Single-Valued Quadripartitioned Neutrosophic Minimal Structure Space

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Abstract

The main objective of this article is to procure the notion of single-valued quadripartitioned neutrosophic minimal structure space (SVQNMSS) and introduce the notion of single-valued quadripartitioned neutrosophic minimal open set and single-valued quadripartitioned neutrosophic minimal continuous function in it. Further, we study several fundamental properties of continuity in SVQNMSS, such as the composition of single-valued quadripartitioned neutrosophic minimal continuous functions and the product of single-valued quadripartitioned neutrosophic minimal functions in product SVQNMSS.

Keywords: Indeterminacy; SVQNTS; Minimal Structure; SVQNMSS; Minimal Open Set

1. Introduction

The concept of crisp set theory proved insufficient to deal with most real-life issues involving uncertainty. Uncertainty is a significant factor in our daily concerns. L.A. Zadeh developed the concept of fuzzy set theory in 1965 to deal with uncertainty while studying truth-membership, which is a significant expansion of two-valued logic. Azad [2] introduced the concept of a fuzzy semi-continuous function and fuzzy weekly continuous function using fuzzy topological space in 1981. Later, K. Atanassov realized that the non-membership of a mathematical statement is equally significant in addressing real-world situations with uncertainty. To do so, he examined the non-membership value in addition to the membership value, and in 1986, he established the concept of an intuitionistic fuzzy set. Following that, Smarandache [19] realized that the current conceptions, such as the fuzzy set and its expansions, are insufficient to cope with all forms of uncertain issues. He next investigated the values of truth-membership, indeterministic-membership, and false-membership and established the concept of neutrosophic set. Later, Smarandache [14] expanded on the concept of a neutrosophic set.

The notion of minimal structure in topological space was introduced by Maki et al. [7]. Many people then explored it from other perspectives. Alimohammady and Roohi [1] established the concepts of fuzzy minimal structure and fuzzy minimal vector space in 2006. Min [9] investigated the concept of fuzzy weaker r-minimal continuity between fuzzy minimal spaces in 2010. Following that, Tripathy and Debnath [16] investigated the concepts of fuzzy minimal-structure, fuzzy minimal-open set, and fuzzy minimal multifunction in 2019.

Salama and Alblowi [18] proposed the concept of neutrosophic topological space. Salama and Alblowi [8] further developed the concepts of generalized neutrosophic set and generalized neutrosophic topological

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space. Following that, Iswarya and Bageerathi [5], Karthika et al. [6], Parimala et al. [11], Parimala et al. [12], later others proposed various notions of open sets, continuity, and connectedness in neutrosophic topological space.

The notion of a single-valued quadripartitioned neutrosophic set was first grounded by Chatterjee et al. [3] in the year 2016. Later on, Das et al. [4] grounded the notion of quadripartitioned neutrosophic topological space. Das and Tripathy [15] defined pairwise neutrosophic b-continuous functions via neutrosophic bitopological space. In 2023, Tripathy et al. [17] proposed a Single-valued quadripartitioned neutrosophic infi pre-open set in single-valued quadripartitioned neutrosophic infi topological space.

In this paper, we introduce SVQNMSS as an extension of the neutrosophic minimal structure space defined by Pal et al. [10]. Furthermore, we introduced the notion of single-valued quadripartitioned neutrosophic minimal open set and single-valued quadripartitioned neutrosophic minimal continuous function through SVQNMSS, and some fundamental results were shown. Further, we investigate the different properties of SVQNMSS.

The remaining part of this article has been divided into the following sections:

Section 2 presents some existing definitions and findings that will be extremely beneficial in preparing the major outcomes of this paper. Section 3 presents the concept of single-valued quadripartitioned neutrosophic minimal open set and single-valued quadripartitioned neutrosophic minimal continuous function through SVQNMSS and proves some fundamental results. Finally, in Section 4, we conclude the work done in this article.

2. Preliminaries

This section collects various pre-existing definitions and findings that will be used throughout the main results of this article.

Definition 2.1. [3] Let X be a non-empty set. A single-valued quadripartitioned neutrosophic set A over X is defined as follows:

$$A = \{(x, T_A(x), C_A(x), I_A(x), F_A(x)): x\in X\},$$

where $T_A(x)$, $C_A(x)$, $I_A(x)$, $F_A(x)$ ($\in[0, 1]$) are the truth, contradiction, unknown, and falsity membership values of $x\in X$. So, $0 \leq T_A(x) + C_A(x) + I_A(x) + F_A(x) \leq 4$, for all $x\in X$.

