Study of Energy and Momentum Coefficients for Compound Transitions in an Open Channels

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Abstract

A series of laboratory experiments was performed in order to investigate momentum and kinetic energy correction coefficients in compound transitions of rectangular channel. Also, In this study, the influence of several independent factors on the kinetic energy and momentum correction coefficients that are connected to the characteristics of the channels transitions is investigated. The Reynolds number varied between 2145.8 and 14388.14. Kinetic energy and momentum correction coefficients, α and β, were computed for different models by using two methods. The first one was based on the calculation of ε and the second method was based on dividing the cross section into slides and using the corresponding velocity for each slides. The results of the present study show that the second one was found to be more accurate than the first method. This method of slides can be used to derive the relationship between both energy and momentum coefficients versus ε values. The values of α and β averaged 1.237 and 1.10, respectively. The energy and momentum coefficients exhibit higher values in the transition zone than the corresponding ones in the upstream section. This is mainly due to turbulence effect. The values of (α-1/β-1) ranges from 2.71 to 2.81, this range don't exceed 4% for different Reynolds number. The error of (α-1/β-1) for different discharges is less than 5% which could be accepted.

Keywords: Compound transition; Open channel; Energy loss; Momentum; Turblent

1. Introduction

Kinetic energy and momentum principles often are used in hydraulic problems. In open channel flow, normally the velocity distributions are not uniform over the cross-section; hence, the velocity head and the momentum flux are generally greater than the values computed by using the average velocity. These values may be corrected by using the so-called energy and momentum correction coefficients. Kinetic energy and momentum correction coefficients, α and β, often are assumed to be unity when the energy and momentum principles are used in the computations (such as Chow, 1959; Streeter and Wylie, 1979; French, 1987; Massey, 1989; Roberson and Crowe, 1998); [1, 9]. On the other hand, because of the non uniform distribution of velocities over a channel cross section, α and β generally are greater than unity. Different theoretical expressions for α and β based on different assumptions and conditions have been derived by many authors (Rouse, 1965; Golubtsov, 1976; Benedict, 1980; Fox and McDonald, 1985; Chen,1991) .Also, kinetic energy and momentum correction coefficients were presented by measuring vertical and lateral velocity distribution in a compound channel cross section, [11]. In asymmetric compound cross-section flumes, the correction coefficients for

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momentum and kinetic energy were examined by [2]. The values of kinetic and momentum coefficients were determined experimentally under varying flow conditions of Froude no. New relations were suggested to determine approximate values of α & β, [7]. The velocity profile at thirteen different sections between two consecutive bend apex was illustrated. The depth-averaged velocity along the path was also shown. Due to the variation in the velocity profile, the changes in the momentum and energy correction coefficients were studied,[13]. The effect of the transition angle and the contraction ratio was investigated on the flow properties and on the scour phenomenon downstream water structures. The study proved that, for an efficient hydraulic performance and economic design, the best transition angle (θ) for the approaches of water structures is 30° with a contraction ratio (r) not less than 0.6, [3]. The kinetic energy and momentum correction factors of non- prismatic compound channels were explored in relation to the effects of several independent variables connected to the properties of the compound channels. Finally, the created models were validated using the data set and a natural river system [10]. Ignition characteristics were analyzed by narrowing (sudden narrowing, transition, and Radius). The investigation demonstrates that the specific energy of the water flow through the sensation of narrowing changes. A sudden constriction type results in the greatest specific energy loss. The experimental data from this and other research studies show a good correlation with the projected results [8].

2. Methodology

Kinetic energy and momentum correction coefficients, α and β, are computed using the following equations.

The true mean velocity head is found by adjusting this term by:

$$\alpha = \frac{1}{U^3 A}\int u^3 dA = \frac{\sum u^3 dA}{AU^3} \qquad (1)$$

for (1/m) power law velocity distribution, the kinetic energy coefficient equal to

$$\alpha = \frac{(m+1)^3}{m^2(m+3)} \qquad (2)$$

For a typical value of m =7, α = 1.045

$$\beta = \frac{1}{U^2 A}\int u^2 dA = \frac{\sum u^2 dA}{AU^2} \qquad (3)$$

for (1/m) power law velocity distribution, the momentum correction coefficient equal to

$$\beta = \frac{(m+1)^2}{m(m+2)} \qquad (4)$$

for a typical value of m = 7, β = 1.016.

