New algebraic extension of interval valued Q-neutrosophic normal subbisemirings of bisemirings

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Abstract

In this research article, we introduce the notions of interval valued Q-neutrosophic subbisemirings (IVQNSSBSs), level sets of an IVQNSSBS and interval valued Q-neutrosophic normal subbisemirings (IVQNSNSBSs) of bisemirings. Let $\overrightarrow{Y}$ be an interval valued Q-neutrosophic set (IVQNS set) in a bisemiring $\mathscr{S}$. Prove that $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$ if and only if all nonempty level set $\overrightarrow{\Xi^{(t,s)}}$ is a subbisemiring (SBS) of $\mathscr{S}$ for $t, s \in D[0,1]$. Let $\overrightarrow{Y}$ be an IVQNSSBS of a bisemiring $\mathscr{S}$ and $\overrightarrow{V}$ be the strongest interval valued Q-neutrosophic relation of $\mathscr{S}$. Prove that $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$ if and only if $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S} \times \mathscr{S}$. We illustrate homomorphic image of IVQNSSBS is an IVQNSSBS. Prove that homomorphic pre-image of IVQNSSBS is an IVQNSSBS. Examples are given to demonstrate our findings.

Keywords: interval valued Q-neutrosophic subbisemiring; interval valued Q-neutrosophic normal subbisemiring; subbisemiring; homomorphism.

1 Introduction

According to Golan, the idea of semirings, which is a generalization of rings, was first proposed by Vandiver in 1935.[10] Because they are the algebraic structure of the set of natural numbers, semirings are a natural subject to explore in algebra. Many areas of mathematics naturally contain semirings. Since the structure of a semiring gives an algebraic framework for modelling and investigating the factors key, semirings have proven effective for solving issues in a variety of fields of applied mathematics and information science. Semirings, for instance, can be used to solve problems in graph theory and optimization as well as in the world of theoretical calculation science. The semiring is one of the structures semigroups and rings. However, the concept of semirings has been evolving since 1950. The fuzzy set (FS) theory published the significant work in 1965 known as Zadeh.[21] According to this definition, FS is a function that can be characterized by a membership value. To deal with the uncertainties, a number of uncertain theories have been presented, including FS,[21] intuitionistic FS (IFS),[4] Pythagorean FS (PFS),[20] picture FS[5] and spherical FS (SFS).[3] A FS is a set with degrees of belongingness

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ranging from 0 to 1, these grades are known as the value of an element's membership in the specific set. Later, Atanassov proposed the concept of an IFS logic, which is categorized by the sum of its MD with NMD is value of not more than 1.[4] We periodically communicate a single issue when the combined grade value for MD and NMD is greater than 1 when employing a decision making approach. Yager invented the novel idea of PFS logic, which is defined by the square sum of its MD and NMD and whose value is not greater than 1, in order to generalize IFS. However, these theories are unable to prove the neutral state (neither favour nor disfavor). The inventor of image FS logic, Cuong and Kreinovich, used three pointers: positive MD, neutral MD, and negative MD, with a maximum combined grade of 1. Finally, for a few applications, it offers more advantages than IFS and PFS. A generalization of the FS and IFS is the neutrosophic set (NSS), in which the truth MD, indeterminacy MD and falsity MD are all independently recorded.

Smarandache[19] developed neutrosophy to address the problems of unclear and inconsistent information because this set contains numerous application related challenges. Recently, a brand new theory called as NSS has been proposed. Neutosophy is the study of neutral cognition, and the major difference between IFS and FS is represented by this neutral. Smarandache is credited with introducing NSS.[19] It is a logic in which the degrees of truth, indeterminacy, and falsity of each assertion are determined. Each component of the universe has a degree of truth, indeterminacy, and falsity, which ranges from $[0, 1]$ in the NSS set. It has been demonstrated that a NSS generalizes a classical set, FS, IVFS, etc. The study of NSS and the expansion of this concept has continued, for example, Al-Tahan et al.[2] defined the notion of single valued neutrosophic sets in ordered groupoids in 2020. A year later, Jagadeeswari et al.[9] initiated Certain Kinds of Bipolar interval valued nentrosphic graphs. In 2022, Iampan et al.[7,8] introduced the concept of IVN sets to ideals/subalgebras of Hilbert algebras. A semiring $(S, +, \cdot)$ is a nonempty set in which S are semigroups both $+$ and $\cdot$, and " $\cdot$ " is distributive over " $+$ ". In 1993, Ahsan et al.[1] introduced the concept of fuzzy semirings. In 2001, Sen and Ghosh introduced in bisemirings. A bisemiring[18] $(\mathscr{S}, +, \circ, \times)$ is an algebraic structure in which $(\mathscr{S}, +, \circ)$ and $(\mathscr{S}, \circ, \times)$ are semirings in which $(\mathscr{S}, +), (\mathscr{S}, \circ)$ and $(\mathscr{S}, \times)$ are semigroups and (1) $\varpi \circ (\tau + \delta) = (\varpi \circ \tau) + (\varpi \circ \delta)$ (2) $(\tau + \delta) \circ \varpi = (\tau \circ \varpi) + (\delta \circ \varpi)$ (3) $\varpi \times (\tau \circ \delta) = (\varpi \times \tau) \circ (\varpi \times \delta)$, and (4) $(\tau \circ \delta) \times \varpi = (\tau \times \varpi) \circ (\delta \times \varpi)$ for all $\varpi, \tau, \delta \in \mathscr{S}$. A nonempty subset Y of a bisemiring $(\mathscr{S}, +, \circ, \times)$ is a SBS if and only if $\varpi + \tau \in Y,\ \ \varpi \circ \tau \in Y$ and $\varpi \times \tau \in Y$ for all $\varpi, \tau \in Y$.[6] Palanikumar et al. discussed various ideals structures and its applications.[11–17]

The goal of this work is to look at many facets of the SBS theory to IVNSQSBS idea and offer findings. The article is made up of the next five principles. The introduction is in Section 1, and the semiring and SBS preparation information is in Section 2 . Its characteristics are presented in Section 3 of IVNSQSBS. In Section 4, the concept of IVNSQNSBS homomorphism is presented. Give some numerical examples for evaluating the IVNSQSBS and IVNSQNSBS.

2 Preliminaries

We will go over the ideas of semirings and bisemirings in this section to make the presentation as full as possible and to make the next talks more convenient.

Definition 2.1. Let $\mathscr{U}$ and Q denote nonempty sets. The mapping $\Xi : \mathscr{U} \times Q \to [0, 1]$ is called a Q-fuzzy set in $\mathscr{U}$.

Definition 2.2. Let $(\mathscr{S}, +, \cdot)$ be semiring. The Q-fuzzy set $\Xi : \mathscr{S} \times Q \to [0, 1]$ is called a Q-fuzzy subsemiring of $\mathscr{S}$ if $\Xi_Y(\varpi + \tau, q) \geq \min\{\Xi_Y(\varpi, q), \Xi_Y(\tau, q)\}$, $\Xi_Y(\varpi \cdot \tau, q) \geq \min\{\Xi_Y(\varpi, q), \Xi_Y(\tau, q)\}$ for all $\varpi, \tau \in \mathscr{S}$ and $q \in Q$.

Definition 2.3. An intuitionistic Q-fuzzy set defined on $\mathscr{U}$ and Q is of the form

$$Y = \{\langle(\varpi, q), \Xi_Y(\varpi, q), \Phi_Y(\varpi, q)\rangle \mid \varpi \in \mathscr{U}, q \in Q\},$$

where $\Xi_Y : \mathscr{U} \times Q \to [0, 1]$ and $\Phi_Y : \mathscr{U} \times Q \to [0, 1]$ define the MD and NMD of the element $\varpi \in \mathscr{U}$, respectively and every $\varpi \in \mathscr{U}$ and $q \in Q$ satisfying $0 \leq \Xi_Y(\varpi, q) + \Phi_Y(\varpi, q) \leq 1$.

