$(x-a)^2 = 4p(y-b)$is equivalent to the direct product of two classi classical parabola.

Proof. Consider the equation $(x-a)^2 = 4p(y-b)$ by computing its direct image with 2-dimensional AH-isometry, we get:

$T(x-a)^2 = 4T(p)T(y-b)$ Thus.

$$\left((x_0-a_0)^2, \left((x_0+x_1)-(a_0+a_1)\right)^2\right) = 4(p_0, p_0+p_1)\left(y_0-b_0, (y_0+y_1)-(b_0+b_1)\right).$$

So that we have:

$$P_1: (x_0-a_0)^2 = 4p_0(y_0-b_0), P_2: \left((x_0+x_1)-(a_0+a_1)\right)^2 = 4(p_0+p_1)[(y_0+y_1)-(b_0+b_1)]$$

Remark 3.2:

If p is invertible, we can write the equation of neutrosophic parabola as follows:

$(x_0-a)^2 = 4p(y_0-b)$.

Now, we should discuss the cases of non-invertible of p.

The p is not invertible, then we have two cases:

1- $p_0 = 0$, $p_0 + p_1 \neq 0$, this means that the neutrosophic parabola will be equivalent to direct product of classical parabola $\left((x_0+x_1)-(a_0+a_1)\right)^2 = 4(p_0+p_1)[(y_0+y_1)-(b_0+b_1)]$ with classical vertical line $x_0 = a_0$.
2- $p_0 \neq 0$, $p_0 + p_1 = 0$, this implies that the neutrosophic parabola will be equivalent of the line $x_0 + x_1 = a_0 + a_1$with the classical parabola $(x_0-a_0)^2 = 4p_0(y_0-b_0)$.
3- If $p_0 = p_0 + p_1 = 0$, then the neutrosophic parabola will be equivalent to the direct product of two lines: $x_0 = a_0$, $x_0 + x_1 = a_0 + a_1$.

Remark 3.3:

(a). The classical Ellipse has the following Cartesian equation:

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

Where $x_0, y_0, a, b \in R$, and $a, b \neq 0$.

It can be written as follows $b^2(x-x_0)^2 + a^2(y-y_0)^2 = a^2b^2$, where $a, b \neq 0$.

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(b). If $a = 0$, then $(x - x_0)^2 = 0$, hence $x - x_0 = 0 \Longrightarrow x = x_0$ which represents a line (vertical line).

(c). If $b = 0$, we get a horizontal line $y - y_0 \Longrightarrow y = y_0$.

Theorem 3.4: Let $\left(R(I)\right)^2$be the neutrosophic plane with two dimensions, let $X = x_1 + x_2 I, Y = y_1 + y_2 I$, $c = c_1 + c_2 I$, $d = d_1 + d_2 I$, $a = a_1 + a_2 I$, $b = b_1 + b_2 I$, then if a, b are invertible, the neutrosophic Ellipse$b^2(x - x_0)^2 + a^2(y - y_0)^2 = a^2 b^2$is equivalent to the direct product of two classical Ellipses.

Proof. Consider the equation $b^2(X - c)^2 + a^2(Y - d)^2 = a^2 b^2$by computing its direct image with the 2-dimensional AH-isometry, we get:$T(b^2)\,T(X - c)^2 + T(a^2)\,T(Y - d)^2 = T(a^2)T(b^2)$, thus

$$\begin{aligned}&\left({b_1}^2, (b_1 + b_2)^2\right).\left((x_1 - c_1)^2, (x_1 + x_2 - c_1 - c_2)^2\right)\\ &\qquad + \left({a_1}^2, (a_1 + a_2)^2\right).\left((y_1 - d_1)^2, (y_1 + y_2 - d_1 - d_2)^2\right)\\ &\qquad = \left({a_1}^2 {b_1}^2, (a_1 + a_2)^2 (b_1 + b_2)^2\right)\end{aligned}$$

so that, ${b_1}^2(x_1 - c_1)^2 + {a_1}^2(y_1 - d_1)^2 = {a_1}^2 {b_1}^2$ and $(b_1 + b_2)^2(x_1 + x_2 - c_1 - c_2)^2 + (a_1 + a_2)^2(y_1 + y_2 - d_1 - d_2)^2 = (a_1 + a_2)^2(b_1 + b_2)^2$, thus the proof is complete.