Example 2.1. Let $X = \{y_1, y_2\}$ be a non-empty set. Clearly, $W = \{(y_1, 0.3, 0.8, 0.4, 0.5), (y_2, 0.5, 0.8, 0.2, 0.9)\}$ is a single-valued quadripartitioned neutrosophic set over X.

Definition 2.2. [3] The null single-valued quadripartitioned neutrosophic set (0_{QN}) and absolute single-valued quadripartitioned neutrosophic set (1_{QN}) over a fixed set X are denoted as follows:

(*i*) $0_{QN} = \{(x, 0, 0, 1, 1) : x\in X\}$;

(*ii*) $1_{QN} = \{(x, 1, 1, 0, 0) : x\in X\}$.

Definition 2.3.[3] Let $A = \{(x, T_A(x), C_A(x), I_A(x), F_A(x)): x\in X\}$ be a single-valued quadripartitioned neutrosophic set over X, then the complement of A is defined as follows:

$$A^c = \{(x, 1\text{-}T_A(x), 1\text{-}C_A(x), 1\text{-}I_A(x), 1\text{-}F_A(x)) : x\in X\}.$$

Example 2.2. Let $X = \{y_1, y_2\}$ be a fixed set. Let $W = \{(y_1, 0.4, 0.6, 0.5, 0.7), (y_2, 0.9, 0.7, 0.5, 0.8)\}$ be a single-valued quadripartitioned neutrosophic set over X. Then, the complement of W is $W^c = \{(y_1, 0.6, 0.4, 0.5, 0.3), (y_2, 0.1, 0.3, 0.5, 0.2)\}$.

Definition 2.4. [3] A single-valued quadripartitioned neutrosophic set $A = \{(x, T_A(x), C_A(x), I_A(x), F_A(x)): x\in X\}$ is contained in the other single-valued quadripartitioned neutrosophic set $B = \{(x, T_B(x), C_B(x), I_B(x), F_B(x)): x\in X\}$ (i.e., $A\subseteq B$) if and only if $T_A(x) \leq T_B(x)$, $C_A(x) \leq C_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$, for each $x\in X$.

Example 2.3. Let $X = \{y_1, y_2\}$ be a fixed set. Let $W = \{(y_1, 0.4, 0.3, 0.4, 0.3), (y_2, 0.5, 0.6, 0.5, 0.6)\}$ and $M = \{(y_1, 0.7, 0.5, 0.7, 0.5), (y_2, 0.6, 0.7, 0.6, 0.7)\}$ be two single-valued quadripartitioned neutrosophic sets over X. Then, it is clearly seen that $W \subseteq M$.

Remark 3.1. For any single-valued quadripartitioned neutrosophic set Q, $0_{QN} \subseteq Q \subseteq 1_{QN}$.

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Definition 2.5. [3] Let $A = \{(x, T_A(x), C_A(x), I_A(x), F_A(x)): x\in X\}$ and $B = \{(x, T_B(x), C_B(x), I_B(x), F_B(x)): x\in X\}$ be any two single-valued quadripartitioned neutrosophic sets over X. Then, $A\cup B$ and $A\cap B$ is defined as follows:

(*i*) $A\cup B = \{(x, T_A(x)\vee T_B(x), C_A(x)\vee C_B(x), I_A(x)\wedge I_B(x), F_A(x)\wedge F_B(x)): x\in X\}$,

(*ii*) $A\cap B = \{(x, T_A(x)\wedge T_B(x), C_A(x)\wedge C_B(x), I_A(x)\vee I_B(x), F_A(x)\vee F_B(x)): x\in X\}$.

Example 2.4. Let $X = \{y_1, y_2\}$ be a fixed set. Let $W = \{(y_1, 0.4, 0.5, 0.6, 0.7), (y_2, 0.5, 0.6, 0.7, 0.8)\}$ and $M = \{(y_1, 0.7, 0.8, 0.9, 1.0), (y_2, 0.5, 0.6, 0.7, 0.8)\}$ be two single-valued quadripartitioned neutrosophic sets over X. Then, we have

(*i*) $W\cup M = \{(y_1, 0.7, 0.8, 0.6, 0.7), (y_2, 0.5, 0.6, 0.7, 0.8)\}$,

(*ii*) $W\cap M = \{(y_1, 0.4, 0.5, 0.9, 1.0), (y_2, 0.5, 0.6, 0.7, 0.8)\}$.

Definition 3.1. [4] Let X be a fixed set. Then, a family τ of single-valued quadripartitioned neutrosophic sets over X is called a single-valued quadripartitioned neutrosophic topology on Ω, if the following conditions hold:

(i) $1_{QN}, 0_{QN} \in \tau$;

(ii) $M_1 \cap M_2 \in \tau$, whenever $M_1, M_2 \in \tau$;

(iii) $\cup M_i \in \tau$, whenever $\{M_i: i\in\Delta\} \subseteq \tau$.

