Some Algebraic Structure of Neutrosophic Matrices

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Abstract

In this paper, we investigate some operations of algebraic structure of neutrosophic matrices related to indeterminacy real numbers.

Keywords: Neutrosophic Matrix; Operations on Neutrosophic Matrix; Rank of Neutrosophic Matrix.

1. Introduction

Mathematical neutrosophic theory is in new branch of mathematical systems. which introduced by Smarandache in 1995 depend on conceptual of indeterminacy in philosophical interpretation of propositional logic see [17]. Smarandache developed intuitionistic fuzzy set theory which presented by Atanassov in 1983 see [4,5] into neutrosophic set see [16]. Kandasamy and Smarandache studied finite neutrosophic complex numbers see [11], next later, neutrosophic vector space, AH-supspace, AHS-subspace, AH-Quotient investigated by Abobala see [1]. After that, Salama with other authors presented some result about neutrosophic square complex matrices see [15]. In this article we study neutrosophic matrices of neutrosophic real numbers with some theorems.

2. Neutrosophic Matrices and Their Operations

Definition 2.1. [10] Let R be the set of real numbers, then $N(R) = \langle R \cup I \rangle = \{a + bI : a, b \in R\}$ is a neutrosophic set where $a + bI$ is a neutrosophic real number and I is indeterminate such that $0.I = 0, I^2 = I$.

Definition 2.2. [10] Let ⟨R ∪ I⟩ be any neutrosophic ring. The collection of all $n \times n$ matrices with entries from $\langle R \cup I \rangle$ is called the neutrosophic matrix ring; i.e. $M_{n \times n} = \{M = (a_{ij}) \,|\, a_{ij} \in \langle R \cup I \rangle\}$. The operations are the usual matrix addition and matrix multiplication.

Definition 2.3. Consider the neutrosophic matrix set $M_{m\times n} = \{a_{ij} + b_{ij}I : a_{ij}, b_{ij} \in R, 0I = 0 \;\&\; I^2 = I\}$ with m rows and n columns , then the scalar entry in the i^{th} neutrosophic row and in j^{th} neutrosophic column of neutrosophic matrix M is denoted by $a_{ij} + b_{ij}I$ and is called the $(i, j) - entry$ of M, the following figure pictorial rectangular neutrosophic matrix M of type $m \times n$.

Column j

$M =$

$[a_{11} + b_{11}I \; a_{12} + b_{12}I \ldots a_{ij} + b_{ij}I \cdots a_{1n} + b_{1n}I \;\vdots\vdots\; \vdots \;\; \vdots \;\; \vdots \;\; a_{i1} + b_{i1}I \;\vdots\; a_{m1} + b_{m1}I \;\; a_{i2} + b_{i2}I \ldots \vdots\; a_{m2} + b_{m2}I \ldots \; a_{ij} + b_{ij}I \;\vdots\; a_{mj} + b_{mj}I \;\cdots \vdots \;\cdots\; a_{in} + b_{in}I \;\vdots\; a_{mn} + b_{mn}I \;]$ Row i

Figure 2.1 of rectangular neutrosophic matrix $M_{m\times n}$.

Definition 2.4. Let M and N be two neutrosophic matrices. Define the equality of M and M' as following:
$M = N$ **iff** $m_{ij} = n_{ij}$ and $m'_{ij} = n'_{ij}$, for all $i = 1,2, \ldots, m$ and $j = 1,2, \ldots, n$.

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Some special types of neutrosophic matrices

Definition 2.5. Any neutrosophic matrix, in which all the neutrosophic elements are neutrosophic zeros is called a neutrosophic zero matrix, and denoted by O, for instance, $O = [0+0I\ 0+0I\ 0+0I\ 0+0I\ 0+0I\ 0+0I\ ]$.

Definition 2.6. Let M be a neutrosophic matrix of size $n \times 1$ (that is, M having a single column), then it's called a column neutrosophic vector (or m-dimensional column neutrosophic vector) and denoted by lowercase letter with omitted the second type subscript, thus:
$m = [m_1 + m'_1 I\ m_2 + m'_2 I\ m_3 + m'_3 I\ \vdots\ m_n + m'_n I\ ]$, which identifies with neutrosophic vector in neutrosophic Euclidian $R^n(I)$.
Definition 2.7. Let N be a neutrosophic matrix of size $1 \times m$ (that is, N having a single row), then it's called a row neutrosophic vector (or $m-dimensional$ row neutrosophic vector) and denoted by lower-case letter with omitted the first type subscript, thus:
$n = [n_1 + n'_1 I\ n_2 + n'_2 I\ n_3 + n'_3 I\ \cdots\ n_m + n'_m I\ ]$, which identifies with neutrosophic vector in neutrosophic Euclidian $R^m(I)$.
Definition 2.8. If $M = [a_{ij} + b_{ij} I]_{n \times n}$. then M is called a square neutrosophic matrix of $nth - order$ and it has a neutrosophic determinates.

