Application of Neutrosophic filters in Lattice implication algebra

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Abstract

Neutrosophic set theory is applied to lattice implication algebras, and the concept of neutrosophic filters and neutrosophic lattice filters in lattice implication algebra are introduced. Several properties are investigated. Characterizations of a neutrosophic filter are discussed. Finally, we prove that every neutrosophic filter is a neutrosophic lattice filter but the converse is invalid.

Keywords: Neutrosophic set (NS); Lattice implication algebra (LIA); Neutrosophic filter(NF); Neutrosophic lattice filter(NLF);

[1] Introduction

Use of neutrosophic sets in decision-making have recently attracted the attention of both scholars and practitioners. From a philosophical standpoint, the neutrosophic set concept was first proposed by Florentin Samarandache in 1998 [3,4,5]. The neutrosophic set generalises the notion of fuzzy sets and intuitionistic fuzzy sets. The applications of the neutrosophic set theory to algebraic structures are comparable to those of the fuzzy set (soft set, rough set) theory in algebraic structures [19,20, 24, and 25], which is used to many scientific domains.

A novel mathematical technique for resolving issues involving erroneous, ambiguous, and inconsistent data is the neutrosophic set. Neutrosophic sets were introduced by Samarandache as a solution to practical issues. A neutrosophic set is made up of the three membership functions truth, indeterminacy, and falsity, where each membership degree is a real standard or non-standard subset of the non-standard unit interval -] 0, 1 + [.

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By zadeh[26], the idea of a fuzzy set was first introduced. Since then, this idea has been applied to several algebraic structures, including topologies, semi groups, groups, modules, rings and vector spaces. The development of the fuzzy set has led to its widespread use in numerous domains. Xu introduced the idea of lattice implication algebra and covered its aspects in [22] in order to investigate the logical system whose propositional value was provided in a lattice. Lattice implication algebra's concept of filters and implicative filters were presented by Xu and Qin, who also looked into their characteristics in [32]. In [30], Xu introduced the idea of fuzzy filters by using the concept of fuzzy sets to lattice implication algebra.

Here we applied neutrosophic set theory to lattice implication algebras, and the concept of neutrosophic filters and neutrosophic lattice filters in lattice implication algebra are introduced. Several properties are investigated. Finally we prove that every neutrosophic filter is a neutrosophic lattice filter but the converse is not true

[2] Related Work

Definition 2.1[4]: Let X be the space of points(objects). A neutrosophic set A in X is characterized by a truth membership function T_A, indeterminate membership function I_A and falsity membership function F_A where T_A, I_A and F_A are real standard elements of [0,1]. It can be written as A=$\{\langle x, (T_A(x), I_A(x), F_A(x)/x \in E, T_A, I_A, F_A \in]0^-, 1^+[\rangle\}$. There is no restriction on the sum of $T_A(x), I_A(x)\, and\, F_A(x)$ and so $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2[22]: By a lattice implication algebra we mean a bounded lattice (L,∨,∧,0,1) with order-reversing involution " ' " and a binary operation "→" satisfying the following axioms:

(I1) $\tau \to (\omega \to \rho) = \omega \to (\tau \to \rho)$

(I2) $\tau \to \tau = 1$,

(I3) $\tau \to \omega = \omega' \to \tau'$,

(I4) $\tau \to \omega = \omega \to \tau = 1 \Rightarrow \tau = \omega$,

(I5) $(\tau \to \omega) \to \omega = (\omega \to \tau) \to \tau$,

(L1) $(\tau \vee \omega) \to \rho = (\tau \to \rho) \wedge (\omega \to \rho)$,

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(L2) $(\tau \wedge \omega) \to \rho = (\tau \to \rho) \vee (\omega \to \rho)$, for all $\tau, \omega, \rho \in L$.

If (L,∨,∧,0,1) satisfies the conditions (I1), (I2), (I3), (I4), and (I5), is called a quasi-lattice implication algebra.

Definition 2.3[22]: We can define a partial ordering $\leq$ on a lattice implication algebra L by $\tau \leq \omega$ if and only if $\tau \to \omega = 1$.

In a lattice implication algebra L, the following hold: for all $\tau, \omega, \rho \in L$,

(1) $0 \to \tau = 1$, $1 \to \tau = \tau$, and $\tau \to 1 = 1$,

(2) $\tau \leq \omega$ implies $\rho \to \tau \leq \rho \to \omega$ and $\tau \to \rho \geq \omega \to \rho$,

(3) $(\tau \to \omega) \to ((\omega \to \rho) \to (\tau \to \rho)) = 1$,

(4) $\tau \to ((\tau \to \omega) \to \omega) = 1$,

(5) $((\tau \to \omega) \to \omega) \to \omega = \tau \to \omega$.

