Generalized Pre-closed Sets in Fermatean Neutrosophic Hypersoft Topological Spaces

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Abstract

The motivation of this article is to develop a new idea namely generalized pre-closed sets in Fermatean Neutrosophic Hypersoft topological spaces. Also, we initiate the idea of generalized pre-open set in Fermatean Neutrosophic Hypersoft topological spaces and its characterizations are analyzed.

Keywords: FNH generalized pre-closed; FNH generalized pre-open; FNH topology.

1. Introduction

Neutrosophic set was defined by Smarandache[11] which was the basic fundamental tool that act with incomplete, indeterminant and inconsistent. Neutrosophic theory describe about truthness, indeterminate and falseness membership functions simulate values between 0 and 1. Undergoing this neutrosophic conditions, numerous researchers have performed on their extension and proposed many results and application. Molodtsov[5] made known us about the concept of soft set which gives a new outlook dealed with uncertainty and recently there is a ascent rise of soft theory beside application in different areas. Smarandache[10] defined generalized hypersoft set from soft set by transmuting into multiargument function from argument function. He also gave mant outcome results from hypersoft sets. Salami and Alblowi[8] proposed neutrosophic topological spaces and further notions namely semiclosed sets, connectedness and generalized closed sets[4] have also advanced .Fuzzy hypersoft set idea was put into fuzzy topological spaces and Ajay and Charisma[1] exposed fuzzy hypersoft topological spaces. Then the idea of neutrosophic hypersoft topological spaces was presented by Ajay and charisma[1]. Fermatean fuzzy set was investigated by Tapan Senapati et.al.[9] in 2020 by comparing with pythagorean fuzzy sets and intuitionistic fuzzy sets. By the elongation of Pythagorean neutrosophic set C.A.C sweety and R. Jansi[2] defined Fermatean neutrosophic set. In recent times, P.Reena Joice and M. Trinita Pricilla [7] has evolved an advanced outcome of Fermatean neutrosophic hypersoft topological spaces.

2.Preliminaries

Definition 2.1[7]

Let $\mathfrak{P}$ be the universal set and $P(\mathfrak{P})$ be the power set of $\mathfrak{P}$. Consider $\mathbb{s}^1, \mathbb{s}^2, \ldots, \mathbb{s}^n$ for $n \geq 1$ be n well defined attributes, whose corresponding attribute values are respectively the set

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$\mathbb{S}^1, \mathbb{S}^2, \ldots, \mathbb{S}^n$ with $\mathbb{S}^i \cap \mathbb{S}^j = \emptyset$, for $i \neq j$ and $i, j \in \{1,2\ldots,n\}$ and their relation $\mathbb{S}^1 \times \mathbb{S}^2 \times \ldots \mathbb{S}^n = \sigma$, then the pair $(\mathbb{R}, \sigma)$ is said to be Fermatean Neutrosophic Hypersoft set (FNHS in short) over $\mathfrak{P}$ where $\mathbb{R}: \mathbb{S}^1 \times \mathbb{S}^2 \times \ldots \times \mathbb{S}^n \rightarrow P(\mathfrak{P})$ and

$$\mathbb{R}(\mathbb{S}^1 \times \mathbb{S}^2 \times \ldots \times \mathbb{S}^n) = \{\langle \mathfrak{p}, \mu_{\mathcal{A}}(\mathbb{R}(\sigma)), \nu_{\mathcal{A}}(\mathbb{R}(\sigma)), \delta_{\mathcal{A}}(\mathbb{R}(\sigma))\rangle, \mathfrak{p} \in \mathfrak{P}\}$$

where

$\mu_{\mathcal{A}}$ is the membership value of truthness, $\nu_{\mathcal{A}}$ is the membership value of indeterminacy and $\delta_{\mathcal{A}}$

is the membership value of falsity such that $\mu_{\mathcal{A}}, \nu_{\mathcal{A}}, \delta_{\mathcal{A}}: \mathfrak{P} \rightarrow [0,1]$,

$$0 \leq \left(\mu_{\mathcal{A}}(\mathbb{R}(\sigma))\right)^3 + \left(\delta_{\mathcal{A}}(\mathbb{R}(\sigma))\right)^3 \leq 1 \text{ and } 0 \leq \left(\nu_{\mathcal{A}}(\mathbb{R}(\sigma))\right)^3 \leq 1.$$

$$\text{Then } 0 \leq \left(\mu_{\mathcal{A}}(\mathbb{R}(\sigma))\right)^3 + \left(\nu_{\mathcal{A}}(\mathbb{R}(\sigma))\right)^3 + \left(\delta_{\mathcal{A}}(\mathbb{R}(\sigma))\right)^3 \leq 2$$

Definition 2.2[7]

Let $(\mathfrak{I}, \mathfrak{Q})$ and $(\mathfrak{K}, \mathfrak{R})$ be two Fermatean neutrosophic hypersoft set of the form

$$(\mathfrak{I}, \mathfrak{Q}) = \{\langle \mathfrak{p}, \mu_{\mathcal{A}}(\mathbb{R}(\sigma)), \nu_{\mathcal{A}}(\mathbb{R}(\sigma)), \delta_{\mathcal{A}}(\mathbb{R}(\sigma))\rangle, \mathfrak{p} \in \mathfrak{P}\} \text{ and}$$

$$(\mathfrak{K}, \mathfrak{R}) = \{\langle \mathfrak{p}, \mu_{\mathcal{B}}(\mathbb{R}(\sigma)), \nu_{\mathcal{B}}(\mathbb{R}(\sigma)), \delta_{\mathcal{B}}(\mathbb{R}(\sigma))\rangle, \mathfrak{p} \in \mathfrak{P}\}. \text{ Then}$$

- $(\mathfrak{I}, \mathfrak{Q}) \subseteq (\mathfrak{K}, \mathfrak{R})$ if and only if $\mu_{\mathcal{A}}(\mathbb{R}(\sigma)) \leq \mu_{\mathcal{B}}(\mathbb{R}(\sigma))$, $\nu_{\mathcal{A}}(\mathbb{R}(\sigma)) \geq \nu_{\mathcal{B}}(\mathbb{R}(\sigma))$, $\delta_{\mathcal{A}}(\mathbb{R}(\sigma)) \geq \delta_{\mathcal{B}}(\mathbb{R}(\sigma))$.
- $(\mathfrak{I}, \mathfrak{Q}) = (\mathfrak{K}, \mathfrak{R})$ if and only if $(\mathfrak{I}, \mathfrak{Q}) \subseteq (\mathfrak{K}, \mathfrak{R})$ and $(\mathfrak{K}, \mathfrak{R}) \subseteq (\mathfrak{I}, \mathfrak{Q})$
- $(\mathfrak{I}, \mathfrak{Q})^c = \{\langle \mathfrak{p}, \delta_{\mathcal{A}}(\mathbb{R}(\sigma)), 1 - \nu_{\mathcal{A}}(\mathbb{R}(\sigma)), \mu_{\mathcal{A}}(\mathbb{R}(\sigma))\rangle, \mathfrak{p} \in \mathfrak{P}\}$
- $(\mathfrak{I}, \mathfrak{Q})) \cup (\mathfrak{K}, \mathfrak{R})$ is defined as

$$\mu((\mathfrak{I}, \mathfrak{Q})\ (\mathfrak{K}, \mathfrak{R})) = \begin{cases} \mu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{Q} - \mathfrak{R} \\ \mu_{\mathfrak{K}(\mathbb{R}(\sigma)))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{R} - \mathfrak{Q} \\ \max\left\{\mu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}), \mu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p})\right\} & \text{if } \sigma \in \mathfrak{Q} \cap \mathfrak{R} \end{cases}$$

$$\nu((\mathfrak{I}, \mathfrak{Q}) \cup (\mathfrak{K}, \mathfrak{R})) = \begin{cases} \nu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{Q} - \mathfrak{R} \\ \nu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{R} - \mathfrak{Q} \\ \min\left\{\nu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}), \nu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p})\right\} & \text{if } \sigma \in \mathfrak{Q} \cap \mathfrak{R} \end{cases}$$

$$\delta\big((\mathfrak{I}, \mathfrak{Q}) \cup (\mathfrak{K}, \mathfrak{R})\big) = \begin{cases} \delta_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{Q} - \mathfrak{R} \\ \delta_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{R} - \mathfrak{Q} \\ \min\left\{\delta_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}), \delta_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p})\right\} & \text{if } \sigma \in \mathfrak{Q} \cap \mathfrak{R} \end{cases}$$

- $(\mathfrak{I}, \mathfrak{Q}) \cap (\mathfrak{K}, \mathfrak{R})$ is defined as

$$\mu((\mathfrak{I}, \mathfrak{Q}) \cap (\mathfrak{K}, \mathfrak{R})) = \begin{cases} \mu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{Q} - \mathfrak{R} \\ \mu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{R} - \mathfrak{Q} \\ \min\left\{\mu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}), \mu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p})\right\} & \text{if } \sigma \in \mathfrak{Q} \cap \mathfrak{R} \end{cases}$$

$$\nu((\mathfrak{I}, \mathfrak{Q})) \cap (\mathfrak{K}, \mathfrak{R})) = \begin{cases} \nu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{Q} - \mathfrak{R} \\ \nu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{R} - \mathfrak{Q} \\ \max\left\{\nu_{\mathfrak{I}(\mathbb{R}(\sigma))}(\mathfrak{p}), \nu_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p})\right\} & \text{if } \sigma \in \mathfrak{Q} \cap \mathfrak{R} \end{cases}$$