Definition 2.4. Let Y and Z be any two intuitionistic Q-fuzzy sets of $\mathscr{U}$. We define two intuitionistic Q-fuzzy sets of $\mathscr{U}$ as follows:

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(1) $Y \cap Z = \{\langle(\varpi, q), \min\{\Xi_Y(\varpi, q), \Xi_Z(\varpi, q)\}, \max\{\Phi_Y(\varpi, q), \Phi_Z(\varpi, q)\}\rangle \mid \varpi \in \mathscr{U}$ and $q \in Q\}$,

(2) $Y \cup Z = \{\langle(\varpi, q), \max\{\Xi_Y(\varpi, q), \Xi_Z(\varpi, q)\}, \min\{\Phi_Y(\varpi, q), \Phi_Z(\varpi, q)\}\rangle \mid \varpi \in \mathscr{U}$ and $q \in Q\}$.

Definition 2.5. Let Y be an intuitionistic Q-fuzzy set of $\mathscr{U}$. For $t, s \in [0,1]$ the level subset of Y is the set $Y_{(t,s)} = \{\varpi \in \mathscr{U} \mid \Xi_Y(\varpi, q) \geq t, \Xi_Y(\varpi, q) \leq s, q \in Q\}$.

Definition 2.6. An interval valued neutrosophic set (IVNS set) $\overrightarrow{Y}$ in $\mathscr{U}$ is of the form

$$\overrightarrow{Y} = \left\{\left\langle \varpi, \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi), \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi)\right\rangle \mid \varpi \in \mathscr{U}\right\},$$

where $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi) = [\Xi_Y^{\mathscr{T}\mathscr{L}}, \Xi_Y^{\mathscr{T}\mathscr{U}}], \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi) = [\Xi_Y^{\mathscr{I}\mathscr{L}}, \Xi_Y^{\mathscr{I}\mathscr{U}}], \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi) = [\Xi_Y^{\mathscr{F}\mathscr{L}}, \Xi_Y^{\mathscr{F}\mathscr{U}}]$, and $\overrightarrow{\Xi_Y^{\mathscr{T}}}, \overrightarrow{\Xi_Y^{\mathscr{I}}}, \overrightarrow{\Xi_Y^{\mathscr{F}}} : \mathscr{U} \to D[0,1]$ represents the truth, indeterminacy and falsity membership function, respectively. For simplicity we define an IVNS set $\overrightarrow{Y} = \left\langle \overrightarrow{\Xi_Y^{\mathscr{T}}}, \overrightarrow{\Xi_Y^{\mathscr{I}}}, \overrightarrow{\Xi_Y^{\mathscr{F}}}\right\rangle$.

Definition 2.7. Let $\overrightarrow{Y} = \left\langle \overrightarrow{\Xi_Y^{\mathscr{T}}}, \overrightarrow{\Xi_Y^{\mathscr{I}}}, \overrightarrow{\Xi_Y^{\mathscr{F}}}\right\rangle$ and $\overrightarrow{Z} = \left\langle \overrightarrow{\Xi_Z^{\mathscr{T}}}, \overrightarrow{\Xi_Z^{\mathscr{I}}}, \overrightarrow{\Xi_Z^{\mathscr{F}}}\right\rangle$ be two IVNS sets of $\mathscr{U}$. We define two IVNS sets in $\mathscr{U}$ as follows:

(1) $\overrightarrow{Y} \cap \overrightarrow{Z} = \left\{\left\langle \varpi, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\varpi)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{I}}}(\varpi)\}, \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{F}}}(\varpi)\}\right\rangle \mid \varpi \in \mathscr{U}\right\}$,

(2) $\overrightarrow{Y} \cup \overrightarrow{Z} = \left\{\left\langle \varpi, \max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\varpi)\}, \max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{I}}}(\varpi)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{F}}}(\varpi)\}\right\rangle \mid \varpi \in \mathscr{U}\right\}$.

Definition 2.8. For any IVNS set $\overrightarrow{Y} = \left\langle \overrightarrow{\Xi_Y^{\mathscr{T}}}, \overrightarrow{\Xi_Y^{\mathscr{I}}}, \overrightarrow{\Xi_Y^{\mathscr{F}}}\right\rangle$ of a set $\mathscr{U}$, we defined a (ζ_1, ζ_2)-cut of as the crisp subset $\left\{\varpi \in \mathscr{U} \mid \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi) \succeq \overrightarrow{\zeta_1}, \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi) \succeq \overrightarrow{\zeta_1}, \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi) \preceq \overrightarrow{\zeta_2}\right\}$ of $\mathscr{U}$.

Definition 2.9. Let $\overrightarrow{Y}$ and $\overrightarrow{Z}$ be the IVNS sets of $\mathscr{U}$. The cartesian product of $\overrightarrow{Y}$ and $\overrightarrow{Z}$ is defined as $\overrightarrow{Y \times Z} = \left\{\left\langle(\varpi, \tau), \overrightarrow{\Xi_{Y\times Z}^{\mathscr{T}}}(\varpi, \tau), \overrightarrow{\Xi_{Y\times Z}^{\mathscr{I}}}(\varpi, \tau), \overrightarrow{\Xi_{Y\times Z}^{\mathscr{F}}}(\varpi, \tau)\right\rangle \mid \varpi, \tau \in \mathscr{U}\right\}$, where

$$\begin{aligned}
\overrightarrow{\Xi_{Y\times Z}^{\mathscr{T}}}(\varpi, \tau) &= \min\left\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\tau)\right\}, \\
\overrightarrow{\Xi_{Y\times Z}^{\mathscr{I}}}(\varpi, \tau) &= \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi) + \overrightarrow{\Xi_Z^{\mathscr{I}}}(\tau)}{2}, \\
\overrightarrow{\Xi_{Y\times Z}^{\mathscr{F}}}(\varpi, \tau) &= \max\left\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi), \overrightarrow{\Xi_Z^{\mathscr{F}}}(\tau)\right\}.
\end{aligned}$$

Definition 2.10. A fuzzy set Y of a bisemiring $(\mathscr{S}, \circledast_1, \circledast_2, \circledast_3)$ is said to be a fuzzy subbisemiring of $\mathscr{S}$ if $\Xi_Y(\varpi \circledast_1 \tau) \succeq \min\{\Xi_Y(\varpi), \Xi_Y(\tau)\}$, $\Xi_Y(\varpi \circledast_2 \tau) \succeq \min\{\Xi_Y(\varpi), \Xi_Y(\tau)\}$, and $\Xi_Y(\varpi \circledast_3 \tau) \succeq \min\{\Xi_Y(\varpi), \Xi_Y(\tau)\}$ for all $\varpi, \tau \in \mathscr{S}$.

Definition 2.11. A fuzzy set Y of a bisemiring $(\mathscr{S}, \circledast_1, \circledast_2, \circledast_3)$ is said to be a fuzzy normal subbisemiring of $\mathscr{S}$ if $\Xi_Y(\varpi \circledast_1 \tau) = \Xi_Y(\tau \circledast_1 \varpi)$, $\Xi_Y(\varpi \circledast_2 \tau) = \Xi_Y(\tau \circledast_2 \varpi)$, and $\Xi_Y(\varpi \circledast_3 \tau) = \Xi_Y(\tau \circledast_3 \varpi)$ for all $\varpi, \tau \in \mathscr{S}$.

Definition 2.12. Let $(\mathscr{S}, +, \cdot, \times)$ and $(\mathscr{T}, \boxminus, \circ, \otimes)$ be the bisemirings. The function $\phi : \mathscr{S} \to \mathscr{T}$ is said to be a homomorphism if $\phi(\varpi + \tau) = \phi(\varpi) \boxminus \phi(\tau)$, $\phi(\varpi \cdot \tau) = \phi(\varpi) \circ \phi(\tau)$, and $\phi(\varpi \times \tau) = \phi(\varpi) \otimes \phi(\tau)$ for all $\varpi, \tau \in \mathscr{S}$.

3 Interval valued Q-neutrosophic SBSs (IVQNSSBSs)

In what follows, let $\mathscr{S}$ denote a bisemiring.

