



New Operators Using Neutrosophic Crisp Open Set

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Abstract

In this paper, we introduce new sets in neutrosophic crisp topology called neutrosophic crisp frontier, neutrosophic crisp border and neutrosophic crisp exterior with the help of neutrosophic crisp open sets in neutrosophic crisp topological space. Also, we discuss the basic and important properties of them and the relations between them. Finally, many examples are presented.

Keywords: Neutrosophic crisp open set; neutrosophic crisp closed set; neutrosophic crisp frontier; neutrosophic crisp border; Neutrosophic crisp exterior.

1. Introduction

Neutrosophy which has many applications in different fields of sciences such as topology introduced by F. Smarandache [1,2], as a new branch of Philosophy. A.S. Mashhour et al. [3] in 1983, as a generalization of topological space was introduced supra topological space. The most generalization of topological spaces, was neutrosophic topological spaces [4] defined in 2014. Also, neutrosophic crisp topological spaces [5].

Recently, P. Iswarya and K. Bageerathi,[6] studied neutrosophic frontier and semi-frontier in neutrosophic topological spaces. Separation axioms in neutrosophic crisp topological space and neutrosophic crisp point were first introduced by A. Alnafee et al. in [7].

Recently, the neutrosophic crisp set have applications in image processing [8],[9], the field of geographic information systems[10] and possible applications to database[11]. Also, neutrosophic sets [12] have applications in the medical field [13-16]. For more details about neutrosophic topology see [17-21].

In this paper, we use the neutrosophic crisp sets to introduce neutrosophic crisp frontier, neutrosophic crisp border and neutrosophic crisp exterior and discuss their properties in neutrosophic crisp topological space. Also, we study some basic properties of this new neutrosophic crisp concepts.

2. Preliminaries

In this part, we recall definitions and which are useful in this paper.

Definition 2.1. [5]

Let $X \neq \emptyset$ be a fixed set. A neutrosophic crisp set $(N_c, S) U$ is an object with the $U = \langle U_1, U_2, U_3 \rangle$ shape ; U_1, U_2 and U_3 are subsets of X .

Definition 24. [5]

A neutrosophic crisp topology (NCT) on a non-empty set χ is a family $\mathbb{T}$ of neutrosophic crisp subsets in χ may be satisfying the following axioms:

1. X_N and $\emptyset_N$ belong to $\mathbb{T}$.
2. $\mathbb{T}$ is closed under finite intersection.
3. $\mathbb{T}$ is closed under arbitrary union.

The pair $(\chi, \mathbb{T})$ is neutrosophic crisp topological space (NCTS) in $\mathbb{T}$. Moreover, the elements in $\mathbb{T}$ are said to be neutrosophic crisp open sets (NCOS). A neutrosophic crisp set F is closed (NCCS) if and only if its complement F^c is neutrosophic crisp open set.

Definition 2.5. [7]

Let χ be a non-empty set. And $x, y, z \in \chi$, then:

- a. $x_{N_1} \square \square \square \{x\}, \emptyset, \emptyset >$ is called a neutrosophic crisp point (NCP_{N_1}) in χ .
- b. $y_{N_2} \square \square \square \emptyset, \{y\}, \emptyset >$ is called a neutrosophic crisp point (NCP_{N_2}) in χ .
- c. $z_{N_3} \square \square \square \emptyset, \emptyset, \{z\} >$ is called a neutrosophic crisp point (NCP_{N_3}) in χ .

The set of all neutrosophic crisp points ($NCP_{N_1}, NCP_{N_2}, NCP_{N_3}$) is denoted by NCP_N

3. Neutrosophic crisp frontier in neutrosophic crisp topological space:

In this section, we introduce neutrosophic crisp frontier and discuss their properties in neutrosophic crisp topological spaces.

Definition 3.1.

Let (X, Ψ_N) be NCTS and H , be a neutrosophic crisp set, then:

The neutrosophic crisp frontier of a neutrosophic crisp subset A ($N_cFr(A)$)

$$N_cFr(H) = N_ccl(H) \cap N_ccl(H^c).$$

Example 3.2.

