Bipolar fuzzy hypersoft set and its application in decision making

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Abstract

Smarandache developed the idea of hypersoft set (HSS) theory as an extension of soft set (SS) theory. HSS provides a general mathematical framework for handling data that can be formulated as several trait-valued disjoint sets which blend to various traits. The major goal of this article is to lay the footing for supplying a new model called bipolar fuzzy hypersoft sets (BFHSSs) by linking both fuzzy sets (FSs) and HSSs under bipolarity property. By using positive and negative membership functions and multi-argument functions, these structures work best for testing uncertainty. This makes them better at solving real-world problems, especially ones that have both good and bad sides. This paper also has different operations for BFHSSs, such as absolute BFHSS, null BFHSS, complement, subset, union, intersection, and their related properties. Moreover, operations like OR and AND for BFHSS have been instituted. Some properties are demonstrated, and some numerical examples are given to illustrate the mechanism of using these tools. Finally, these tools are applied in the decision-making process based on an algorithm that is built.

Kewords: Bipolar Fuzzy Set; Fuzzy Set;Fuzzy Soft Set; Fuzzy Hypersoft Set; Hypersoft Set; Soft Set.

1 introduction

Scholars from all over the world came up with many mathematical ideas to deal with problems in everyday life that involve uncertain and imprecise information. These ideas can be used to solve problems in a flexible way that is not hard or complicated. The first of those scholars was Zadah,[1] who established FS in 1965 as a way of growing crisp sets to face complicated issues in real-life situations that are soaked in uncertainties. In FS, there is one true membership (TM) function that works to manage the uncertainty data by giving every object in universal a numerical degree belonging to closed interval $[0, 1]$. Later FSs have been generalized by many scholars into powerful models like intuitionistic fuzzy set (IFS),[2] complex fuzzy set (CFS),[3] Q-complex

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fuzzy set (Q-CFS),[4] and neutrosophic set (NS)[5] in order to counteract the complexity of the utilities of data uncertainty in our daily lives.

In 1999, Molodtsov[6] pointed out that there is a shortage of these concepts, as he indicated that these concepts do not have the ability to deal with uncertain data in a parameterized way. To handle this shortage, he suggested a SS to address uncertainty in a parameterized form. The SS attracted the attention of researchers and kept them motivated to introduce more contributions; for instance, Maji et al.[7] presented the idea of fuzzy soft set (FSS) by incorporating SS and FS. Çağman and Karataş[8] constructed a generalized algorithm of decision-making based on intuitionistic fuzzy soft set (IFSS). Deli and Broumi[9] defined a relation between neutrosophic soft sets (NSSs) that allows the composition of two NSSs. Thirunavukarasu and Suresh[10] developed the FSS from a real part to a complex part and showed its fundamental set theory. Following them, Al-Sharqi et al.[11–13] invented new models to handle the uncertainty that maintains a periodic nature by merging both SS and CNS into an interval form. In addition to other contributions in various fields such as economics, medicine, engineering, programming, computer science, etc., see.[14–19]

In some practical situations, traits that give the alternatives more elaboration should be sub-divided into trait values for clearer insight. In response to this purpose, Samarandache[20] proposed the HSS as a generalization of the SS. Saeed et al.[21] prepared basic operations on HSS. Decision-making applications on fuzzy hypersoft (FHSS) were presented by Yolcu and Ozturk.[22]

On the other hand, the idea of bipolar emerged to cover real-world problems, which are characterized by two aspects, positive and negative, such as profit and loss, return and progress, black and white. The idea of bipolar fuzzy set (BFS) was first proposed by Zhang[23] when he extended the range of the TM of FS from [0,1] to [-1,1]. Some algebraic properties of bipolar fuzzy soft set (BFSS) were specified by Naz and Shabir.[24] Recently, Musa and Asaad[25] grown the idea of a bipolar hypersoft set(BHSS) and presented its essential algebraic properties. Following this tendency, and with the goal of allowing researchers to make greater research contributions, in this work we will conceptualize a new hybrid model, namely BFHSS, by combining BFS with HSS.
The rest of the research is prepared thus: In the next Section, we briefly presented some fundamental definitions of the SSs, FSs, and BFSs. HSS, FHSS, and some operational principles for BFSs. Part 2 presents a basic, comprehensive definition of BFHSS along with some related operations and theories. In addition, numerical examples are given to illustrate how the concept works more clearly in Part 3. In Part 4, these tools are applied in the decision-making process based on an algorithm that is built. Finally, the conclusion and scope of future studies are summarized and examined.

2 Preliminaries

In this part, we outline some previous ideas like BFS, SS, BFS, HSS and FHSS suitable to this work is proposed.