Definition 2.9. A neutrosophic square matrix is called a neutrosophic diagonal matrix, if all its non-diagonal neutrosophic elements are neutrosophic zero, and denoted by: $ND = \{a_{ij} + b_{ij} I = 0, i \neq j \quad a_{ij} + b_{ij} I \neq 0, for\ some\ i = j$

For instance, $ND = [3+2I\ 0+0I\ 0+0I\ 0+0I\ 1+4I\ 0+0I\ 0+0I\ 0+0I\ 1-5I\ ]$.

Definition 2.10. A neutrosophic square matrix of $nth - order$ is called a neutrosophic identity if all diagonal elements are neutrosophic unity and non-diagonal neutrosophic elements are neutrosophi**c** zero, denoted by:

$$NI_n = \{a_{ij} + b_{ij} I = 0, if\ \ i \neq j \qquad a_{ij} + b_{ij} I = 1 + 0I, if\ \ i = j$$

For example,$NI_3 = [1+0I\ 0+0I\ 0+0I\ 0+0I\ 1+0I\ 0+0I\ 0+0I\ 0+0I\ 1+0I\ ]$.

Operations on neutrosophic matrices

Definition 2.11. Let M and N be two neutrosophic matrices. Define the addition and differences of M and N as following:

1. M and N have the same size or order or capacity, and
2. $M \pm N = (m_{ij} + m'_{ij} I) \pm (n_{ij} + n'_{ij} I) = \left((m_{ij} \pm n_{ij}) + ((m'_{ij} \pm n'_{ij}) I\right)$.

Definition 2.12. Let M be neutrosophic matrix and $x = x_1 + x_2 I$ is a scalar neutrosophic number of $R(I)$. Then $x.M = (x_1 + x_2 I)[m_{ij} + m'_{ij} I] = \left[(x_1.m_{ij}) + \left((x_1.m'_{ij}) + (x_2.m_{ij}) + (x_2.m'_{ij})\right) I\right]$, for all i and j.
Example 2.1. Consider the following two neutrosophic matrices $M = [1+2I\ 0\ -3+4I\ 2-2I\ 1\ 2+2I\ 3\ 0\ 5+I\ ]$ and
$M' = [0\ 3+6I\ 2\ 5+I\ 5\ 0+I\ -2\ -3+I\ 1+2I\ ]$, then:
$M + M' = [1+2I\ 0\ -3+4I\ 2-2I\ 1\ 2+2I\ 3\ 0\ 5+I\ ] + [0\ 3+6I\ 2\ 5+I\ 5\ 0+I\ -2\ -3+I\ 1+2I\ ]$

$= [((1+2I) + (0+0I)\ (0+0I+(3+6I))\ ((-3+4I)+2+0I)\ ((2-2I)+(5+I))\ (1+0I+5+0I)\ ((2+2I)+(0+I))\ (3+0I+(-2)+0I)\ (0+0.I+(-3+I))\ (5+I)+(1+2I)\ ]$

$= [1+2I\ 3+6I\ -1+4I\ 7-I\ 6\ 2+3I\ 1\ -3+I\ 6+3I\ ]$

By similar manner, we get:

$M - M' = [1+2I\ -3-6I\ -5+4I\ -3-3I\ -4\ 2+I\ 5\ -3-I\ 4-I\ ]$

Now, If $x = 2+3I$, then $x.M = [2+13I\ 0\ -6+11I\ 4-4I\ 2+3I\ 4+16I\ 6+9I\ 0\ 10+20I\ ]$.