Let $\chi = \{a, b, c\}$, $\mathbb{T} = \{\emptyset_N, X_N, A\}$, $A = \{< \{a\}, \emptyset, \emptyset >\}$.

Then $(\chi, \mathbb{T})$ neutrosophic crisp topological space, let $H = \{< \{a\}, \{a, b\}, \{a, c\} >\}$

$$N_cFr(H) = N_ccl(H) \cap N_ccl(H^c) = X_N \cap \emptyset_N = \emptyset_N.$$

Example 3.3.

Let (X, Ψ_N) be NCTS, then:

$$N_cFr(X_N) = N_ccl(X_N) \cap N_ccl(X_N^c) = N_ccl(X_N) \cap N_ccl(\emptyset_N) = X_N \cap \emptyset_N = \emptyset_N.$$

$$N_cFr(\emptyset_N) = N_ccl(\emptyset_N) \cap N_ccl(\emptyset_N^c) = N_ccl(\emptyset_N) \cap N_ccl(X_N) = \emptyset_N \cap X_N = \emptyset_N.$$

Remark 3.4.

Let (X, Ψ_N) be NCTS and H be a neutrosophic crisp subset, then:

$N_cFr(H)$ is NCCS.

Proof:

Since $N_cFr(H) = N_ccl(H) \cap N_ccl(H^c)$, then $N_cFr(H)$ is NCCS.

Theorem 3.5.

Let (X, Ψ_N) be NCTS and H be a neutrosophic crisp subset, then:

$$N_c Fr(H) = N_c Fr(H^c).$$

Proof.

Let H be a neutrosophic crisp subset in NCTS (X, Ψ_N) . Then by Definition 3.1 $N_c Fr(H) = N_c cl(H) \cap N_c cl(H^c) = N_c cl(H^c) \cap N_c cl(H) = N_c cl(H^c) \cap (N_c cl(H^c)^c)$. But, by Definition 3.1 this is equal to $N_c Fr(H^c)$.

Hence $N_c Fr(H) = N_c Fr(H^c)$.

Theorem 3.6.

Let (X, Ψ_N) be NCTS and H be a neutrosophic crisp subset, then:

$$N_c Fr(H) = N_c cl(H) - N_c int(H).$$

Proof.

Let H be a neutrosophic crisp subset in NCTS. Since $(N_c cl(H))^c = N_c int(H^c)$, then $(N_c cl(H^c))^c = N_c int(H)$

We have that, $N_c Fr(H) = N_c cl(H) \cap (N_c cl(H^c))^c = N_c cl(H) \cap (N_c int(H))^c$. By using

$$N - M = N \cap M^c, N_c Fr(H) = N_c cl(H) - N_c int(H). \text{ Hence } N_c Fr(H) = N_c cl(H) - N_c int(H).$$

Theorem 3.7.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

A is NCCS set in X if and only if $N_c Fr(A) \subseteq A$.

Proof.

$\Rightarrow$: Let A be a NCCS set in X . Then by Definition 3.1, $N_c Fr(A) = N_c cl(A) \cap N_c cl(A^c) \subseteq N_c cl(A) = A$. Therefore $N_c Fr(A) \subseteq A$.

$\Leftarrow$: Let $N_c Fr(A) \subseteq A$. Then $N_c cl(A) - N_c int(A) \subseteq A$. Since $N_c int(A) \subseteq A$,

we conclude that $N_c cl(A) = A$ and hence A is NCCS.

Theorem 3.8.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

If A is a NCOS set in X , then $N_c Fr(A) \subseteq A^c$.

Proof.

Let A be a NCOS set in X . then A^c is NCCS set in X . By Theorem 3.6,

$N_c Fr(A^c) \subseteq A^c$ and by Theorem 3.5, we get $N_c Fr(A) \subseteq A^c$.

Theorem 3.9.

Let $A \subseteq B$ and B be any NCCS set in X . Then $N_c Fr(A) \subseteq B$.