In 1998 Nieuwstadt defined the friction velocity related to bed stress as (Vermaas, 2009), [12]

$$u^* = \sqrt{\frac{\tau_b}{\rho}} \qquad (5)$$

in which τ_b is the bed shear stress.

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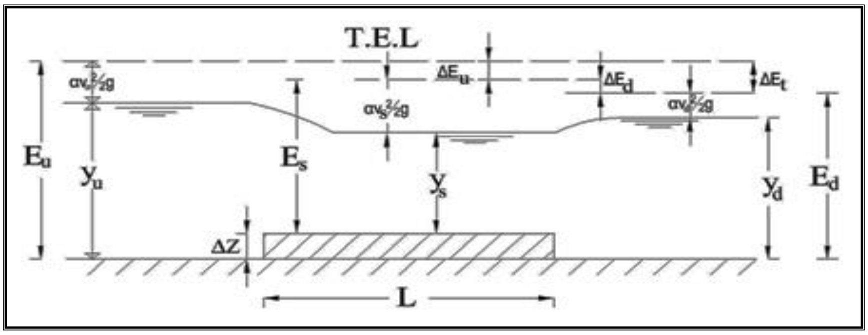
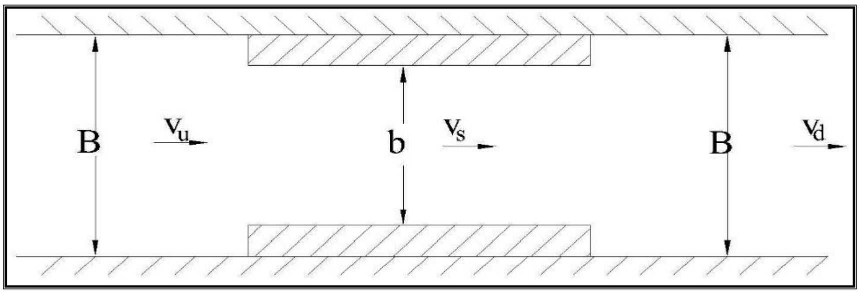
3. Experimental work

A. Experimental Facilities

A series of laboratory experiments was performed in the laboratory of irrigation and hydraulics Dept. A rectangular flume with 10 m long, 0.4 m width and 0.40 m depth was used. The width of the flume is contracted from 0.4 m to 0.3 m by wood. The wooden model of transitions was placed in the flume. At the upstream of the flume, the first and second head tanks were installed which received water from the delivery pipe and conduit it to the approaching channel. Upstream the flume, a rectangular weir was installed to measure the flow discharge. Figure 1 shows a plan and a view of the flume arrangement [5]. Water leaves the flume and falling into an underground water tank from which water was re-circulated to the flume. The flume was provided with a centrifugal pump to recirculate water from the underground tank to feed the flume with the required flow.



Elevation



Plan

Figure 1: Model of present study

The tested models were placed at the middle of the flume. The model consisted of different plates. The plate was made of a hardwood of 2.5, 5.0 and 7.5 cm thick. It was rigidly fixed between the wall sides of the flume, Figure 2.

Figure 2: Experimental setup

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B. Experimental Models

Table 1 shows the dimensions of all used models.

Table 1: Dimensions of the used models in (cm)

Compound transition					
$\Delta Z_1 = 2.5$	$\Delta b_1 = 5$	$\Delta Z_2 = 5$	$\Delta b_1 = 5$	$\Delta Z_3 = 7.5$	$\Delta b_1 = 5$
	$\Delta b_2 = 10$		$\Delta b_2 = 10$		$\Delta b_2 = 10$
	$\Delta b_3 = 15$		$\Delta b_3 = 15$		$\Delta b_3 = 15$

C. Measuring Devices

1- Measuring water depth

A digital point gauge is used to measure water depths. The digital point gauge moves on the carriage in the transverse direction. It is equipped with a vertical scale and vernier to read the depth of water. The vertical scale is graduated to measure values to the nearest 1.0 mm, and the vernier is graduated to read the level nearest to 0.10 mm.