Theorem 2.4[22]: In a lattice implication algebra L, the following hold: for all $\tau, \omega, \rho \in L$,

1) $1 \to \tau = \tau$;

2) $\tau \to 1 = 1$;

3) $0 \to \tau = 1$;

4) $\tau \leq \omega$ implies $\rho \to \tau \leq \rho \to \omega$,

5) $(\tau \to \omega) \to (\omega \to \rho) \to (\tau \to \rho)) = 1$;

6) $\tau \to ((\tau \to \omega) \to \omega) = 1$;

7) $((\tau \to \omega) \to \omega) \to \omega = \tau \to \omega$

Definition 2.5 [29]: Let L be a lattice implication algebra. Then a binary operation $\otimes$ (read as"times closed") on L defined $\tau \otimes \omega = (\tau \to \omega')'$ for any $\tau, \omega \in L$.

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Theorem 2.6 [30]: Let L be lattice implication algebra. Then the following holds for any $\tau, \omega \in$L,

1) $(\tau \otimes \omega')' = \tau \rightarrow \omega$

2) $\tau \otimes (\tau \rightarrow \omega) = (\tau \wedge \omega) \leq \omega$

3) $\tau \otimes \omega = \omega \otimes \tau$

4) $(\tau \rightarrow \omega) \otimes \tau \leq \omega$

5) $\tau \rightarrow (\omega \rightarrow (\tau \otimes \omega))=1$

Definition 2.7[19]: Let (L,∨,∧, ,→) be a lattice implication algebra. A subset F of L is called a filter of L if it satisfies for all $\tau, \omega \in$ L

(F1) 1 ∈ F,

(F2) $\tau \in$ F and $\tau \rightarrow \omega \in$ F imply $\omega \in$ F.

[3] Neutrosophic filters in lattice implication algebra:

Definition 3.1: Let the universe be X. A NS $\mathbb{A} = \{\langle \tau: T_A(\tau), I_A(\tau), F_A(\tau) / \tau \in X\}\rangle$ of a $\mathcal{LJA}$ L is referred to as Neutrosophic filter of L if it satisfies

(i) $T_A(1) \geq T_A(\tau)$, $I_A(1) \geq I_A(\tau)$, $F_A(1) \leq F_A(\tau)$

(ii) $T_A(\omega) \geq \min\{T_A(\tau), T_A(\tau \longrightarrow \omega)\}$

(iii) $I_A(\omega) \geq \min\{I_A(\tau), I_A(\tau \longrightarrow \omega)\}$

(iv) $F_A(\omega) \leq \max\{F_A(\tau), F_A(\tau \longrightarrow \omega)\}$

Example 3.2: Let L= $\{0, a, b, c, 1\}$. Consider the partial order on L as 0< $a < b < c$ <1 and define $\tau \wedge \omega = \min\{\tau, \omega\}$ and $\tau \vee \omega = \max\{\tau, \omega\}$ for all $\tau, \omega \in L$ and " ' "and "⟶" as follows

Table1: Representation of Complement

τ	0	a	b	c	1
τ'	1	c	b	a	0

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Table 2: Representation of Implication

$\longrightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Table 3: Representation of truth values of a Neutrosophic set A

L	0	a	b	c	1
T_A	0.1	0.2	0.3	0.4	0.5
I_A	0.2	0.4	0.3	0.2	0.2
F_A	0.5	0.4	0.3	0.2	0.1

Hence $(L, \vee, \wedge, ', \longrightarrow)$ is $\mathcal{LIA}$.

Therefore A is a NF on L.

Theorem 3.3: Let L be a $\mathcal{LIA}$ and $A = \{\langle \tau: T_A(\tau), I_A(\tau), F_A(\tau)/\tau \in X\rangle\}$ is a NF of L,

then $\omega \geq \tau \Rightarrow T_A(\omega) \geq T_A(\tau), I_A(\omega) \geq I_A(\tau), F_A(\omega) \leq F_A(\tau)$ for all $\tau, \omega \in L$,

Proof: Consider $T_A(\omega) \geq \min\{T_A(\tau), T_A(\tau \longrightarrow \omega)\} \geq \min\{T_A(\tau), T_A(1)\} = T_A(\tau)$

Consider $I_A(\omega) \geq \min\{I_A(\tau), I_A(\tau \longrightarrow \omega)\} \geq \min\{I_A(\tau), I_A(1)\} = I_A(\tau)$

Consider $F_A(\omega) \leq \max\{F_A(\tau), F_A(\tau \longrightarrow \omega)\} \leq \max\{F_A(\tau), F_A(1)\} = F_A(\tau)$

Theorem 3.4: Let L be $\mathcal{LIA}$ and $A = \{\langle \tau; T_A(\tau), I_A(\tau), F_A(\tau)/\tau \in X\}\rangle$ is a NF of L iff it satisfies

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(i) $T_A(1) \geq T_A(\tau)$, $I_A(1) \geq I_A(\tau), J_A(1) \leq J_A(\tau)$

(ii) $T_A(\tau \longrightarrow \rho) \geq \min\{T_A(\tau \longrightarrow \omega), T_A(\omega \longrightarrow \rho)\}$

(iii) $I_A(\tau \longrightarrow \rho) \geq \min\{I_A(\tau \longrightarrow \omega), I_A(\omega \longrightarrow \rho)\}$

(iv) $J_A(\tau \longrightarrow \rho) \leq \max\{J_A(\tau \longrightarrow \omega), J_A(\omega \longrightarrow \rho)\}$

Proof: Consider A to be NF of L.