Proof.

Since $A \subseteq B$, then $N_c cl(A) \subseteq N_c cl(B)$. By Definition 3.1, $N_c Fr(A) = N_c cl(A) \cap N_c cl(A^c) \subseteq N_c cl(B) \cap N_c cl(A^c) \subseteq N_c cl(B)$. Since B be any NCCS set, then $N_c cl(B) = B$. Hence $N_c Fr(A) \subseteq B$.

Theorem 3.10.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset in X , then $N_c cl(N_c Fr(A)) \subseteq N_c Fr(A)$.

Proof.

Let A be the neutrosophic crisp subset in X . Then by Definition 3.1, $N_c cl(N_c Fr(A)) = N_c cl(N_c cl(A) \cap (N_c cl(A^c))) \subseteq (N_c cl(N_c cl(A))) \cap (N_c cl(N_c cl(A^c)))$. Since $N_c cl(A)$ and $N_c cl(A^c)$ are NCCS, then $N_c cl(N_c Fr(A)) \subseteq N_c cl(A) \cap (N_c cl(A^c)) = N_c Fr(A)$.

Theorem 3.11.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c Fr(N_c int(A)) \subseteq N_c Fr(A).$$

Proof. Let A be the neutrosophic crisp subset in X . Then

$$\begin{aligned} N_c Fr(N_c int(A)) &= N_c cl(N_c int(A)) \cap (N_c cl(N_c int(A)))^c \text{ [by Definition 3.1]} \\ &= N_c cl(N_c int(A)) \cap (N_c cl(N_c cl(A^c)))^c [(N_c int(A))^c = N_c cl(A^c)] \\ &= N_c cl(N_c int(A)) \cap (N_c cl(A^c)) [N_c cl(A^c) \text{ is NCCS}] \\ &\subseteq N_c cl(A) \cap N_c cl(A^c) [N_c int(A) \subseteq A] \\ &= N_c Fr(A) \text{ [by Definition 3.1].} \end{aligned}$$

$$\text{Hence } N_c Fr(N_c \delta int(A)) \subseteq (N_c Fr(A)).$$

Theorem 3.12.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c Fr(N_c cl(A)) \subseteq N_c Fr(A).$$

Proof.

Let A be a neutrosophic crisp subset in X . Then

$$\begin{aligned} N_c Fr(N_c cl(A)) &= N_c cl(N_c cl(A)) \cap (N_c cl(N_c cl(A)))^c \text{ [by Definition 3.1]} \\ &= N_c cl(A) \cap (N_c cl(N_c int(A^c)))^c [(N_c cl(A))^c = N_c int(A^c)] \\ &\subseteq N_c cl(A) \cap N_c cl(A^c) [N_c int(A^c) \subseteq A^c] \\ &= N_c Fr(A) \text{ [by Definition 3.1]} \end{aligned}$$

$$\text{Hence } N_c Fr(N_c cl(A)) \subseteq N_c Fr(A).$$

Theorem 3.13.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c int(A) \subseteq A - N_c Fr(A)$$

Proof.

Let A be a neutrosophic crisp subset in X . Now by Definition 3.1,

$$\begin{aligned} A - N_c Fr(A) &= A \cap (N_c Fr(A))^c \\ &= A \cap [N_c cl(A) \cap N_c cl(A^c)]^c \\ &= A \cap [N_c int(A^c) \cup N_c int(A)] \\ &= [A \cap N_c int(A^c)] \cup [A \cap N_c int(A)] \end{aligned}$$

$$=[A \cap N_c \text{int}(A^c)] \cup N_c \text{int}(A) \dots (1)$$

$$\text{But } N_c \text{int}(A) \subseteq [A \cap N_c \text{int}(A^c)] \cup N_c \text{int}(A) \dots (2)$$

Hence $N_c \text{int}(A) \subseteq A - N_c \text{Fr}(A)$ [by (1) and (2)].

Theorem 3.14.