2- Measuring water discharge

The rate of flow was measured by two methods. The first method was using the volume per certain time. The other one was using a rectangular weir fixed at the outlet of the second head tank to measure the flow discharge.
The discharge equation of the rectangular weir:

$$Q = \frac{2}{3} C_d \, W \sqrt{2g} \, H^{1.5} \tag{6}$$

in which: Q is the discharge in cm3/s; Cd is the discharge coefficient; w is the weir width cm; g is the gravitational acceleration cm/sec2; H is the water head over the crest in cm and equals to (hn-ho); hn is the water surface elevation in cm; and ho is the crest elevation in cm. Width of the weir crest w is equal to 42.1 cm and the coefficient of discharge, Cd is found to be 0.613 from the weir calibration shown For the present study, equation (6) can be rearranged to:

$$Q = 762.08 \, H^{1.5} \tag{7}$$

A vertical scale fixed on the sidewall of the tank was used to read hn and ho.

3- Measuring water velocity

A caustic doppler velocimeter ADV (The Sontek 10 MHZ- type) was used to measure the velocities. It was a versatile, high-precision instrument used to measure 3D water velocity as shown, figure 3.

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Figure 3: Acoustic Doppler Velocimeter.

Two minute sampling intervals were deemed sufficient since it was a standard time used in previous experiments (Tritico and Hotchkiss 2005). Optional Features for ADV were Sensor mounted on a 100 cm cable, Built-in rechargeable battery for splash proof systems, Compass/Tilt sensor, several probe configurations available, strain-gage pressure sensor (0.1%) and resonant pressure transducer (0.01%).

SNR is the most important quality control data provided and is a measure of the strength of the reflected acoustic signal relative to ambient noise level of the unit. This value should be greater than 10 decibels (dB) on the best operating conditions but the Son-Tek may be used when the SNR is lower. Field and laboratory data analyses have indicated that velocity data are unreliable when the SNR is below 4 (dB). The filtering process technique involves two main steps. First, using ADV signal correlation (COR) and signal-to-noise ratio, the time series' individual low quantity values are eliminated (SNR). The second step entails finding the spikes and replacing them with the mean value. The statistical indicator of how tightly connected the pulse pear is is the signal correlation (COR) (outgoing and incoming). As the noise and signal are also equal, low SNR can indicate faulty data. When the correlation values are below the threshold, the velocity values are replaced by the signal correlation (COR) methods. The most similar approach is SNR.

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4. Results And Analysis

Two different methods have been used for calculating energy and momentum coefficients. The first was based on the calculation of ε

$$\alpha=1+3\varepsilon^2-2\varepsilon^3 \quad (8)$$

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β=1+ε2 (9)

The second method was based on dividing the cross section into slides and using the corresponding velocity for each strip.

A. The first Method

Figure 4 shows a relationship between energy coefficient (α) and momentum (β) for every discharge value (2.5, 5.2, 8.2, and 11.2 L/s) at the upstream section. It can be observed from this figure that the family of curves could be represented by polynomial function. It was found that, the value of (α) increases with the increasing value of (β).

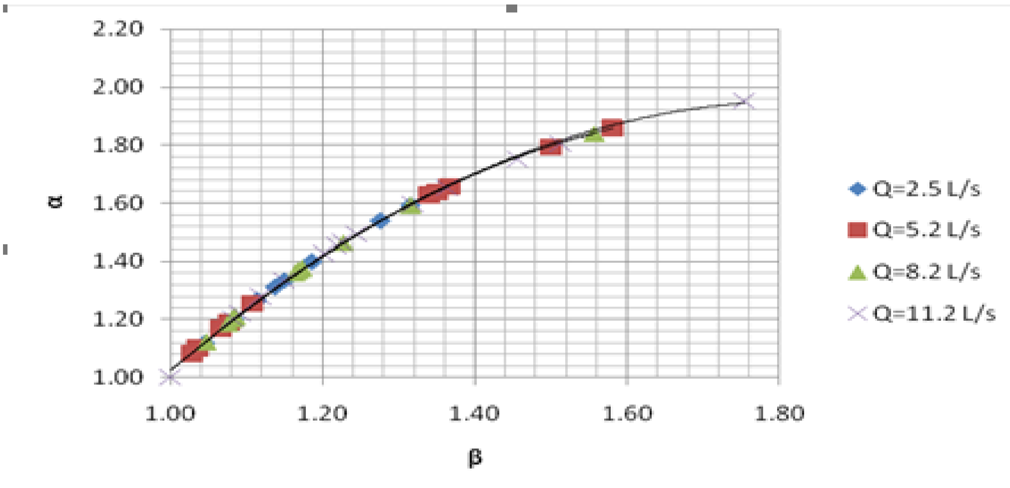

Figure 4: Relationship between α and β at upstream section

Figure 5 shows a variation of energy coefficient (α) and momentum (β) for different contraction ratios Δb/B of 0.17, 0.33 and 0.5. This curve exhibit the relationship between α and β at transition sections. It can be observed from this figure that the family of curves could be represented by polynomial curves. It was found that, the values of both (α) and (β) increase with the increasing value contraction ratio Δb/B.