Then (i) is obvious.

To prove $T_A(\tau \longrightarrow \rho) \geq \min\{T_A(\tau \longrightarrow \omega), T_A(\omega \longrightarrow \rho)\}$

$T_A(\tau \longrightarrow \rho) \geq \min\{T_A(\omega \longrightarrow \rho), T_A((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))\}$…….(1)

Consider$(\tau \longrightarrow \omega) \longrightarrow ((\omega \longrightarrow \rho) \longrightarrow (\omega \longrightarrow \rho))$

$= (\tau \longrightarrow \omega) \longrightarrow ((\rho \longrightarrow \omega) \longrightarrow (\tau \longrightarrow \omega))$

$=(\rho \longrightarrow \omega) \longrightarrow ((\tau \longrightarrow \omega) \longrightarrow (\tau \longrightarrow \omega))$

$=(\rho \longrightarrow \omega) \longrightarrow 1=1$

$\Longrightarrow (\tau \longrightarrow \omega) \leq ((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))$

$\Longrightarrow T_A(\tau \longrightarrow \omega) \leq T_A((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))$……..(2)

From (1) and (2) $T_A(\tau \longrightarrow \rho) \geq \min\{T_A(\tau \longrightarrow \omega), T_A(\omega \longrightarrow \rho)\}$

To prove $I_A(\tau \longrightarrow \rho) \geq \min\{I_A(\tau \longrightarrow \omega), I_A(\omega \longrightarrow \rho)\}$

$I_A(\tau \longrightarrow \rho) \geq \min\{I_A(\omega \longrightarrow \rho), I_A((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))\}$……..(3)

Since$(\tau \longrightarrow \omega) \leq ((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))$

$\Longrightarrow I_A(\tau \longrightarrow \omega) \leq I_A((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))$

$\Longrightarrow I_A(\tau \longrightarrow \rho) \geq \min\{I_A(\tau \longrightarrow \omega), I_A(\omega \longrightarrow \rho)\}$

To prove $F_A(\tau \longrightarrow \rho) \leq \max\{F_A(\tau \longrightarrow \omega), F_A(\omega \longrightarrow \rho)\}$

$F_A(\tau \longrightarrow \rho) \leq \max\{F_A(\omega \longrightarrow \rho), F_A((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))\}$

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Since $(\tau \longrightarrow \omega) \leq ((\omega \longrightarrow \rho) \longrightarrow (\omega \longrightarrow \rho))$

$\Longrightarrow F_A(\tau \longrightarrow \omega) \geq F_A((\omega \longrightarrow \rho) \longrightarrow (\tau \longrightarrow \rho))$

Therefore $F_A(\tau \longrightarrow \rho) \leq \max\{F_A(\tau \longrightarrow \omega), F_A(\omega \longrightarrow \rho)\}$.

Theorem 3.5: Let A be a NS and L is a $\mathscr{L}\mathscr{J}\mathscr{A}$.Then $A = \{\langle\tau; T_A(\tau), I_A(\tau), J_A(\tau)/\tau \in X\}\rangle$ is a NF of L iff it satisfies

(i) $T_A(1) \geq T_A(\tau)$, $I_A(1) \geq I_A(\tau), J_A(1) \leq J_A(\tau)$

(ii) $T_A(\rho) \geq \min\{T_A(\tau), T_A(\omega), T_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

(iii) $I_A(\rho) \geq \min\{I_A(\tau), I_A(\omega), I_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

(iv) $J_A(\rho) \leq \max\{J_A(\tau), J_A(\omega), J_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

Proof: Consider A be a NF of L.

Then (i) is obvious for any $\tau, \omega, \rho \in L$.

Since $T_A(\rho) \geq \min\{T_A(\tau), T_A(\tau \longrightarrow \rho)\}$ and $T_A(\tau \longrightarrow \rho) \geq \min\{T_A(\omega), T_A(\omega \longrightarrow (\tau \longrightarrow \rho)\}$

Hence $T_A(\rho) \geq \min\{T_A(\tau), T_A(\omega), T_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

Since $I_A(\rho) \geq \min\{I_A(\tau), I_A(\tau \longrightarrow \rho)\}$ and $I_A(\tau \longrightarrow \rho) \geq \min\{I_A(\omega), I_A(\omega \longrightarrow (\tau \longrightarrow \rho)\}$

Hence $I_A(\rho) \geq \min\{I_A(\tau), I_A(\omega), I_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

Since $F_A(\rho) \geq max\{F_A(\tau), F_A(\tau \longrightarrow \rho)\}$ and $F_A(\tau \longrightarrow \rho) \geq \max\{F_A(\omega), F_A(\omega \longrightarrow (\tau \longrightarrow \rho)\}$

Hence $F_A(\rho) \geq max\{F_A(\tau), F_A(\omega), F_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$.