Let (X, Ψ_N) be NCTS and A, B be a neutrosophic crisp subsets, then:

$$N_c \text{Fr}(A \cup B) \subseteq N_c \text{Fr}(A) \cup N_c \text{Fr}(B).$$

Proof. Let A and B be neutrosophic crisp subsets in X . Then

$$N_c \text{Fr}(A \cup B) = N_c \text{cl}(A \cup B) \cap N_c \text{cl}(A \cup B)^c \text{ [by Definition 3.1]}$$

$$= N_c \text{cl}(A \cup B) \cap N_c \text{cl}(A^c \cap B^c)$$

$$\subseteq [(N_c \text{cl}(A) \cup N_c \text{cl}(B)) \cap ((N_c \text{cl}(A^c)) \cap (N_c \text{cl}(B^c)))]$$

$$= [(N_c \text{cl}(A) \cup (N_c \text{cl}(B))) \cap (N_c \text{cl}(A^c))] \cap [(N_c \text{cl}(A) \cup (N_c \text{cl}(B))) \cap (N_c \text{cl}(B^c))]$$

$$= [(N_c \text{cl}(A) \cap N_c \text{cl}(A^c))] \cup [(N_c \text{cl}(B) \cap (N_c \text{cl}(A^c)))] \cap [(N_c \text{cl}(A) \cap (N_c \text{cl}(B^c)))]$$

$$\cup [(N_c \text{cl}(B) \cap (N_c \text{cl}(B^c)))]$$

$$= [N_c \text{Fr}(A) \cup (N_c \text{cl}(B)) \cap (N_c \text{cl}(A^c))] \cap [(N_c \text{cl}(A) \cap (N_c \text{cl}(B^c)) \cup (N_c \text{Fr}(B))]$$

[by Definition 3.1]

$$= (N_c \text{Fr}(A) \cup N_c \text{Fr}(B)) \cap [(N_c \text{cl}(B) \cap (N_c \text{cl}(A^c))) \cup ((N_c \text{cl}(A) \cap N_c \text{cl}(B^c)))]$$

$$\subseteq N_c \text{Fr}(A) \cup N_c \text{Fr}(B).$$

Hence, $N_c \text{Fr}(A \cup B) \subseteq N_c \text{Fr}(A) \cup N_c \text{Fr}(B)$.

Theorem 3.15.

Let (X, Ψ_N) be NCTS and A, B be a neutrosophic crisp subsets, then:

$$N_c \text{Fr}(A \cap B) \subseteq (N_c \text{Fr}(A) \cap (N_c \text{cl}(B))) \cup (N_c \text{Fr}(B) \cap N_c \text{cl}(A)).$$

Proof.

Let A and B be neutrosophic crisp subsets in X . Then

$$N_c \text{Fr}(A \cap B) = N_c \text{cl}(A \cap B) \cap (N_c \text{cl}(A \cap B))^c \text{ [by Definition 3.1]}$$

$$= N_c \text{cl}(A \cap B) \cap (N_c \text{cl}(A^c \cup B^c))$$

$$\subseteq (N_c \text{cl}(A) \cap N_c \text{cl}(B)) \cap (N_c \text{cl}(A^c) \cup N_c \text{cl}(B^c))$$

$$= [(N_c \text{cl}(A) \cap N_c \text{cl}(B)) \cap N_c \text{cl}(A^c)] \cup [(N_c \text{cl}(A) \cap N_c \text{cl}(B)) \cap N_c \text{cl}(B^c)]$$

$$= (N_c \text{Fr}(A) \cap N_c \text{cl}(B)) \cup (N_c \text{Fr}(B) \cap N_c \text{cl}(A)) \text{ [by Definition 3.1].}$$

$$\text{Hence } N_c \text{Fr}(A \cap B) \subseteq ((N_c \text{Fr}(A) \cap (N_c \text{cl}(B))) \cup (N_c \text{Fr}(B) \cap (N_c \text{cl}(A)))).$$

Theorem 3.16.