Remark 3.5:

If a, b are invertible, we can write the equation of neutrosophic Ellipse as follows:

$$\frac{(X - c)^2}{a^2} + \frac{(Y - d)^2}{b^2} = 1$$

Now, we should discuss the cases of non-invertibity of a, b.

If a is not invertible, then we have three possible cases.

Case1. $a_1 = 0$and $a_1 + a_2 \neq 0$, this means that the neutrosophic Ellipse will be equivalent to the direct product of the classical Ellipse:

$(b_1 + b_2)^2(x_1 + x_2 - c_1 - c_2)^2 + (a_1 + a_2)^2(y_1 + y_2 - d_1 - d_2)^2 = (a_1 + a_2)^2(b_1 + b_2)^2$with the following classical vertical line$x_1 = c_1$.

Case2. $a_1 \neq 0$and $a_1 + a_2 = 0$, this means that the neutrosophic Ellipse will be equivalent to the direct product of the line $x_1 + x_2 = c_1 - c_2$ with the classical Ellipse ${b_1}^2(x_1 - c_1)^2 + {a_1}^2(y_1 - d_1)^2 = {a_1}^2 {b_1}^2$.

Case3. $a_1 = a_2 = 0$, then the neutrosophic Ellipse will be equivalent to the origin point $(0,0)$.

By a similar argument, we can find the form of a neutrosophic Ellipse if b is invertible.

Example 3.6:consider the following neutrosophic Ellipse:

$$\frac{(X - 1 - I)^2}{(2 + I)^2} + \frac{(Y - I)^2}{(3 - I)^2} = 1$$

It is equivalent to the following direct product $E_1 \times E_2$, where.

$E_1: 9(x_1 - 1)^2 + 4(y_1)^2 = (4)(9) = 36$

$E_2: 4(x_1 + x_2 - 2)^2 + 9(y_1 + y_2 - 2)^2 = (9)(4) = 36$

Remark 3.7:

(a). The classical Hyperbola has the following cartesian equation:

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1$$

Where $x_0, y_0, a, b \in R$, and $a, b \neq 0$.

It can be written as follows $b^2(x - x_0)^2 - a^2(y - y_0)^2 = a^2 b^2$, where $a, b \neq 0$.

(b). If $a = 0$, it becomes a vertical line$x = x_0$.

(c). If $b = 0$, , it becomes a horizontal line $y = y_0$.

Theorem 3.8: Let $(R(I))^2$ be the neutrosophic plane with two dimensions, let $X = x_1 + x_2 I, Y = y_1 + y_2 I$, $c = c_1 + c_2 I$, $d = d_1 + d_2 I$, $a = a_1 + a_2 I$, $b = b_1 + b_2 I$, then if a, b are invertible, the neutrosophic Hyperbola $b^2(X - x_0)^2 - a^2(Y - y_0)^2 = a^2 b^2$ is equivalent to the direct product of the following two classical Hyperbolas.

$\mathrm{H}_1: {b_1}^2(x_1 - c_1)^2 - {a_1}^2(y_1 - d_1)^2 = {a_1}^2{b_1}^2$

$\mathrm{H}_2: (b_1 + b_2)^2(x_1 + x_2 - c_1 - c_2)^2 - (a_1 + a_2)^2(y_1 + y_2 - d_1 - d_2)^2 = (a_1 + a_2)^2(b_1 + b_2)^2$

The proof is similar to that Theorem3.4.

Example 3.9: Let**.**

$$\frac{(X - I)^2}{(1 + I)^2} - \frac{(Y - 2I)^2}{(2 + 5I)^2} = 1$$

be a neutrosophic Hyperbola, it is equivalent to the direct product $H_1 \times H_2$, where.

$H_1: 9(x_1)^2 - (y_1)^2 = (4)(1) = 4$

$H_2: 49(x_1 + x_2 - 1)^2 - 4(y_1 + y_2 - 2)^2 = (4)(49) = 196$

Remark 3.10: If a or b is not invertible, we can discuss it by a similar way of the neutrosophic Ellipse.

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