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Definition 3.1. An interval valued Q-neutrosophic set (IVQNS set) $\overrightarrow{Y}$ in $\mathscr{U}$ is of the form

$$\overrightarrow{Y}=\left\{\left\langle(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi,q)\right\rangle \mid \varpi\in\mathscr{U},q\in Q\right\},$$

where $\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q)=[\Xi^{\mathscr{T}\mathscr{L}}_{Y},\Xi^{\mathscr{T}\mathscr{U}}_{Y}]$, $\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi,q)=[\Xi^{\mathscr{I}\mathscr{L}}_{Y},\Xi^{\mathscr{I}\mathscr{U}}_{Y}]$, $\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi,q)=[\Xi^{\mathscr{F}\mathscr{L}}_{Y},\Xi^{\mathscr{F}\mathscr{U}}_{Y}]$, and $\overrightarrow{\Xi^{\mathscr{T}}_{Y}},\overrightarrow{\Xi^{\mathscr{I}}_{Y}},\overrightarrow{\Xi^{\mathscr{F}}_{Y}}:\mathscr{U}\times Q\to D[0,1]$ represents the truth, indeterminacy and falsity membership function, respectively. For simplicity we define an IVQNS set $\overrightarrow{Y}=\left\langle\overrightarrow{\Xi^{\mathscr{T}}_{Y}},\overrightarrow{\Xi^{\mathscr{I}}_{Y}},\overrightarrow{\Xi^{\mathscr{F}}_{Y}}\right\rangle$.

Definition 3.2. The IVQNS set $\overrightarrow{Y}$ of $\mathscr{S}$ is said to be an IVQNSSBS of $\mathscr{S}$ if

$$\left\{\begin{array}{l}\max\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi\circledast_1\tau,q)\}\succeq\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\tau,q)\}\\ \max\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi\circledast_2\tau,q)\}\succeq\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\tau,q)\}\\ \max\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi\circledast_3\tau,q)\}\succeq\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\tau,q)\}\end{array}\right\}\left\{\begin{array}{r}\max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi\circledast_1\tau,q)\}\succeq\dfrac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\tau,q)}{2}\\ \text{or}\\ \max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi\circledast_2\tau,q)\}\succeq\dfrac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\tau,q)}{2}\\ \text{or}\\ \max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi\circledast_3\tau,q)\}\succeq\dfrac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\tau,q)}{2}\end{array}\right\}$$

$$\left\{\begin{array}{l}\min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi\circledast_1\tau,q)\}\preceq\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\tau,q)\}\\ \min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi\circledast_2\tau,q)\}\preceq\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\tau,q)\}\\ \min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi\circledast_3\tau,q)\}\preceq\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\tau,q)\}\end{array}\right\}$$

for all $\varpi,\tau\in\mathscr{S}$ and $q\in Q$.

Example 3.3. Let $\mathscr{S}=\{t_1,t_2,t_3,t_4\}$ be a bisemiring with the following Cayley tables:

$\circledast_1$	t_1	t_2	t_3	t_4
t_1	t_1	t_1	t_1	t_1
t_2	t_1	t_2	t_1	t_2
t_3	t_1	t_1	t_3	t_3
t_4	t_1	t_2	t_3	t_4

$\circledast_2$	t_1	t_2	t_3	t_4
t_1	t_1	t_2	t_3	t_4
t_2	t_2	t_2	t_4	t_4
t_3	t_3	t_4	t_3	t_4
t_4	t_4	t_4	t_4	t_4

$\circledast_3$	t_1	t_2	t_3	t_4
t_1	t_1	t_1	t_1	t_1
t_2	t_1	t_2	t_3	t_4
t_3	t_4	t_4	t_4	t_4
t_4	t_4	t_4	t_4	t_4

	$t=t_1$	$t=t_2$	$t=t_3$	$t=t_4$
$\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(t)$	$[0.9,0.95]$	$[0.85,0.9]$	$[0.65,0.75]$	$[0.75,0.8]$
$\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(t)$	$[0.7,0.75]$	$[0.6,0.65]$	$[0.4,0.5]$	$[0.55,0.6]$
$\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(t)$	$[0.45,0.55]$	$[0.7,0.8]$	$[0.85,0.9]$	$[0.8,0.85]$

Then $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$ for all $q\in Q$.

Theorem 3.4. *The intersection of a family of IVQNSSBSs of $\mathscr{S}$ is an IVQNSSBS of $\mathscr{S}$.*

Proof. Let $\{\overrightarrow{V_i}\mid i\in I\}$ be a family of IVQNSSBSs of $\mathscr{S}$ and $\overrightarrow{Y}=\bigcap_{i\in I}\overrightarrow{V_i}$. Let ϖ and τ in $\mathscr{S}$. Then

$$\begin{aligned}\max\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi\circledast_1\tau,q)\}&=\inf_{i\in I}\{\overrightarrow{\Xi^{\mathscr{T}}_{V_i}}(\varpi\circledast_1\tau,q)\}\\&\succeq\inf_{i\in I}\{\min\{\overrightarrow{\Xi^{\mathscr{T}}_{V_i}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{V_i}}(\tau,q)\}\}\\&=\min\{\inf_{i\in I}\{\overrightarrow{\Xi^{\mathscr{T}}_{V_i}}(\varpi,q)\},\inf_{i\in I}\{\overrightarrow{\Xi^{\mathscr{T}}_{V_i}}(\tau,q)\}\}\\&=\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\tau,q)\}.\end{aligned}$$

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Similarly,

$$\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi \divideontimes_2 \tau, q)\} \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau, q)\}$$

and

$$\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi \divideontimes_3 \tau, q)\} \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau, q)\}.$$

Now,

$$\begin{aligned}
\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi \divideontimes_1 \tau, q)\} &= \inf_{i\in I}\{\overrightarrow{\Xi_{V_i}^{\mathscr{I}}}(\varpi \divideontimes_1 \tau, q)\} \\
&\succeq \inf_{i\in I}\left\{\frac{\overrightarrow{\Xi_{V_i}^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_{V_i}^{\mathscr{I}}}(\tau, q)}{2}\right\} \\
&= \frac{\inf_{i\in I}\{\overrightarrow{\Xi_{V_i}^{\mathscr{I}}}(\varpi, q)\} + \inf_{i\in I}\{\overrightarrow{\Xi_{V_i}^{\mathscr{I}}}(\tau, q)\}}{2} \\
&= \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau, q)}{2}.
\end{aligned}$$

Similarly,

$$\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi \divideontimes_2 \tau, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau, q)}{2}$$

and

$$\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi \divideontimes_3 \tau, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau, q)}{2}.$$

Now,

$$\begin{aligned}
\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi \divideontimes_1 \tau, q)\} &= \sup_{i\in I}\{\overrightarrow{\Xi_{V_i}^{\mathscr{F}}}(\varpi \divideontimes_1 \tau, q)\} \\
&\preceq \sup_{i\in I}\{\max\{\overrightarrow{\Xi_{V_i}^{\mathscr{F}}}(\varpi, q), \overrightarrow{\Xi_{V_i}^{\mathscr{F}}}(\tau, q)\}\} \\
&= \max\{\sup_{i\in I}\{\overrightarrow{\Xi_{V_i}^{\mathscr{F}}}(\varpi, q)\}, \sup_{i\in I}\{\overrightarrow{\Xi_{V_i}^{\mathscr{F}}}(\tau, q)\}\} \\
&= \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau, q)\}.
\end{aligned}$$

Similarly,

$$\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi \divideontimes_2 \tau, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau, q)\}$$

and

$$\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi \divideontimes_3 \tau, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau, q)\}.$$

Hence, $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$. □

Theorem 3.5. *If $\overrightarrow{Y}$ and $\overrightarrow{Z}$ are any two IVQNSSBSs of bisemirings $\mathscr{S}_1$ and $\mathscr{S}_2$, respectively, then $\overrightarrow{Y \times Z}$ is an IVQNSSBS of $\mathscr{S}_1 \times \mathscr{S}_2$.*