Let (X, Ψ_N) be NCTS and A, B be a neutrosophic crisp subset, then:

$$N_c \text{Fr}(A \cap B) \subseteq N_c \text{Fr}(A) \cup N_c \text{Fr}(B).$$

Proof.

Let A and B be neutrosophic subsets in the NCTS (X, Ψ_N) . Then

$$\begin{aligned} N_c Fr(A \cap B) &= N_c cl(A \cap B) \cap (N_c cl(A \cap B))^c \text{ [by Definition 3.1]} \\ &= N_c cl(A \cap B) \cap (N_c cl(A^c \cup B^c)) \\ &\subseteq (N_c cl(A) \cap N_c cl(B)) \cap (N_c cl(A^c) \cup N_c cl(B^c)) \\ &= (N_c cl(A) \cap N_c cl(B)) \cap (N_c cl(A^c) \cup (N_c cl(A) \cap N_c cl(B)) \cap (N_c cl(B^c))) \\ &= (N_c Fr(A) \cap N_c cl(B)) \cup (N_c cl(A) \cap N_c Fr(B)) \text{ [by Definition 3.1]} \\ &\subseteq N_c Fr(A) \cup (N_c Fr(B)). \end{aligned}$$

Hence $N_c Fr(A \cap B) \subseteq N_c Fr(A) \cup N_c Fr(B)$.

Theorem 3.17.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c Fr(N_c Fr(A)) \subseteq N_c Fr(A),$$

Proof.

Let A be a neutrosophic crisp subset in X . Then

$$\begin{aligned} N_c Fr(N_c Fr(A)) &= N_c cl(N_c Fr(A)) \cap N_c cl(N_c Fr(A))^c \text{ [by Definition 3.1]} \\ &= N_c cl(N_c cl(A) \cap (N_c cl(A^c)) \cap (N_c cl(N_c cl(A)) \cap (N_c cl(A^c))^c)) \text{ [by Definition 3.1]} \\ &\subseteq (N_c cl(N_c cl(A)) \cap (N_c cl(N_c cl(A^c))) \cap (N_c cl(N_c int(A^c))) \cup (N_c int(A))) \\ &= (N_c cl(A) \cap (N_c cl(A^c)) \cap (N_c cl(N_c cl(A) \cup N_c int(A)))) \\ &\subseteq N_c cl(A) \cap N_c cl(A^c) \\ &= N_c Fr(A) \text{ [by Definition 3.1]}. \end{aligned}$$

Therefore $N_c Fr(N_c Fr(A)) \subseteq N_c Fr(A)$.

(b) Again, $N_c Fr(N_c Fr(N_c Fr(A))) \subseteq N_c Fr(N_c Fr(A))$.

Remark 3.18.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

(a) $N_c Fr(N_c Fr(A)) \subseteq N_c Fr(A)$,

(b) $N_c Fr(N_c Fr(N_c Fr(A))) \subseteq N_c Fr(N_c Fr(A))$.

Proof.

By Theorem 3.17. $N_c Fr(N_c Fr(A)) \subseteq N_c Fr(A)$ then, $N_c Fr(N_c Fr(N_c Fr(A))) \subseteq N_c Fr(N_c Fr(A))$.

Theorem 3.19.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$(N_c Fr(A))^c = N_c int(A) \cup N_c int(A^c).$$

Proof.

Let A be a neutrosophic crisp subset in X . Then by Definition 3.1, $(N_c Fr(A))^c =$

$(N_c cl(A) \cap N_c cl(A^c))^c = ((N_c cl(A))^c \cup (N_c cl(A^c))^c = N_c int(A^c) \cup N_c int(A)$. Hence $(N_c Fr(A))^c = N_c int(A) \cup N_c int(A^c)$.

4. Neutrosophic crisp border and neutrosophic crisp exterior in NCTS:

In this section, we introduce the neutrosophic crisp border, neutrosophic crisp exterior using neutrosophic crisp open sets and their properties are discussed in NCTS.

Definition 4.1.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

the set $N_c Br(A) = A - N_c int(A)$ is called the neutrosophic crisp border of A .

Example 4.2.