Conversely Since $T_A(\rho) \geq \min\{T_A(\tau), T_A(\omega), T_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

If we take $\omega = \tau$ then $T_A(\rho) \geq \min\{T_A(\tau), T_A(\tau), T_A(\tau \longrightarrow (\tau \longrightarrow \rho)\} = \min\{T_A(\tau), T_A(\tau \longrightarrow (\tau \longrightarrow \rho)\}$

Since $((\tau \longrightarrow \rho) \longrightarrow (\tau \longrightarrow (\tau \longrightarrow \rho)) = I$ then $((\tau \longrightarrow \rho) \leq (\tau \longrightarrow (\tau \longrightarrow \rho)$

Implies $T_A(\tau \longrightarrow \rho) \leq T_A(\tau \longrightarrow (\tau \longrightarrow \rho)$

Therefore $T_A(\rho) \geq \min\{T_A(\tau), T_A(\tau \longrightarrow \rho)\}$

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Since $I_A(\rho) \geq \min\{ I_A(\tau), I_A(\omega), I_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

If we take $\omega = \tau$ then $I_A(\rho) \geq \min\{ I_A(\tau), I_A(\tau), I_A(\tau \longrightarrow (\tau \longrightarrow \rho)\} = \min\{I_A(\tau), I_A(\tau \longrightarrow (\tau \longrightarrow \rho)\}$

Since $((\tau \longrightarrow \rho) \longrightarrow (\tau \longrightarrow (\tau \longrightarrow \rho)) = I$ then $((\tau \longrightarrow \rho) \leq (\tau \longrightarrow (\tau \longrightarrow \rho)$

$\Longrightarrow I_A(\tau \longrightarrow \rho) \leq I_A(\tau \longrightarrow (\tau \longrightarrow \rho)$

Therefore $I_A(\rho) \geq \min\{ I_A(\tau), I_A(\tau \longrightarrow \rho)\}$

Since $F_A(\rho) \geq \max\{ F_A(\tau), F_A(\omega), F_A(\tau \longrightarrow (\omega \longrightarrow \rho)\}$

If we take $\omega = \tau$ then $F_A(\rho) \geq \max\{F_A(\tau), F_A(\tau), F_A(\tau \longrightarrow (\tau \longrightarrow \rho)\} = \max\{F_A(\tau), F_A(\tau \longrightarrow (\tau \longrightarrow \rho)\}$

Since $((\tau \longrightarrow \rho) \longrightarrow (\tau \longrightarrow (\tau \longrightarrow \rho)) = I$ then $((\tau \longrightarrow \rho) \leq (\tau \longrightarrow (\tau \longrightarrow \rho)$

Implies $F_A(\tau \longrightarrow \rho) \leq F_A(\tau \longrightarrow (\tau \longrightarrow \rho)$

$F_A(\rho) \leq max\{F_A(\tau), F_A(\tau \longrightarrow \rho)\}$

Then A is a NF of L.

Theorem 3.6: Let A be a NS of L. A is a NF of L iff

(i) $T_A(1) \geq T_A(\tau)$, $I_A(1) \geq I_A(\tau), J_A(1) \leq J_A(\tau)$

(ii) $T_A(\tau \longrightarrow \rho) \geq \min\{ T_A\big((\rho \longrightarrow \omega) \longrightarrow \tau\big), T_A(\omega)\}$

(iii) $I_A(\tau \longrightarrow \rho) \geq \min\{ I_A\big((\rho \longrightarrow \omega) \longrightarrow \tau\big), I_A(\omega)\}$

(iv) $F_A(\tau \longrightarrow \rho) \leq \max\{F_A\big((\rho \longrightarrow \omega) \longrightarrow \tau\big), F_A(\omega)\}$ for any $\tau, \omega, \rho \in L$.

Proof: Consider A be a NF of L then for any $\tau, \omega, \rho \in L$

$T_A(\rho \longrightarrow \tau) \geq \min\{ T_A\big(\omega \longrightarrow (\rho \longrightarrow \tau)\big), T_A(\omega)\}$

$I_A(\rho \longrightarrow \tau) \geq \min\{ I_A\big(\omega \longrightarrow (\rho \longrightarrow \tau)\big), I_A(\omega)\}$

$F_A(\rho \longrightarrow \tau) \leq \max\{F_A\big(\omega \longrightarrow (\rho \longrightarrow \tau)\big), F_A(\omega)\}$

For any $\tau, \omega, \rho \in L$,$((\rho \longrightarrow \omega) \longrightarrow \tau) \longrightarrow ((\rho \longrightarrow \omega) \longrightarrow (\rho \longrightarrow \tau))$