Proof. Let $\overrightarrow{Y}$ and $\overrightarrow{Z}$ be two IVQNSSBSs of $\mathscr{S}_1$ and $\mathscr{S}_2$, respectively. Let $\varpi_1, \varpi_2 \in \mathscr{S}_1$ and $\tau_1, \tau_2 \in \mathscr{S}_2$. Then (ϖ_1, τ_1) and (ϖ_2, τ_2) are in $\mathscr{S}_1 \times \mathscr{S}_2$ and for all $q \in Q$. Then

$$\begin{aligned}
\max\{\overrightarrow{\Xi_{Y\times Z}^{\mathscr{T}}}((\varpi_1, \tau_1) \divideontimes_1 (\varpi_2, \tau_2), q)\} &= \max\{\overrightarrow{\Xi_{Y\times Z}^{\mathscr{T}}}((\varpi_1 \divideontimes_1 \varpi_2, \tau_1 \divideontimes_1 \tau_2), q)\} \\
&\succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_1 \varpi_2, q), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\tau_1 \divideontimes_1 \tau_2, q)\} \\
&\succeq \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q)\}, \min\{\overrightarrow{\Xi_Z^{\mathscr{T}}}(\tau_1, q), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\tau_2, q)\}\} \\
&= \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\tau_1, q)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q), \overrightarrow{\Xi_Z^{\mathscr{T}}}(\tau_2, q)\}\} \\
&= \min\{\overrightarrow{\Xi_{Y\times Z}^{\mathscr{T}}}((\varpi_1, \tau_1), q), \overrightarrow{\Xi_{Y\times Z}^{\mathscr{T}}}((\varpi_2, \tau_2), q)\}.
\end{aligned}$$

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Also,

$$\max\{\overrightarrow{\Xi^{\mathscr{T}}_{Y\times Z}}[(\varpi_1,\tau_1)\circledast_2(\varpi_2,\tau_2),q]\}\succeq\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y\times Z}}((\varpi_1,\tau_1),q),\overrightarrow{\Xi^{\mathscr{T}}_{Y\times Z}}((\varpi_2,\tau_2),q)\}$$

and

$$\max\{\overrightarrow{\Xi^{\mathscr{T}}_{Y\times Z}}[(\varpi_1,\tau_1)\circledast_3(\varpi_2,\tau_2),q]\}\succeq\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y\times Z}}((\varpi_1,\tau_1),q),\overrightarrow{\Xi^{\mathscr{T}}_{Y\times Z}}((\varpi_2,\tau_2),q)\}.$$

Now,

$$\begin{aligned}\max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_1,\tau_1)\circledast_1(\varpi_2,\tau_2),q)\}&=\max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}(\varpi_1\circledast_1\varpi_2,\tau_1\circledast_1\tau_2)\}\\&=\frac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi_1\circledast_1\varpi_2,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Z}}(\tau_1\circledast_1\tau_2,q)}{2}\\&\succeq\frac{1}{2}\left(\frac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi_1,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi_2,q)}{2}+\frac{\overrightarrow{\Xi^{\mathscr{I}}_{Z}}(\tau_1,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Z}}(\tau_2,q)}{2}\right)\\&=\frac{1}{2}\left(\frac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi_1,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Z}}(\tau_1,q)}{2}+\frac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi_2,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Z}}(\tau_2,q)}{2}\right)\\&=\frac{1}{2}(\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_1,\tau_1),q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_2,\tau_2),q)).\end{aligned}$$

Also,

$$\max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_1,\tau_1)\circledast_2(\varpi_2,\tau_2),q)\}\succeq\frac{1}{2}(\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_1,\tau_1),q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_2,\tau_2),q))$$

and

$$\max\{\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_1,\tau_1)\circledast_3(\varpi_2,\tau_2),q)\}\succeq\frac{1}{2}(\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_1,\tau_1),q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y\times Z}}((\varpi_2,\tau_2),q)).$$

Now,

$$\begin{aligned}\min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_1,\tau_1)\circledast_1(\varpi_2,\tau_2),q)\}&=\min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_1\circledast_1\varpi_2,\tau_1\circledast_1\tau_2),q)\}\\&\preceq\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi_1\circledast_1\varpi_2,q),\overrightarrow{\Xi^{\mathscr{F}}_{Z}}(\tau_1\circledast_1\tau_2,q)\}\\&\preceq\max\{\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi_1,q),\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi_2,q)\},\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Z}}(\tau_1,q),\overrightarrow{\Xi^{\mathscr{F}}_{Z}}(\tau_2,q)\}\}\\&=\max\{\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi_1,q),\overrightarrow{\Xi^{\mathscr{F}}_{Z}}(\tau_1,q)\},\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi_2,q),\overrightarrow{\Xi^{\mathscr{F}}_{Z}}(\tau_2,q)\}\}\\&=\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_1,\tau_1),q),\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_2,\tau_2),q)\}.\end{aligned}$$

Also,

$$\min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}[(\varpi_1,\tau_1)\circledast_2(\varpi_2,\tau_2),q]\}\preceq\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_1,\tau_1),q),\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_2,\tau_2),q)\}$$

and

$$\min\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}[(\varpi_1,\tau_1)\circledast_3(\varpi_2,\tau_2),q]\}\preceq\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_1,\tau_1),q),\overrightarrow{\Xi^{\mathscr{F}}_{Y\times Z}}((\varpi_2,\tau_2),q)\}.$$

Hence, $\overrightarrow{Y\times Z}$ is an IVQNSSBS of $\mathscr{S}$. □

Corollary 3.6. *If $\overrightarrow{Y_1},\overrightarrow{Y_2},...,\overrightarrow{Y_n}$ are IVQNSSBSs of bisemirings $\mathscr{S}_1,\mathscr{S}_2,...,\mathscr{S}_n$, respectively, then $\overrightarrow{Y_1\times Y_2\times...\times Y_n}$ is an IVQNSSBS of $\mathscr{S}_1\times\mathscr{S}_2\times...\times\mathscr{S}_n$.*

Definition 3.7. Let $\overrightarrow{Y}$ be an IVQNS set in $\mathscr{S}$, the strongest IVNS relation on $\mathscr{S}$, that is an IVQNS relation on $\overrightarrow{Y}$ is $\overrightarrow{V}$ given by $\max\{\overrightarrow{\Xi^{\mathscr{T}}_{V}}((\varpi,\tau),q)\}=\min\{\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{T}}_{Y}}(\tau,q)\}$, $\max\{\overrightarrow{\Xi^{\mathscr{I}}_{V}}((\varpi,\tau),q)\}=\frac{\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\varpi,q)+\overrightarrow{\Xi^{\mathscr{I}}_{Y}}(\tau,q)}{2}$, $\min\{\overrightarrow{\Xi^{\mathscr{F}}_{V}}((\varpi,\tau),q)\}=\max\{\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\varpi,q),\overrightarrow{\Xi^{\mathscr{F}}_{Y}}(\tau,q)\}$ for all $\varpi,\tau\in\mathscr{S}$ and $q\in Q$.

Theorem 3.8. *Let $\overrightarrow{Y}$ be an IVQNSSBS of $\mathscr{S}$ and $\overrightarrow{V}$ be the strongest IVNS relation of $\mathscr{S}$. Then $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$ if and only if $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S}\times\mathscr{S}$.*

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Proof. Suppose that $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$ and $\overrightarrow{V}$ is the strongest IVNS relation of $\mathscr{S}$. Let for any $\varpi = (\varpi_1, \varpi_2)$ and $\tau = (\tau_1, \tau_2)$ are in $\mathscr{S} \times \mathscr{S}$. Then

$$\begin{aligned}
\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi \circledast_1 \tau, q)\} &= \max\{\overrightarrow{\Xi_V^{\mathscr{T}}}((\varpi_1, \varpi_2) \circledast_1 (\tau_1, \tau_2), q)\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{T}}}((\varpi_1 \circledast_1 \tau_1, \varpi_2 \circledast_1 \tau_2), q)\} \\
&= \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \circledast_1 \tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2 \circledast_1 \tau_2, q)\} \\
&\succeq \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_2, q)\}\} \\
&= \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_2, q)\}\} \\
&= \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}((\varpi_1, \varpi_2), q), \overrightarrow{\Xi_V^{\mathscr{T}}}((\tau_1, \tau_2), q)\} \\
&= \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi, q), \overrightarrow{\Xi_V^{\mathscr{T}}}(\tau, q)\}.
\end{aligned}$$

Also,

$$\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi \circledast_2 \tau, q)\} \succeq \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi, q), \overrightarrow{\Xi_V^{\mathscr{T}}}(\tau, q)\}$$

and

$$\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi \circledast_3 \tau, q)\} \succeq \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi, q), \overrightarrow{\Xi_V^{\mathscr{T}}}(\tau, q)\}.$$

