



n-Refined Indeterminacy of Some Modules

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Abstract

This article presents the notion of n-refined neutrosophic modules such as cyclic, simple, and finitely generated modules. n-refined neutrosophic is a generalization of neutrosophic properties. This paper presents new relations among n-refined neutrosophic modules. Finally, several examples and properties have been studied about the relations between these modules.

Keywords: Simple module; cyclic module; finitely generated module; neutrosophic set; neutrosophic Modul.

1. Introduction

The neutrosophic notion was founded by Smarandache [1] in 1998 as an extension of fuzzy [2] and intuitionistic fuzzy [3] notions to deal with indeterminacy in real-life issues. This idea has been expanded and applied to various fields of mathematics, such as complex space [4-7], topology space [8-10], statistics, probability [11,12]. In neutrosophic algebra, several useful studies have emerged that have worked to highlight the role of indeterminacy in numerous algebraic structures. The neutrosophic triplet group was introduced by Smarandache and Ali [13]. Jun et al. [14] constructed neutrosophic quadruple BCK/BCI-numbers. Abobala [15] devoted some linear equations over the neutrosophic field. Abed et al. [16] introduced some new results of the neutrosophic multiplication module. Al-Hamido [17] discussed a new approach to a neutrosophic algebraic structure called neutrosophic groupoid. Zail et al. [18] studied some new results of a generalization of BCK-algebra (Ω -BCK-algebra). and other contributions, see [19-22].

A refined neutrosophic structure [23] was proposed by dividing the indeterminacy part into two levels of subindeterminacies. Recently, the idea of n-refined neutrosophic structures was defined as a generalization of Refined neutrosophic, and it was used by [24,25]. In this paper, we give some new results on n-Refined Indeterminacy of some Modules based on n-refined neutrosophic idea, where we present new relations of n-refined neutrosophic Modules.

2. Preliminaries

This section is about some of the vital definitions, remarks and examples which are used later in this paper.

Definition 2.1. [2]

Let X be a nonempty set. A fuzzy set $A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$ is characterized by a membership function $\mu_A: X \rightarrow [0, 1]$, where $\mu_A(x)$ is interpreted as the degree of membership of the element x in the fuzzy subset A for any $x \in X$.

Definition 2.2. [1]

Let A be a universal set. The neutrosophic A , in short $NE(A)$ is defined as

$$B = \{(\xi, tH(\xi), iH(\xi), fH(\xi)) : \xi \in A\} \ni tH, iH, fH : A \rightarrow [0, 1].$$

Note that there is an equivalent definition to Definition 2.2 and by the following:

Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \rightarrow]-0, 1+[$ and an indeterminacy function $\sigma_A : X \rightarrow]-0, 1+[$ and a non-membership function $\nu_A : X \rightarrow]-0, 1+[$ which satisfy the condition: $-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3+$, for any $x \in X$.

Definition 2.3. [14]

Let (G, \times) be a group. Then $NE(G) = (\langle G \cup I \rangle, *)$ is said to be neutrosophic group and generated by G and I with binary operation $*$, where $\langle G \cup I \rangle = \{a_1 + a_2 I \mid a_1, a_2 \in G\}$.

Remark 2.2.

- 1- We denote I to neutrosophic element such That $I^2 = I$
- 2- The inverse of I does not exists.
- 3- All integers $n, n + I, nI$ and $0, I, 0.I^{-1}$ are neutrosophic element.
- 4- $NE(G)$ is commutative If $a_1 a_2 = a_2 a_1 \forall a_1, a_2 \in NE(G)$.
- 5- $NE(R) = \langle G \cup I \rangle$ is not a group.
- 6- Any group $(G, *) \subseteq NE(G)$.

Definition 2.4. [15]

Let $(R, +, \cdot)$ be a ring. Then $NE(R) = (\langle R \cup I \rangle, +, \cdot)$ is said to be neutrosophic ring and generate by R and I with two binary operation $(+)$ and (\cdot) .

Remarks 2.5.

- 1- $I^2 = I, 0.I = 0, I^{-1}, n, n + I$ and nI are neutrosophic elements.
- 2- I^{-1} not exists

Example 2.6.