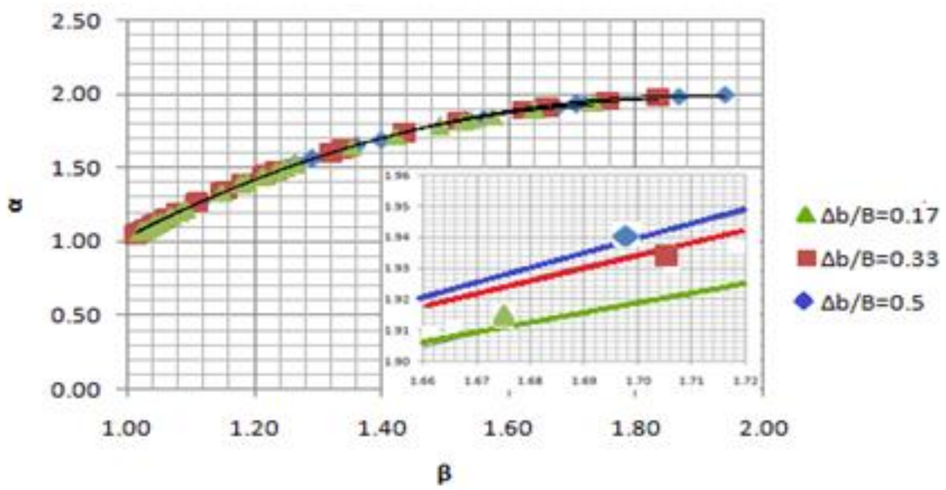

Figure 5: Relationship between α and β at transition section

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Figure 6 shows a relationship of energy coefficient (α) and momentum coefficient (β) for discharge values of 11.2 L/s, 8.2 L/s, 5.2 L/s and 2.5 L/s. These curves demonstrate the relationship between α and β at transition sections. It can be observed from this figure based on statistical analysis that the best fit of this curve is polynomial function of second degree. It could be noticed that, the value of (α) increases with the increasing value of (β). The reason of this increase could be due to the turbulence effect in the transition zone. The value of (α) increases with the increasing value of discharge (Q).

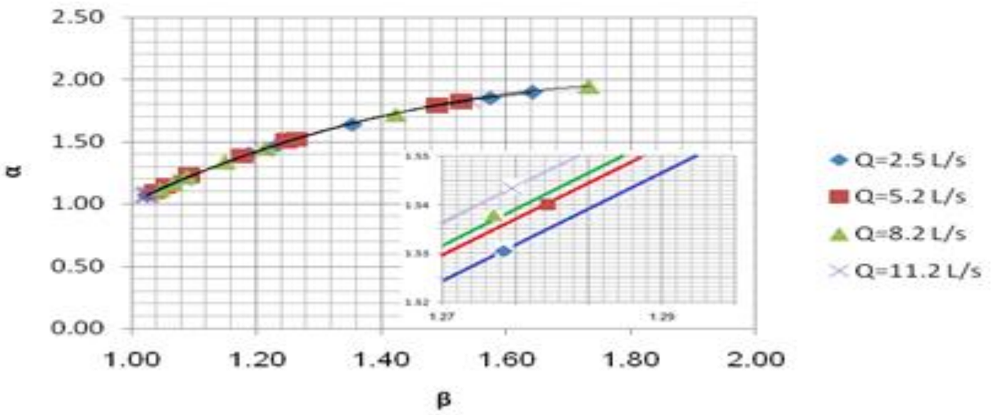

Figure 6: Relationship between α and β at transition section for different discharge

Figure 7 shows a variation of energy coefficient (α) and momentum (β) for every change in relative height of bed ΔZ/L equals to 0.025, 0.05, and 0. 075.This curve show that the relationship between α and β at transition sections. The trend of variation of the energy coefficient with momentum coefficient is a polynomial function of second degree with increasing trend. It is found that, the value of (α) increases with the increasing value of (β) and the value of α decreases with the increasing value of relative hump ΔZ/L. The value of ΔZ/L in this range has a negligible influence on this relationship.