Now,

$$\begin{aligned}
\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi \circledast_1 \tau, q)\} &= \max\{\overrightarrow{\Xi_V^{\mathscr{I}}}((\varpi_1, \varpi_2) \circledast_1 (\tau_1, \tau_2), q)\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{I}}}((\varpi_1 \circledast_1 \tau_1, \varpi_2 \circledast_1 \tau_2), q)\} \\
&= \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}((\varpi_1 \circledast_1 \tau_1, q)) + \overrightarrow{\Xi_Y^{\mathscr{I}}}((\varpi_2 \circledast_1 \tau_2, q))}{2} \\
&\succeq \frac{1}{2}\left(\frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q)}{2} + \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_2, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_2, q)}{2}\right) \\
&= \frac{1}{2}\left(\frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_2, q)}{2} + \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_2, q)}{2}\right) \\
&= \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}((\varpi_1, \varpi_2), q) + \overrightarrow{\Xi_V^{\mathscr{I}}}((\tau_1, \tau_2), q)}{2} \\
&= \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_V^{\mathscr{I}}}(\tau, q)}{2}.
\end{aligned}$$

Also,

$$\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi \circledast_2 \tau, q)\} \succeq \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_V^{\mathscr{I}}}(\tau, q)}{2}$$

and

$$\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi \circledast_3 \tau, q)\} \succeq \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_V^{\mathscr{I}}}(\tau, q)}{2}.$$

Similarly, $\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\varpi \circledast_1 \tau, q)\} \preceq \max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\varpi, q), \overrightarrow{\Xi_V^{\mathscr{F}}}(\tau, q)\}, \min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\varpi \circledast_2 \tau, q)\}\} \preceq \max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\varpi, q),$ $\overrightarrow{\Xi_V^{\mathscr{F}}}(\tau, q)\}$, and $\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\varpi \circledast_3 \tau, q)\} \preceq \max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\varpi, q), \overrightarrow{\Xi_V^{\mathscr{F}}}(\tau, q)\}$. Hence, $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S} \times \mathscr{S}$.

Conversely, assume that $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S} \times \mathscr{S}$. Let for any $\varpi = (\varpi_1, \varpi_2)$ and $\tau = (\tau_1, \tau_2)$ are in

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$\mathscr{S} \times \mathscr{S}$. Then

$$\begin{aligned}
&\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}((\varpi_1 \divideontimes_1 \tau_1), q), \overrightarrow{\Xi_Y^{\mathscr{T}}}((\varpi_2 \divideontimes_1 \tau_2), q)\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{T}}}((\varpi_1 \divideontimes_1 \tau_1), (\varpi_2 \divideontimes_1 \tau_2), q)\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{T}}}((\varpi_1, \varpi_2) \divideontimes_1 (\tau_1, \tau_2), q)\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi \divideontimes_1 \tau, q)\} \\
&\succeq \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\varpi, q), \overrightarrow{\Xi_V^{\mathscr{T}}}(\tau, q)\} \\
&= \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}((\varpi_1, \varpi_2), q)\}, \overrightarrow{\Xi_V^{\mathscr{T}}}((\tau_1, \tau_2), q)\} \\
&= \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_2, q)\}\}.
\end{aligned}$$

If $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_1 \tau_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2 \divideontimes_1 \tau_2, q)$, then $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q)$ and $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_2, q)$. We get $\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_1 \tau_1, q) \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q)\}$ for all $\varpi_1, \tau_1 \in \mathscr{S}$, and $\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_2 \tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2 \divideontimes_2 \tau_2, q)\} \succeq \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_2, q)\}\}$.

If $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_2 \tau_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2 \divideontimes_2 \tau_2, q)$, then $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_2 \tau_1, q) \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q)\}$. We get $\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_3 \tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2 \divideontimes_3 \tau_2, q)\} \succeq \min\{\min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2, q)\}, \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_2, q)\}\}$.

If $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_3 \tau_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_2 \divideontimes_3 \tau_2, q)$, then $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1 \divideontimes_3 \tau_1, q) \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau_1, q)\}$. Now,

$$\begin{aligned}
\frac{1}{2}\left(\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1 \divideontimes_1 \tau_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_2 \divideontimes_1 \tau_2, q)\right) &= \max\{\overrightarrow{\Xi_V^{\mathscr{I}}}((\varpi_1 \divideontimes_1 \tau_1, \varpi_2 \divideontimes_1 \tau_2), q)\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{I}}}[((\varpi_1, \varpi_2), q) \divideontimes_1 ((\tau_1, \tau_2), q)]\} \\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi \divideontimes_1 \tau, q)\} \\
&\succeq \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}(\varpi, q) + \overrightarrow{\Xi_V^{\mathscr{I}}}(\tau, q)}{2} \\
&= \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}((\varpi_1, \varpi_2), q) + \overrightarrow{\Xi_V^{\mathscr{I}}}((\tau_1, \tau_2), q)}{2} \\
&= \frac{1}{2}\left(\frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_2, q)}{2} + \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_2, q)}{2}\right).
\end{aligned}$$

If $\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1 \divideontimes_1 \tau_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_2 \divideontimes_1 \tau_2, q)$, then $\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_2, q)$ and $\overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q) \preceq \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_2, q)$. We get $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1 \divideontimes_1 \tau_1, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q)}{2}$. Similarly, $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1 \divideontimes_2 \tau_1, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q)}{2}$ and $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1 \divideontimes_3 \tau_1, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi_1, q) + \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau_1, q)}{2}$. Similarly $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_1 \tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2 \divideontimes_1 \tau_2, q)\} \preceq \max\{\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2, q)\}, \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_2, q)\}\}$.

If $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_1 \tau_1, q) \succeq \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2 \divideontimes_1 \tau_2, q)$, then $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q) \succeq \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2, q)$ and $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q) \succeq \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_2, q)$. We get $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_1 \tau_1, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q)\}$. Also, $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_2 \tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2 \divideontimes_2 \tau_2, q)\} \preceq \max\{\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2, q)\}, \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_2, q)\}\}$.

If $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_2 \tau_1, q) \succeq \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2 \divideontimes_2 \tau_2, q)$, then $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_2 \tau_1, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q)\}$. We get $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_3 \tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2 \divideontimes_3 \tau_2, q)\} \preceq \max\{\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2, q)\}, \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_2, q)\}\}$.

If $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_3 \tau_1, q) \succeq \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_2 \divideontimes_3 \tau_2, q)$, then $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1 \divideontimes_3 \tau_1, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau_1, q)\}$.

Hence, $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$. □

Theorem 3.9. *Let $\overrightarrow{Y}$ be an IVQNS set in $\mathscr{S}$. Then $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$ if and only if all nonempty level set $\overrightarrow{\Xi^{(t,s)}}$ is a SBS of $\mathscr{S}$ for $t, s \in D[0,1]$.*

Proof. Assume that $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$. Let $t, s \in D[0,1]$ and $\upsilon_1, \upsilon_2 \in \overrightarrow{\Xi^{(t,s)}}$. Then $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1, q) \succeq \overrightarrow{t}, \overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_2, q) \succeq \overrightarrow{t}$ and $\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1, q) \succeq \overrightarrow{t}, \overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_2, q) \succeq \overrightarrow{t}$ and $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1, q) \preceq \overrightarrow{s}, \overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_2, q) \preceq \overrightarrow{s}$. Now, $\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_2, q)\} \succeq \overrightarrow{t}$ and $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1,q)+\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_2,q)}{2} \succeq \frac{\overrightarrow{t}+\overrightarrow{t}}{2} = \overrightarrow{t}$ and $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_2, q)\} \preceq \overrightarrow{s}$. This implies that $\upsilon_1 \circledast_1 \upsilon_2 \in \overrightarrow{\Xi^{(t,s)}}$. Similarly, $\upsilon_1 \circledast_2 \upsilon_2 \in \overrightarrow{\Xi^{(t,s)}}$ and $\upsilon_1 \circledast_3 \upsilon_2 \in \overrightarrow{\Xi^{(t,s)}}$. Therefore $\overrightarrow{\Xi^{(t,s)}}$ is a SBS of $\mathscr{S}$ for each $t, s \in D[0,1]$.