A pair $\langle Z \cup I \rangle$ is a neutrosophic ring, where Z is the integer numbers.

Now we can present neutrosophic Module on based neutrosophic group and neutrosophic ring.

Definition 2.7. [20]

A commutative group M is called R -Module With the ring R where $*$: $R \times M \rightarrow M$ define by: $f(r_1 m) = r m \forall r \in R, m \in M$ such that: 1- $r(m_1 + m_2) = r m_1 + r m_2 \forall r \in R, m_1, m_2 \in M$.

$$2- (r_1 + r_2) m = r_1 m + r_2 m \forall r_1 \in R, r_2 \in R, m \in M.$$

$$3- r_1 (r_2 m) = (r_1 r_2) m$$

$$4- 1.m = m = m.1$$

Definition 2.8. [21]

Let M be an R -Module and let I be indeterminacy (neutrosophic). Then $(M(I), +, \cdot)$ is called neutrosophic Module over $R(I)$ Where $R(I)$ is a neutrosophic ring.

Remark 2.9.

Any element in $M(I)$ has form $a + b \ni a, b \in M$ ($M(I) = M + MI$).

3. Main Results.**Definition 3.1.**

We say $M_n(I)$ is n -refined neutrosophic Module where $M_n(I) = M + MI_1 + \dots + MI_n = \{a_0 + a_1I + a_2I + \dots + a_nI_n\}$.

Remark 3.2. $R_n(I)$ is called n -refined neutrosophic ring and $G_n(I)$ is called n -refined neutrosophic group.

Definition 3.3.

Any R -Module M is called cyclic if $\forall () \in M$, then $M = Rx$ ($(x) = M$). So $M(I)$ is called cyclic neutrosophic Module if $M_c(I) = R(I)x$ and $M_e(I) = R(I)x = \{(rx)I : r \in R(I), x \in M(I)\}$.

Example 3.4.

If Z_e as a neutrosophic $Z(I)$ -Modules, then $Z_e(I)$ is cyclic neutrosophic Module, because there exists $2I \in Z_e(I) \ni$

$$(2I) = 2Z(I) = \{2Z(I) : z \in Z(I)\} = Z_e(I).$$

Example 3.5.

Let $Q(I)$ as a $Z(I)$ -Module. Then $Q(I)$ is not cyclic neutrosophic Module, because there is no $xI \in Q(I) \ni (xI) = Q(I)$.

Note that from **Example 3.4**; we say $Z_{e(n)}(I)$ is n -refined cyclic neutrosophic Module because $2I_n = 2Z(I) = Z_{e(n)}(I)$. Also, since $\nexists xI \in Q(I) \ni (xI) = Q(I)$, then $Q(I)$ is not n -refined neutrosophic cyclic Module.

Theorem 3.6.

Let $R^2(I)$ be an neutrosophic Module over neutrosophic ring $R(I)$. Then $R^2(I)$ is not cyclic neutrosophic Module.

Proof.

If xI generating $R^2(I)$, then xI has at least two neutrosophic elements like $\{(1, 0)I_1, (0, 1)I_2\}$. Hence $\nexists xI \in R^2(I) \ni (xI) = R^2(I)$.

Remarks 3.7.

- 1- $Z_n(I)$ as a $Z_n(I)$ Module is n -refined cyclic neutrosophic Module.
- 2- $R^2(I)$ as a $R(I)$ -Module is not n -refined cyclic neutrosophic Module.
- 3- $K_{Z_n}(I)$ is n -refined cyclic submodule of $Z_n(I)$, because $k \in K_{Z_n}(I) \ni (k) = kZ_n(I)$ where n -refined neutrosophic submodule define in general by:

Definition 3.8.

Let $M_n(I)$ be n -refined neutrosophic Module over the n -refined neutrosophic ring $R_n(I)$. Then $\phi \neq N_n(I) \subseteq M_n(I)$ is called n -refined neutrosophic submodule of $M_n(I)$ if $N_n(I) \leq M_n(I)$.

Theorem 3.9.