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$$\delta((\mathfrak{J},\mathfrak{Q}) \cap (\mathfrak{K},\mathfrak{R})) = \begin{cases} \delta_{\mathfrak{J}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{Q}-\mathfrak{R} \\ \delta_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p}) & \text{if } \sigma \in \mathfrak{R}-\mathfrak{Q} \\ \max\left\{\delta_{\mathfrak{J}(\mathbb{R}(\sigma))}(\mathfrak{p}),\ \delta_{\mathfrak{K}(\mathbb{R}(\sigma))}(\mathfrak{p})\right\} & \text{if } \sigma \in \mathfrak{Q}\cap\mathfrak{R} \end{cases}$$

Definition 2.3[7]

Let FNHS $(\mathfrak{P},\mathbb{G})$ be the family of all fermatean neutrosophic hypersoft sets over the universe set $\mathfrak{P}$ and $\tau \subseteq \text{FNHS}(\mathfrak{P},\mathbb{G})$. Then τ is said to be a fermatean neutrosophic hypersoft topology on $\mathfrak{P}$ if

1. $0_{(\mathfrak{P}_{FNH},\mathbb{G})}, 1_{(\mathfrak{P}_{FNH},\mathbb{G})}$ belongs to τ
2. the union of any number of fermatean neutrosophic hypersoft sets in τ belongs to τ
3. the intersection of finite number of fermatean neutrosophic hypersoft sets in τ belongs to τ.

Then $(\mathfrak{P},\mathbb{G},\tau)$ is said to be a fermatean neutrosophic hypersoft topological space over $\mathfrak{P}$. Each members of τ is said to be fermatean neutrosophic hypersoft open set.

Definition 2.4[7]

Let $(\mathfrak{P},\mathbb{G},\tau)$ be a fermatean neutrosophic hypersoft topological space over $\mathfrak{P}$ and $(\mathfrak{J},\mathfrak{Q}) \in \text{FNHS}(\mathfrak{P},\mathbb{G})$ be a fermatean neutrosophic hypersoft set. Then, the fermatean neutrosophic hypersoft interior of $(\mathfrak{J},\mathfrak{Q})$, denoted $\text{FNHint}(\mathfrak{J},\mathfrak{Q})$ is defined as the fermatean neutrosophic hypersoft union of all fermatean neutrosophic hypersoft open subsets of $(\mathfrak{J},\mathfrak{Q})$. Clearly, $\text{FNHint}(\mathfrak{J},\mathfrak{Q})$ is the biggest fermatean neutrosophic hypersoft open set that is contained by $(\mathfrak{J},\mathfrak{Q})$.

Let $(\mathfrak{P},\mathbb{G},\tau)$ be a fermatean neutrosophic hypersoft topological space over $\mathfrak{P}$ and $(\mathfrak{J},\mathfrak{Q}) \in \text{FNHS}(\mathfrak{P},\mathbb{G})$ be a fermatean neutrosophic hypersoft set. Then, the fermatean neutrosophic hypersoft closure of $(\mathfrak{J},\mathfrak{Q})$, denoted $\text{FNHScl}(\mathfrak{J},\mathfrak{Q})$ is defined as the fermatean neutrosophic hypersoft intersection of all fermatean neutrosophic hypersoft closed super sets of $(\mathfrak{J},\mathfrak{Q})$. Clearly, $\text{FNHScl}(\mathfrak{J},\mathfrak{Q})$ is the smallest fermatean neutrosophic hypersoft closed set that containing $(\mathfrak{J},\mathfrak{Q})$.

3. Fermatean Neutrosophic Hypersoft Generalized Pre- Closed Set

In this portion we initiate the concept of Fermatean Neutrosophic Hypersoft generalized pre-closed set and few of its properties are examined.

Definition 3.1

A FNHS $(\mathfrak{J},\mathfrak{Q}) = \{\langle \mathfrak{p}, \mu_{\mathcal{A}}(\mathbb{R}(\sigma)), \nu_{\mathcal{A}}(\mathbb{R}(\sigma)), \delta_{\mathcal{A}}(\mathbb{R}(\sigma))\rangle, \mathfrak{p} \in \mathfrak{P}\}$ in a FNHTS $(\mathfrak{P},\tau)$ is said to be a

(i) Fermatean Neutrosophic Hypersoft semi closed set(FNHSCS) if

$\text{FNHint}(\text{FNHcl}(\mathfrak{J},\mathfrak{Q})) \subseteq (\mathfrak{J},\mathfrak{Q})$.

(ii) Fermatean Neutrosophic Hypersoft semi open set(FNHSOS) if

$(\mathfrak{J},\mathfrak{Q}) \subseteq \text{FNHcl}(\text{FNHint}((\mathfrak{J},\mathfrak{Q})))$.

(iii)Fermatean Neutrosophic Hypersoft pre closed set(FNHPCS) if

$\text{FNHcl}(\text{FNHint}\ (\mathfrak{J},\mathfrak{Q})) \subseteq (\mathfrak{J},\mathfrak{Q})$.

(iv) Fermatean Neutrosophic Hypersoft pre open set(FNHPOS) if

$(\mathfrak{J},\mathfrak{Q}) \subseteq \text{FNHint}(\text{FNHcl}((\mathfrak{J},\mathfrak{Q})))$.

(v) Fermatean Neutrosophic Hypersoft α closed set(FNHαCS) if

$\text{FNHcl}(\text{FNHint}(\text{FNHcl}(\mathfrak{J},\mathfrak{Q}))) \subseteq (\mathfrak{J},\mathfrak{Q})$.

(vi) Fermatean Neutrosophic Hypersoft pre open set(FNHαOS) if

$(\mathfrak{J},\mathfrak{Q}) \subseteq \text{FNHint}(\text{FNHcl}(\text{FNHint}(\mathfrak{J},\mathfrak{Q})))$.

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(vii) Fermatean Neutrosophic Hypersoft regular closed set(FNHRCS) if

$(\mathfrak{J}, \mathfrak{Q})$ =FNHcl(FNHint$(\mathfrak{J}, \mathfrak{Q})$).

(viii) Fermatean Neutrosophic Hypersoft regular open set(FNHROS) if

$(\mathfrak{J}, \mathfrak{Q})$ =FNHint(FNHcl$(\mathfrak{J}, \mathfrak{Q})$).

Definition 3.2

A FNHS $(\mathfrak{J}, \mathfrak{Q})=\{\langle \mathfrak{p}, \mu_{\mathcal{A}}(\mathbb{R}(\sigma)), \nu_{\mathcal{A}}(\mathbb{R}(\sigma)), \delta_{\mathcal{A}}(\mathbb{R}(\sigma))\rangle, \mathfrak{p} \in \mathfrak{P}\}$ of a FNHTS $(\mathfrak{P},\tau)$ is a

(i) Fermatean Neutrosophic Hypersoft generalized closed set(FNHGCS) if

FNHcl$(\mathfrak{J}, \mathfrak{Q}) \subseteq \mathfrak{G}$ whenever $(\mathfrak{J}, \mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $\mathfrak{P}$.

(ii)Fermatean Neutrosophic Hypersoft α generalized closed set(FNHαGCS) if

FNHαcl$(\mathfrak{J}, \mathfrak{Q}) \subseteq \mathfrak{G}$ whenever$(\mathfrak{J}, \mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $\mathfrak{P}$.

(iii)Fermatean Neutrosophic Hypersoft generalized semi closed set(FNHGSCS) if

FNHscl$(\mathfrak{J}, \mathfrak{Q}) \subseteq \mathfrak{G}$ whenever $(\mathfrak{J}, \mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $\mathfrak{P}$.

Definition 3.3

Let $(\mathfrak{J}, \mathfrak{Q})$ be a FNHS of a FNHTS $(\mathfrak{P},\tau)$. Then the fermatean neutrosophic hypersoft pre closure of $(\mathfrak{J}, \mathfrak{Q})$FNHpcl$(\mathfrak{J}, \mathfrak{Q})$) in short) is defined as FNHpcl$(\mathfrak{J}, \mathfrak{Q})=\cap\{(\mathfrak{H}, \mathfrak{E})/(\mathfrak{H}, \mathfrak{E})$ is a FNHPCS in $\mathfrak{P}$ and $(\mathfrak{J}, \mathfrak{Q})\subseteq (\mathfrak{H}, \mathfrak{E})\}$.

Definition 3.4

Let $(\mathfrak{J}, \mathfrak{Q})$ be a FNHS of a FNHTS $(\mathfrak{P},\tau)$. Then the fermatean neutrosophic hypersoft pre interior of $(\mathfrak{J}, \mathfrak{Q})$ (FNHpint$(\mathfrak{J}, \mathfrak{Q})$in short) is defined as FNHpint$(\mathfrak{J}, \mathfrak{Q})=\cup\{(\mathfrak{H}, \mathfrak{E}))/(\mathfrak{H}, \mathfrak{E})$ is a FNHPOS in $\mathfrak{P}$ and $(\mathfrak{H}, \mathfrak{E}) \subseteq (\mathfrak{J}, \mathfrak{Q})\}$.

Definition 3.5

Let $(\mathfrak{J}, \mathfrak{Q})$ be a FNHS of a FNHTS $(\mathfrak{P},\tau)$. Then the fermatean neutrosophic hypersoft semi closure of $(\mathfrak{J}, \mathfrak{Q})$ (FNHscl$(\mathfrak{J}, \mathfrak{Q})$ in short) is defined as FNHscl$(\mathfrak{J}, \mathfrak{Q}) = \cap\{(\mathfrak{H}, \mathfrak{E})/(\mathfrak{H}, \mathfrak{E})$ is a FNHSCS in $\mathfrak{P}$ and $(\mathfrak{J}, \mathfrak{Q}) \subseteq (\mathfrak{H}, \mathfrak{E})\}$.