Then, (X, τ) is called a single-valued quadripartitioned neutrosophic topological space. Every member of τ is called a single-valued quadripartitioned neutrosophic open set. If $Y\in \tau$, then Y^c is called a single-valued quadripartitioned neutrosophic closed set.

Remark 3.2. [4] In every single-valued quadripartitioned neutrosophic topological space, 0_{QN} and 1_{QN} are both single-valued quadripartitioned neutrosophic open set and single-valued quadripartitioned neutrosophic closed set.

Example 2.5. Let $X = \{y, x\}$. Let $M = \{(y, 0.4, 0.6, 0.4, 0.6), (x, 0.3, 0.1, 0.3, 0.1): y, x \in X\}$ and $N = \{(y, 0.9, 0.8, 0.1, 0.5), (x, 0.4, 0.5, 0.1, 0.1): y, x \in X\}$ be two single-valued quadripartitioned neutrosophic sets over X. Then, $\Im = \{0_{QN}, 1_{QN}, M, N\}$ form a single-valued quadripartitioned neutrosophic topology on X.

Definition 2.7.[4] Let (X, τ) be a single-valued quadripartitioned neutrosophic topological space and U be a single-valued quadripartitioned neutrosophic set in X. Then, the single-valued quadripartitioned neutrosophic interior (QN_{int}) and single-valued quadripartitioned neutrosophic closure (QN_{cl}) of Y are defined by

$$QN_{int}(Y) = \cup\{D : D \text{ is a } SVQNOS \text{ in } X \text{ and } D\subseteq Y\},$$

$$\text{and } QN_{cl}(Y) = \cap\{S : S \text{ is a } SVQNCS \text{ in } X \text{ and } Y\subseteq S\}.$$

Example 2.6. Assume that (X, τ) be a single-valued quadripartitioned neutrosophic topological space as shown in **Example 2.5.** Let $U = \{(y, 0.4, 0.4, 0.6, 0.6), (x, 0.3, 0.3, 0.1, 0.1): y, x \in X\}$ be a single-valued quadripartitioned neutrosophic set over X. Then, $QN_{int}(U) = 0_{QN}$ and $QN_{cl}(U) = 1_{QN}$.

Remark 2.1.[4] Clearly, $QN_{int}(U)$ is the largest single-valued quadripartitioned neutrosophic open set over X which is contained in U and $QN_{cl}(U)$ is the smallest single-valued quadripartitioned neutrosophic closed set over X which contains U.

Definition 2.8. [4] Let (X, τ) be a single-valued quadripartitioned neutrosophic topological space, and G be a single-valued quadripartitioned neutrosophic set over X. Then, G is called as

(*i*) single-valued quadripartitioned neutrosophic semi-open set (*SVQNSOS*) if and only if $G\subseteq QN_{cl}(QN_{int}(G))$;

(*ii*) single-valued quadripartitioned neutrosophic pre-open set (*SVQNPOS*) if and only if $G\subseteq QN_{int}(QN_{cl}(G))$.

The collection of all *SVQNSOS* and *SVQNPOS* in (X,τ) are denoted by SVQNSOS(X, τ) and SVQNPOS(X, τ) respectively.

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Definition 2.9.[4] Let (X, τ) be a single-valued quadripartitioned neutrosophic topological space. Then, a single-valued quadripartitioned neutrosophic set W over X is said to be a single-valued quadripartitioned neutrosophic α-open set (SVQN-α-OS) if and only if $W \subseteq QN_{int}(QN_{cl}(QN_{int}(W)))$.

Remark 2.2.[4] Let (X, τ) be a single-valued quadripartitioned neutrosophic topological space. Then,

(i) Complement of an SVQN-α-OS is called a single-valued quadripartitioned neutrosophic α-closed set (SVQN-α-CS).

(ii) Every SVQNOS is an SVQN-α-OS.

(iii) Every SVQNCS is an SVQN-α-CS.

Definition 2.10. [4] A single-valued quadripartitioned neutrosophic set G is called a single-valued quadripartitioned neutrosophic b-open set (SVQN-b-OS) in an SVQNTS (X, τ) if and only if $G \subseteq QN_{int}(QN_{cl}(G)) \cup QN_{cl}(QN_{int}(G))$. A single-valued quadripartitioned neutrosophic set H is said to be a single-valued quadripartitioned neutrosophic b-closed set (SVQN-b-CS) if its complement i.e., H^c is an SVQN-b-OS in (X, τ).