Let $M_n(I)$ be n -refined neutrosophic Module and let $(xI_n) \in M_n(I)$. Then $(xI_n) = R_n(xI_n) = \{rxI_n : r \in R_n(I)\}$ is n -refined neutrosophic submodule of $M_n(I)$.

Proof.

Since $(0I_n) \in M_n(I)$ and $(0x) = 0$, then $0 \in (xI_n)$. So $(xI_n) \neq \emptyset$. Now, let $r_1xI_n, r_2xI_n \in (xI_n) \forall r_1, r_2 \in R_n(I)$. Then $r_1xI_n + r_2xI_n = (r_1+r_2)xI_n, r_1 + r_2 \in R_n(I) = r_3xI_n \in (xI_n)$.

Also, let $rxI_n \in (xI_n)$ and let $s \in R_n(I)$, so $S(rxI_n) = (Sr)xI_n$, ($sr \in R_n(I)$, $t = sr$). So $(sr)xI_n = txI_n \in (xI_n)$. Thus $(xI_n) = R_n(I)x \leq M_n(I)$.

Definition 3.10.

Let M be an R -Module. Then M is called simple if $\emptyset \neq M$ and M have only two submodules $\{0\}$ and M .

Definition 3.11.

Let $M_n(I)$ be n -refined neutrosophic Module over n -refined neutrosophic ring $R_n(I)$. Then $M_n(I)$ is called n -refined simple neutrosophic Module if $\emptyset \neq M_n(I)$ and $M_n(I)$ have only two n -refined neutrosophic submodules $0I$ and $M_n(I)$.

Example 3.12.

Let $Z_{n(p)}(I)$ as a $Z_n(I)$ -Module is n -refined simple neutrosophic Module. Where p is neutrosophic prime number, because there are only two n -refined neutrosophic submodules are $0I$ and $Z_{n(p)}(I)$.

Example 3.13.

$Z(I)$ as a $Z_n(I)$ -Module is not n -refined simple neutrosophic Module, because $(2I_1) \leq Z_6(I)$ and $3I_2 \leq Z_6(I)$, but $(2I_1)$ and $(3I_2)$ are proper submodules of Z_6 .

Theorem 3.14.

Let $M_n(I)$ be n -refined neutrosophic Module. If $M_n(I)$ is n -refined simple neutrosophic Module, then it is n -refined cyclic neutrosophic Module.

Proof.

Let $M_n(I)$ be n -refined simple neutrosophic Module and let $0 \neq xI$ be a neutrosophic element of $M_n(I)$. So $(xI) \leq M_n(I)$, because $xI \in M_n(I)$, so $(xI) = R_n(I)xI = \{rxI : r \in R_n(I) \leq M_n(I)\}$. Since $M_n(I)$ is n -refined simple neutrosophic Module, then $(xI) = 0$ or $xI = M_n(I)$. We have $xI \neq 0$. Then $xI = M_n(I)$. Thus $M_n(I)$ is n -refined cyclic neutrosophic Module.

Remark 3.15.

The converse Theorem 3-14 is not true in general, for example; Z_6 as $Z_n(I)$ -Module is n -refined cyclic neutrosophic Module,

but Z_6 is not n -refined simple neutrosophic Module because; $(2I) = \{0I_1, 2I_2, 4I_3\}$ and $(3I) = \{0I, 3I\}$ are proper n -refined neutrosophic submodule of Z_6 .

Corollary 3.16.

Let $M_n(I)$ be n -refined neutrosophic Module. Then $M_n(I)$ is n -refined Simple neutrosophic Module iff $M_n(I) \neq 0I$ and $\forall 0I \neq mI \in M_n(I)$, $mI \neq 0$, so $(mI)R_n(I) = M_n(I)$.

Proof.

\Rightarrow Let $mI \neq 0$, so $mI = mI \cdot 1 \in mIR_n(I)$. Then $mIR_n(I) \neq 0I$. Hence $mIR_n(I) = M_n(I)$.

\Leftarrow Let $0I \leq N_n(I)$ and $0I \neq mI \in N_n(I)$. Then $mIR_n(I) = M_n(I)$.

Definition 3.17.