$=((\rho \longrightarrow \omega) \longrightarrow \tau) \longrightarrow (\rho \longrightarrow ((\rho \longrightarrow \omega) \longrightarrow \tau))$

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$= \rho \rightarrow (((\rho \rightarrow \omega) \rightarrow \tau) \rightarrow ((\rho \rightarrow \omega) \rightarrow \tau)) = I$

$\Rightarrow ((\rho \rightarrow \omega) \rightarrow \tau) \leq ((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

$\Rightarrow T_A((\rho \rightarrow \omega) \rightarrow \tau) \leq T_A((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

$I_A((\rho \rightarrow \omega) \rightarrow \tau) \leq I_A((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

$F_A((\rho \rightarrow \omega) \rightarrow \tau) \geq F_A((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

We know that $((\rho \rightarrow (\omega \rightarrow \tau)) \leq ((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

Implies $T_A((\rho \rightarrow (\omega \rightarrow \tau)) \leq T_A((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

$I_A((\rho \rightarrow (\omega \rightarrow \tau)) \leq I_A((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

$F_A\left((\rho \rightarrow (\omega \rightarrow \tau)\right) \geq F_A((\rho \rightarrow \omega) \rightarrow (\rho \rightarrow \tau))$

Hence $T_A((\rho \rightarrow (\omega \rightarrow \tau) \geq T_A((\rho \rightarrow \omega) \rightarrow \tau)$

$I_A((\rho \rightarrow (\omega \rightarrow \tau)) \geq I_A((\rho \rightarrow \omega) \rightarrow \tau))$

$F_A\left((\rho \rightarrow (\omega \rightarrow \tau)\right) \leq F_A((\rho \rightarrow \omega) \rightarrow \tau))$

Therefore $T_A(\tau \rightarrow \rho) \geq \min\{T_A\left((\rho \rightarrow \omega) \rightarrow \tau\right), T_A(\omega)\}$

$I_A(\tau \rightarrow \rho) \geq \min\{I_A\left((\rho \rightarrow \omega) \rightarrow \tau\right), I_A(\omega)\}$

$F_A(\varsigma \rightarrow \rho) \leq \max\{F_A\left((\rho \rightarrow \omega) \rightarrow \tau\right), F_A(\omega)\}$

Conversely put z = 1 in (ii), (iii) and (iv) and from (i)

(i) $T_A(1) \geq T_A(\tau)$, $I_A(1) \geq I_A(\tau)$, $F_A(1) \leq J_A(\tau)$

(ii) $T_A(\omega) \geq \min\{T_A(\tau), T_A(\tau \rightarrow \omega)\}$

(iii) $I_A(\omega) \geq \min\{I_A(\tau), I_A(\tau \rightarrow \omega)\}$

(iv) $F_A(\omega) \leq \max\{F_A(\tau), F_A(\tau \rightarrow \omega)\}$

Theorem 3.7: Let N be a NF of L if $\tau \leq \omega \rightarrow \rho$ for any $\tau, \omega, \rho \in L$ then

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(i) $T_A(\rho) \geq \min\{T_A(\tau), T_A(\omega)\}$

(ii) $I_A(\rho) \geq \min\{I_A(\tau), I_A(\omega)\}$

(iii) $J_A(\rho) \leq \max\{J_A(\tau), J_A(\omega)\}$.

Proof: Let N be a NF of L.

$T_A(\rho) \geq \min\{T_A(\omega), T_A(\omega \longrightarrow \rho)\}$

$I_A(\rho) \geq \min\{I_A(\omega), I_A(\omega \longrightarrow \rho)\}$

$F_A(\rho) \leq \max\{F_A(\omega), F_A(\omega \longrightarrow \rho)\}$

If $\tau \leq \omega \longrightarrow \rho$ then $T_A(\tau) \leq T_A(\omega \longrightarrow \rho)$

$I_A(\tau) \leq I_A(\omega \longrightarrow \rho)$

$F_A(\tau) \geq F_A(\omega \longrightarrow \rho)$

Hence $T_A(\rho) \geq \min\{T_A(\tau), T_A(\omega)\}$

$I_A(\rho) \geq \min\{I_A(\tau), I_A(\omega)\}$

$F_A(\rho) \leq \max\{F_A(\tau), F_A(\omega)\}$.