Definition 3.6

Let $(\mathfrak{J}, \mathfrak{Q})$ be a FNHS of a FNHTS $(\mathfrak{P},\tau)$. Then the fermatean neutrosophic hypersoft semi interior of $(\mathfrak{J}, \mathfrak{Q})$ (FNHsint$(\mathfrak{J}, \mathfrak{Q})$ in short) is defined as FNHsint$(\mathfrak{J}, \mathfrak{Q}) = \cup \{(\mathfrak{H}, \mathfrak{E})/(\mathfrak{H}, \mathfrak{E})$ is a FNHSOS in $\mathfrak{P}$ and $(\mathfrak{H}, \mathfrak{E}) \subseteq (\mathfrak{J}, \mathfrak{Q})\}$.

Definition 3.7

Let $(\mathfrak{J}, \mathfrak{Q})$ be a FNHS of a FNHTS $(\mathbb{X},\tau)$. Then the fermatean neutrosophic hypersoft alpha closure of $(\mathfrak{J}, \mathfrak{Q})$ (FNHαcl$(\mathfrak{J}, \mathfrak{Q})$ in short) is defined as FNHαcl$(\mathfrak{J}, \mathfrak{Q}) = \cap\{(\mathfrak{H}, \mathfrak{E})/(\mathfrak{H}, \mathfrak{E})$ is a FNHαCS in $\mathfrak{P}$ and $(\mathfrak{J}, \mathfrak{Q})\subseteq (\mathfrak{H}, \mathfrak{E})\}$.

Definition 3.8

Let $(\mathfrak{J}, \mathfrak{Q})$ be a FNHS of a FNHTS $(\mathfrak{P},\tau)$. Then the fermatean neutrosophic hypersoft alpha interior of $(\mathfrak{J}, \mathfrak{Q})$ (FNHαint$(\mathfrak{J}, \mathfrak{Q})$ in short) is defined as FNHαint$(\mathfrak{J}, \mathfrak{Q}) = \cup\{(\mathfrak{H}, \mathfrak{E})/(\mathfrak{H}, \mathfrak{E})$ is a FNHαOS in $\mathfrak{P}$ and $(\mathfrak{H}, \mathfrak{E}) \subseteq (\mathfrak{J}, \mathfrak{Q})\}$.

Definition 3.9

Let $(\mathfrak{J}, \mathfrak{Q})$) be a FNHS in $(\mathfrak{P},\tau)$ then

(i)FNHpcl$(\mathfrak{J}, \mathfrak{Q})=(\mathfrak{J}, \mathfrak{Q})\cup$ FNHcl(FNHint$(\mathfrak{J}, \mathfrak{Q})$)

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(ii)FNHpint$(\mathfrak{J},\mathfrak{Q})$= $(\mathfrak{J},\mathfrak{Q})$ ∩ FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$)

(iii)FNHscl$(\mathfrak{J},\mathfrak{Q})$=$(\mathfrak{J},\mathfrak{Q})$∪ FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$)

(iv)FNHsint$(\mathfrak{J},\mathfrak{Q})$= $(\mathfrak{J},\mathfrak{Q})$ ∩FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$)

(v)FNHαcl$(\mathfrak{J},\mathfrak{Q})$=$(\mathfrak{J},\mathfrak{Q})$∪ FNHcl(FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))

(vi)FNHαint$(\mathfrak{J},\mathfrak{Q})$=$(\mathfrak{J},\mathfrak{Q})$∩ FNHint(FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$))

Definition 3.10

A FNHS $(\mathfrak{J},\mathfrak{Q})$ is said to be a Fermatean Neutrosophic Hypersoft generalized pre-closed set (FNHGPCS in short) in $(\mathfrak{P},\tau)$ if FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ whenever $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $\mathfrak{P}$. The family of all FNHGPCSs of a FNHTS $(\mathfrak{P},\tau)$ is denoted by FNHGPC$(\mathfrak{P})$.

Example 3.11

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be a sets of attributes. Attributes are given as;

$$\mathbb{S}_1= \{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2= \{\eta_1,\eta_2\},\ \mathbb{S}_3= \{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let τ = {0, $(\mathbb{P},\mathbb{R})$, 1} be an FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{\mathfrak{q}_1}{[0.1,0.5,0.6]},\frac{\mathfrak{q}_2}{[0.5,0.5,0.6]},\frac{\mathfrak{q}_3}{[0.3,0.6,0.4]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{\mathfrak{q}_1}{,[0.3,0.5,0.4]},\frac{\mathfrak{q}_2}{[0.8,0.1,0.2]},\frac{\mathfrak{q}_3}{[0.5,0.4,0.1]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{\mathfrak{q}_1}{[0.8,0.4,0.3]},\frac{\mathfrak{q}_2}{[0.6,0.4,0.1]},\frac{\mathfrak{q}_3}{[0.4,0.2,0.8]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{\mathfrak{q}_1}{[0.8,0.7,0.4]},\frac{\mathfrak{q}_2}{[0.8,0.3,0.4]},\frac{\mathfrak{q}_3}{[0.6,0.7,0.8]}\right\}\rangle\end{array}\right\}$$

Then the FNHS $(\mathfrak{J},\mathfrak{Q})$ = $\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{\mathfrak{q}_1}{[0.1,0.7,0.7]},\frac{\mathfrak{q}_2}{[0.4,0.8,0.6]},\frac{\mathfrak{q}_3}{[0.2,0.7,0.4]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{\mathfrak{q}_1}{,[0.2,0.7,0.8]},\frac{\mathfrak{q}_2}{[0.5,0.9,0.8]},\frac{\mathfrak{q}_3}{[0.1,0.7,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{\mathfrak{q}_1}{[0.4,0.5,0.7]},\frac{\mathfrak{q}_2}{[0.1,0.7,0.7]},\frac{\mathfrak{q}_3}{[0.3,0.6,0.8]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{\mathfrak{q}_1}{[0.3,0.7,0.8]},\frac{\mathfrak{q}_2}{[0.2,0.6,0.7]},\frac{\mathfrak{q}_3}{[0.1,0.8,0.8]}\right\}\rangle\end{array}\right\}$ is a FNHGPCS in $\mathfrak{P}$.

Theorem 3.12

Every FNHCS is a FNHGPCS but not conversely.

Proof: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHCS in $\mathfrak{P}$ and let $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $(\mathfrak{P}, \tau)$. Since FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq$ FNHcl$(\mathfrak{J},\mathfrak{Q})$ and $(\mathfrak{J},\mathfrak{Q})$ is a FNHCS in $\mathfrak{P}$, FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq$ FNHcl$(\mathfrak{J},\mathfrak{Q}) = (\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$. Therefore $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPCS in $\mathfrak{P}$.

Example 3.13

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1= \{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2= \{\eta_1,\eta_2\},\ \mathbb{S}_3= \{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let τ = {0, $(\mathbb{P},\mathbb{R})$, 1} be a FNHT on $\mathfrak{P}$. Where,

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$$(\mathbb{P},\mathbb{R})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.2,0.3]},\frac{q_2}{[0.7,0.6,0.8]},\frac{q_3}{[0.2,0.4,0.6]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.6,0.5,0.7]},\frac{q_2}{[0.4,0.5,0.6]},\frac{q_3}{[0.7,0.8,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.3,0.2,0.6]},\frac{q_2}{[0.6,0.2,0.8]},\frac{q_3}{[0.6,0.9,0.3]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.4,0.6,0.8]},\frac{q_2}{[0.5,0.2,0.9]},\frac{q_3}{[0.3,0.6,0.2]}\right\}\rangle\end{array}\right\}$$

Then the FNHS $(\mathfrak{J},\mathfrak{Q})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.8,0.8]},\frac{q_2}{[0.5,0.6,0.7]},\frac{q_3}{[0.9,0.6,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.1,0.7,0.5]},\frac{q_2}{[0.5,0.1,0.3]},\frac{q_3}{[0.1,0.7,0.7]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.7,0.8,0.4]},\frac{q_2}{[0.2,0.8,0.4]},\frac{q_3}{[0.3,0.6,0.9]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.5,0.7,0.6]},\frac{q_2}{[0.2,0.4,0.6]},\frac{q_3}{[0.3,0.1,0.6]}\right\}\rangle\end{array}\right\}$

is a FNHGPCS in $\mathfrak{P}$ but not a FNHCS in $\mathfrak{P}$.

Theorem 3.14

Every FNHαCS is a FNHGPCS but not conversely.

Proof: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHαCS in $\mathfrak{P}$ and let $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $(\mathfrak{P}, \tau)$. By hypothesis, FNHcl(FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))⊆ $(\mathfrak{J},\mathfrak{Q})$. Since $(\mathfrak{J},\mathfrak{Q}) \subseteq$ FNHcl$(\mathfrak{J},\mathfrak{Q})$, FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$)⊆ FNHcl(FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))⊆ $(\mathfrak{J},\mathfrak{Q})$. Hence FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq (\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$. Therefore $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPCS in $\mathfrak{P}$.