Conversely, assume that $\overrightarrow{\Xi^{(t,s)}}$ is a SBS of $\mathscr{S}$ for each $t, s \in D[0,1]$. Suppose if there exist $\upsilon_1, \upsilon_2 \in \mathscr{S}$ such that $\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \prec \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_2, q)\}$, $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \prec \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1,q)+\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_2,q)}{2}$ and $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \succ \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_2, q)\}$. Select $t, s \in D[0,1]$ such that $\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \prec \overrightarrow{t} \preceq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_2, q)\}$ and $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \prec \overrightarrow{t} \preceq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1,q)+\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_2,q)}{2}$ and $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \succ \overrightarrow{s} \succeq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_2, q)\}$. Then $\upsilon_1, \upsilon_2 \in \overrightarrow{\Xi^{(t,s)}}$ but $\upsilon_1 \circledast_1 \upsilon_2 \notin \overrightarrow{\Xi^{(t,s)}}$. This contradicts to that $\overrightarrow{\Xi^{(t,s)}}$ is a SBS of $\mathscr{S}$. Hence, $\max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{T}}}(\upsilon_2, q)\}$, $\max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_1,q)+\overrightarrow{\Xi_Y^{\mathscr{I}}}(\upsilon_2,q)}{2}$ and $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1 \circledast_1 \upsilon_2, q)\} \preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_1, q), \overrightarrow{\Xi_Y^{\mathscr{F}}}(\upsilon_2, q)\}$. Similarly to prove other parts. Hence, $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}$. □

Definition 3.10. Let $(\mathscr{S}_1, \boxminus_1, \boxminus_2, \boxminus_3)$ and $(\mathscr{S}_2, \ominus_1, \ominus_2, \ominus_3)$ be any two bisemirings. Let $\Upsilon : \mathscr{S}_1 \to \mathscr{S}_2$ be any function and $\overrightarrow{Y}$ be any IVQNSSBS in $\mathscr{S}_1$, $\overrightarrow{V}$ be any IVQNSSBS in $\Upsilon(\mathscr{S}_1) = \mathscr{S}_2$. If $\overrightarrow{\Xi_Y} = \left\langle \overrightarrow{\Xi_Y^{\mathscr{T}}}, \overrightarrow{\Xi_Y^{\mathscr{I}}}, \overrightarrow{\Xi_Y^{\mathscr{F}}} \right\rangle$ is an IVQNS set in $\mathscr{S}_1$, then $\overrightarrow{\Xi_V}$ is an IVQNS set in $\mathscr{S}_2$, defined by for all $x \in \mathscr{S}_1$ and $y \in \mathscr{S}_2$,

$$\overrightarrow{\Xi_V^{\mathscr{T}}}(\tau, q) = \begin{cases} \sup\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi, q)\} & \text{if } (\varpi, q) \in \Upsilon^{-1}(\tau, q) \\ 0 & \text{otherwise} \end{cases}$$

$$\overrightarrow{\Xi_V^{\mathscr{I}}}(\tau, q) = \begin{cases} \sup\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi, q)\} & \text{if } (\varpi, q) \in \Upsilon^{-1}(\tau, q) \\ 0 & \text{otherwise} \end{cases}$$

$$\overrightarrow{\Xi_V^{\mathscr{F}}}(\tau, q) = \begin{cases} \inf\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi, q)\} & \text{if } (\varpi, q) \in \Upsilon^{-1}(\tau, q) \\ 1 & \text{otherwise,} \end{cases}$$

which is called the image of Ξ_Y under Υ. Similarly, if $\overrightarrow{\Xi_V} = \left\langle \overrightarrow{\Xi_V^{\mathscr{T}}}, \overrightarrow{\Xi_V^{\mathscr{I}}}, \overrightarrow{\Xi_V^{\mathscr{F}}} \right\rangle$ is an IVQNS set in $\mathscr{S}_2$, then IVQNS set $\overrightarrow{\Xi_Y} = \Upsilon \circ \overrightarrow{\Xi_V}$ in $\mathscr{S}_1$ (i.e., the IVQNS set defined by $\overrightarrow{\Xi_Y}(\varpi, q) = \overrightarrow{\Xi_V}(\Upsilon(\varpi, q)))$ is called the pre-image of $\overrightarrow{\Xi_V}$ under Υ.

Theorem 3.11. *Let $(\mathscr{S}_1, \boxminus_1, \boxminus_2, \boxminus_3)$ and $(\mathscr{S}_2, \ominus_1, \ominus_2, \ominus_3)$ be any two bisemirings. The homomorphic image of IVQNSSBS of $\mathscr{S}_1$ is an IVQNSSBS of $\mathscr{S}_2$.*

Proof. Let $\Upsilon : \mathscr{S}_1 \to \mathscr{S}_2$ be any homomorphism. Then $\Upsilon(\varpi \boxminus_1 \tau, q) = \Upsilon(\varpi, q) \ominus_1 \Upsilon(\tau, q)$, $\Upsilon(\varpi \boxminus_2 \tau, q) = \Upsilon(\varpi, q) \ominus_2 \Upsilon(\tau, q)$ and $\Upsilon(\varpi \boxminus_3 \tau, q) = \Upsilon(\varpi, q) \ominus_3 \Upsilon(\tau, q)$ for all $\varpi, \tau \in \mathscr{S}_1$. Let $\overrightarrow{V} = \Upsilon(\overrightarrow{Y})$, where $\overrightarrow{Y}$ is any IVQNSSBS of $\mathscr{S}_1$. Let $\Upsilon(\varpi, q), \Upsilon(\tau, q) \in \mathscr{S}_2$. Let $(\varpi, q) \in \Upsilon^{-1}(\Upsilon(\varpi, q))$ and $(\tau, q) \in \Upsilon^{-1}(\Upsilon(\tau, q))$

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be such that $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi,q) = \sup_{(\delta,q)\in\Upsilon^{-1}(\Upsilon(\varpi,q))}\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\delta,q)\}$ and $\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau,q) = \sup_{(\delta,q)\in\Upsilon^{-1}(\Upsilon(\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\delta,q)\}$. Now,

$$\begin{aligned}
\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\} &= \sup_{(\delta',q)\in\Upsilon^{-1}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\delta',q)\}\\
&= \sup_{(\delta',q)\in\Upsilon^{-1}(\Upsilon(\varpi\boxminus_1\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\delta',q)\}\\
&= \max\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi\boxminus_1\tau,q)\}\\
&\succeq \min\{\overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau,q)\}\\
&= \min\{\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\tau,q)\}.
\end{aligned}$$

Thus $\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\}\succeq\min\{\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\tau,q)\}$. Similarly,

$$\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\Upsilon(\varpi,q)\ominus_2\Upsilon(\tau,q))\}\succeq\min\{\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\tau,q)\}$$

and

$$\max\{\overrightarrow{\Xi_V^{\mathscr{T}}}(\Upsilon(\varpi,q)\ominus_3\Upsilon(\tau,q))\}\succeq\min\{\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{T}}}\Upsilon(\tau,q)\}.$$

Let $(\varpi,q)\in\Upsilon^{-1}(\Upsilon(\varpi,q))$ and $(\tau,q)\in\Upsilon^{-1}(\Upsilon(\tau,q))$ be such that $\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi,q) = \sup_{(\delta,q)\in\Upsilon^{-1}(\Upsilon(\varpi,q))}\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\delta,q)\}$ and $\overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau,q) = \sup_{(\delta,q)\in\Upsilon^{-1}(\Upsilon(\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\delta,q)\}$. Now,

$$\begin{aligned}
\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\} &= \sup_{(\delta',q)\in\Upsilon^{-1}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\delta',q)\}\\
&= \sup_{(\delta',q)\in\Upsilon^{-1}(\Upsilon(\varpi\boxminus_1\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\delta',q)\}\\
&= \max\{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi\boxminus_1\tau,q)\}\\
&\succeq \frac{\overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi,q)+\overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau,q)}{2}\\
&= \frac{\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\varpi,q)+\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\tau,q)}{2}.
\end{aligned}$$