Theorem2.1. Let A, B, and C be three neutrosophic matrices of the same capacity, and consider x and y are two neutrosophic scalars, then:

i. $A + B = B + A$;
ii. $(A + B) + C = A + (B + C)$" " associative law";

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iii. $A + 0 = A$;
iv. $x(A + B) = xA + xB$;
v. $(x + y)A = xA + yA$;
vi. $x(yA) = (xy)A$; and

Proof.

i. Consider the left-hand side: $A + B = \left[\left(a_{ij} + a'_{ij}I\right) + \left(b_{ij} + b'_{ij}I\right)\right]$

$$= \left(\left(a_{ij} + b_{ij}\right) + ((a'_{ij} + b'_{ij})I\right)$$

$$= \left(\left(b_{ij} + a_{ij}\right) + ((b'_{ij} + a'_{ij})I\right)$$

$$= \left[\left(b_{ij} + b'_{ij}I\right) + \left(a_{ij} + a'_{ij}I\right)\right]$$

$$= B + A : \text{R.H.S.}$$

ii. Consider the left-hand side: $(A + B) + C = \left[\left(a_{ij} + a'_{ij}I\right) + \left(b_{ij} + b'_{ij}I\right)\right] + (c_{ij} + c'_{ij}I)$

$$= \left[\left(a_{ij} + b_{ij}\right) + \left(a'_{ij} + b'_{ij}\right)I\right] + (c_{ij} + c'_{ij}I)$$

$$= \left[\left(\left(a_{ij} + b_{ij}\right) + c_{ij}\right) + \left(\left(a'_{ij} + b'_{ij}\right) + c'_{ij}\right)I\right]$$

$$= \left[\left(a_{ij} + \left(b_{ij} + c_{ij}\right)\right) + \left(a'_{ij} + \left(b'_{ij} + c'_{ij}\right)\right)I\right]$$

$$= \left(a_{ij} + a'_{ij}I\right) + \left[\left(b_{ij} + c_{ij}\right) + \left(\left(b'_{ij} + c'_{ij}\right)\right)I\right]$$

$$= \left(a_{ij} + a'_{ij}I\right) + \left[\left(b_{ij} + b'_{ij}I\right) + \left(+c_{ij} + c'_{ij}\right)I\right]$$

$$= A + (B + C).$$

iii. $A + 0 = \left[\left(a_{ij} + a'_{ij}I\right) + (0 + 0.I)\right] = \left(a_{ij} + 0\right) + \left(a'_{ij} + 0\right)I\right) = a_{ij} + a'_{ij}I = A.$

iv. $x(A + B) = (x_1 + x_2I).\left[\left(a_{ij} + b_{ij}\right) + \left(a'_{ij} + b'_{ij}\right)I\right]$

$$= \left[\left(x_1.\left(a_{ij} + b_{ij}\right)\right) + \left(\left(x_1.\left(a'_{ij} + b'_{ij}\right)\right) + \left(x_2.\left(a_{ij} + b_{ij}\right)\right) + \left(x_2.\left(a'_{ij} + b'_{ij}\right)\right)\right)I\right]$$

$$= \left[\left(\left(x_1.a_{ij}\right) + \left(x_1 + b_{ij}\right)\right) + \left(\left(\left(x_1.a'_{ij}\right) + \left(x_1.b'_{ij}\right)\right) + \left(\left(x_2.a_{ij}\right) + \left(x_2.b_{ij}\right)\right) + \left(\left(x_2.a'_{ij}\right) + \left(x_2.b'_{ij}\right)\right)\right)I\right]$$

$$= \left[\left(x_1.a_{ij}\right) + \left(\left(\left(x_1.a'_{ij}\right) + \left(x_2.a_{ij}\right) + \left(x_2.a'_{ij}\right)\right)I\right) + \left(x_1.b_{ij}\right) + \left(\left(\left(x_1.b'_{ij}\right) + \left(x_2.b_{ij}\right) + \left(x_2.b'_{ij}\right)\right)I\right)\right]$$

$$= (x_1 + x_2I).\left(a_{ij} + a'_{ij}\right)I + (x_1 + x_2I).\left(b_{ij} + b'_{ij}\right)I = x.A + x.B.$$

v. $x(yA) = (xy)A$. By similar method.

vi. $1.A = A$. Obvious $\blacksquare$.

Definition 2.13. Let $M = [m_{ik} + m'_{ik}I]_{m\times r}$ and $N = [n_{kj} + n'_{kj}I]_{r\times n}$ be two neutrosophic matrices such that the neutrosophic number of neutrosophic columns of M is equal to the neutrosophic number of neutrosophic rows of N, where, then the product of neutrosophic matrices $MN = C = [c_{ij} + c'_{ij}I]$, where,