Let $M_n(I)$ be n -refined neutrosophic Module. We say $M_n(I)$ is n -refined finitely generated neutrosophic Module if it is generated by a finite subset $X_n(I)$, that is $M_n(I) = \langle X_n(I) \rangle$.

Remarks and Examples 3.18.

1) $M_n(I) = \langle x_1I, x_2I, \dots, x_kI \rangle = \{ \sum (r_iI)(x_iI) : r_iI \in R_n(I), x_iI \in X_n(I), kI \in Z_n(I) \}$.

2) $M_n(I) = \langle xI, yI \rangle = rIxI + rIyI$, Module generate by two elements xI, yI .

3) $M_n(I) = \langle xI, yI, zI \rangle = rIxI + sIyI + tIzI$, Module generated by three elements.

4) Let $M_n(I) = Z_4(I) = \{0I, 1I, 2I, 3I\}$ as $Z_n(I)$ -Module and $X_n(I) = \{0I, 1I, 2I\}$. Since $1I + 2I = 3I$, then $M_n(I) = \langle X_n(I) \rangle = \{0I, 1I, 2I, 3I\} = Z_4(I)$. So $Z_4(I)$ is n -refined finitely generated neutrosophic Module.

- 5) The rational numbers $Q_n(I)$ as $Z_n(I)$ -Module is not n -refined finitely generated neutrosophic Module, because there is no finite n -refined neutrosophic subset $X_n(I)$ such that $Q_n(I) = \langle X_n(I) \rangle$ as a $Z_n(I)$ -Module.
- 6) If $X_n(I)$ is n -refined neutrosophic subset of $R_n(I)$ -Module, then $\langle X_n(I) \rangle$ will denote the n -refined neutrosophic intersection of all submodules of $M_n(I)$ which contains $X_n(I)$ ($\langle X_n(I) \rangle = (\cap A_n(I), X_n(I) \subseteq A_n(I)$ with $A_n(I) \leq M_n(I)$).

Theorem 3.19.

Let $X_n(I)$ be n -refined neutrosophic subset of n -refined neutrosophic $R_n(I)$ -Module $M_n(I)$. Then $\langle X_n(I) \rangle$ is the Smallest n -refined neutrosophic submodule of $M_n(I)$ that Contains $X_n(I)$.

Proof.

since $\langle X_n(I) \rangle \leq M_n(I)$ with $\langle X_n(I) \rangle = \cap A_n(I)$, $X_n(I) \subseteq A_n(I)$, $A_n(I) \leq M_n(I)$, so if $N_n(I)$ is n -refined neutrosophic submodule of $M_n(I)$ ($N_n(I) \leq M_n(I)$), such that $X_n(I) \subseteq N_n(I)$, then $\langle X_n(I) \rangle \subseteq N_n(I)$. Hence $\langle X_n(I) \rangle$ is the smallest n -refined neutrosophic submodule of $M_n(I)$, $X_n(I) \subseteq M_n(I)$. **Example 3.20.** Let $X_n(I) = \{4I, 8I\}$. Then $\langle 4I, 8I \rangle = 4IZ_n(I)$.

Theorem 3.21.

Every n -refined cyclic neutrosophic Module is n -refined finitely generated neutrosophic Module.

Proof.

Let $M_n(I)$ be n -refined neutrosophic $R_n(I)$ -Module. Suppose that $M_n(I)$ is a cyclic. Then

$$\exists xI \in M_n(I) \ni \langle xI \rangle = M_n(I).$$

Since $\{xI\}$ is neutrosophic singleton set, then $\{xI\}$ is finite neutrosophic subset of $M_n(I)$ and $\langle \{xI\} \rangle = M_n(I)$. Thus $M_n(I)$ is n -refined finitely generated neutrosophic Module.

Remark 3.22.

The converse of Theorem 3.11. is not true for example, let $M_n(I)$ as $Z_n(I)$ -Module $\ni R^2 = M_n(I) = Z_3(I) \oplus Z_6(I)$. Then $M_n(I)$ is n -refined finitely generated neutrosophic Module and $M_n(I) = \langle X_n(I) \rangle$, $X_n(I) = \{(1, 0)I, (0, 1)I\}$. But this means $M_n(I)$ generate by two neutrosophic element and there is no one neutrosophic element $xI \in M_n(I) \ni M_n(I) = \langle xI \rangle$. Thus $M_n(I)$ is not n -refined cyclic neutrosophic Module.