Let $\chi = \{a, b, c\}$, $T = \{\emptyset_N, X_N, A\}$, $A = \{< \{a\}, \{b\}, \{a, c\} >\}$.

Then (χ, T) neutrosophic crisp topological space.

$N_c Fr(A) = A - N_c int(A) = A - A = \emptyset_N$.

Theorem 4.3.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

A is a NCOS set if and only if $N_c Br(A) = \phi_N$.

Proof.

$\Rightarrow$: Suppose A is NCOS. Then $N_c int(A) = A$. Now, $N_c Br(A) =$

$A - N_c int(A) = A - A = \phi_N$.

$\Leftarrow$: Suppose $N_c Br(A) = \phi_N$. This implies, $A - N_c int(A) = \phi_N$. Therefore $A = N_c int(A)$

and hence A is NCOS.

Remark 4.4.

Let (X, Ψ_N) be NCTS, then:

$N_c Br(\phi_N) = \phi_N$ and $N_c Br(1_N) = \phi_N$.

Proof.

Since ϕ_N and 1_N are NCOS, by Theorem 4.3, $N_c Br(\phi_N) = \phi_N$ and $N_c Br(1_N) = \phi_N$.

Theorem 4.5.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$N_c Br(N_c int(A)) = \phi_N$.

Proof.

By the definition of N_c border, $N_c Br(N_c int(A)) = N_c int(A) - N_c int(N_c int(A))$. But, $N_c int(N_c int(A)) = N_c int(A)$ hence $N_c Br(N_c int(A)) = \phi_N$.

Theorem 4.6.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

the following statements are equivalent

- (i) A is NCOS.
- (ii) $A = N_c \text{int}(A)$.
- (iii) $N_c \text{Br}(A) = \phi_N$.

Proof.

(i) $\rightarrow$ (ii) Obvious.

(ii) $\rightarrow$ (iii). Suppose that $A = N_c \text{int}(A)$. Then by Definition, $N_c \text{Br}(A) = N_c \text{int}(A) - N_c \text{int}(A) = \phi_N$.

(iii) $\rightarrow$ (i). Let $N_c \text{Br}(A) = \phi_N$. Then by Definition 4.1, $A - N_c \text{int}(A) = \phi_N$ and hence $A = N_c \text{int}(A)$.

Theorem 4.7.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c \text{Br}(A) = A \cap N_c \text{cl}(A^c).$$

Proof.

$$\text{Since } N_c \text{Br}(A) = A - N_c \text{int}(A) = A \cap (N_c \text{int}(A))^c = A \cap N_c \text{cl}(A^c).$$

Theorem 4.8.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c \text{Br}(A) \subseteq N_c \text{Fr}(A).$$

Proof.

Since $A \subseteq N_c \text{cl}(A)$, $A - N_c \text{int}(A) \subseteq N_c \text{cl}(A) - N_c \text{int}(A)$. That implies, $N_c \text{Br}(A) \subseteq N_c \text{Fr}(A)$.

Definition 4.9.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

The neutrosophic crisp interior of A^c is called the neutrosophic crisp exterior of A and it is denoted by $N_c \text{Ext}(A)$. That is, $N_c \text{Ext}(A) = N_c \text{int}(A^c)$.

Theorem 4.10.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c \text{Ext}(A) = (N_c \text{cl}(A))^c.$$

Proof.

We know that, $(N_c \text{cl}(A))^c = N_c \text{int}(A^c)$, then $N_c \text{Ext}(A) = N_c \text{int}(A^c) = (N_c \text{cl}(A))^c$.

Example 4.11.

Let $X = \{a, b, c\}$, $T = \{\emptyset_N, X_N, A\}$, $A = \{< \{a\}, \emptyset, \emptyset >\}$.