Example 3.15

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2 = \{\eta_1,\eta_2\},\ \mathbb{S}_3 = \{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let τ = {0, $(\mathbb{P},\mathbb{R})$, 1} be an FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.3,0.6,0.5]},\frac{q_2}{[0.4,0.5,0.6]},\frac{q_3}{[0.4,0.5,0.6]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.3,0.5,0.9]},\frac{q_2}{[0.5,0.3,0.2]},\frac{q_3}{[0.5,0.6,0.7]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.2,0.6,0.1]},\frac{q_2}{[0.6,0.2,0.8]},\frac{q_3}{[0.6,0.9,0.3]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.5,0.4,0.3]},\frac{q_2}{[0.6,0.2,0.9]},\frac{q_3}{[0.1,0.4,0.6]}\right\}\rangle\end{array}\right\}$$

Then the FNHS $(\mathfrak{J},\mathfrak{Q}) = \left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\gamma_1),\left\{\frac{q_1}{[0.8,0.1,0.8]},\frac{q_2}{[0.4,0.6,0.8]},\frac{q_3}{[0.9,0.6,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.4,0.7,0.6]},\frac{q_2}{[0.5,0.1,0.3]},\frac{q_3}{[0.7,0.1,0.7]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.7,0.8,0.4]},\frac{q_2}{[0.8,0.2,0.4]},\frac{q_3}{[0.1,0.7,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.4,0.8,0.3]},\frac{q_2}{[0.2,0.4,0.6]},\frac{q_3}{[0.6,0.3,0.1]}\right\}\rangle\end{array}\right\}$ is a FNHGPCS in $\mathfrak{P}$

but not a FNHαCS in $\mathfrak{P}$.

Theorem 3.16

Every FNHGCS is a FNHGPCS but not conversely.

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Proof: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHGCS in $\mathfrak{P}$ and let $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $(\mathfrak{P}, \tau)$. Since FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq$ FNHcl$(\mathfrak{J},\mathfrak{Q})$ and by hypothesis, FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$. Therefore $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPCS in $\mathfrak{P}$.

Example 3.17

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P}, \mathbb{R}), 1\}$ be an FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.5,0.6]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.5]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.4]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{,[0.3,0.5,0.4]}, \frac{\mathfrak{q}_2}{[0.8,0.1,0.3]}, \frac{\mathfrak{q}_3}{[0.5,0.4,0.1]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.8,0.4,0.3]}, \frac{\mathfrak{q}_2}{[0.6,0.5,0.2]}, \frac{\mathfrak{q}_3}{[0.4,0.2,0.8]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.8,0.7,0.4]}, \frac{\mathfrak{q}_2}{[0.7,0.4,0.3]}, \frac{\mathfrak{q}_3}{[0.6,0.7,0.8]}\right\}\rangle \end{array}\right\}$$

Then the FNHS $(\mathbb{M}, \mathbb{A})$ $= \left\{\begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.8,0.8]}, \frac{\mathfrak{q}_2}{[0.4,0.8,0.6]}, \frac{\mathfrak{q}_3}{[0.2,0.7,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{,[0.2,0.7,0.9]}, \frac{\mathfrak{q}_2}{[0.5,0.9,0.8]}, \frac{\mathfrak{q}_3}{[0.1,0.7,0.6]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.2,0.6,0.8]}, \frac{\mathfrak{q}_2}{[0.1,0.7,0.7]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.8]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.3,0.7,0.8]}, \frac{\mathfrak{q}_2}{[0.2,0.6,0.7]}, \frac{\mathfrak{q}_3}{[0.1,0.8,0.8]}\right\}\rangle \end{array}\right\}$ is a FNHGPCS in $\mathfrak{P}$

but not a FNHGCS in $\mathfrak{P}$.

Theorem 3.18

Every FNHRCS is an FNHGPCS but not conversely.

Proof: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHRCS in $\mathfrak{P}$. By Definition 3.1, $(\mathfrak{J},\mathfrak{Q})$ = FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$). This implies FNHcl$(\mathfrak{J},\mathfrak{Q})$ = FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$). Therefore FNHcl$(\mathfrak{J},\mathfrak{Q}) = (\mathfrak{J},\mathfrak{Q})$. That is $(\mathfrak{J},\mathfrak{Q})$ is a FNHCS in $\mathfrak{P}$. By Theorem 3.12, $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPCS in $\mathfrak{P}$.

Example 3.19

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P}, \mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.5,0.3,0.6]}, \frac{\mathfrak{q}_2}{[0.7,0.6,0.8]}, \frac{\mathfrak{q}_3}{[0.4,0.5,0.6]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{,[0.6,0.5,0.7]}, \frac{\mathfrak{q}_2}{[0.4,0.8,0.3]}, \frac{\mathfrak{q}_3}{[0.7,0.8,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.3,0.2,0.6]}, \frac{\mathfrak{q}_2}{[0.6,0.2,0.8]}, \frac{\mathfrak{q}_3}{[0.5,0.7,0.2]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.3,0.5,0.4]}, \frac{\mathfrak{q}_2}{[0.5,0.2,0.9]}, \frac{\mathfrak{q}_3}{[0.1,0.4,0.6]}\right\}\rangle \end{array}\right\}$$

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Then the FNHS $(\mathfrak{J},\mathfrak{Q}) = \left\{ \begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.8,0.8]},\frac{q_2}{[0.8,0.6,0.4]},\frac{q_3}{[0.9,0.6,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.6,0.7,0.4]},\frac{q_2}{[0.5,0.1,0.3]},\frac{q_3}{[0.1,0.7,0.7]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.7,0.8,0.4]},\frac{q_2}{[0.2,0.8,0.4]},\frac{q_3}{[0.5,0.6,0.1]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.4,0.8,0.3]},\frac{q_2}{[0.6,0.4,0.2]},\frac{q_3}{[0.3,0.1,0.6]}\right\}\rangle \end{array} \right\}$

is a FNHGPCS in $\mathfrak{P}$ but not a FNHRCS in $\mathfrak{P}$.

Theorem 3.20

Every FNHPCS is a FNHGPCS but not conversely.

Proof: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHPCS in $\mathfrak{P}$ and let $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $(\mathfrak{P},\tau)$. By Definition 3.1, FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$) $\subseteq (\mathfrak{J},\mathfrak{Q})$. This implies FNHpcl$(\mathfrak{J},\mathfrak{Q}) = (\mathfrak{J},\mathfrak{Q})$ ∪FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$) $\subseteq (\mathfrak{J},\mathfrak{Q})$. Therefore FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$. Hence $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPCS in $\mathfrak{P}$.

Example 3.21

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be a initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2 = \{\eta_1,\eta_2\},\ \mathbb{S}_3 = \{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P},\mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{ \begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.5,0.6]},\frac{q_2}{[0.4,0.6,0.5]},\frac{q_3}{[0.3,0.6,0.4]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.2,0.5,0.6]},\frac{q_2}{[0.8,0.1,0.3]},\frac{q_3}{[0.5,0.4,0.1]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.8,0.4,0.3]},\frac{q_2}{[0.6,0.5,0.2]},\frac{q_3}{[0.5,0.2,0.7]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.8,0.7,0.4]},\frac{q_2}{[0.7,0.4,0.3]},\frac{q_3}{[0.6,0.7,0.8]}\right\}\rangle \end{array} \right\}$$

Then the FNHS $(\mathfrak{J},\mathfrak{Q})) = \left\{ \begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.6,0.5,0.1]},\frac{q_2}{[0.8,0.5,0.4]},\frac{q_3}{[0.9,0.5,0.3]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.5,0.4,0.3]},\frac{q_2}{[0.9,0.1,0.3]},\frac{q_3}{[0.8,0.3,0.1]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.9,0.3,0.2]},\frac{q_2}{[0.9,0.4,0.2]},\frac{q_3}{[0.7,0.1,0.6]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.9,0.6,0.3]},\frac{q_2}{[0.8,0.3,0.2]},\frac{q_3}{[0.7,0.6,0.7]}\right\}\rangle \end{array} \right\}$ is a FNHGPCS in $\mathfrak{P}$

but not a FNHPCS in $\mathfrak{P}$.

Theorem 3.22

Every FNHαGCS is a FNHGPCS but not conversely.

Proof: Let $(\mathfrak{J},\mathfrak{Q})$ be an FNHαGCS in $\mathfrak{P}$ and let $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $(\mathfrak{P},\tau)$. By Definition 3.2, $(\mathfrak{J},\mathfrak{Q})$ ∪ FNHcl(FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))$\subseteq \mathfrak{G}$. This implies FNHcl(FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))$\subseteq \mathfrak{G}$ and FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$)$\subseteq \mathfrak{G}$. Therefore FNHpcl$(\mathfrak{J},\mathfrak{Q})$ = $(\mathfrak{J},\mathfrak{Q})$ ∪ FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})$) $\subseteq \mathfrak{G}$. Hence $(\mathfrak{J},\mathfrak{Q})$ is an FNHGPCS in $\mathfrak{P}$.