$$(c_{ij} + c'_{ij}I) = \Big\{\left(\left(m_{i1}.n_{1j}\right) + \left(\left(m_{i1}.\ n'_{1j}\right) + \left(m'_{i1}.n_{1j}\right) + \left(m'_{i1}.n'_{1j}\right)I\right)\right) + \left(\left(m_{i2}.n_{2j}\right) + \left(\left(m_{i2}.n'_{2j}\right) + \left(m'_{i2}.n_{2j}\right) + \left(m'_{i2}.n'_{2j}\right)I\right)\right) + \left(\left(m_{i3}.n_{3j}\right) + \left(\left(m_{i3}.n'_{3j}\right) + \left(m'_{i3}.n_{3j}\right) + \left(m'_{i3}.n'_{3j}\right)I\right)\right) \vdots$$
$$+ \left(\left(m_{ir}.n_{rj}\right) + \left(\left(m_{ir}.m'_{rj}\right) + \left(m'_{ir}.n_{rj}\right) + \left(m'_{ir}.m'_{rj}\right)I\right)\right) \Big\}$$

$$= \sum_{k=1}^{r} \left(\left(m_{ik} + m'_{ik}I\right).\left(b_{kj} + b'_{kj}I\right)\right).$$

Example 2.2. Consider the following two neutrosophic matrices $A = [1 + 2I\ 3 + I\ 2 - I\ 0\ ]_{2\times 2}$ and

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$B = [-3+I\ 0\ 2-I\ 5+4I\ 4+2I\ -4+I\ ]_{2\times 3}$, Consider $C = AB = [1+2I\ 3+I\ 2-I\ 0\ ][-3+I\ 0\ 2-I\ 5+4I\ 4+2I\ -4+I\ ]$, where

$$C = [c_{ij}+c'_{ij}I] = [c_{11}+c'_{11}I\ c_{12}+c'_{12}I\ c_{13}+c'_{13}I\ c_{21}+c'_{21}I\ c_{22}+c'_{22}I\ c_{23}+c'_{23}I\ ]$$

To compute , $c_{11}+c'_{11}I = \Big(\big((a_{11}.b_{11}) + ((a_{11}.\ b'_{11}) + (a'_{11}.b_{11}) + (a'_{11}.b'_{11})I)\big) + \big((a_{12}.b_{21}) + ((a_{12}.b'_{21}) + (a'_{12}.b_{21}) + (a'_{12}.b'_{21})I)\big)\Big)$

$= \Big(\big((1.-3) + ((1.\ 1) + (2.-3) + (2.1)I)\big) + \big((3.5)\ + ((3.4) + (1.5) + (1.4)I)\big)\Big)$

$= ((-3 + (1-6+2)I)\ +\ (15\ + (12+5+4)I)\ ) = ((-3-3I)\ + (15\ + 21I)\ ) = (12+18I)$.

$$c_{23}+c'_{23}I = \Big(\big((a_{i1}.b_{1j}) + ((a_{i1}.\ b'_{1j}) + (a'_{i1}.b_{1j}) + (a'_{i1}.b'_{1j})I)\big) + \big((a_{i2}.b_{2j})\ + ((a_{i2}.b'_{2j}) + (a'_{i2}.b_{2j}) + (a'_{i2}.b'_{2j})I)\big)\Big)$$

$= \Big(\big((a_{21}.b_{13}) + ((a_{21}.\ b'_{13}) + (a'_{21}.b_{13}) + (a'_{21}.b'_{13})I)\big) + \big((a_{22}.b_{23})\ + ((a_{22}.b'_{23}) + (a'_{22}.b_{23}) + (a'_{22}.b'_{23})I)\big)\Big)$

$= \Big(\big((2.2) + ((2.-1) + (-1.2) + (-1.-1)I)\big) + \big((0.-4)\ + ((0.1) + (0.-4) + (0.1)I)\big)\Big)$

$= ((4 + (-2-2+1)I)\ +\ (0\ + (0+0+0)I)\ ) = ((4-3I)\ +\ (0\ + 0I)\ ) = (4-3I)$. By the same manner, we can compute neutrosophic elements $c_{12}+c'_{12}I, c_{13}+c'_{13}I, c_{21}+c'_{21}I$ and $c_{22}+c'_{22}I$ to get the neutrosophic matrix

$$C = [12+18I\ 12+12I\ -10+I\ -6+4I\ 0\ 4-3I\ ].$$

In general, the neutrosophic multiplication is not commutative.

Example 2.3. Consider the two neutrosophic matrices $A = [1+2I\ 2+3I\ 0\ 4+1\ ]$ and

$B = [2-2I\ 0\ 3+3I\ 5+2I\ ]$, then $AB \neq BA$.