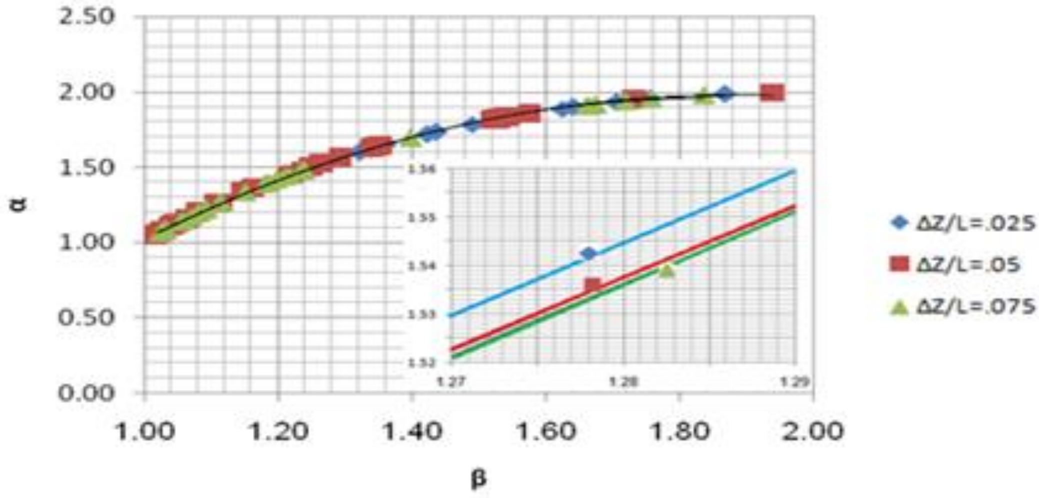

Figure 7: Relationship between α and β at transition section

for different relative hump

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Figure 8 shows a variation of both the energy coefficient (α) and the momentum coefficient (β) with different values of contraction ratio, height of bed, and discharge. It was found that, the value of (α) increases with the increasing value of (β). It could be observed from the figure that the family of curves is fitted to polynomial function of second degree which may be expressed as:

$\alpha = -1.242\beta^2 + 4.645\beta - 2.371$ at upstream $R^2 = 0.999$ (10)

$\alpha = -1.275\beta^2 + 4.747\beta - 2.449$ at transition $R^2 = 0.999$ (11)

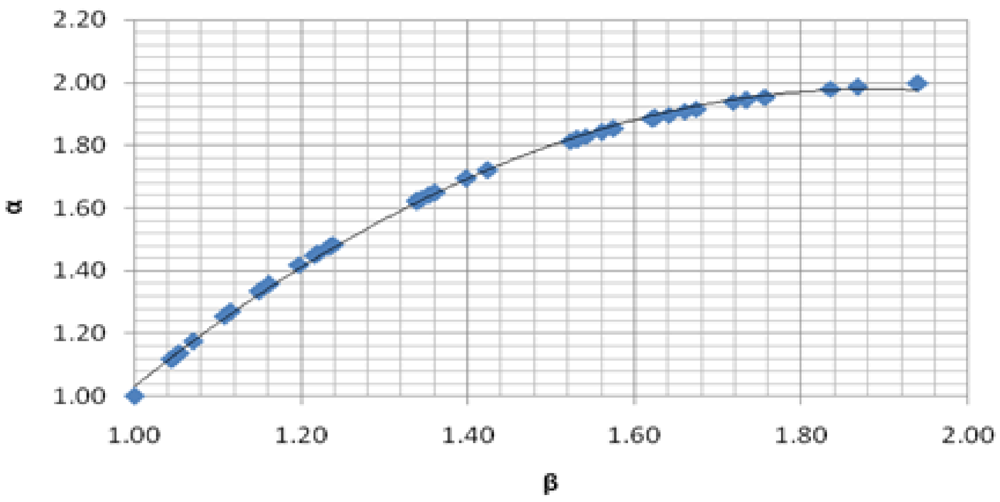

Figure 8: Relationship between α and β at transition section

Figure 9 shows a variation of both the energy coefficient (α-1) and the momentum coefficient (β-1) with different values of contraction ratio, height of bed, and discharge. It was found that, the value of (α-1) increases with the increasing value of (β-1). It could be observed from the figure that the family of curves is fitted by polynomial function of second degree. It may be expressed by an equation:

$\alpha = -1.171\beta^2 + 4.44\beta - 2.23$ $R^2=0.999$ (12)

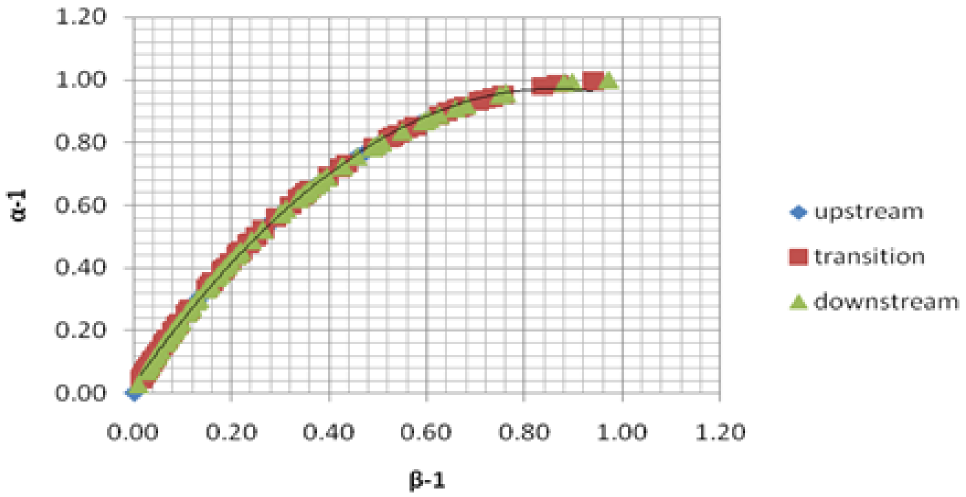

Figure 9: Relationship between (α-1) and (β-1) at different cases

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B. The second method

Figure 10 shows a variation of both the energy coefficient (α) and the momentum coefficient (β) with different values of contraction ratio, height of bed, and discharge at the upstream and transition zone in the second method. It was found that, the value of (α) increases with the increasing value of (β). It could be observed from this figure that the family of curves is fitted to linear form. It may be expressed by an equation:

$\alpha = 3.065\beta - 2.084$ $R2 = 0.991$ (13)

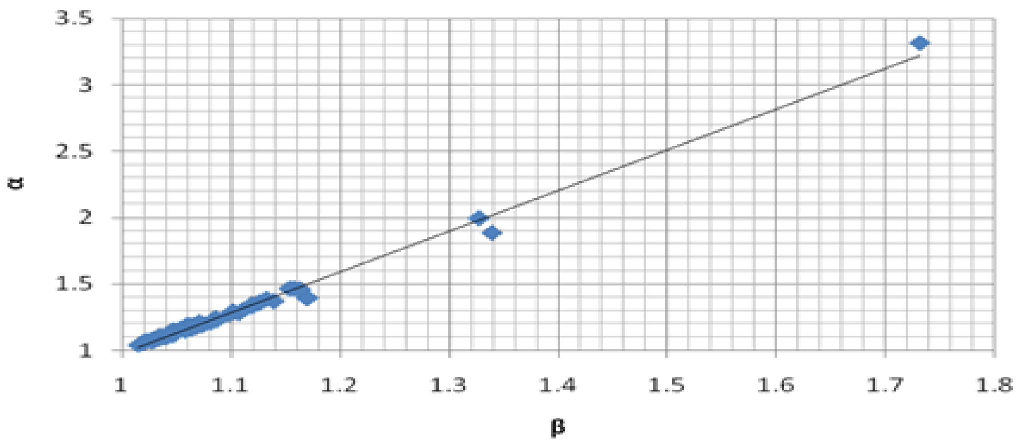

Figure 10: Relationship between α and β for all models.

Figure 11 and figure 12 demonstrate the relationship between (α-1) and (β-1) for the models under study at the upstream and transition sections, respectively. It is found that, the value of (α-1) increases with the increasing value of (β-1). It can be deduced from these two figures the following equations.