3. Single-Valued Quadripartitioned Neutrosophic Minimal Structure Space

In this section, we procure the notion of SVQNMSS, and introduce the notion of single-valued quadripartitioned neutrosophic minimal open set and single-valued quadripartitioned neutrosophic minimal continuous function in it. Besides, we study several basic properties of continuity in SVQNMSS, such as the composition of single-valued quadripartitioned neutrosophic minimal continuous functions, product of single-valued quadripartitioned neutrosophic minimal functions in product space of SVQNMSS.

Definition 3.1. A collection M of single-valued quadripartitioned neutrosophic sub-sets of X if $M \subseteq P(X)$, where $P(X)$ is the power set of X, is called a single-valued quadripartitioned neutrosophic minimal structure on X if and only if 0_{QN}, $1_{QN} \in M$. Then, the structure (X, M) is called a single-valued quadripartitioned neutrosophic minimal structure space.

Example 3.1. Let P, Q and R be three SVQNSs over a fixed set $X = \{p, q, r\}$ such that:

$P = \{(p, 0.5, 0.8, 0.6, 0.5), (q, 0.6, 0.4, 0.3, 0.5), (r, 0.8, 0.5, 0.3, 0.5): p, q, r \in X\}$;

$Q = \{(p, 0.3, 0.4, 0.7, 0.5), (q, 0.4, 0.6, 0.9, 0.2), (r, 1.0, 0.4, 0.8, 0.7): p, q, r \in X\}$;

$R = \{(p, 0.4, 0.6, 0.7, 1.0), (q, 0.5, 0.1, 0.3, 0.5), (r, 0.7, 0.5, 0.3, 1.0): p, q, r \in X\}$.

Then, the family $M = \{0_{QN}, 1_{QN}, P, Q, R\}$ form a single-valued quadripartitioned neutrosophic minimal structure on X. So, the pair (X, M) is a single-valued quadripartitioned neutrosophic minimal structure space.

Remark 3.1. Every SVQNTS is a single-valued quadripartitioned neutrosophic minimal structure space. But, every single-valued quadripartitioned neutrosophic minimal structure space may not be an SVQNTS in general. This follows from the following example.

Example 3.2. Let P, Q and R be three single-valued quadripartitioned neutrosophic sets over a fixed set $X = \{p, q, r\}$ such that:

$P = \{(p, 0.6, 0.4, 0.5, 0.7), (q, 0.5, 0.5, 0.6, 0.6), (r, 0.9, 0.9, 0.7, 0.7): p, q, r \in X\}$;

$Q = \{(p, 0.7, 0.3, 0.3, 0.8), (q, 0.6, 0.4, 0.9, 0.9), (r, 0.5, 0.7, 0.7, 0.9): p, q, r \in X\}$;

$R = \{(p, 0.5, 0.9, 0.6, 0.7), (q, 0.3, 0.8, 0.6, 0.5), (r, 0.5, 0.5, 0.6, 0.6): p, q, r \in X\}$.

Clearly, the collection $M = \{0_{QN}, 1_{QN}, P, Q, R\}$ is a single-valued quadripartitioned neutrosophic minimal structure on X, and the pair (X, M) is a single-valued quadripartitioned neutrosophic minimal structure space. But (X, M) is not an SVQNTS.

Definition 3.2. Let (X, M) be a single-valued quadripartitioned neutrosophic minimal structure space. If $E \in M$, then E is called a single-valued quadripartitioned neutrosophic minimal-open set, and its complement is called a single-valued quadripartitioned neutrosophic minimal-closed set in (X, M).

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Example 3.3. Let (X, M) be a single-valued quadripartitioned neutrosophic minimal structure space as shown in **Example 3.1.** Clearly, 0_{QN}, 1_{QN}, P, Q, R are single-valued quadripartitioned neutrosophic minimal-open sets in (X, M), and their complements 1_{QN}, 0_{QN}, $P^c = \{(p, 0.5, 0.2, 0.4, 0.5), (q, 0.4, 0.6, 0.7, 0.5), (r, 0.2, 0.5, 0.7, 0.5): p, q, r\in X\}$, $Q^c = \{(p, 0.7, 0.6, 0.3, 0.5), (q, 0.6, 0.4, 0.1, 0.8), (r, 0.0, 0.6, 0.2, 0.3): p, q, r\in X\}$, $R^c = \{(p, 0.6, 0.4, 0.3, 0.0), (q, 0.5, 0.9, 0.7, 0.5), (r, 0.3, 0.5, 0.7, 0.0): p, q, r\in X\}$ are single-valued quadripartitioned neutrosophic minimal-closed sets in (X, M).