Theorem2.2. Let A, B, and C be three neutrosophic matrices which are defined under multiplication, with x is a neutrosophic scalars, then:

i. $(AB)C = A(BC)$ " associative law";
ii. $A(B+C) = AB + AC$ " left distributive law";
iii. $(B+C)A = BA + CA$ "right distributive law" and
iv. $x(AB) = (xA)B = A(xB)$.
v. $0A. = 0,\ B.0 = 0$. Where 0 is a neutrosophic zero matrix.

Proof. Suppose that A, B, and C are three neutrosophic matrices which are defined under multiplication, with x is a neutrosophic scalars. Then the picture of them like: $A = [a_{ij}+a'_{ij}I]_{m\times r}, B = [b_{jk}+b'_{jk}I]_{r\times n}$ and

$C = [c_{kl}+c'_{kl}I]_{n\times s}$. Let $M = (AB)$

$[m_{ik}+m'_{ik}I] = [a_{ij}+a'_{ij}I][b_{jk}+b'_{jk}I] =$, where,

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$= \Big\{ \Big((a_{i1}.b_{1j}) + \big((a_{i1}.\ b'_{1j}) + (a'_{i1}.b_{1j}) + (a'_{i1}.b'_{1j})I \big) \Big) + \Big((a_{i2}.b_{2j}) + \big((a_{i2}.b'_{2j}) + (a'_{i2}.b_{2j}) + (a'_{i2}.b'_{2j})I \big) \Big) + \Big((a_{i3}.b_{3j}) + \big((a_{i3}.b'_{3j}) + (a'_{i3}.b_{3j}) + (a'_{i3}.b'_{3j})I \big) \Big) \vdots + \Big((a_{ir}.b_{rj}) + \big((a_{ir}.b'_{rj}) + (a'_{ir}.b_{rj}) + (a'_{ir}.b'_{rj})I \big) \Big) \Big\}$

$$= \sum_{j=1}^{r} \big((a_{ik} + a'_{ik}I).(b_{kj} + b'_{kj}I) \big). \qquad (1).$$

Now multiplying (1): $M = (AB)$ by C, then the $il - entry$ is given by:

$$\begin{aligned}
(m_{ik} + m'_{ik}I)(c_{kl} + c'_{kl}I) \\
&= \Big\{ \Big((m_{i1}.c_{1j}) + \big((m_{i1}.\ c'_{1j}) + (m'_{i1}.c_{1j}) + (m'_{i1}.c'_{1j})I \big) \Big) \\
&+ \Big((m_{i2}.c_{2j}) + \big((m_{i2}.c'_{2j}) + (m'_{i2}.c_{2j}) + (m'_{i2}.c'_{2j})I \big) \Big) \\
&+ \Big((m_{i3}.c_{3j}) + \big((m_{i3}.c'_{3j}) + (m'_{i3}.c_{3j}) + (m'_{i3}.c'_{3j})I \big) \Big) \\
&\vdots + \Big((m_{ir}.c_{rj}) + \big((m_{ir}.c'_{rj}) + (m'_{ir}.c_{rj}) + (m'_{ir}.c'_{rj})I \big) \Big) \Big\} \\
&= \sum_{k=1}^{n} (m_{ik} + m'_{ik}I).(c_{kl} + c'_{kl}I) \\
&= \sum_{k=1}^{n} \sum_{j=1}^{r} \big((a_{ik} + a'_{ik}I).(b_{kj} + b'_{kj}I) \big).(c_{kl} + c'_{kl}I). \qquad (2).
\end{aligned}$$

From other hand, Let $N = (BC)$

$[n_{jl} + n'_{jl}I] = [b_{jk} + b'_{jk}I][c_{kl} + c'_{kl}I]$

$= \Big\{ \Big((b_{i1}.c_{1j}) + \big((b_{i1}.\ c'_{1j}) + (b'_{i1}.c_{1j}) + (b'_{i1}.c'_{1j})I \big) \Big) + \Big((b_{i2}.c_{2j}) + \big((b_{i2}.c'_{2j}) + (b'_{i2}.c_{2j}) + (b'_{i2}.c'_{2j})I \big) \Big) + \Big((b_{i3}.c_{3j}) + \big((b_{i3}.c'_{3j}) + (b'_{i3}.c_{3j}) + (b'_{i3}.c'_{3j})I \big) \Big) \vdots + \Big((b_{ir}.c_{rj}) + \big((b_{ir}.c'_{rj}) + (b'_{ir}.c_{rj}) + (b'_{ir}.c'_{rj})I \big) \Big) \Big\}$

$$= \sum_{k=1}^{n} \big((b_{jk} + b'_{jk}I).(c_{kl} + c'_{kl}I) \big) \qquad (3).$$