Example 3.23

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be a initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2 = \{\eta_1,\eta_2\},\ \mathbb{S}_3 = \{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P},\mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

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$$(\mathbb{P},\mathbb{R})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.5,0.6]},\frac{q_2}{[0.4,0.6,0.5]},\frac{q_3}{[0.3,0.5,0.3]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.3,0.5,0.4]},\frac{q_2}{[0.4,0.6,0.8]},\frac{q_3}{[0.5,0.7,0.7]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.6,0.6,0.7]},\frac{q_2}{[0.3,0.5,0.7]},\frac{q_3}{[0.1,0.9,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.6,0.7,0.9]},\frac{q_2}{[0.2,0.7,0.5]},\frac{q_3}{[0.6,0.7,0.8]}\right\}\rangle\end{array}\right\}$$

Then the FNHS $(\mathfrak{J},\mathfrak{Q})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.6,0.7]},\frac{q_2}{[0.3,0.7,0.6]},\frac{q_3}{[0.2,0.7,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.2,0.5,0.5]},\frac{q_2}{[0.3,0.6,0.8]},\frac{q_3}{[0.4,0.7,0.7]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.5,0.7,0.8]},\frac{q_2}{[0.2,0.6,0.7]},\frac{q_3}{[0.1,0.9,0.6]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.4,0.7,0.9]},\frac{q_2}{[0.1,0.7,0.8]},\frac{q_3}{[0.3,0.7,0.8]}\right\}\rangle\end{array}\right\}$ is a FNHGPCS in $\mathfrak{P}$ but not a FNHαGCS in $\mathfrak{P}$.

Proposition 3.24

FNHSCS and FNHGPCS are independent to each other.

Example 3.25

Let $\mathfrak{P}=\{q_1,q_2,q_3\}$ be a initial universe and $\mathbb{S}_1,\mathbb{S}_2,\mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1=\{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2=\{\eta_1,\eta_2\},\ \mathbb{S}_3=\{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau=\{0,(\mathbb{P},\mathbb{R}),1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.5,0.6]},\frac{q_2}{[0.4,0.6,0.5]},\frac{q_3}{[0.3,0.6,0.4]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.3,0.5,0.4]},\frac{q_2}{[0.4,0.6,0.8]},\frac{q_3}{[0.5,0.7,0.7]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.6,0.6,0.8]},\frac{q_2}{[0.3,0.5,0.7]},\frac{q_3}{[0.1,0.9,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.6,0.7,0.9]},\frac{q_2}{[0.2,0.7,0.8]},\frac{q_3}{[0.6,0.7,0.8]}\right\}\rangle\end{array}\right\}$$

Then the FNHS $(\mathfrak{J},\mathfrak{Q})=(\mathbb{P},\mathbb{R})$ is a FNHSCS in $\mathfrak{P}$ but not a FNHGPCS in $\mathfrak{P}$.

Example 3.26

Let $\mathfrak{P}=\{q_1,q_2,q_3\}$ be an initial universe and $\mathbb{S}_1,\mathbb{S}_2,\mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1=\{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2=\{\eta_1,\eta_2\},\ \mathbb{S}_3=\{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau=\{0,(\mathbb{P},\mathbb{R}),1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R})=\left\{\begin{array}{l}\langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.3,0.6,0.5]},\frac{q_2}{[0.7,0.6,0.8]},\frac{q_3}{[0.5,0.4,0.3]}\right\}\rangle,\\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{,[0.6,0.4,0.5]},\frac{q_2}{[0.5,0.3,0.2]},\frac{q_3}{[0.7,0.8,0.5]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.3,0.2,0.6]},\frac{q_2}{[0.7,0.1,0.5]},\frac{q_3}{[0.6,0.9,0.3]}\right\}\rangle,\\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.2,0.3,0.7]},\frac{q_2}{[0.5,0.2,0.9]},\frac{q_3}{[0.1,0.4,0.6]}\right\}\rangle\end{array}\right\}$$

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Then the FNHS $(\mathfrak{I},\mathfrak{Q}) = \left\{\begin{array}{l} \left\langle (\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.8,0.8]}, \frac{\mathfrak{q}_2}{[0.5,0.7,0.6]}, \frac{\mathfrak{q}_3}{[0.9,0.6,0.5]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.5,0.6,0.4]}, \frac{\mathfrak{q}_2}{[0.5,0.1,0.3]}, \frac{\mathfrak{q}_3}{[0.1,0.7,0.7]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.7,0.8,0.4]}, \frac{\mathfrak{q}_2}{[0.4,0.7,0.1]}, \frac{\mathfrak{q}_3}{[0.1,0.7,0.5]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.4,0.8,0.3]}, \frac{\mathfrak{q}_2}{[0.2,0.4,0.6]}, \frac{\mathfrak{q}_3}{[0.6,0.8,0.2]}\right\}\right\rangle \end{array}\right\}$

is a FNHGPCS in $\mathfrak{P}$ but not a FNHSCS in $\mathfrak{P}$.

Proposition 3.27

FNHGSCS and FNHGPCS are independent to each other.

Example 3.28

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P},\mathbb{R}), 1\}$ be an FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{array}{l} \left\langle (\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.5,0.6]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.5]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.4]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.3,0.5,0.4]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.8]}, \frac{\mathfrak{q}_3}{[0.5,0.7,0.7]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.6,0.6,0.8]}, \frac{\mathfrak{q}_2}{[0.3,0.5,0.7]}, \frac{\mathfrak{q}_3}{[0.1,0.9,0.5]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.6,0.7,0.9]}, \frac{\mathfrak{q}_2}{[0.2,0.7,0.8]}, \frac{\mathfrak{q}_3}{[0.6,0.7,0.8]}\right\}\right\rangle \end{array}\right\}$$

Then the FNHS $(\mathfrak{I},\mathfrak{Q}) = (\mathbb{P},\mathbb{R})$ is a FNHGSCS in $\mathfrak{P}$ but not a FNHGPCS in $\mathfrak{P}$.

Example 3.29

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as; $\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\}$, $\mathbb{S}_2 = \{\eta_1, \eta_2\}$, $\mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$ are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P},\mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{array}{l} \left\langle (\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.5,0.6]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.5]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.4]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.3,0.5,0.4]}, \frac{\mathfrak{q}_2}{[0.8,0.1,0.3]}, \frac{\mathfrak{q}_3}{[0.5,0.4,0.1]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.8,0.4,0.3]}, \frac{\mathfrak{q}_2}{[0.6,0.5,0.2]}, \frac{\mathfrak{q}_3}{[0.4,0.2,0.8]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.8,0.7,0.4]}, \frac{\mathfrak{q}_2}{[0.7,0.4,0.3]}, \frac{\mathfrak{q}_3}{[0.6,0.7,0.8]}\right\}\right\rangle \end{array}\right\}$$

Then the FNHS $(\mathfrak{I},\mathfrak{Q}) = \left\{\begin{array}{l} \left\langle (\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.8,0.8]}, \frac{\mathfrak{q}_2}{[0.4,0.8,0.6]}, \frac{\mathfrak{q}_3}{[0.2,0.7,0.5]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.2,0.7,0.9]}, \frac{\mathfrak{q}_2}{[0.5,0.9,0.8]}, \frac{\mathfrak{q}_3}{[0.1,0.7,0.6]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.2,0.6,0.8]}, \frac{\mathfrak{q}_2}{[0.1,0.7,0.7]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.8]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.3,0.7,0.8]}, \frac{\mathfrak{q}_2}{[0.2,0.6,0.7]}, \frac{\mathfrak{q}_3}{[0.1,0.8,0.8]}\right\}\right\rangle \end{array}\right\}$

is a FNHGPCS in $\mathfrak{P}$ but not a FNHGSCS in $\mathfrak{P}$.

The following implications are true

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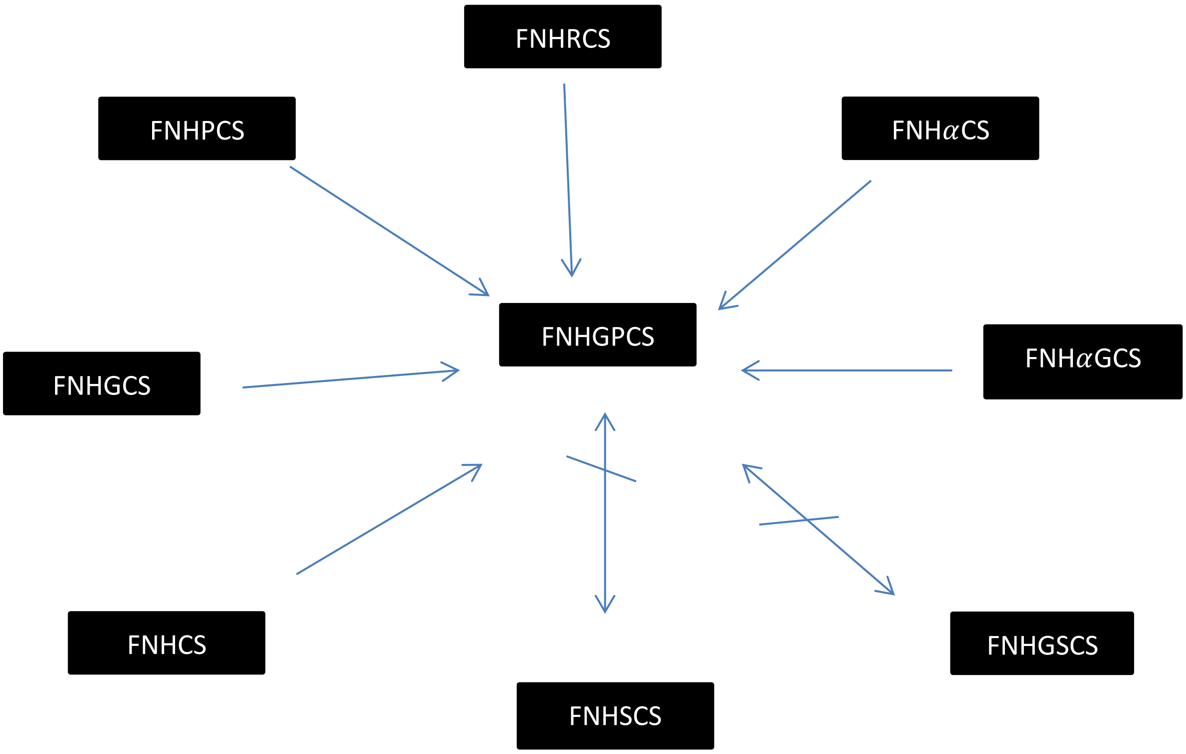

In this diagram "$\mathbb{U} \rightarrow \mathbb{V}$ "we mean $\mathbb{U}$ implies $\mathbb{V}$ but not conversely and "$\mathbb{U} \nleftrightarrow \mathbb{V}$" are independent of each other.