Then (X, T) neutrosophic crisp topological space, let $H = \{< \{a\}, \{a, b\}, \{a, c\} >\}$

$$N_c \text{Ext}(A) = (N_c \text{cl}(A))^c = (X_N)^c = \emptyset_N.$$

Theorem 4.12.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c \text{Ext}(N_c \text{Ext}(A)) = N_c \text{int}(N_c \text{cl}(A)) \supseteq N_c \text{int}(A)$$

Proof.

$$\text{Now, } N_c \text{Ext}(N_c \text{Ext}(A)) = N_c \text{Ext}(N_c \text{int}(A^c)) = N_c \text{int}((N_c \text{int}(A^c))^c) = N_c \text{int}(N_c \text{cl}(A)) \supseteq N_c \text{int}(A).$$

Theorem 4.13.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

If $A \subseteq B$, then $N_c \text{Ext}(B) \subseteq N_c \text{Ext}(A)$.

Proof. Suppose $A \subseteq B$. Now, $N_c \text{Ext}(B) = N_c \text{int}(B^c) \subseteq N_c \text{int}(A^c) = N_c \text{Ext}(A)$.

Theorem 4.14.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c \text{Ext}(X_N) = \phi_N \text{ and } N_c \text{Ext}(\phi_N) = X_N.$$

Proof.

Now, $N_c \text{Ext}(X_N) = N_c \text{int}((X_N)^c) = N_c \text{int}(\phi_N)$ and $N_c \text{Ext}(\phi_N) = N_c \text{int}((\phi_N)^c) = N_c \text{int}(X_N)$. Since ϕ_N and X_N are NCOS sets, then $N_c \text{int}(\phi_N) = \phi_N$ and $N_c \text{int}(X_N) = X_N$. Hence $N_c \text{Ext}(\phi_N) = X_N$ and $N_c \text{Ext}(X_N) = \phi_N$.

Theorem 4.15.

Let (X, Ψ_N) be NCTS and A be a neutrosophic crisp subset, then:

$$N_c \text{Ext}(A) = N_c \text{Ext}((N_c \text{Ext}(A))^c).$$

Proof.

$$\text{Now, } N_c \text{Ext}((N_c \text{Ext}(A))^c) = N_c \text{Ext}((N_c \text{int}(A^c))^c) = N_c \text{int}((((N_c \text{int}(A^c))^c))^c) = N_c \text{int}(N_c \text{int}(A^c)) = N_c \text{int}(A^c) = N_c \text{Ext}(A).$$

Theorem 4.16.

Let (X, Ψ_N) be NCTS and A, B be a neutrosophic crisp subsets, then:

$$(i) \ N_c \text{Ext}(A \cup B) \subseteq N_c \text{Ext}(A) \cap N_c \text{Ext}(B).$$

$$(ii) \ N_c \text{Ext}(A \cap B) \supseteq N_c \text{Ext}(A) \cup N_c \text{Ext}(B).$$

Proof.

$$(i) \ N_c \text{Ext}(A \cup B) = N_c \text{int}((A \cup B)^c) = N_c \text{int}((A^c) \cap (B^c)) \subseteq N_c \text{cl}(A^c) \cap N_c \text{cl}(B^c) = N_c \text{Ext}(A) \cap N_c \text{Ext}(B).$$

$$(ii) \ N_c \text{Ext}(A \cap B) = N_c \text{int}((A \cap B)^c) = N_c \text{int}((A^c) \cup (B^c)) \supseteq N_c \text{cl}(A^c) \cup N_c \text{cl}(B^c) = N_c \text{Ext}(A) \cup N_c \text{Ext}(B).$$

5. Conclusion

In this paper, we have defined a new concepts in neutrosophic crisp topological space by using neutrosophic crisp sets. This new concepts called neutrosophic crisp frontier, neutrosophic crisp border and neutrosophic crisp exterior in NCTS. Also we studied some of their basic properties and their relationship with each other. In the future, using these notions, various classes of mappings, separation axioms, and many researchers can be studied in NCTS.