Theorem 3.8: Let A be a NS of L. If A is a NF of L if and only if for any $\tau, \omega, \rho \in L$, $\tau \longrightarrow (\omega \longrightarrow \rho)$= I implies $\min\{T_A(\tau), T_A(\omega)\} \leq T_A(\rho)$, $\min\{I_A(\tau), I_A(\omega)\} \leq I_A(\rho)$,$\max\{F_A(\tau), F_A(\omega)\} \geq F_A(\rho)$

Proof: Since $\tau \longrightarrow (\omega \longrightarrow \rho)$= I then $\tau \leq (\omega \longrightarrow \rho)$

$\tau \leq (\omega \longrightarrow \rho)$ if and only if $(\tau \otimes \omega) \leq \rho$

Implies $T_A(\tau \otimes \omega) \leq T_A(\rho)$

$I_A(\tau \otimes \omega) \leq I_A(\rho)$

$F_A(\tau \otimes \omega) \geq F_A(\rho)$

Implies $T_A(\rho) \geq T_A(\tau \otimes \omega) \geq \min\{T_A(\tau), T_A(\omega)\}$

$I_A(\rho) \geq I_A(\tau \otimes \omega) \geq \min\{I_A(\tau), I_A(\omega)\}$

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$F_A(\rho) \leq F_A(\tau \otimes \omega) \leq \max\{F_A(\tau), F_A(\omega)\}$

Hence $T_A(\rho) \geq \min\{T_A(\tau), T_A(\omega)\}$

$I_A(\rho) \geq \min\{I_A(\tau), I_A(\omega)\}$

$F_A(\rho) \leq \max\{F_A(\tau), F_A(\omega)\}$

Theorem 3.9: Let A be a NF of L. Then A[a]={ $\varsigma \in L / T_A(a) \leq T_A(\tau), I_A(a) \leq I_A(\tau), F_A(a) \geq F_A(\tau)$} is a filter of L.

Proof: Since $T_A(1) \geq T_A(\tau)$, $I_A(1) \geq I_A(\tau)$, $F_A(1) \leq F_A(\tau)$

Then 1∈ $A[a]$.

Suppose $\tau, \varrho \in L$ and $\tau \longrightarrow \omega \in A[a]$,$\omega \in A[a]$

Then $T_A(a) \leq T_A(\tau \longrightarrow \omega)$, $I_A(a) \leq I_A(\tau \longrightarrow \omega)$, $F_A(a) \geq F_A(\tau \longrightarrow \omega)$

and $T_A(a) \leq T_A(\tau)$, $I_A(a) \leq I_A(\tau)$, $F_A(a) \geq F_A(\tau)$

Hence $\min\{T_A(\tau \longrightarrow \omega), T_A(\tau)\} \geq T_A(a)$

$\min\{I_A(\tau \longrightarrow \omega), I_A(\tau)\} \geq I_A(a)$

$\max\{F_A(\tau \longrightarrow \omega), F_A(\tau)\} \leq F_A(a)$

since A is a NF of L then

$T_A(\omega) \geq \min\{T_A(\tau), T_A(\tau \longrightarrow \omega)\} \geq T_A(a)$

$I_A(\omega) \geq \min\{I_A(\tau), I_A(\tau \longrightarrow \omega)\} \geq I_A(a)$

$F_A(\omega) \leq \max\{F_A(\tau), F_A(\tau \longrightarrow \omega)\} \geq J_A(a)$

Then $\omega \in A[a]$

Hence $A[a]$ is a filter of L.

Theorem 3.10: Let A be a NS of L. A is a NF of L if and only if it satisfies the following conditions for any $\tau, \omega \in L$

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(i) If $\varsigma \leq \omega \Longrightarrow T_A(\tau) \leq T_A(\omega), I_A(\tau) \leq I_A(\omega), F_A(\tau) \geq F_A(\omega)$,

(ii) $T_A(\tau \otimes \omega) \geq \min\{T_A(\tau), T_A(\omega)\}$

(iii) $I_A(\tau \otimes \omega) \geq \min\{I_A(\tau), I_A(\omega)\}$

(iv) $F_A(\tau \otimes \omega) \leq \max\{F_A(\tau), F_A(\omega)\}$

Proof: Consider N be a NF of L.

Then (i) is obvious for any $\tau, \omega \in L$

$T_A(\tau \otimes \omega) \geq \min\{T_A(\tau), T_A(\tau \longrightarrow (\tau \otimes \omega))\}$

$I_A(\tau \otimes \omega) \geq \min\{I_A(\tau), I_A(\tau \longrightarrow (\tau \otimes \omega))\}$

$F_A(\tau \otimes \omega) \leq \max\{F_A(\tau), F_A(\tau \longrightarrow (\tau \otimes \omega))\}$

$\omega \longrightarrow (\tau \longrightarrow (\tau \otimes \omega)) = \omega \longrightarrow (\tau' \vee \omega) = (\omega \longrightarrow \tau') \vee (\omega \longrightarrow \omega) = (\omega \longrightarrow \tau)) \vee I = I$

$(\tau \longrightarrow (\tau \otimes \omega))) \geq \omega$

Implies $T_A\ (\tau \longrightarrow (\tau \otimes \omega)) \geq T_A(\omega)$

$I_A\ (\tau \longrightarrow (\tau \otimes \omega)) \geq I_A(\omega)$

$F_A\ (\tau \longrightarrow (\tau \otimes \omega)) \leq F_A(\omega)$

Hence $T_A(\tau \otimes \omega) \geq \min\{T_A(\tau), T_A(\omega)\}$

$I_A(\tau \otimes \omega) \geq \min\{I_A(\tau), I_A(\omega)\}$

$F_A(\tau \otimes \omega) \leq \max\{F_A(\tau), F_A(\omega)\}$

Conversely if A is satisfied (i),(ii),(iii) and (iv) then $T_A(\tau \otimes \omega) \geq \min\{T_A(\tau), T_A(\omega)\}$