Remark 3.30

The union of any two FNHGPCSs is not a FNHGPCS in general as seen in the following example.

Example 3.31

Let $\mathfrak{P} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3\}$ be a initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P}, \mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{array}{c} \left\langle (\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.2,0.5,0.6]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.5]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.4]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{,[0.3,0.5,0.4]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.8]}, \frac{\mathfrak{q}_3}{[0.5,0.7,0.7]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.6,0.6,0.8]}, \frac{\mathfrak{q}_2}{[0.4,0.5,0.7]}, \frac{\mathfrak{q}_3}{[0.1,0.9,0.5]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.6,0.7,0.9]}, \frac{\mathfrak{q}_2}{[0.2,0.7,0.8]}, \frac{\mathfrak{q}_3}{[0.6,0.7,0.8]}\right\}\right\rangle \end{array}\right\}$$

Then the FNHSs $(\mathfrak{J}, \mathfrak{Q}) = \left\{\begin{array}{c} \left\langle (\zeta_1,\eta_1,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.1,0.5,0.8]}, \frac{\mathfrak{q}_2}{[0.4,0.6,0.5]}, \frac{\mathfrak{q}_3}{[0.3,0.6,0.6]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_1,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{,[0.1,0.7,0.8]}, \frac{\mathfrak{q}_2}{[0.3,0.7,0.8]}, \frac{\mathfrak{q}_3}{[0.5,0.7,0.7]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_1), \left\{\frac{\mathfrak{q}_1}{[0.6,0.6,0.8]}, \frac{\mathfrak{q}_2}{[0.1,0.6,0.7]}, \frac{\mathfrak{q}_3}{[0.1,0.9,0.5]}\right\}\right\rangle, \\ \left\langle (\zeta_1,\eta_2,\vartheta_2), \left\{\frac{\mathfrak{q}_1}{[0.5,0.7,0.9]}, \frac{\mathfrak{q}_2}{[0.2,0.7,0.8]}, \frac{\mathfrak{q}_3}{[0.5,0.7,0.8]}\right\}\right\rangle \end{array}\right\}$ and

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$$(\mathfrak{K},\mathfrak{R}) = \left\{\begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.2,0.5,0.6]},\frac{q_2}{[0.2,0.7,0.7]},\frac{q_3}{[0.3,0.6,0.4]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{[0.3,0.5,0.4]},\frac{q_2}{[0.4,0.6,0.8]},\frac{q_3}{[0.4,0.7,0.8]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.5,0.7,0.8]},\frac{q_2}{[0.4,0.5,0.7]},\frac{q_3}{[0.1,0.9,0.6]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.6,0.7,0.9]},\frac{q_2}{[0.1,0.7,0.8]},\frac{q_3}{[0.6,0.7,0.8]}\right\}\rangle \end{array}\right\} \text{ is a FNHGPCSs in } \mathfrak{P}$$

but $(\mathfrak{I},\mathfrak{Q}) \cup (\mathfrak{K},\mathfrak{R})$ is not a FNHGPCS in $\mathfrak{P}$.

4. Fermatean Neutrosophic Hypersoft Generalized Pre-Open Sets

In this section we initiate the idea of Fermatean Neutrosophic Hypersoft generalized pre-open set and its properties are examined.

Definition 4.1

A FNHS $(\mathfrak{I},\mathfrak{Q})$ is said to be an Fermatean Neutrosophic Hypersoft generalized pre-open set (FNHGPOS in short) in $(\mathfrak{P}, \tau)$ if the complement $(\mathfrak{I},\mathfrak{Q})^c$ is an IFGPCS in $\mathfrak{P}$. The family of all FNHGPOSs of an FNHTS $(\mathfrak{P}, \tau)$ is denoted by FNHGPO($\mathfrak{P}$).

Example 4.2

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1,\zeta_2,\zeta_3\},\ \mathbb{S}_2 = \{\eta_1,\eta_2\},\ \mathbb{S}_3 = \{\vartheta_1,\vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P},\mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.3,0.6,0.5]},\frac{q_2}{[0.7,0.6,0.8]},\frac{q_3}{[0.4,0.5,0.6]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{[0.6,0.5,0.7]},\frac{q_2}{[0.5,0.3,0.2]},\frac{q_3}{[0.7,0.8,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.3,0.2,0.6]},\frac{q_2}{[0.6,0.2,0.8]},\frac{q_3}{[0.6,0.9,0.3]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.5,0.4,0.3]},\frac{q_2}{[0.5,0.2,0.9]},\frac{q_3}{[0.1,0.4,0.6]}\right\}\rangle \end{array}\right\}$$

$$\text{Then the FNHS } (\mathfrak{I},\mathfrak{Q}) = \left\{\begin{array}{l} \langle(\zeta_1,\eta_1,\vartheta_1),\left\{\frac{q_1}{[0.1,0.8,0.8]},\frac{q_2}{[0.4,0.8,0.6]},\frac{q_3}{[0.9,0.6,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2),\left\{\frac{q_1}{[0.4,0.7,0.6]},\frac{q_2}{[0.5,0.1,0.3]},\frac{q_3}{[0.1,0.7,0.7]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1),\left\{\frac{q_1}{[0.7,0.8,0.4]},\frac{q_2}{[0.2,0.8,0.4]},\frac{q_3}{[0.1,0.7,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2),\left\{\frac{q_1}{[0.4,0.8,0.3]},\frac{q_2}{[0.2,0.4,0.6]},\frac{q_3}{[0.3,0.1,0.6]}\right\}\rangle \end{array}\right\} \text{ is a FNHGPOS in } \mathfrak{P}.$$

Theorem 4.3

For any FNHTS $(\mathfrak{P}, \tau)$ we have the following:

- Every FNHOS is an FNHGPOS
- Every FNHSOS is an FNHGPOS
- Every FNHαOS is an FNHGPOS
- Every FNHPOS is an FNHGPOS. But the converses are not true in general.

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Proof: Straight forward.

The converse of the above statements need not be true which can be seen from the following examples.

Example 4.4

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P}, \mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{matrix} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{q_1}{[0.3,0.6,0.5]}, \frac{q_2}{[0.7,0.6,0.8]}, \frac{q_3}{[0.1,0.2,0.3]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{q_1}{,[0.3,0.6,0.8]}, \frac{q_2}{[0.5,0.3,0.2]}, \frac{q_3}{[0.7,0.8,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{q_1}{[0.3,0.2,0.6]}, \frac{q_2}{[0.6,0.2,0.8]}, \frac{q_3}{[0.6,0.9,0.3]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{q_1}{[0.5,0.4,0.3]}, \frac{q_2}{[0.4,0.1,0.6]}, \frac{q_3}{[0.1,0.4,0.6]}\right\}\rangle, \end{matrix}\right\}$$

Then the FNHS $(\mathfrak{J}, \mathfrak{Q}) = \left\{\begin{matrix} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{q_1}{[0.1,0.6,0.5]}, \frac{q_2}{[0.4,0.6,0.8]}, \frac{q_3}{[0.9,0.6,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{q_1}{,[0.4,0.7,0.6]}, \frac{q_2}{[0.5,0.1,0.3]}, \frac{q_3}{[0.1,0.7,0.7]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{q_1}{[0.7,0.8,0.4]}, \frac{q_2}{[0.8,0.4,0.5]}, \frac{q_3}{[0.1,0.7,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{q_1}{[0.4,0.8,0.3]}, \frac{q_2}{[0.2,0.4,0.6]}, \frac{q_3}{[0.7,0.5,0.4]}\right\}\rangle \end{matrix}\right\}$ is a FNHGPOS in $\mathfrak{P}$

but not a FNHOS, FNHSOS, FNHαOS in $\mathfrak{P}$.

Example 4.5

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be an initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P}, \mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P},\mathbb{R}) = \left\{\begin{matrix} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{q_1}{[0.1,0.6,0.9]}, \frac{q_2}{[0.3,0.7,0.8]}, \frac{q_3}{[0.3,0.9,0.7]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{q_1}{,[0.2,0.7,0.8]}, \frac{q_2}{[0.8,0.3,0.6]}, \frac{q_3}{[0.4,0.9,0.6]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{q_1}{[0.7,0.5,0.6]}, \frac{q_2}{[0.5,0.6,0.7]}, \frac{q_3}{[0.2,0.3,0.8]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{q_1}{[0.7,0.8,0.5]}, \frac{q_2}{[0.4,0.7,0.9]}, \frac{q_3}{[0.3,0.8,0.8]}\right\}\rangle \end{matrix}\right\}$$

Then the FNHS $(\mathfrak{J}, \mathfrak{Q}) = \left\{\begin{matrix} \langle(\zeta_1,\eta_1,\vartheta_1), \left\{\frac{q_1}{[0.1,0.8,0.8]}, \frac{q_2}{[0.4,0.8,0.8]}, \frac{q_3}{[0.3,0.6,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_1,\vartheta_2), \left\{\frac{q_1}{,[0.4,0.7,0.6]}, \frac{q_2}{[0.5,0.7,0.8]}, \frac{q_3}{[0.1,0.7,0.7]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_1), \left\{\frac{q_1}{[0.6,0.8,0.8]}, \frac{q_2}{[0.2,0.8,0.6]}, \frac{q_3}{[0.1,0.7,0.5]}\right\}\rangle, \\ \langle(\zeta_1,\eta_2,\vartheta_2), \left\{\frac{q_1}{[0.4,0.8,0.8]}, \frac{q_2}{[0.2,0.4,0.6]}, \frac{q_3}{[0.3,0.4,0.6]}\right\}\rangle \end{matrix}\right\}$ is a FNHGPOS in $\mathfrak{P}$

but not a FNHPOS in $\mathfrak{P}$.