References

- [1] F. Smarandache, (1988). "Neutrosophy / Neutrosophic probability, set, and logic". American Research Press, See also: <http://gallup.unm.edu/~smarandache/NeutLog.txt>.
- [2] F. Smarandache, (2002). "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics". University of New Mexico, Gallup, NM 87301, USA.
- [3] S. Mashhour, A. A. Allam, F. S. Mahmoud, and F. H. Khedr, (1983). "On Supra topological spaces". Indian Jr. Pure and Appl. Math, No.4, 14, 502-510.
- [4] A. A. Salama, F. Smarandache and Kroumov. (2012). "Neutrosophic Set and Neutrosophic Topological Spaces". IOSR Journal of Mathematics, 3, 31-35.
- [5] A. A. Salama, F. Smarandache and Kroumov, (2014). "Neutrosophic crisp Sets and Neutrosophic crisp Topological Spaces" Neutrosophic Sets and Systems Vlo.2, 25-30.
- [6] P. Iswarya, and K. Bageerathi, (2017). "A Study on Neutrosophic Frontier and Neutrosophic Semi-frontier in Neutrosophic Topological Spaces", Neutrosophic Sets and Systems, Vol 16, pp 6-15.
- [7] A. B. AL-Nafee, R. K. Al-Hamido, F. Smarandache, (2019). "Separation Axioms In Neutrosophic Crisp Topological Spaces", Neutrosophic Sets and Systems, vol. 25, 25-32
- [8] A. A Salama, I. M Hanafy, Hewayda Elghawalby Dabash M.S, Neutrosophic Crisp Closed Region and Neutrosophic Crisp Continuous Functions, New Trends in Neutrosophic Theory and Applications.
- [9] A. A Salama, Hewayda Elghawalby, M.S. Dabash, A. M. NASR , (2018).Retrac Neutrosophic Crisp System For Gray Scale Image, Asian Journal of Mathematics and Computer Research, Vol. 24, 104-117-22.
- [10] A. A Salama, (2015). Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology. Neutrosophic Sets and Systems, Vol. 7, 18-22.
- [11] A. A Salama, F. Smarandache, (2014).Neutrosophic Crisp Set Theory, Neutrosophic Sets and Systems, Vol. 5, 1-9.
- [12] F. Smarandache, (2002). "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM 87301, USA.
- [13] M. Abdel-Basset, M. Mai, E. Mohamed, C. Francisco, H. Z. Abd El-Nasser, (2019). "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases" Artificial Intelligence in Medicine Vol. 101 , 101735.
- [14] M. Abdel-Basset, E. Mohamed, G. Abdullallah, and F. Smarandache, (2019). "A novel model for evaluation Hospital medical care systems based on plithogenic sets" Artificial intelligence in medicine 100, 101710.
- [15] M. Abdel-Basset, G. Gunasekaran Mohamed, G. Abdullallah. C. Victor, (2019). "A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT" IEEE Internet of Things Journal, Vol. 7.
- [16] M. Abdel-Basset, G. Abdullallah, G. Gunasekaran, L. Hoang Viet, (2019). "A novel group decision making model based on neutrosophic sets for heart disease diagnosis" Multimedia Tools and Applications, 1-26.
- [17] R. K. Al-Hamido, (2018). "Neutrosophic Crisp Bi-Topological Spaces", Neutrosophic Sets and Systems, Vol 21, pp66-73.
- [18] R. K. Al-Hamido, (2018). "Neutrosophic Crisp Supra Bi-Topological Spaces", International Journal of Neutrosophic Science, Vol 1, pp66-73.
- [19] R. K. Al-Hamido, (2018). " A New Approach Of Neutrosophic Topological space ", International Journal of Neutrosophic Science, Vol 1, pp66-73, 2018.
- [20] R. K. Al-Hamido, Q. H. Imran, K. A. Alghurabi, T. Gharibah, (2018). "On Neutrosophic Crisp Semi Alpha Closed Sets", Neutrosophic Sets and Systems", vol. 21, 28-35.
- [21] Q. H. Imran, F. Smarandache, R. K. Al-Hamido, R. Dhavasselan, (2017) "On Neutrosophic Semi Alpha open Sets", Neutrosophic Sets and Systems, vol. 18, 37-42.