The notion of single-valued quadripartitioned neutrosophic minimal interior and single-valued quadripartitioned neutrosophic minimal closure of a single-valued quadripartitioned neutrosophic set in a single-valued quadripartitioned neutrosophic minimal structure space is defined as follows:

Definition 3.3. Let (X, M) be a single-valued quadripartitioned neutrosophic minimal structure space. Let U be a single-valued quadripartitioned neutrosophic set over X. Then, the single-valued quadripartitioned neutrosophic minimal interior ($QN_{m\text{-}int}$) and single-valued quadripartitioned neutrosophic minimal closure ($QN_{m\text{-}cl}$) of U are defined as follows:

$QN_{m\text{-}int}(U) = \cup\{E : E$ is an SVQN-m-OS in X and $E\subseteq U\}$,

and $QN_{m\text{-}cl}(U) = \cap\{F : F$ is an SVQN-m-CS in X and $U\subseteq F\}$.

Example 3.4. Let (X, M) be a single-valued quadripartitioned neutrosophic minimal structure space as defined in **Example 3.1.** Then, the single-valued quadripartitioned neutrosophic minimal interior and single-valued quadripartitioned neutrosophic minimal closure of $U = \{(p, 0.3, 0.3, 0.3, 0.3), (q, 0.4, 0.4, 0.4, 0.4), (r, 0.5, 0.5, 0.5, 0.5)\}$ are $QN_{m\text{-}int}(U) = \{(p, 0, 0, 1, 1), (q, 0, 0, 1, 1), (r, 0, 0, 1, 1)\}$ and $QN_{m\text{-}cl}(U) = \{(p, 1, 1, 0, 0), (q, 1, 1, 0, 0), (r, 1, 1, 0, 0)\}$ respectively.

Example 3.5. From the above definitions, it is clear that every single-valued quadripartitioned neutrosophic pre-open sets, single-valued quadripartitioned neutrosophic semi-open sets, single-valued quadripartitioned neutrosophic b-open sets are single-valued quadripartitioned neutrosophic minimal-open sets of (X, M), where $M = \tau \cup NPO(\tau)\cup NSO(\tau)\cup N\text{-}b\text{-}O(\tau)$.

Example 3.6. Let P, Q and R be three single-valued quadripartitioned neutrosophic sets over a fixed set $X = \{p, q\}$ such that:

$P = \{(p, 0.8, 0.8, 0.6, 0.4), (q, 0.6, 0.6, 0.4, 0.1): p, q\in X\}$;

$Q = \{(p, 0.7, 0.7, 0.7, 0.8), (q, 0.6, 0.6, 0.7, 0.2): p, q\in X\}$;

$R = \{(p, 0.6, 0.6, 0.7, 0.9), (q, 0.5, 0.5, 0.8, 0.3): p, q\in X\}$.

Then, $\tau = \{0_{QN}, 1_{QN}, P, Q, R\}$ forms a single-valued quadripartitioned neutrosophic topology on X, and so (X, τ) is a single-valued quadripartitioned neutrosophic topological space.

Let $M = \tau\cup NPO(\tau)\cup NSO(\tau)\cup N\text{-}b\text{-}O(\tau)$, then (X, M) is a single-valued quadripartitioned neutrosophic minimal structure space. Now, from the above it is clear that, every single-valued quadripartitioned neutrosophic pre-open sets, single-valued quadripartitioned neutrosophic semi-open sets, single-valued quadripartitioned neutrosophic b-open sets in (X, τ) are single-valued quadripartitioned neutrosophic minimal-open sets in (X, M). Further, it is also seen that, every single-valued quadripartitioned neutrosophic minimal-open set in (X, M) is also a single-valued quadripartitioned neutrosophic pre-open set, single-valued quadripartitioned neutrosophic semi-open set, single-valued quadripartitioned neutrosophic b-open set in (X, τ).

Lemma 3.1. Let $f : X\rightarrow Y$ be a mapping, and $\{W_i : i\in\Delta\}$ be a collection of neutrosophic subsets of Y, then

(*i*) $f^{-1}(\cup_{i\in\Delta} W_i) = \cup_{i\in\Delta} f^{-1}(W_i)$,

(*ii*) $f^{-1}(\cap_{i\in\Delta} W_i) = \cap_{i\in\Delta} f^{-1}(W_i)$.

Proposition 3.1. If $f_i : X_i\rightarrow Y_i$ is a mapping, and W_i is a neutrosophic set of Y for $i = 1, 2$, then

$(f_1 x f_2)^{-1}(W_1 x W_2) = f_1^{-1}(W_1) \text{ x } f_2^{-1}(W_2)$.