$\alpha = 3.06\ \beta -2.081$	$R2 = 0.987$	upstream	(14)
$\alpha = 3.046\ \beta -2.063$	$R2 = 0.985$	transition	(15)

For transition section, (α-1/β-1) ranges from 2.71 to 2.81, this range don't exceed 4% for different Reynolds number varies between 4059 and 1378. It is noticed that the value of (α-1/β-1) in the transition section is the bigger than the corresponding one for the upstream section. Then the error of (α-1/β-1) for different discharges is less than 5% which could be accepted. This conclusion comparable very well with an equation (α-1/β-1) ranges between 2.7 and 2.8 as given by Henderson, (1966), [6].

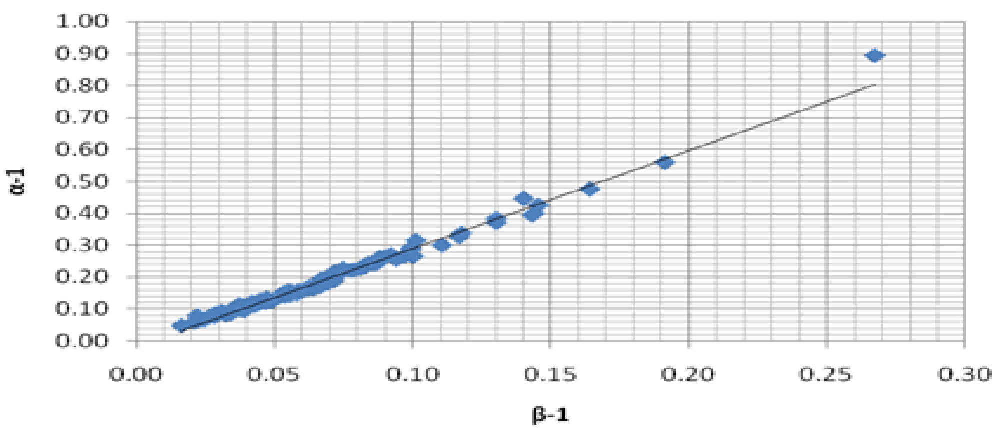

Figure 11: Relationship between (α-1) and (β-1) for upstream section

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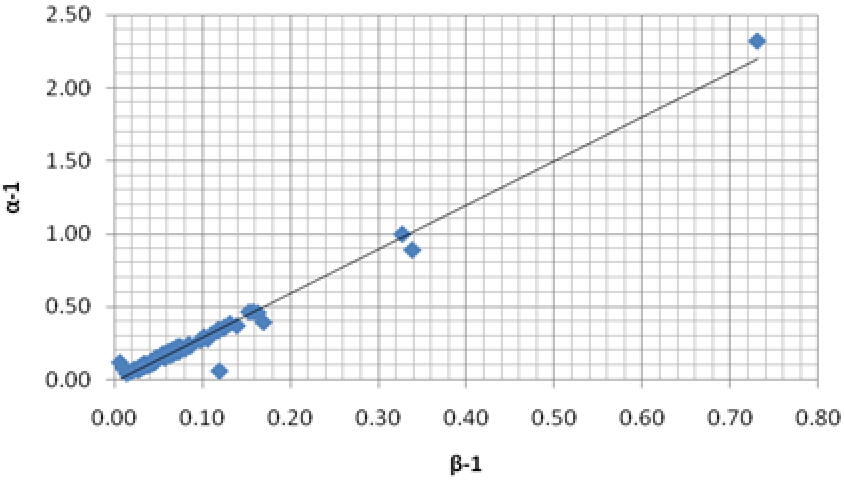

Figure 12: Relationship between (α-1) and (β-1) for transition section.

Figure 13 and figure 14 show a relationship between both energy coefficient and momentum coefficient versus ε for different values of used discharges. It is found that, the values of α and β increase with the increasing value of ε. It could be observed from these figures that the family of curves is fitted to polynomial curves of first degree. It may be expressed by the following equations:

$\alpha = 0.706\ \varepsilon2 - 0.356\varepsilon + 1.187$ R2 = 0.797 (16)

$\beta = 0.213\ \varepsilon2 - 0.096\varepsilon + 1.063$ R2 = 0.739 (17)