Corollary 3:23.

The $R_n(I)$ -Module $M_n(I)$ is n -finitely generated neutrosophic Module if and only if there is in every Set $\{A_{in}(I) : i \in I\}$ of n -refined neutrosophic submodules ($A_{in}(I) \leq M_n(I)$) with $\sum_{i \in I} A_{in}(I) = M_n(I)$ a finite neutrosophic subset $\{A_{in}(I) : i \in I_0\}$ such that $\sum_{i \in I_0} A_{in}(I) = M_n(I)$ with $I_0 \subseteq I$ and I_0 is finite.

Proof.

\Rightarrow Let $M_n(I)$ be n -refined finitely generated neutrosophic Module. So,

$M_n(I) = m_1IR_n(I) + m_2IR_n(I) + \dots + m_rIR_n(I)$. Since $\sum A_{in}(I) = M_n(I)$, $i \in I$, then every m_{in} is finite sum of neutrosophic elements from $A_n(I)$, So there exists a finite neutrosophic subset $I_0 \subset I \ni M_{1n}I, M_{2n}I, M_{3n}I, \dots, M_{kn}I \in \sum A_{in}(I)$, $i \in I_0$. Then $M_n(I) \leq \sum_{i \in I_0} A_{in}(I) \leq M_n(I)$.

Thus $M_n(I) = \sum_{i \in I_0} A_{in}(I)$, $i \in I_0$.

\Leftarrow Consider the n -refined neutrosophic set of n -refined neutrosophic Submodules $\{rI \in R_n(I), mI \in M_n(I)$. Hence there exists n -refined neutrosophic subset $\{m_1IR_n(I), m_2IR_n(I), \dots, m_nIR_n(I)\}$ such that

$m_1IR_n(I) + m_2IR_n(I) + \dots + m_nIR_n(I) = M_n(I)$. Thus $M_n(I)$ is n -refined finitely generated neutrosophic Module.

Proposition 3.24.

Let $M_n(I)$ be n -refined Noetherian neutrosophic Module. Then $M_n(I)$ is n -refined f. generated neutrosophic Module.

Proof.

Suppose that $F_n(A)$ the family represents to all n -refined f -generated neutrosophic submodule in $M_n(I)$. So $F_n(A)$ has n -refined neutrosophic maximal element and satisfied neutrosophic ascending chain condition. Assume that $mI \in M_n(I)$. Hence $xI + (mI)R_n(I)$ is n -refined f -generated neutrosophic submodule of $M_n(I)$. Therefore, $x_0I + (mI)R_n(I)$ (neutrosophic maximally of x_0I). So $xI \in x_0I$ and hence $M_n(I) = x_0I$. Then $M_n(I)$ is n -refined f -generated neutrosophic Module.

Definition 3.24.

Any Module $M_n(I)$ is called n -refined Noetherian neutrosophic module if it Satisfies n -refined neutrosophic ascending Chain Condition (ACC) of submodules:

$$0 = A_0(I) \subseteq A_1(I) \subseteq A_2(I) \subseteq \dots \subseteq A_n(I) = R_n(I).$$

Corollary 3.26.

If $M_n(I)$ is n -refined neutrosophic module has finite length property. Then $M_n(I)$ is n -refined f -generated neutrosophic module.

Proof.

Suppose that $M_n(I)$ is n -refined neutrosophic finite length. So $M_n(I)$ is n -refined Noetherian neutrosophic module. Thus $M_n(I)$ is n -refined neutrosophic f -generated.

4. Conclusion

In this work, we have employed the idea of the refined neutrosophic set to produce some modules, such as cyclic, simple, and finitely generated modules. Also, we showed the new relations of n -refined neutrosophic modules as well as several examples and properties that have been studied about the relations. Finally, we hope that this study will show the importance of neutrosophic ideas in strengthening different algebraic structures.

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