Improving Karmarker Algorithm to Obtain Optimal Solution

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Abstract

In this research, the Karmarker's method of linear programming was improved using the eigenvector of the starting point with all iterations.Where the improvement showed that Karmarker's method can be reduced in a theoretical way by direct method without iterations and access to the optimal solution. The procedure was also Comparison of the two methods and the results of the proposed method were faster and better to reach.

Keywords: Karmarker's algorithm; optimal solution; linear programming.

The perfect solution

Linear programming currently occupies a prestigious position in the field of research. The processes have wide applications in many scientific fields and over time have been. The development of scientific methods used to solve the dilemmas of linear programming after the war. This use of linear programming has appeared under the name of Operations Research. Linear programming methods are applied to find the best use of limited resources, and the word. Programming means using a certain method to get the best possible solutions to a problem Finite resource relation, a linear adjective used to describe the relationship between two variables or more, the solution that fits the constraints of the problem and the objective function is called the optimal solution. A new algorithm (Karmarker1984), to solve problems of all kinds of linear and nonlinear problems , which was much faster. His method consisted of repeated applications. (Simplex) from the simple solo method and each of them is followed by a Homomorphism through the spherical body, (Projection) of projective transitions described for the purpose of creating a sequence of Interior points that approximates towards the optimal solution in the case of a polynomial. Since the Karmarker's method was found, point methods have attracted internal interest of some famous experts in operations research and in mathematics

The canonical form of the Karmarker's method in solving linear programming

The (LP) problem needs the following:

[1] The centre a_0 in Δ i.e. $a_0 \in \Omega$.

[2] The optimized solution of goal function is zero in Ω.

[3] The matrix $\begin{bmatrix} A \\ e^T \end{bmatrix}$ has order m+1.

[4] $q > 0$ has the property
$\frac{e^T x}{e^T a_0} \leq 2^{-q}$, then x is the solution.

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$min\ e^T x, x \in R^n$

$$s.t.$$

$$x \in \Omega \cap \Delta$$

where

$$\Omega = \{x \in R^n; Ax = 0\}$$

$$\Delta = \{x \in R^n; e^T x = 1, x \geq 0\}$$

$e^T = [1,1, \dots ,1] \in R^n, A \in R^{m \times n}, c \in R^n, n \geq 2.$

Karamarker algorithm:

Step 1: we start at
$x_0 = \frac{e}{n}, k = 0, \varepsilon > 0.$

Step 2: we stop at $e^T x_k < \varepsilon$ other wise go to step 3.

Step 3: find the point $y_k = x_k - \theta r d_k$, where
$\theta = \frac{n-1}{3n}, r = \frac{1}{\sqrt{n(n-1)}}.$

$d_k = \frac{P_k}{\|P_k\|}$

$P_k = (I - B_k^T (B_k B_k^T)^{-1} B_k) D_k c$

$D_k = \text{diag}\ (x^k)$

$B_k = \begin{bmatrix} A_k \\ e^T \end{bmatrix}$

$A_k = \text{A}D_k.$

Step 4:
$x_{k+1} = \frac{D_k y_k}{e^T D_k y_k}.$

Then, we add k unit, then going back to step 2.

The points we get, are in the useful area:

$\Omega \cap \Delta = \{x \in R^n: Ax = 0, e^T x = 1, x \geq 0\}$

$= \left\{x \in R^n: \begin{bmatrix} A \\ e^T \end{bmatrix} x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x \geq 0\right\}$

$= \left\{x \in R^n: \text{B}_0 x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x \geq 0\right\}$

$\text{B}_0 \in R^{(m+1) \times n}.$

Suggested method:

The main differences between the suggested method and Karamarker method that the suggested method uses one vector d_0 with starting point x_0.

Then use $\theta = \frac{n}{3n-1}$ instead of $\theta = \frac{n-1}{3n}.$

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$x_0 = \frac{e}{n}, k = 0, \varepsilon > 0.$

Step 2: we stop at $e^T x_k < \varepsilon$ other wise go to step 3.