Proof. Let f_i: $X_i\rightarrow Y_i$ be a mapping for $i = 1, 2$. Let $W_1 = \{(T_1, I_1, F_1)(p_1) : p_1 \in X_1\}$ and $W_2 = \{(T_2, I_2, F_2)(p_2) : p_2 \in X_2\}$ be neutrosophic subsets in Y_1 and Y_2 respectively. Then, for (p_1, p_2) in $X_1 x X_2$, we have

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$(f_1 x f_2)^{-1}(T_1 x T_2)\ (p_1, p_2) = (T_1 x T_2)\ (f_1(p_1), f_2(p_2))$

$= \min\ \{T_1 f_1(p_1), T_2\ f_2(p_2)\}$

$= \min\ \{f_1^{-1}(T_1(p_1)), f_2^{-1}(T_2(p_2))\}$

$= (f_1^{-1}(T_1), f_2^{-1}(T_2))\ (p_1, p_2).$

Following the above argument, we can show that

$(f_1 x f_2)^{-1}(I_1 x I_2)\ (p_1, p_2) = (f_1^{-1}(I_1), f_2^{-1}(I_2))\ (p_1, p_2)$

and $(f_1 x f_2)^{-1}(F_1 x F_2)\ (p_1, p_2) = (f_1^{-1}(F_1), f_2^{-1}(F_2))\ (p_1, p_2)$.

Definition 3.4. A one-to-one and onto function $f : (X, M_1)\longrightarrow(Y, M_2)$ is called a single-valued quadripartitioned neutrosophic minimal-continuous function if $f^{-1}(U)$ is a single-valued quadripartitioned neutrosophic minimal-open set, whenever U is a single-valued quadripartitioned neutrosophic minimal-open set in M_2.

Definition 3.5. A mapping $f : (X, M_1)\longrightarrow(Y, M_2)$ is called a single-valued quadripartitioned neutrosophic weakly minimal-continuous function if for each single-valued quadripartitioned neutrosophic point x_0 and each single-valued quadripartitioned neutrosophic minimal-open set V with $f(x_0)\in V$, there exists a single-valued quadripartitioned neutrosophic minimal-open set U such that $x_0\in U$ and $f(U)\subseteq QN_{m\text{-}cl}(V)$.

Theorem 3.1. Let $f:(X, M_1)\longrightarrow(Y, M_2)$ and $g:(X, M_2)\longrightarrow(Y, M_3)$ be two single-valued quadripartitioned neutrosophic minimal-continuous functions. Then, the composition function $gof:(X, M_1)\longrightarrow(Y, M_3)$ is also a single-valued quadripartitioned neutrosophic minimal-continuous.

Proof. Let $f:(X, M_1)\longrightarrow(Y, M_2)$ and $g:(Y, M_2)\longrightarrow(Z, M_3)$ be two single-valued quadripartitioned neutrosophic minimal-continuous functions. Let U be a single-valued quadripartitioned neutrosophic minimal-open set in M_3. Since $g:(Y, M_2)\longrightarrow(Z, M_3)$ is a single-valued quadripartitioned neutrosophic minimal-continuous function, so $g^{-1}(U)$ is a single-valued quadripartitioned neutrosophic minimal-open set in M_2. Further, since $f:(X, M_1)\longrightarrow(Y, M_2)$ is a single-valued quadripartitioned neutrosophic minimal-continuous function, so $f^{-1}(g^{-1}(U)) = gof^{-1}(U)$ is a single-valued quadripartitioned neutrosophic minimal-open set in M_1. Hence, $gof^{-1}(U)$ is a single-valued quadripartitioned neutrosophic minimal-open set in M_1, whenever U be a single-valued quadripartitioned neutrosophic minimal-open set in M_3. Therefore, the composition function $gof:(X, M_1)\longrightarrow(Z, M_3)$ is a single-valued quadripartitioned neutrosophic minimal-continuous function.

Theorem 3.2. Let (Y, M_2) be a single-valued quadripartitioned neutrosophic minimal structure space, and $f:X\longrightarrow(Y, M_2)$ be a single-valued quadripartitioned neutrosophic minimal function. Then, there exists a single-valued quadripartitioned neutrosophic weaker minimal structure M_1 on X for which f is a single-valued quadripartitioned neutrosophic minimal-continuous function.

Proof: Let $f:X\longrightarrow(Y, M_2)$ be a single-valued quadripartitioned neutrosophic minimal function such that (Y, M_2) be a single-valued quadripartitioned neutrosophic minimal structure space. Suppose that $M_1\subseteq P(X)$ is defined by $M_1=\{f^{-1}(V) : V\in M_2\}$. Hence, (X, M_1) is a single-valued quadripartitioned neutrosophic minimal structure on X. From the definition of single-valued quadripartitioned neutrosophic minimal-continuous function and construction of M_1, it follows that $f:(X, M_1)\longrightarrow(Y, M2)$ is a single-valued quadripartitioned neutrosophic minimal-continuous function. Further, by the definition of single-valued quadripartitioned neutrosophic weaker minimal structure and construction of M_1, it follows that M_1 is a single-valued quadripartitioned neutrosophic weaker minimal structure on X.