Now multiplying (3): A by $N = (BC)$, then the $il - entry$ is given by:

$$\begin{aligned}
(a_{ij} + a'_{ij}I)(n_{jl} + n'_{jl}I) = \\
&= \Big\{ \Big((a_{i1}.n_{1j}) + \big((a_{i1}.\ n'_{1j}) + (a'_{i1}.n_{1j}) + (a'_{i1}.n'_{1j})I \big) \Big) \\
&+ \Big((a_{i2}.n_{2j}) + \big((a_{i2}.n'_{2j}) + (a'_{i2}.n_{2j}) + (a'_{i2}.n'_{2j})I \big) \Big) \\
&+ \Big((a_{i3}.n_{3j}) + \big((a_{i3}.n'_{3j}) + (a'_{i3}.n_{3j}) + (a'_{i3}.n'_{3j})I \big) \Big) \\
&\vdots + \Big((a_{ir}.n_{rj}) + \big((a_{ir}.n'_{rj}) + (a'_{ir}.n_{rj}) + (a'_{ir}.n'_{rj})I \big) \Big) \Big\} \\
&= \sum_{j=1}^{r} (a_{ij} + a'_{ij}I)(n_{jl} + n'_{jl}I) \\
&= \sum_{j=1}^{r} \sum_{k=1}^{n} (a_{ij} + a'_{ij}I)\big((b_{jk} + b'_{jk}I).(c_{kl} + c'_{kl}I) \big). \qquad (4).
\end{aligned}$$

From (3) and (4). We deduced that the associative law is hold, since the summation are equal $\blacksquare$.

Definition 2.14. Let $M = [m_{ik} + m'_{ik}I]_{m\times r}$ be neutrosophic matrix , the transpose of neutrosophic matrix M is obtained by writing the columns of M ,in order as rows and written by $M^t = [m_{ik} + m'_{ik}I]_{r\times m}$ and $N = [n_{kj} + n'_{kj}I]_{r\times n}$.

Theorem2.3. Let A and, B be two neutrosophic matrices and x is a neutrosophic scalars, then:

i. $(A + B)^t = A^t + B^t$;

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ii. $(AB)^t = B^t . A^t$;
iii. $(xB)^t = x . A^t$ and
iv. $(A^t)^t = A$

Proof. (1). Suppose that A and, B be two neutrosophic matrices and x is a neutrosophic scalars.

$$(A+B)^t = \Big([a_{11}+a'_{11}I\ a_{12}+a'_{12}I \ldots a_{ij}+a'_{ij}I \cdots a_{1n}+a'_{1n}I \ \vdots\vdots \ \vdots \quad \vdots \quad \vdots \quad a_{i1}+a'_{i1}I$$
$$\vdots\ a_{m1}+a'_{m1}I\ \ a_{i2}+a'_{i2}I \ldots \vdots\ a_{m2}+a'_{m2}I \ldots\ a_{ij}+a'_{ij}I\ \vdots\ a_{mj}+a'_{mj}I\ \cdots$$
$$\vdots\ \cdots\ a_{in}+a'_{in}I\ \vdots\ a_{mn}+a'_{mn}I\ ]$$
$$+\ [b_{11}+b'_{11}I\ b_{12}+b'_{12}I \ldots b_{ij}+b'_{ij}I \cdots b_{1n}+b'_{1n}I\ \vdots\vdots\ \vdots \quad \vdots$$
$$\vdots \quad b_{i1}+b'_{i1}I\ \vdots\ b_{m1}+b'_{m1}I\ \ b_{i2}+b'_{i2}I \ldots \vdots\ b_{m2}+b'_{m2}I \ldots\ b_{ij}+b'_{ij}I$$
$$\vdots\ b_{mj}+b'_{mj}I\ \cdots \vdots\ \cdots\ b_{in}+b'_{in}I\ \vdots\ b_{mn}+b'_{mn}I\ ]\Big)^t$$

$$= \Big([(a_{11}+a'_{11}I)+(b_{11}+b'_{11}I) \ldots (a_{ij}+a'_{ij}I)+(b_{ij}+b'_{ij}I) \ldots (a_{1n}+a'_{1n}I)+(b_{1n}+b'_{1n}I)\ \vdots\vdots$$
$$\vdots\ (a_{i1}+a'_{i1}I)+(b_{i1}+b'_{i1}I) \ldots$$
$$\vdots\ (a_{m1}+a'_{m1}I)+(b_{m1}+b'_{m1}I) \ldots\ (a_{ij}+a'_{ij}I)+(b_{ij}+b'_{ij}I) \ldots$$
$$\vdots\ (a_{mj}+a'_{mj}I)+(b_{mj}+b'_{mj}I) \ldots\ (a_{in}+a'_{in}I)+(b_{in}+b'_{in}I)$$
$$\vdots\ (a_{mn}+a'_{mn}I)+(b_{mn}+b'_{mn}I)\ ]\Big)^t$$