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Step 3: find the point $y_k = x_k - \theta r d_k$, where

$$\theta = \frac{n}{3(n-1)}, r = \frac{1}{\sqrt{n(n-1)}}$$

$$d_0 = \frac{P_0}{\|P_0\|}$$

$$P_0 = (I - B_0^T(B_0B_0^T)^{-1}B_0)D_0c$$

$$D_0 = \text{diag}\,(x_0)$$

$$B_0 = \begin{bmatrix} A_0 \\ e^T \end{bmatrix}$$

$$A_0 = \text{A}D_0$$

step 4:

$$x_{k+1} = \frac{D_0 y_k}{e^T D_0 y_k}.$$

Then, we get $x_1, x_2, \ldots\ldots, x_n$ and $x \in \Omega \cap \Delta$.

Then, we get:

$$y_0 = x_0 - \theta r d_0$$

$$x_1 = \frac{D_0 y_0}{e^T D_0 y_0} = y_0$$

$$d_0 = \frac{P_0}{\|P_0\|}$$

$$P_0 = (I - B_0^T(B_0B_0^T)^{-1}B_0)D_0c$$

$$D_0 = \text{diag}\,(x_0)$$

$$B_0 = \begin{bmatrix} A_0 \\ e^T \end{bmatrix}$$

$$A_0 = \text{A}D_0 = \left[[a_1 \quad a_2] \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \right] = [0.5a_1 \quad 0.5a_2]$$

$$B_0 = \begin{bmatrix} AD_0 \\ e^T \end{bmatrix} = \begin{bmatrix} 0.5a_1 & 0.5a_2 \\ 1 & 1 \end{bmatrix}$$

$$B_0B_0^T = \begin{bmatrix} 0.25({a_1}^2 + {a_2}^2) & 0.5(a_1 + a_2) \\ 0.5(a_1 + a_2) & 2 \end{bmatrix}$$

Thus,

$$B_0B_0^T = \begin{bmatrix} 0.25({a_1}^2 + {a_2}^2) & 0 \\ 0 & 2 \end{bmatrix}$$

$$(B_0B_0^T)^{-1} = \begin{bmatrix} \frac{4}{{a_1}^2 + {a_2}^2} & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$B_0^T(B_0B_0^T)^{-1}B_0 = \frac{1}{2({a_1}^2 + {a_2}^2)} \begin{bmatrix} 3{a_1}^2 + {a_2}^2 & (a_1 + a_2)^2 \\ (a_1 + a_2)^2 & 3{a_1}^2 + {a_2}^2 \end{bmatrix}$$

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$$=\frac{1}{2({a_1}^2+{a_2}^2)}\begin{bmatrix}3{a_1}^2+{a_2}^2 & 0\\ 0 & 3{a_1}^2+{a_2}^2\end{bmatrix}$$

$$(B_0B_0^T)^{-1}B_0=\begin{bmatrix}\frac{{a_2}^2-{a_1}^2}{2({a_1}^2+{a_2}^2)} & 0\\ 0 & \frac{{a_1}^2-{a_2}^2}{2({a_1}^2+{a_2}^2)}\end{bmatrix}$$

$$D_0c=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix}=\begin{bmatrix}0.5c_1\\ 0.5c_2\end{bmatrix}$$

$$P_0=(I-B_0^T(B_0B_0^T)^{-1}B_0)D_0c=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$d_0=\begin{bmatrix}0\\ 0\end{bmatrix}.$$

$$y_0=x_0-\theta rd_0=\begin{bmatrix}0.5\\ 0.5\end{bmatrix}-\begin{bmatrix}0\\ 0\end{bmatrix}=\begin{bmatrix}0.5\\ 0.5\end{bmatrix}$$

$$x_1=\frac{D_0y_0}{e^TD_0y_0}=\frac{\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}\begin{bmatrix}0.5\\ 0.5\end{bmatrix}}{\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}\begin{bmatrix}0.5\\ 0.5\end{bmatrix}}=\begin{bmatrix}0.5\\ 0.5\end{bmatrix}=y_0.$$

Also,

$y_1=x_1-\theta rd_0=y_0-\theta rd_0=x_0-\theta rd_0-\theta rd_0=x_0-2\theta rd_0,\ and$

$$x_2=y_1=x_0-2\theta rd_0$$

$$x_3=y_2=x_0-3\theta rd_0$$

$$x_k=y_{k-1}=x_0-\ k\theta rd_0.$$

Testing results:

Test 1:

$$Min\ z=2x_2-x_3$$

$$\text{s.t.}$$

$$x_1-2x_2+x_3=0$$

$$x_1+x_2+x_3=1$$

$$x_1,x_2,x_3\geq 0$$

Test 2:

$$Min\ z=x_1+2x_2-x_3$$

$$\text{s.t.}$$

$$x_1-x_3=0$$

$$x_1+x_2+x_3=1$$

$$x_1,x_2,x_3\geq 0$$

Test 3:

$$Min\ z=x_1-2x_2+6x_3$$

$$\text{s.t.}$$

$$x_1-x_2=0$$

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$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Test 4:

$$Min\ z = 2x_1 + x_2 - 2x_3$$

$$\text{s.t.}$$

$$x_1 - x_3 = 0$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Test 5:

$$Min\ z = x_1 + x_2 - x_3$$

$$\text{s.t.}$$

$$x_2 - x_3 = 0$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Test 6:

$$Min\ z = -x_1 + 2x_2$$

$$\text{s.t.}$$

$$x_1 - 2x_2 - x_3 = 0$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Test 7:

$$Min\ z = x_1 - 3x_2 + 3x_3$$

$$\text{s.t.}$$

$$x_1 - 3x_2 + 2x_3 = 0$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Test 8:

$$Max\ z = -4x_1 + x_3 - x_4$$

$$\text{s.t.}$$

$$-2x_1 + 2x_2 + x_3 - x_4 = 0$$

$$x_1 + x_2 + x_3 + x_4 = 1$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Test 9:

$$Max\ z = 5x_1 + 5x_2 - 23x_5$$

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$$\text{s.t.}$$

$$3x_1 + 8x_2 + 3x_3 - x_4 - 13x_5 = 0$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

We get the following tables:

Table 1

		x_1	x_2	x_3	Z
Karamarker	1	0.26918	0.33333	0.39748	0.2692
	2	0.15496	0.32549	0.51955	0.1314
	3	0.030331	0.2685	0.70117	−0.1642
Suggested method	$k = 2$	0.044658	0.33333	0.62201	0.0447

2

		x_1	x_2	x_3	Z
Karamarker	1	0.37037	0.25926	0.37037	0.5185
	2	0.43137	0.13726	0.43137	0.2745
	3	0.48949	0.021011	0.48949	0.0420
Suggested method	$k = 2$	0.5	0.000	0.5	0.000

3

		x_1	x_2	x_3	Z
Karamarker	1	0.37037	0.37037	0.25926	0.5556
	2	0.43137	0.43137	0.13726	0.8235
	3	0.48949	0.48949	0.021011	0.1261
Suggested method	$k = 2$	0.5	0.5	0.000	−0.5

4

		x_1	x_2	x_3	Z
Karamarker	1	0.37037	0.25926	0.25926	0.2593
	2	0.43137	0.13726	0.13726	0.1373
	3	0.48949	0.021011	0.021011	0.0210
Suggested method	$k = 2$	0.5	0.000	0.5	0.000

5

		x_1	x_2	x_3	Z
Karamarker	1	0.2593	0.37037	0.25926	0.2593

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	2	0.1373	0.43137	0.13726	0.1373
	3	0.0210	0.48949	0.021011	0.0210
Suggested method	$k = 2$	0.000	0.5	0.5	0.000

6

		x_1	x_2	x_3	Z
Karamarker	1	0.39748	0.33333	0.26918	0.2692
	2	0.51955	0.32549	0.15496	0.1314
	3	0.70117	0.2685	0.030331	−0.1642
Suggested method	$k = 2$	0.62201	0.33333	0.044658	0.0447

7

		x_1	x_2	x_3	Z
Karamarker	1	0.40333	0.31933	0.27734	0.2773
	2	0.54266	0.28541	0.17193	0.2022
Suggested method	$k = 2$	0.6483	0.27034	0.081357	0.0814

8

		x_1	x_2	x_3	x_4	Z
Karamarker	1	0.21392	0.21392	0.28608	0.21392	−0.8557
	2	0.14608	0.14608	0.35392	0.35392	−0.5843
	3	0.052136	0.052136	0.44786	0.44786	−0.2085
Suggested method	$k = 3$	0.05755	0.05755	0.44245	0.44245	−0.2302

9

		x_1	x_2	x_3	x_4	x_5	Z
Karamarker	1	0.22917	0.16866	0.18609	0.2345	0.18158	−2.1873
	2	0.28445	0.11268	0.15208	0.30628	0.14451	−1.3382
	3	0.36219	0.039824	0.081861	0.43825	0.077872	0.2190

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