$I_A(\tau \otimes \omega) \geq \min\{I_A(\tau), I_A(\omega)\}$

$F_A(\tau \otimes \omega) \leq \max\{F_A(\tau), F_A(\omega)\}$

Implies min $\{T_A(\tau), T_A(\tau \longrightarrow \omega)\} \leq T_A(\tau \otimes (\tau \longrightarrow \omega))$

$(\tau \otimes (\tau \longrightarrow \omega)) \longrightarrow \omega = (\tau \wedge \omega) \longrightarrow \omega = (\tau \longrightarrow \omega) \vee (\omega \longrightarrow \omega) = I$

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$\omega \geq (\tau \otimes (\tau \longrightarrow \omega))$ then $T_A(\omega) \geq T_A\ (\tau \otimes (\tau \longrightarrow \omega))$

It follows that min $\{T_A(\tau), T_A(\tau \longrightarrow \omega)\} \leq\ T_A(\tau \otimes (\tau \longrightarrow \omega)) \leq\ T_A(\omega)$

i.e $T_A(\omega) \geq \min\{\,T_A(\tau), T_A(\tau \longrightarrow \omega)\}$

Hence A is a NF of L.

Theorem 3.11: Let A be a NF of a $\mathscr{LJA}$ L, if $\tau \leq \omega$ for any $\tau, \omega \in L$ then

(i) $T_A(1) \geq\ T_A(\tau)\ , I_A(1) \geq I_A(\tau), F_A(1) \leq F_A(\tau)$

(ii) $T_A(\tau\ \vee \omega) \geq \min\{T_A(\tau), T_A(\tau \oplus \omega)$

(iii) $I_A(\tau\ \vee \omega) \geq \min\{I_A(\tau), I_A(\tau \oplus \omega)$

(iv) $F_A(\tau\ \vee \omega) \leq \max\{F_A(\tau), F_A(\tau \oplus \omega)$

Proof: Consider A be a NF of a $\mathscr{LJA}$ L then (i) is obvious for any $\tau, \omega \in L$ and

$T_A(\tau\ \vee \omega) \geq \min\{T_A(\tau), T_A\big(\tau \rightarrow (\tau\ \vee \omega)\big)\}$ …….(1)

To prove (ii) we have to show that $T_A(\tau \oplus \omega) \leq T_A\big(\tau \rightarrow (\tau\ \vee \omega)\big)$

To prove this we have to show $(\tau \oplus \omega) \rightarrow \big(\tau \rightarrow (\tau\ \vee \omega)\big) = 1$

Hence $(\tau \oplus \omega) \rightarrow \big(\tau \rightarrow (\tau\ \vee \omega)\big) = (\tau \oplus \omega) \rightarrow ((\ \omega \rightarrow\ \tau) \rightarrow (\tau \rightarrow\ \tau))$

$= (\tau \oplus \omega) \rightarrow ((\ \omega \rightarrow\ \tau) \rightarrow 1) = (\tau \oplus \omega) \rightarrow (\omega \rightarrow (\tau \rightarrow 1))$

$= (\tau \oplus \omega) \rightarrow (\omega \rightarrow 1) = (\tau \oplus \omega) \rightarrow 1) = (\tau \rightarrow \omega) \rightarrow 1) = 1$ since $\tau \leq \omega$

Hence $(\tau \oplus \omega) \leq \big(\tau \rightarrow (\tau\ \vee \omega)\big)$

There fore $T_A(\tau \oplus \omega) \leq T_A\big(\tau \rightarrow (\tau\ \vee \omega)\big)$…….(2)

From (1) and (2) $T_A(\tau \vee \omega) \geq \min\{T_A(\tau), T_A\big(\tau \rightarrow (\tau\ \vee \omega)\big)\}$

$I_A(\tau\ \vee \omega) \geq \min\{I_A(\tau), I_A(\tau \oplus \omega)$

$F_A(\tau\ \vee \omega) \leq \max\{F_A(\tau), F_A(\tau \oplus \omega)\}$

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[4] Neutrosophic lattice filters:

Definition 4.1: A NS A=$\{\langle\tau; T_A(\tau), I_A(\tau), F_A(\tau)/\tau \in X\}\rangle$ is on L is known as a Neutrosophic lattice filter if it satisfies for all $\tau,\omega \in L$,

$T_A(\tau\wedge\omega) \geq \min\{ T_A(\tau),\ T_A(\omega)\}$

$I_A(\tau\wedge\omega) \geq min\{I_A(\tau), I_A(\omega)\}$

$F_A(\tau\wedge\omega) \leq \max\{F_A(\tau), F_A(\omega)\}$

Example 4.2: For the $\mathscr{LJA}$ taken in the example 3.2.