$$= \Big([(a_{11}+b_{11})+(a'_{11}+b'_{11})I \ldots (a_{ij}+b_{ij})+(a'_{ij}+b'_{ij})I \ldots (a_{1n}+b_{1n})+(a'_{1n}+b'_{1n})I\ \vdots\vdots$$
$$\vdots\ (a_{i1}+b_{i1})+(a'_{i1}+b'_{i1})I \ldots$$
$$\vdots\ (a_{m1}+b_{m1})+(a'_{m1}+b'_{m1})I \ldots\ (a_{ij}+b_{ij})+(a'_{ij}+b'_{ij})I \ldots$$
$$\vdots\ (a_{mj}+b_{mj})+(a'_{mj}+b'_{mj})I \ldots\ (a_{in}+b_{in})+(a'_{in}+b'_{in})I$$
$$\vdots\ (a_{mn}+b_{mn})+(a'_{mn}+b'_{mn})I\ ]\Big)^t$$

$$= [(a_{11}+b_{11})+(a'_{11}+b'_{11})I \ldots (a_{i1}+b_{i1})+(a'_{i1}+b'_{i1})I \ldots (a_{m1}+b_{m1})+(a'_{m1}+b'_{m1})I\ \vdots\vdots$$
$$\vdots\ (a_{ij}+b_{ij})+(a'_{ij}+b'_{ij})I \ldots$$
$$\vdots\ (a_{1n}+b_{1n})+(a'_{1n}+b'_{1n})I \ldots\ (a_{ij}+b_{ij})+(a'_{ij}+b'_{ij})I \ldots$$
$$\vdots\ (a_{in}+b_{in})+(a'_{in}+b'_{in})I \ldots\ (a_{mj}+b_{mj})+(a'_{mj}+b'_{mj})I$$
$$\vdots\ (a_{mn}+b_{mn})+(a'_{mn}+b'_{mn})I\ ]$$

By similar method, we get

$$A^t + B^t = [(a_{11}+b_{11})+(a'_{11}+b'_{11})I \ldots (a_{i1}+b_{i1})+(a'_{i1}+b'_{i1})I \ldots (a_{m1}+b_{m1})+(a'_{m1}+b'_{m1})I\ \vdots\vdots\vdots\ (a_{ij}+b_{ij})+(a'_{ij}+b'_{ij})I \ldots \vdots\ (a_{1n}+b_{1n})+(a'_{1n}+b'_{1n})I \ldots\ (a_{ij}+b_{ij})+(a'_{ij}+b'_{ij})I \ldots \vdots\ (a_{in}+b_{in})+(a'_{in}+b'_{in})I \ldots\ (a_{mj}+b_{mj})+(a'_{mj}+b'_{mj})I\ \vdots\ (a_{mn}+b_{mn})+(a'_{mn}+b'_{mn})I\ ]$$

For ii, iii and v. by same technique of arguments $\blacksquare$.

Example 2.4. Consider the two neutrosophic matrices $A = [2+I\ 5-I\ 1+4I\ 1+2I\ ]$ and $B = [3-I\ 6-4I\ 3+3I\ 2-4I\ ]$, then

$A^t = [2+I\ 1+4I\ 5-I\ 1+2I\ ]$ and $B^t = [3-I\ 3+3I\ 6-4I\ 2-4I\ ]$, we see that:

$A^t + B^t = [2+I\ 1+4I\ 5-I\ 1+2I\ ] + [3-I\ 3+3I\ 6-4I\ 2-4I\ ] = [5\ 4+7I\ 11-5I\ 3-2I\ ]$, while

$A + B = [2+I\ 5-I\ 1+4I\ 1+2I\ ] + [3-I\ 6-4I\ 3+3I\ 2-4I\ ] = [5\ 11-5I\ 4+7I\ 3-2I\ ]$ and

$(A+B)^t = [5\ 11-5I\ 4+7I\ 3-2I\ ]^t = [5\ 4+7I\ 11-5I\ 3-2I\ ]$. Therefore $(A+B)^t = A^t + B^t$. On the other hand,

$AB = [2+I\ 5-I\ 1+4I\ 1+2I\ ][3-I\ 6-4I\ 3+3I\ 2-4I\ ] = [21+5I\ 22-24I\ 6+22I\ 8-4I\ ]$, so

$(AB)^t = [21+5I\ 22-24I\ 6+22I\ 8-4I\ ]^t = [21+5I\ 6+22I\ 22-24I\ 8-4I\ ]$, and

$B^t A^t = [3-I\ 3+3I\ 6-4I\ 2-4I\ ][2+I\ 1+4I\ 5-I\ 1+2I\ ] = [21+5I\ 6+22I\ 22-24I\ 8-4I\ ]$, hence $(AB)^t = B^t A^t$.