Lemma 3.2. Let (X, M) be a single-valued quadripartitioned neutrosophic minimal structure space, and $Y\subseteq X$. Then, the structure $(Y, M\cap Y)$ is also a single-valued quadripartitioned neutrosophic minimal structure space. Further, for $(X, M\cap Y)$ there is a single-valued quadripartitioned neutrosophic weaker minimal structure space.

Theorem 3.3. Let (X, M_1) be a single-valued quadripartitioned neutrosophic minimal structure space, and $Y\subseteq X$. Then, there is a single-valued quadripartitioned neutrosophic weaker minimal structure on Y say M_2 such the map $i_f : (Y, M_2)\longrightarrow(X, M_1\cap Y)$ is a single-valued quadripartitioned neutrosophic minimal-continuous.

Proof: In view of the above **Theorem 3.2** on considering the identity function, we can have the map i_f to be *a* single-valued quadripartitioned neutrosophic minimal-continuous function.

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Remark 3.2. The above result is true for the inclusion function $i : (Y, M_2) \longrightarrow (X, M_1)$. In this case, M_2 is called the induced single-valued quadripartitioned neutrosophic minimal structure on Y.

Theorem 3.4. Let $Y \subseteq X$, and $f:(X, M_1) \longrightarrow (Z, N_1)$ be a single-valued quadripartitioned neutrosophic minimal-continuous function. Then, $f|_Y:(Y, M_2) \longrightarrow (Z, N_1 \cap f(Y))$ is a single-valued quadripartitioned neutrosophic minimal-continuous, where Y is endowed with M_2, induced minimal structure.

Proof: By the above **Remark 3.2** and **Theorem 3.1**, we have $f|_Y = f o i$ (or $f o i_f$), and hence $f|_Y$ is N_m– continuous.

Theorem 3.5. Let $\{(X_i, M_i) : i \in \Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal structure spaces, where Δ being the index set, and $\{f_i : X \longrightarrow (X_i, M_i) : i \in \Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal-continuous functions. Then, there is a single-valued quadripartitioned neutrosophic weakest minimal structure M on X such that f_i's are single-valued quadripartitioned neutrosophic minimal-continuous function.

Proof: Let $\{(X_i, M_i) : i \in \Delta\}$, where Δ is the index set by a family of single-valued quadripartitioned neutrosophic minimal structure spaces, and $\{f_i:X \longrightarrow (X_i, M_i) : i \in \Delta\}$ be a family function. Let $E_i = f^{-1}(M_i) = \{f^{-1}(V) : V \in M_i\}$ for $i \in \Delta$. Consider $M = \cup_{i \in \Delta} E_i$. Then, (X, M) is a single-valued quadripartitioned neutrosophic minimal structure space by a known definition. Further, from the construction of M, it is clear that $f_i:(X, M) \longrightarrow (X_i, M_i)$ is a single-valued quadripartitioned neutrosophic minimal-continuous. Since we have considered the union while considering the single-valued quadripartitioned neutrosophic minimal structure M on X, it will include all other single-valued quadripartitioned neutrosophic minimal structures on X, so it is the weakest single-valued quadripartitioned neutrosophic minimal structure on X.

Theorem 3.6. Let $\{f_i:X \longrightarrow (X_i, M_i) : i \in \Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal-continuous functions, where (X_i, M_i) are single-valued quadripartitioned neutrosophic minimal structure spaces. Let the single-valued quadripartitioned neutrosophic minimal structure M in X be generated by $\{f_i : i \in \Delta\}$. Then, the function $f : (Y, N) \longrightarrow (X, M)$ is a single-valued quadripartitioned neutrosophic continuous if and only if $f_i o f$ is a single-valued quadripartitioned neutrosophic minimal-continuous function for all $i \in \Delta$.

Proof: Let $\{f_i: X \longrightarrow (X_i, M_i) : i \in \Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal-continuous functions, and $f : (Y, N) \longrightarrow (X, M)$ be a single-valued quadripartitioned neutrosophic minimal-continuous. Then, by **Proposition 3.1**, $f_i o f$ is a single-valued quadripartitioned neutrosophic minimal-continuous function.

Next, let $f_i o f$ be a single-valued quadripartitioned neutrosophic minimal-continuous functions for each $i \in \Delta$, but f is not a single-valued quadripartitioned neutrosophic minimal-continuous function. Thus, we have $B \in M$ such that $f^{-1}(B) \notin N$. Then, we have the following possibilities:

Case-1: there exists $i_0 \in \Delta$ and $B_{i_0} \in M_{i_0}$ such that $B = f_{i_0}^{-1}(B_{i_0})$.