Thus $\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\}\succeq\frac{\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\varpi,q)+\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\tau,q)}{2}$. Similarly,

$$\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\Upsilon(\varpi,q)\ominus_2\Upsilon(\tau,q))\}\succeq\frac{\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\varpi,q)+\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\tau,q)}{2}$$

and

$$\max\{\overrightarrow{\Xi_V^{\mathscr{I}}}(\Upsilon(\varpi,q)\ominus_3\Upsilon(\tau,q))\}\succeq\frac{\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\varpi,q)+\overrightarrow{\Xi_V^{\mathscr{I}}}\Upsilon(\tau,q)}{2}.$$

Let $\Upsilon(\varpi,q),\Upsilon(\tau,q)\in\mathscr{S}_2$. Let $(\varpi,q)\in\Upsilon^{-1}(\Upsilon(\varpi,q))$ and $(\tau,q)\in\Upsilon^{-1}(\Upsilon(\tau,q))$ be such that $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q) = \inf_{(\delta,q)\in\Upsilon^{-1}(\Upsilon(\varpi,q))}\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\delta,q)\}$ and $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q) = \inf_{z\in\Upsilon^{-1}(\Upsilon(\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\delta,q)\}$. Now,

$$\begin{aligned}
\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\} &= \inf_{(\delta',q)\in\Upsilon^{-1}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\delta',q)\}\\
&= \inf_{(\delta',q)\in\Upsilon^{-1}(\Upsilon(\varpi\boxminus_1\tau,q))}\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\delta',q)\}\\
&= \min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}\\
&\preceq \max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}\\
&= \max\{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)\}.
\end{aligned}$$

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Thus $\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\}\preceq\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)\}$. Similarly,

$$\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_2\Upsilon(\tau,q))\}\preceq\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)\}$$

and

$$\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_3\Upsilon(\tau,q))\}\preceq\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)\}.$$

Hence, $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S}_2$. □

Theorem 3.12. *Let $(\mathscr{S}_1,\boxminus_1,\boxminus_2,\boxminus_3)$ and $(\mathscr{S}_2,\ominus_1,\ominus_2,\ominus_3)$ be any two bisemirings. The homomorphic pre-image of IVQNSSBS of $\mathscr{S}_2$ is an IVQNSSBS of $\mathscr{S}_1$.*

Proof. Let $\Upsilon:\mathscr{S}_1\to\mathscr{S}_2$ be any homomorphism. Then $\Upsilon(\varpi\boxminus_1\tau,q)=\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q),\Upsilon(\varpi\boxminus_2\tau,q)=\Upsilon(\varpi,q)\ominus_2\Upsilon(\tau,q)$, and $\Upsilon(\varpi\boxminus_3\tau,q)=\Upsilon(\varpi,q)\ominus_3\Upsilon(\tau,q)$ for all $\varpi,\tau\in\mathscr{S}_1$. Let $\overrightarrow{V}=\Upsilon(\overrightarrow{Y})$, where $\overrightarrow{V}$ is any IVQNSSBS of $\mathscr{S}_2$. Let $\varpi,\tau\in\mathscr{S}_1$. Now, $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}=\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi\boxminus_1\tau,q))\}=\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\}\succeq\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)\}=\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}$. Thus $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}\succeq\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}$. Now, $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}=\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi\boxminus_1\tau,q))\}=\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\}\succeq\frac{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q)+\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)}{2}=\frac{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)+\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)}{2}$. Thus $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}\succeq\frac{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)+\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)}{2}$. Now, $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}=\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi\boxminus_1\tau,q))\}=\min\{\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\}\preceq\max\{\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\varpi,q),\overrightarrow{\Xi_V^{\mathscr{F}}}\Upsilon(\tau,q)\}=\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}$. Therefore, $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\preceq\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}$. Hence, $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}_1$. □

Theorem 3.13. *Let $(\mathscr{S}_1,\boxminus_1,\boxminus_2,\boxminus_3)$ and $(\mathscr{S}_2,\ominus_1,\ominus_2,\ominus_3)$ be any two bisemirings. If $\Upsilon:\mathscr{S}_1\to\mathscr{S}_2$ is a homomorphism, then $\Upsilon(\overrightarrow{Y_{(t,s)}})$ is a level SBS of IVQNSSBS $\overrightarrow{V}$ of $\mathscr{S}_2$.*

Proof. Let $\Upsilon:\mathscr{S}_1\to\mathscr{S}_2$ be any homomorphism. Then $\Upsilon(\varpi\boxminus_1 y,q)=\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q),\Upsilon(\varpi\boxminus_2\tau,q)=\Upsilon(\varpi,q)\ominus_2\Upsilon(\tau,q)$, and $\Upsilon(\varpi\boxminus_3\tau,q)=\Upsilon(\varpi,q)\ominus_3\Upsilon(\tau,q)$ for all $\varpi,\tau\in\mathscr{S}_1$. Let $\overrightarrow{V}=\Upsilon(\overrightarrow{Y})$, where $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}_1$. By Theorem 3.11, $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S}_2$. Let $\overrightarrow{Y_{(t,s)}}$ be any level SBS of $\overrightarrow{Y}$. Suppose that $\varpi,\tau\in\overrightarrow{Y_{(t,s)}}$. Then $\Upsilon(\varpi\boxminus_1\tau,q),\Upsilon(\varpi\boxminus_2\tau,q),\Upsilon(\varpi\boxminus_3\tau,q)\in\overrightarrow{Y_{(t,s)}}$. Now, $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q))=\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)\succeq\overrightarrow{t},\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\tau,q))=\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\succeq\overrightarrow{t}$. Thus $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\succeq\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\succeq\overrightarrow{t}$. Now, $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q))=\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)\succeq\overrightarrow{t},\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\tau,q))=\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\succeq\overrightarrow{t}$. Thus $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\succeq\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\succeq\overrightarrow{t}$. Now, $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q))=\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)\preceq\overrightarrow{s}$ and $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\tau,q))=\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\preceq\overrightarrow{s}$. Thus $\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q))\preceq\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\preceq\overrightarrow{s}$ for all $\Upsilon(\varpi,q),\Upsilon(\tau,q)\in\mathscr{S}_2$. Hence, $\Upsilon(\overrightarrow{Y_{(t,s)}})$ is a level SBS of IVQNSSBS $\overrightarrow{V}$ of $\mathscr{S}_2$. □

Theorem 3.14. *Let $(\mathscr{S}_1,\boxminus_1,\boxminus_2,\boxminus_3)$ and $(\mathscr{S}_2,\ominus_1,\ominus_2,\ominus_3)$ be any two bisemirings. If $\Upsilon:\mathscr{S}_1\to\mathscr{S}_2$ is any homomorphism, then $\overrightarrow{Y_{(t,s)}}$ is a level SBS of IVQNSSBS $\overrightarrow{Y}$ of $\mathscr{S}_1$.*

Proof. Let $\Upsilon:\mathscr{S}_1\to\mathscr{S}_2$ be any homomorphism. Then $\Upsilon(\varpi\boxminus_1\tau,q)=\Upsilon(\varpi,q)\ominus_1\Upsilon(\tau,q),\Upsilon(\varpi\boxminus_2\tau,q)=\Upsilon(\varpi,q)\ominus_2\Upsilon(\tau,q)$, and $\Upsilon(\varpi\boxminus_3\tau,q)=\Upsilon(\varpi,q)\ominus_3\Upsilon(\tau,q)$ for all $\varpi,\tau\in\mathscr{S}_1$. Let $\overrightarrow{V}=\Upsilon(\overrightarrow{Y})$, where $\overrightarrow{V}$ is an IVQNSSBS of $\mathscr{S}_2$. By Theorem 3.12, $\overrightarrow{Y}$ is an IVQNSSBS of $\mathscr{S}_1$. Let $\Upsilon(\overrightarrow{Y_{(t,s)}})$ be a level SBS of $\overrightarrow{V}$. Suppose that $\Upsilon(\varpi,q),\Upsilon(\tau,q)\in\Upsilon(\overrightarrow{Y_{(t,s)}})$. Then $\Upsilon(\varpi\boxminus_1\tau,q),\Upsilon(\varpi\boxminus_2\tau,q),\Upsilon(\varpi\boxminus_3\tau,q)\in\Upsilon(\overrightarrow{Y_{(t,s)}})$. Now, $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)=\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q))\succeq\overrightarrow{t},\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)=\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\tau,q))\succeq\overrightarrow{t}$. Thus $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}\succeq\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}\succeq\overrightarrow{t}$. Now, $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)=\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q))\succeq\overrightarrow{t},\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)=\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\tau,q))\succeq\overrightarrow{t}$. Thus $\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}\succeq\frac{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)+\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)}{2}\succeq\overrightarrow{t}$. Now, $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q)=\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\varpi,q))\preceq\overrightarrow{s}$ and $\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)=\overrightarrow{\Xi_V^{\mathscr{F}}}(\Upsilon(\tau,q))\preceq\overrightarrow{s}$. Thus $\min\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi\boxminus_1\tau,q)\}=\max\{\overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi,q),\overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau,q)\}\preceq\overrightarrow{s}$ for all $\varpi,\tau\in\mathscr{S}_1$. Hence, $\overrightarrow{Y_{(t,s)}}$ is a level SBS of IVQNSSBS $\overrightarrow{Y}$ of $\mathscr{S}_1$. □