Table 4: Representation of A= (T_A, I_A, F_A) for LIA L

L	0	*a*	*b*	*c*	1
T_A	0.5	0.5	0.7	0.7	0.7
I_A	0.3	0.3	0.5	0.5	0.5
F_A	0.7	0.7	0.7	0.5	0.5

Hence A is a NLF on L.

Theorem 4.3: A NS A= $\{\langle\tau; T_A(\tau), I_A(\tau), F_A(\tau)/\tau \in X\}\rangle$ is on L is a NF then A is a NLF on L.

Proof: Consider $T_A(\tau\wedge\omega) \geq \min\{\ T_A(\tau),\ T_A((\tau\wedge\omega) \longrightarrow \omega)\} = \min\{\ T_A(\tau),\ T_A((\tau \longrightarrow \omega) \vee (\omega \longrightarrow \omega)\}$

$= \min\{\ T_A(\tau),\ T_A((\tau \longrightarrow \omega) \vee 1\} = \min\{ T_A(\tau),\ T_A(1)\} \geq\ \min\{ T_A(\tau),\ T_A(\omega)\}$

$I_A(\tau\wedge\omega) \geq \min\{ I_A(\tau), I_A((\tau\wedge\omega) \longrightarrow \omega)\} = \min\{ I_A(\tau), I_A((\tau \longrightarrow \omega) \vee (\omega \longrightarrow \omega)\}$

$= \min\{ I_A(\tau), I_A((\tau \longrightarrow \omega) \vee 1\} = \min\{I_A(\tau), I_A(1)\} \geq \min\{I_A(\tau), I_A(\omega)\}$

$F_A(\tau\wedge\omega) \leq \max\{ F_A(\tau),\ F_A((\tau\wedge\omega) \longrightarrow \omega)\} = \max\{ F_A(\tau),\ F_A((\tau \longrightarrow \omega) \vee (\omega \longrightarrow \omega)\}$

$= \max\{\ F_A(\tau), F((\tau \longrightarrow \omega) \vee 1\} = \max\{ F_A(\tau),\ F_A(1)\} \leq\ \max\{ F_A(\tau), F_A(\omega)\}$

Then A is a NLF on L.

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Note: The opposite of the fore mentioned theorem is untrue. It is shown by below example

Example 4.4: The Hasse diagram and cayley tables of a poset L = {0, a, b, c, d, 1} are given below

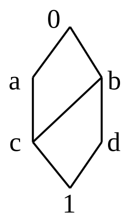

Figure1:Hasse diagram

Table 5: Representation of complement table

τ	0	a	b	c	d	1
τ'	1	c	d	a	b	0

Table 6: Representation of Implication table

$\rightarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	c	1	b	c	b	1
b	d	a	1	b	a	1
c	a	a	1	1	a	1
d	b	1	1	b	1	1
1	0	a	b	c	d	1

The operations V and Λ on L are defined as follows

$\tau \vee \omega = (\tau \rightarrow \omega) \rightarrow \omega$, $\tau \wedge \omega = ((\tau' \rightarrow \omega') \rightarrow \omega')'$,forall $\tau, \omega \in L$.

Clearly we can observe that L is a Lattice implication algebra.

Assume A =(T_A, I_A , F_A)is a MBJ-Neutrosophic set on L defined by Table .

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Table 7: Representation of A = (T_A, I_A,F_A)

L	0	a	b	c	d	1
T_A	0.2	0.6	0.6	0.6	0.4	0.6
I_A	0.2	0.3	0.3	0.3	0.3	0.8
F_A	0.9	0.5	0.5	0.7	0.5	0.5

In general we verify that A =(T_A, I_A ,F_A)∈NLF(L).

Here A is a Neutrosophic lattice filter on L but not a Neutrosophic filter on L .

Since $T_A(d) \ngeq \min\{T_A(a), T_A(a \longrightarrow d)\}$

$T_A(d) \ngeq \min\{T_A(a), T_A(b)\}$

$0.4 \ngeq \min\{0.6, 0.6\}$

$0.4 \ngeq 0.6$

Property of Neutrosophic filter is not satisfied. Therefore converse of the theorem 4.3 is not true.

6. Conclusion

We have applied the notion of neutrosophic set theory to lattice implication algebras. We have introduced the concepts of neutrosophic filters and neutrosophic lattice filters of lattice implication algebra, and investigated several properties. We have proved every neutrosophic filter is neutrosophic lattice filter but the converse is not true.

Probing more profound, the results in this paper also provide a strong foundation for future work in logical algebraic structure and in neutrosophic set. One area of future work is in combining some other kind of sub algebra like implicative filter, positive implicative filter and associative filter etc with neutrosophic sets. Another area is in applying the results studied here to the other algebraic structures like BCI/BCK algebras. Future work will be in these two areas

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