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Theorem 4.6

Let $(\mathfrak{P}, \tau)$ be a FNHTS. If $(\mathfrak{J}, \mathfrak{Q}) \in$ FNHGPO($\mathfrak{P}$) then $(\mathfrak{C}, \mathfrak{D}) \subseteq$ FNHint(FNHcl$(\mathfrak{J}, \mathfrak{Q})$) whenever $(\mathfrak{C}, \mathfrak{D}) \subseteq (\mathfrak{J}, \mathfrak{Q})$ and $(\mathfrak{C}, \mathfrak{D})$is FNHCS in $\mathfrak{P}$.

Proof: Let $(\mathfrak{J}, \mathfrak{Q}) \in$ FNHGPO($\mathfrak{P}$). Then $(\mathfrak{J}, \mathfrak{Q})^c$ is a FNHGPCS in $\mathfrak{P}$. Therefore FNHpcl($(\mathfrak{J}, \mathfrak{Q})^c$)$\subseteq \mathfrak{G}$ whenever $(\mathfrak{J}, \mathfrak{Q})^c \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $\mathfrak{P}$. That is FNHcl(FNHint($(\mathfrak{J}, \mathfrak{Q})^c$))$\subseteq \mathfrak{G}$. This implies $\mathfrak{G}^c \subseteq$ FNHint(FNHcl$(\mathfrak{J}, \mathfrak{Q})$) whenever $\mathfrak{G}^c \subseteq (\mathfrak{J}, \mathfrak{Q})$ and $\mathfrak{G}^c$ is FNHCS in $\mathfrak{P}$. Replacing $\mathfrak{G}^c$ by $(\mathfrak{C}, \mathfrak{D})$, we get $(\mathfrak{C}, \mathfrak{D}) \subseteq$ FNHint(cl$(\mathfrak{J}, \mathfrak{Q})$) whenever $(\mathfrak{C}, \mathfrak{D}) \subseteq (\mathfrak{J}, \mathfrak{Q})$ and $(\mathfrak{C}, \mathfrak{D})$ is FNHCS in $\mathfrak{P}$.

Theorem 4.7

Let $(\mathfrak{P}, \tau)$ be an FNHTS. Then for every $(\mathfrak{J}, \mathfrak{Q}) \in$FNHGPO($\mathfrak{P}$) and for every $(\mathfrak{K}, \mathfrak{R}) \in$ FNHS($\mathfrak{P}$), FNHpint$(\mathfrak{J}, \mathfrak{Q}) \subseteq (\mathfrak{K}, \mathfrak{R}) \subseteq (\mathfrak{J}, \mathfrak{Q})$ implies $(\mathfrak{K}, \mathfrak{R}) \in$ FNHGPO($\mathfrak{P}$).

Proof: By hypothesis $(\mathfrak{J}, \mathfrak{Q})^c \subseteq (\mathfrak{K}, \mathfrak{R})^c \subseteq$ (FNHpint$(\mathfrak{J}, \mathfrak{Q})$)^c. Let $(\mathfrak{K}, \mathfrak{R})^c \subseteq \mathfrak{G}$ and $\mathfrak{G}$ be a FNHOS. Since $(\mathfrak{J}, \mathfrak{Q})^c \subseteq (\mathfrak{K}, \mathfrak{R})^c$, $(\mathfrak{J}, \mathfrak{Q})^c \subseteq \mathfrak{G}$. But $(\mathfrak{J}, \mathfrak{Q})^c$ is an FNHGPCS, FNHpcl $(\mathfrak{J}, \mathfrak{Q})^c \subseteq \mathfrak{G}$. Also $(\mathfrak{K}, \mathfrak{R})^c \subseteq$ (FNHpint$(\mathfrak{J}, \mathfrak{Q})$)^c= FNHpcl($(\mathfrak{J}, \mathfrak{Q})^c$). Therefore FNHpcl($(\mathfrak{K}, \mathfrak{R})^c$)$\subseteq$ FNHpcl($(\mathfrak{J}, \mathfrak{Q})^c$)$\subseteq \mathfrak{G}$. Hence $(\mathfrak{K}, \mathfrak{R})^c$ is an FNHGPCS. Which implies $(\mathfrak{K}, \mathfrak{R})$ is an FNHGPOS of $\mathfrak{P}$.

Remark 4.8

The intersection of any two FNHGPOSs is not an FNHGPOS in general.

Example 4.9

Let $\mathfrak{P} = \{q_1, q_2, q_3\}$ be a initial universe and $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3$ be sets of attributes. Attributes are given as;

$$\mathbb{S}_1 = \{\zeta_1, \zeta_2, \zeta_3\},\ \mathbb{S}_2 = \{\eta_1, \eta_2\},\ \mathbb{S}_3 = \{\vartheta_1, \vartheta_2\}$$

are subset of $\mathbb{S}_i$ for each i=1,2,3. Let $\tau = \{0, (\mathbb{P}, \mathbb{R}), 1\}$ be a FNHT on $\mathfrak{P}$. Where,

$$(\mathbb{P}, \mathbb{R}) = \left\{\begin{array}{l} \left\langle(\zeta_1, \eta_1, \vartheta_1), \left\{\frac{q_1}{[0.1,0.5,0.6]}, \frac{q_2}{[0.4,0.6,0.5]}, \frac{q_3}{[0.3,0.6,0.4]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_1, \vartheta_2), \left\{\frac{q_1}{,[0.3,0.5,0.4]}, \frac{q_2}{[0.4,0.6,0.8]}, \frac{q_3}{[0.5,0.7,0.7]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_2, \vartheta_1), \left\{\frac{q_1}{[0.6,0.6,0.8]}, \frac{q_2}{[0.3,0.5,0.7]}, \frac{q_3}{[0.1,0.9,0.5]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_2, \vartheta_2), \left\{\frac{q_1}{[0.6,0.7,0.9]}, \frac{q_2}{[0.2,0.7,0.8]}, \frac{q_3}{[0.6,0.7,0.8]}\right\}\right\rangle \end{array}\right\}$$

Then the FNHSs $(\mathfrak{J}, \mathfrak{Q}) = \left\{\begin{array}{l} \left\langle(\zeta_1, \eta_1, \vartheta_1), \left\{\frac{q_1}{[0.6,0.5,0.1]}, \frac{q_2}{[0.5,0.4,0.4]}, \frac{q_3}{[0.4,0.4,0.3]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_1, \vartheta_2), \left\{\frac{q_1}{,[0.7,0.5,0.3]}, \frac{q_2}{[0.8,0.4,0.4]}, \frac{q_3}{[0.8,0.3,0.4]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_2, \vartheta_1), \left\{\frac{q_1}{[0.8,0.4,0.6]}, \frac{q_2}{[0.8,0.4,0.3]}, \frac{q_3}{[0.5,0.1,0.1]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_2, \vartheta_2), \left\{\frac{q_1}{[0.9,0.4,0.2]}, \frac{q_2}{[0.8,0.3,0.2]}, \frac{q_3}{[0.9,0.2,0.4]}\right\}\right\rangle \end{array}\right\}$ and

$$(\mathfrak{K}, \mathfrak{R}) = \left\{\begin{array}{l} \left\langle(\zeta_1, \eta_1, \vartheta_1), \left\{\frac{q_1}{[0.7,0.4,0.1]}, \frac{q_2}{[0.6,0.3,0.3]}, \frac{q_3}{[0.6,0.4,0.3]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_1, \vartheta_2), \left\{\frac{q_1}{,[0.4,0.5,0.3]}, \frac{q_2}{[0.8,0.3,0.2]}, \frac{q_3}{[0.7,0.3,0.5]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_2, \vartheta_1), \left\{\frac{q_1}{[0.8,0.3,0.5]}, \frac{q_2}{[0.7,0.5,0.3]}, \frac{q_3}{[0.7,0.1,0.1]}\right\}\right\rangle, \\ \left\langle(\zeta_1, \eta_2, \vartheta_2), \left\{\frac{q_1}{[0.9,0.7,0.6]}, \frac{q_2}{[0.8,0.3,0.1]}, \frac{q_3}{[0.8,0.3,0.6]}\right\}\right\rangle \end{array}\right\}$$

is a FNHGPCSs in $\mathfrak{P}$ but $(\mathfrak{J}, \mathfrak{Q}) \cap (\mathfrak{K}, \mathfrak{R})$ is not a FNHGPCS in $\mathfrak{P}$.