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4 Interval valued Q-neutrosophic SBSs (IVQNSNSBSs)

Definition 4.1. An IVQNS set $\overrightarrow{Y}$ of $\mathscr{S}$ is said to be an IVQNSNSBS of $\mathscr{S}$ if

$$\begin{Bmatrix} \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi \circledast_1 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau \circledast_1 \varpi, q) \\ \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi \circledast_2 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau \circledast_2 \varpi, q) \\ \overrightarrow{\Xi_Y^{\mathscr{T}}}(\varpi \circledast_3 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{T}}}(\tau \circledast_3 \varpi, q) \end{Bmatrix} \begin{Bmatrix} \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi \circledast_1 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau \circledast_1 \varpi, q) \\ \text{or} \\ \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi \circledast_2 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau \circledast_2 \varpi, q) \\ \text{or} \\ \overrightarrow{\Xi_Y^{\mathscr{I}}}(\varpi \circledast_3 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{I}}}(\tau \circledast_3 \varpi, q) \end{Bmatrix}$$

$$\begin{Bmatrix} \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi \circledast_1 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau \circledast_1 \varpi, q) \\ \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi \circledast_2 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau \circledast_2 \varpi, q) \\ \overrightarrow{\Xi_Y^{\mathscr{F}}}(\varpi \circledast_3 \tau, q) = \overrightarrow{\Xi_Y^{\mathscr{F}}}(\tau \circledast_3 \varpi, q) \end{Bmatrix}$$

for all $\varpi, \tau \in \mathscr{S}$.

Corollary 4.2. *(1) The intersection of a family of IVQNSNSBSs of $\mathscr{S}$ is an IVQNSNSBS of $\mathscr{S}$.*

(2) If $\overrightarrow{Y_1}, \overrightarrow{Y_2}, ..., \overrightarrow{Y_n}$ are IVQNSNSBSs of bisemirings $\mathscr{S}_1, \mathscr{S}_2, ..., \mathscr{S}_n$, respectively, then $\overrightarrow{Y_1 \times Y_2 \times ... \times Y_n}$ is an IVQNSNSBS of $\mathscr{S}_1 \times \mathscr{S}_2 \times ... \times \mathscr{S}_n$.

(3) Let $\overrightarrow{Y}$ be any IVQNSNSBS of $\mathscr{S}$ and $\overrightarrow{V}$ be the strongest IVNS relation of $\mathscr{S}$. Then $\overrightarrow{Y}$ is an IVQNSNSBS of $\mathscr{S}$ if and only if $\overrightarrow{V}$ is an IVQNSNSBS of $\mathscr{S} \times \mathscr{S}$.

(4) The homomorphic image of any IVQNSNSBS of $\mathscr{S}_1$ is an IVQNSNSBS of $\mathscr{S}_2$.

(5) The homomorphic pre-image of any IVQNSNSBS of $\mathscr{S}_2$ is an IVQNSNSBS of $\mathscr{S}_1$.

5 Conclusion

We created the IVQNSSBS level set concepts. We presented a method for using IVQNSNSBS over bisemiring. The main goal of this study is to present a homomorphism to IVQNSSBS of bisemirings. Therefore, we consider the uses of cubic Q-neutrosophic SBS in the future.

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References

[1] J. Ahsan, K. Saifullah, F. Khan, Fuzzy semirings, Fuzzy Sets and systems, vol. 60, pp. 309-320, 1993.

[2] M. Al-Tahan, B. Davvaz, M. Parimala, A note on single valued neutrosophic sets in ordered groupoids, International Journal of Neutrosophic Science, vol. 10, no. 2, pp. 73-83, 2020.

[3] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision making problems, Journal of Intelligent and Fuzzy Systems, vol. 36, pp. 2829-2844, 2019.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87-96, 1986.

Received: July 08, 2022 Accepted: December 22, 2022

[5] B. C. Cuong, V. Kreinovich, Picture fuzzy sets a new concept for computational intelligence problems, Proceedings of 2013 Third World Congress on Information and Communication Technologies (WICT 2013), IEEE, pp. 1-6, 2013.

[6] F. Hussian, R. M. Hashism, A. Khan, M. Naeem, Generalization of bisemirings, International Journal of Computer Science and Information Security, vol. 14, no. 9, pp. 275-289, 2016.

[7] A. Iampan, P. Jayaraman, S. D. Sudha, N. Rajesh, Interval-valued neutrosophic ideals of Hilbert algebras, International Journal of Neutrosophic Science, vol. 18, no. 4, pp. 223-237, 2022.

[8] A. Iampan, P. Jayaraman, S. D. Sudha, N. Rajesh, Interval-valued neutrosophic subalgebras of Hilbert algebras, Asia Pacific Journal of Mathematics, vol. 9, Article no. 16, 2022.

[9] L. Jagadeeswari, V. J. Sudhakar, V. Navaneethakumar, S. Broumi, Certain kinds of bipolar interval valued neutrosophic graphs, International Journal of Neutrosophic Science, vol. 16, no. 1, pp. 49-61, 2021.

[10] S. J. Golan, Semirings and their applications, Kluwer Academic Publishers, London, 1999.

[11] M. Palanikumar, K. Arulmozhi, On various ideals and its applications of bisemirings, Gedrag and Organisatie Review, vol. 33, no. 2, pp. 522-533, 2020.

[12] M. Palanikumar, K. Arulmozhi, On intuitionistic fuzzy normal subbisemirings of bisemirings, Nonlinear Studies, vol. 28, no. 3, pp. 717-721, 2021.

[13] M. Palanikumar, K. Arulmozhi, On new ways of various ideals in ternary semigroups, Matrix Science Mathematic, vol. 4, no. 1, pp. 6-9, 2020.

[14] M. Palanikumar, K. Arulmozhi, (α, β)-Neutrosophic subbisemiring of bisemiring, Neutrosophic Sets and Systems, vol. 48, pp. 368-385, 2022.

[15] M. Palanikumar, K. Arulmozhi, On various tri-ideals in ternary semirings, Bulletin of the International Mathematical Virtual Institute, vol. 11, no. 1, pp. 79-90, 2021.

[16] M. Palanikumar, K. Arulmozhi, On Pythagorean normal subbisemiring of bisemiring, Annals of Communications in Mathematics, vol. 4, no. 1, pp. 63-72, 2021.

[17] M. Palanikumar, K. Arulmozhi, On various almost ideals of semirings, Annals of Communications in Mathematics, vol. 4, no. 1, 17-25, 2021.

[18] M. K. Sen, S. Ghosh, An introduction to bisemirings, Asian Bulletin of Mathematics, vol. 28, no. 3, pp. 547-559, 2001.

[19] F. Smarandache, A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.

[20] R. R. Yager, Pythagorean membership grades in multi criteria decision-making, IEEE Transactions on Fuzzy Systems, vol. 22, no. 4, pp. 958-965, 2014.

[21] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8, no. 3, pp. 338-353, 1965.

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