Case-2: for every $i \in \Delta$ and every $B_i \in M_i$, $B \neq f_{i_0}^{-1}(B_{i_0})$.

Consider Case-1, we have $B = f_{i_0}^{-1}(B_{i_0})$, implies $f^{-1}(f_{i_0}^{-1}(B_{i_0})) = (f_{i_0} \circ f)^{-1}(B_{i_0})$. Thus, for $B_{i_0} \in M_{i_0}$, we have $(f_{i_0} \circ f)^{-1}(B_{i_0}) \notin N$, which shows that $f_{i_0} \circ f$ is not a single-valued quadripartitioned neutrosophic minimal-continuous function. Hence, we arrive at a contradiction. Thus, our supposition is wrong.

For Case-2, we have $f^{-1}(0_{N,X}) = 0_{N,Y}$ and $f^{-1}(1_{N,X}) = 1_{N,Y}$, which leads to $B(0_{N,Y}, 1_{N,Y})$. Hence, M \ $\{B\}$ is a single-valued quadripartitioned neutrosophic minimal structure on X. Thus for each $i \in \Delta$, $f_i:(X, M \setminus \{B\}) \longrightarrow (X_i, M_i)$, we have $f_i o f : (Y, N) \longrightarrow (X_i, M_i)$ is a single-valued quadripartitioned neutrosophic minimal-continuous function for each $i \in \Delta$. This leads to a contradiction to the choice of the single-valued quadripartitioned neutrosophic minimal structure M on X. Thus, f is a single-valued quadripartitioned neutrosophic minimal-continuous function whenever $f_i o f$ is a single-valued quadripartitioned neutrosophic minimal-continuous function for each $i \in \Delta$.

Remark 3.3. Let $\{(X_i, M_i): i \in \Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal structure spaces, then the product space is defined by $\Pi_{i \in \Delta} X_i$. It can be easily verified that $(\Pi_{i \in \Delta} X_i , \Pi_{i \in \Delta} M_i)$ is a single-valued quadripartitioned neutrosophic minimal structure on $\Pi_{i \in \Delta} X_i$. Again, $M = \Pi_{i \in \Delta} M_i$ is the

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weakest single-valued quadripartitioned neutrosophic minimal structure on $\Pi_{i\in\Delta} X_i$. One can easily verify that for each $j\in\Delta$, the canonical projection $\pi_j : \Pi_{i\in\Delta} X_i \longrightarrow X_j$ is a single-valued quadripartitioned neutrosophic minimal-continuous function.

Corollaries 3.1. Let $\{(X_i, M_i): i\in\Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal structure spaces, and $X = \Pi_{i\in\Delta} X_i$ exists.

Lemma 3.3. Let $\{(X_i, M_i): i\in\Delta\}$ be a family of single-valued quadripartitioned neutrosophic minimal structure spaces, and $X = \Pi_{i\in\Delta} X_i$. Let the minimal structure on X be generated by $\Pi_{i\in\Delta} X_i$. Then, $f : (Y, N)\longrightarrow(X, M)$ is a single-valued quadripartitioned neutrosophic minimal-continuous function if and only if $\pi_i o f$ is a single-valued quadripartitioned neutrosophic minimal-continuous function for all $i\in\Delta$.

Proposition 3.2. Let $f : (X, M)\longrightarrow(Y, N)$ and $g: (X, M)\longrightarrow(Z, Q)$ be a single-valued quadripartitioned neutrosophic minimal-continuous functions. Then, the function fx$g : (X, M)\longrightarrow(YxZ, NxQ)$ defined by $(f$x$g)(x)=(f(x), g(x))$ is a single-valued quadripartitioned neutrosophic minimal-continuous function.

4. Conclusion

In this paper, we have presented the notion of single-valued quadripartitioned neutrosophic minimal-open set and single-valued quadripartitioned neutrosophic minimal-continuous function, and also studied the notion of the product of single-valued quadripartitioned neutrosophic minimal spaces. In addition, we investigated several fundamental properties of single-valued quadripartitioned neutrosophic minimal-continuous functions in single-valued quadripartitioned neutrosophic minimal structure spaces, such as composition and product of single-valued quadripartitioned neutrosophic minimal-continuous functions, among others. Furthermore, these ideas can be extended to the fields of Bipolar Quadripartitioned Neutrosophic Set, Bipolar Pentapartitioned Neutrosophic Set, Neutrosophic Multiset Topological Space, Neutrosophic Bitopological Space, Pentapartitioned Neutrosophic Topological Space, and Generalized Neutrosophic Topological Space.

Conflicts of Interest: "The authors declare no conflict of interest."

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