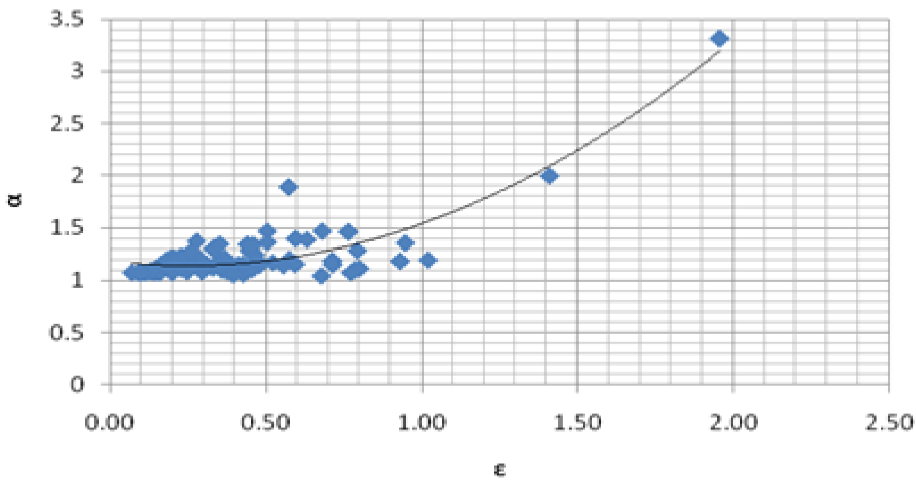

Figure 13: Relationship between (α) and (ε) for transition section

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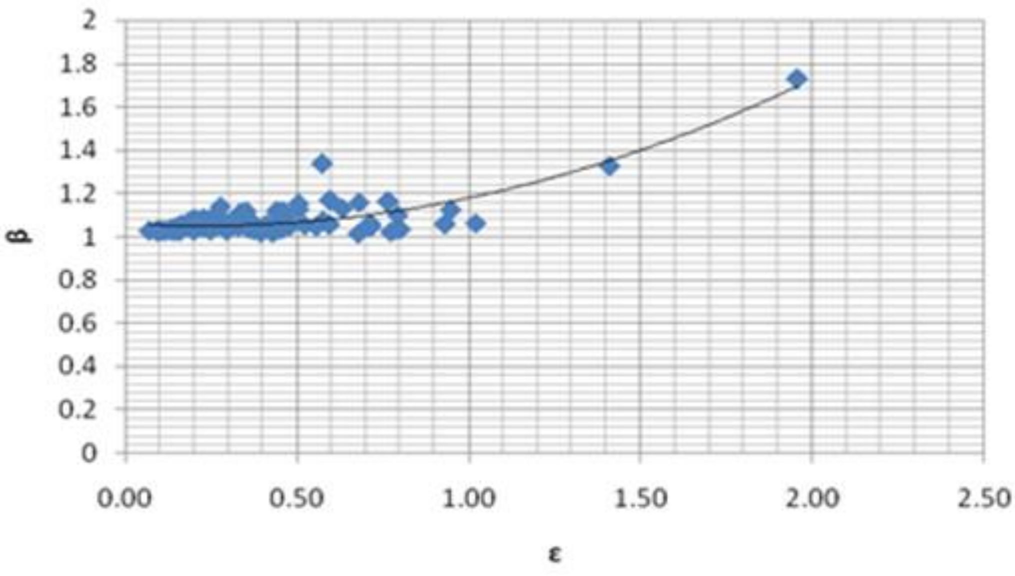

Figure 14: Relationship between (β) and (ε) for transition section.

5. Conclusion

Kinetic energy and momentum correction coefficients, α and β, are often used to be unity when the energy and momentum principles are used in the hydraulic computations. However, because of non uniform distribution of velocities over a channel section, α and β, are generally greater than unity. Two different methods have been used for calculating energy and momentum coefficients. The first one was based on the calculation of ε. the second method was based on dividing the cross section into slides and using the corresponding velocity for each strip. The second one was found to be more accurate than the first method. This method of slides can be used to derive the relationship between both energy and momentum coefficients versus ε values. Also, the results show that α and β individually are strongly depend on the kind of velocity distribution, but the ratio (α-1/β-1) varies slightly in the region (2.7:2.8) as the flow changes from laminar to turbulent. The energy and momentum coefficients exhibit higher values in the transition zone than the corresponding ones in the upstream section. This is mainly due to turbulence effect. For approximate values, the energy and momentum coefficients can be computed using the following formula

$\alpha = 0.706\ \varepsilon 2-0.356\varepsilon+1.187$

$\beta = 0.213\ \varepsilon 2-0.096\varepsilon+1.063$

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