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Definition 2.15. Let $M = [m_{ij} + m'_{ij}I]_{n\times n}$ be a square neutrosophic matrix, then neutrosophic diagonal of M consists of the neutrosophic elements with the same subscribe: $m_{11} + m'_{11}I, m_{22} + m'_{22}I, \dots, m_{nn} + m'_{nn}I$.

Definition 2.16. Let $M = [m_{ij} + m'_{ij}I]_{n\times n}$ be a square neutrosophic matrix, then neutrosophic trace of M is denoted by $NTr(M)$ and given by $NTr(M) = (m_{11} + m'_{11}I) + (m_{22} + m'_{22}I), \dots + (m_{nn} + m'_{nn}I)$.

$$NTr(M) = \sum_{i=1}^{n} (m_{ii} + m'_{ii}I)$$

Example 2.5. Consider the following two neutrosophic matrix $M = [1 + 2I\ 0\ -3 + 4I\ 2 - 2I\ 1\ 2 + 2I\ 3\ 0\ 5 + I\ ]$, then

neutrosophic diagonal of $M, 1 + 2I, 1{,}5 + I$, and $NTr(M) = 7 + 3I$.

Theorem2.4. Let $A = [a_{ij} + a'_{ij}I]$ and $B = [b_{ij} + b'_{ij}I]$, be two neutrosophic m -square matrices and consider x is a neutrosophic scalars, then:

i. $NTr(A + B) = NTr(A) + NTr(B)$;
ii. $NTr(AB) = NTr(BA)$;
iii. $NTr(xA) = x.NTr(A)$ and
iv. $NTr(A^t) = NTr(A)$.

Proof. Suppose that $A = [a_{ij} + a'_{ij}I]$ and $B = [b_{ij} + b'_{ij}I]$, are two neutrosophic m -square matrices and consider x is a neutrosophic scalars, then:

i. $$NTr(A + B) = NTr\left((a_{ij} + a'_{ij}I) + (b_{ij} + b'_{ij}I)\right)$$
$$= NTr\left((a_{ij} + b_{ij}) + (a'_{ij} + b'_{ij})I)\right)$$
$$= \sum_{i=1}^{n} \left((a_{ii} + b_{ii}) + (a'_{ii} + b'_{ii})I)\right) \qquad (1).$$

Take R.H.S: $NTr(A) + NTr(B) = NTr([a_{ij} + a'_{ij}I]) + NTr([b_{ij} + b'_{ij}I])$

$$= NTr([a_{ij} + a'_{ij}I]) + NTr([b_{ij} + b'_{ij}I])$$
$$= \sum_{i=1}^{n} (a_{ii} + a'_{ii}I) + \sum_{i=1}^{n} (b_{ii} + b'_{ii}I)$$
$$= \sum_{i=1}^{n} \left((a_{ii} + b_{ii}) + (a'_{ii} + b'_{ii})I)\right) \qquad (2).$$

From (1) and (2). $NTr(A + B) = NTr(A) + NTr(B)$.

ii. $$NTr(AB) = NTr\left((a_{ij} + a'_{ij}I)(b_{jk} + b'_{jk}I)\right)$$
$$= NTr\left(\sum_{j=1}^{r} \left((a_{ij} + a'_{ij}I).(b_{jk} + b'_{jk}I)\right)\right)$$
$$= \sum_{i=1}^{r} \left(\sum_{j=1}^{r} \left((a_{ij} + a'_{ij}I).(b_{ji} + b'_{ji}I)\right)\right).$$
$$= \sum_{i=1}^{r} \left(\sum_{j=1}^{r} \left((b_{ji} + b'_{ji}I)(a_{ij} + a'_{ij}I).\right)\right).$$
$$= NTr\left(\sum_{i=1}^{r} \left((b_{ji} + b'_{ji}I)(a_{ik} + a'_{ik}I)\right)\right).$$
$$= NTr(BA).$$

(3) and (4). By the similar argument ■.

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