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Theorem 4.10

A FNHS $(\mathfrak{J},\mathfrak{Q})$ of a FNHTS $(\mathfrak{P}, \tau)$ is an FNHGPOS if and only if $(\mathfrak{T},\mathfrak{W}) \subseteq$ FNHpint $(\mathfrak{J},\mathfrak{Q})$ whenever $(\mathfrak{T},\mathfrak{W})$ is a FNHCS and $(\mathfrak{T},\mathfrak{W}) \subseteq (\mathfrak{J},\mathfrak{Q})$.

Proof: Necessity: Suppose $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPOS in $\mathfrak{P}$. Let $(\mathfrak{T},\mathfrak{W})$ be a FNHCS and $(\mathfrak{T},\mathfrak{W}) \subseteq (\mathfrak{J},\mathfrak{Q})$. Then $(\mathfrak{T},\mathfrak{W})^c$ is a FNHOS in $\mathfrak{P}$ such that $(\mathfrak{J},\mathfrak{Q})^c \subseteq (\mathfrak{T},\mathfrak{W})^c$. Since $(\mathfrak{J},\mathfrak{Q})^c$ is a FNHGPCS, we have FNHpcl$((\mathfrak{J},\mathfrak{Q})^c) \subseteq (\mathfrak{T},\mathfrak{W})^c$. Hence $(FNHpint(\mathfrak{J},\mathfrak{Q}))^c \subseteq (\mathfrak{T},\mathfrak{W})^c$. Therefore$(\mathfrak{T},\mathfrak{W}) \subseteq$ FNHpint$(\mathfrak{J},\mathfrak{Q})$).

Sufficiency: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHS of $\mathfrak{P}$ and let $(\mathfrak{T},\mathfrak{W}) \subseteq$ FNHpint$(\mathfrak{J},\mathfrak{Q})$ whenever $(\mathfrak{T},\mathfrak{W})$ is a FNHCS and $(\mathfrak{T},\mathfrak{W}) \subseteq (\mathfrak{J},\mathfrak{Q})$. Then $(\mathfrak{J},\mathfrak{Q})^c \subseteq (\mathfrak{T},\mathfrak{W})^c$ and $(\mathfrak{T},\mathfrak{W})^c$ is a FNHOS. By hypothesis, $(FNHpint(\mathfrak{J},\mathfrak{Q}))^c \subseteq (\mathfrak{T},\mathfrak{W})^c$. Which implies FNHpcl$((\mathfrak{J},\mathfrak{Q})^c) \subseteq (\mathfrak{T},\mathfrak{W})^c$. Therefore $(\mathfrak{J},\mathfrak{Q})^c$ is a FNHGPCS of $\mathfrak{P}$. Hence $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPOS of $\mathfrak{P}$.

Corollary 4.11

A FNHS $(\mathfrak{J},\mathfrak{Q}$ of a FNHTS $(\mathfrak{P}, \tau)$ is a FNHGPOS if and only if $(\mathfrak{T},\mathfrak{W}) \subseteq$ FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$) whenever $(\mathfrak{T},\mathfrak{W})$ is a FNHCS and $(\mathfrak{T},\mathfrak{W}) \subseteq (\mathfrak{J},\mathfrak{Q})$.

Proof: Necessity: Suppose $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPOS in $\mathfrak{P}$. Let $(\mathfrak{T},\mathfrak{W})$ be a FNHCS and $(\mathfrak{T},\mathfrak{W}) \subseteq (\mathfrak{J},\mathfrak{Q})$. Then $(\mathfrak{T},\mathfrak{W})^c$ is a FNHOS in $\mathfrak{P}$ such that $(\mathfrak{J},\mathfrak{Q})^c \subseteq (\mathfrak{T},\mathfrak{W})^c$. Since $(\mathfrak{J},\mathfrak{Q})^c$ is a FNHGPCS, we have FNHpcl$((\mathfrak{J},\mathfrak{Q})^c) \subseteq (\mathfrak{T},\mathfrak{W})^c$. Therefore FNHcl(FNHint$(\mathfrak{J},\mathfrak{Q})^c$) $\subseteq (\mathfrak{T},\mathfrak{W})^c$. Hence (FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))$^c \subseteq (\mathfrak{T},\mathfrak{W})^c$. This implies $(\mathfrak{T},\mathfrak{W}) \subseteq$ FNHint(cl$(\mathfrak{J},\mathfrak{Q})$).

Sufficiency: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHS of $\mathfrak{P}$ and let $(\mathfrak{T},\mathfrak{W}) \subseteq$ FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$) whenever $(\mathfrak{T},\mathfrak{W})$ is an FNHCS and $(\mathfrak{T},\mathfrak{W}) \subseteq (\mathfrak{J},\mathfrak{Q})$. Then $(\mathfrak{J},\mathfrak{Q})^c \subseteq (\mathfrak{T},\mathfrak{W})^c$ and $(\mathfrak{T},\mathfrak{W})^c$ is an FNHOS. By hypothesis, (FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$))$^c \subseteq (\mathfrak{T},\mathfrak{W})^c$. Hence FNHcl(FNHint$((\mathfrak{J},\mathfrak{Q})^c$)) $\subseteq (\mathfrak{T},\mathfrak{W})^c$, which implies FNHpcl$((\mathfrak{J},\mathfrak{Q})^c) \subseteq (\mathfrak{T},\mathfrak{W})^c$. Hence $(\mathfrak{J},\mathfrak{Q})$ is an FNHGPOS of $\mathfrak{P}$.

Theorem 4.12

For a FNHS $(\mathfrak{J},\mathfrak{Q})$, $(\mathfrak{J},\mathfrak{Q})$ is a FNHOS and a FNHGPCS in $\mathfrak{P}$ if and only if $(\mathfrak{J},\mathfrak{Q})$ is a FNHROS in $\mathfrak{P}$.

Proof: Necessity: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHOS and a FNHGPCS in $\mathfrak{P}$. Then FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq (\mathfrak{J},\mathfrak{Q})$. This implies FNHCl(FNHint$(\mathfrak{J},\mathfrak{Q})$) $\subseteq (\mathfrak{J},\mathfrak{Q})$. Since $(\mathfrak{J},\mathfrak{Q})$) is a FNHOS, it is an FNHPOS. Hence $(\mathfrak{J},\mathfrak{Q}) \subseteq$ FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$). Therefore $(\mathfrak{J},\mathfrak{Q})$ = FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$). Hence $(\mathfrak{J},\mathfrak{Q})$) is a FNHROS in $\mathfrak{P}$.

Sufficiency: Let $(\mathfrak{J},\mathfrak{Q})$ be a FNHROS in $\mathfrak{P}$. Therefore $(\mathfrak{J},\mathfrak{Q})$ = FNHint(FNHcl$(\mathfrak{J},\mathfrak{Q})$). Let $(\mathfrak{J},\mathfrak{Q}) \subseteq \mathfrak{G}$ and $\mathfrak{G}$ is a FNHOS in $\mathfrak{P}$. This implies FNHpcl$(\mathfrak{J},\mathfrak{Q}) \subseteq (\mathfrak{J},\mathfrak{Q})$. Hence $(\mathfrak{J},\mathfrak{Q})$ is a FNHGPCS in $\mathfrak{P}$.

5. Conclusion

In this paper we have discussed about the concept of fermatean neutrosophic hypersoft generalized pre closed sets in fermatean neutrosophic hypersoft topological spaces and their characterizations are analyzed in these areas.

References

[1] Ajay D and Charisma J. J, "Neutrosophic hypersoft topological spaces", Neutrosophic Sets and Systems, vol. 40, pp. 179–194, 2021.

[2] Antony Crispin Sweety and Jansi R, "Fermatean neutrosophic sets", International Journal of Advance Research in Computer and Communication Engineering, vol. 10, pp. 24–27, 2021.

[3] Broumi S, Sundareswaran R, Shanmugapriya M, Nordo G, Talea M, Bakali A and Smarandache F, Interval valued Fermatean Neutrosophic Graphs, Decision making: Applications in Management and Engineering, 5(2), 176-200, 2022.

[4] Iswarya P and Bageerathi K, "On neutrosophic semi-open sets in neutrosophic topological spaces", International Journal of Mathematics Trends and Technology, vol. 37, pp. 214–223,

Doi: https://doi.org/10.54216/IJNS.200308
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2016.
[5] Molodtsov D, "Soft set theory-first results", Computers & Mathematics with Applications, vol. 37, no. 4-5, pp. 19–31, 1999.
[6] Rajarajeswari P and Senthil Kumar L, "Generalized Pre- Closed sets in Intuitionistic Fuzzy topological spaces". International Journal of Fuzzy Mathematics and Systems, vol. 1, pp. 253– 262, 2011.
[7] Reena Joice P and Trinita Pricilla M, "An introduction to Fermatean neutrosophic hypersoft topological spaces", International Journal of Creative and Research Thoughts, vol. 10, pp. 424–433, 2022.
[8] Salama A. A and Alblowi S. A, "Neutrosophic set and neutrosophic topological spaces", IOSR Journal of Mathematics, vol. 3, no. 4, pp. 31–35, 2012.
[9] Senapati T and Yager R. R, "Fermatean fuzzy sets", Journal of Ambient Intelligence and Humanized Computing 11, 663-674, 2020.
[10] Smarandache F, "Extension of soft set to hypersoft set and then to plithogenic hypersoft set", Neutrosophic Sets and Systems, vol. 22, pp. 168–170, 2018.
[11] Smarandache F, "A unifying Field of Logics. Neutrosophy: Neutrosophic Probability, Set and Logic", American Research Press, Rehoboth, MA, USA, 1998.

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