Definition 2.1. [23] A BFS over a fixed non empty set $\hat{\mathcal{Z}}$ is defined as the following structure:

$$\bar{\bar{\mathcal{N}}} = \left\{ \left(\hat{z}, \mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}}(\hat{z}) \right) \middle| \hat{z} \in \hat{\mathcal{Z}} \right\}$$

where $\mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}}(\hat{z}) : \hat{\mathcal{Z}} \to [0,1]$ and $\mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}}(\hat{z}) : \hat{\mathcal{Z}} \to [-1,0]$ denote the positive TM and negative TM of $\hat{z} \in \hat{\mathcal{Z}}$ with standard condition $-1 \leq \mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}}(\hat{z}) + \mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}}(\hat{z}) \leq 1$.

Definition 2.2. [23] Let $\bar{\bar{\mathcal{N}}}_1 = \left\{ \left(\hat{z}, \mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}) \right) \middle| \hat{z} \in \hat{\mathcal{Z}} \right\}$ and $\bar{\bar{\mathcal{N}}}_2 = \left\{ \left(\hat{z}, \mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}_2}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}_2}(\hat{z}) \right) \middle| \hat{z} \in \hat{\mathcal{Z}} \right\}$ be two BFSs. Then we have .

1. $\bar{\bar{\mathcal{N}}}_1^c = c\left(\bar{\bar{\mathcal{N}}}_1\right) = \left\{ \left(\hat{z}, 1 - \mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}), -1 - \mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}) \right) \middle| \hat{z} \in \hat{\mathcal{Z}} \right\}$.

2. $\bar{\bar{\mathcal{N}}}_1 \subseteq \bar{\bar{\mathcal{N}}}_2$ iff $\mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}) \leq \mathcal{F}^{+}_{\bar{\bar{\mathcal{N}}}_2}(\hat{z})$ and $\mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}) \geq \mathcal{F}^{-}_{\bar{\bar{\mathcal{N}}}_2}(\hat{z})$; $\forall \hat{z} \in \hat{\mathcal{Z}}$.

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$.\bar{\bar{\mathcal{N}}}_1 \tilde{\cup} \bar{\bar{\mathcal{N}}}_2 = \left\{ \hat{z}, \left(\max \left(\mathcal{F}^+_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}), \mathcal{F}^+_{\bar{\bar{\mathcal{N}}}_2}(\hat{z}) \right), \min \left(\mathcal{F}^-_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}), \mathcal{F}^-_{\bar{\bar{\mathcal{N}}}_2}(\hat{z}) \right) \right) \middle| \hat{z} \in \hat{\mathcal{Z}} \right\}.$

$4.\bar{\bar{\mathcal{N}}}_1 \tilde{\cap} \bar{\bar{\mathcal{N}}}_2 = \left\{ \hat{z}, \left(\min \left(\mathcal{F}^+_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}), \mathcal{F}^+_{\bar{\bar{\mathcal{N}}}_2}(\hat{z}) \right), \max \left(\mathcal{F}^-_{\bar{\bar{\mathcal{N}}}_1}(\hat{z}), \mathcal{F}^-_{\bar{\bar{\mathcal{N}}}_2}(\hat{z}) \right) \right) \middle| \hat{z} \in \hat{\mathcal{Z}} \right\}.$

Definition 2.3. [6] A structure $(\bar{\bar{\varphi}}, \bar{\bar{\nu}})$ called SS, where $\bar{\bar{\varphi}} : \bar{\bar{\nu}} \to \widehat{\mathbb{P}}\left(\hat{\mathcal{Z}}\right)$, $\widehat{\mathbb{P}}\left(\hat{\mathcal{Z}}\right)$ is the power set of $\hat{\mathcal{Z}}$ and here $\hat{\mathcal{Z}}$, $\bar{\bar{\nu}} \subset \bar{\bar{\mathfrak{B}}}$ refer to both the universal set and the attributes set respectively.

Definition 2.4. [24] A BFSS is a structure that combines BFS and SS and is defined on a non-empty fixed set and the attributes set as follows:

$$\left(\bar{\bar{\Psi}}, \bar{\bar{\nu}}\right) = \bar{\bar{\Psi}}(\mathfrak{b}_j) = \left\{ \left(\hat{z}_k, \mathfrak{F}^+_{\bar{\bar{\Psi}}}(\hat{z}_k), \mathfrak{F}^-_{\bar{\bar{\Psi}}}(\hat{z}_k) \right) \middle| \forall \hat{z}_k \in \hat{\mathfrak{Z}}, \mathfrak{b}_j \in \bar{\bar{\nu}} \subseteq \bar{\bar{\mathfrak{B}}}, j = 1, 2, ..., n, \ \ k = 1, 2, ..., m \right\}.$$

Example 2.5. Assume that $\hat{\mathcal{Z}} = \{\hat{z}_1, \hat{z}_2\}$ is the set of two smart-phones and $\bar{\bar{\nu}} = \{\mathfrak{b}_1 = Costly, \mathfrak{b}_2 = Battery, \mathfrak{b}_3 = lightweight\} \subseteq \bar{\bar{\mathcal{B}}}$ is the set of attributes.Then the BFSS is analyzed as follows:

$$\left(\bar{\bar{\Psi}}, \bar{\bar{\nu}}\right) = \begin{Bmatrix} \bar{\bar{\Psi}}(\mathfrak{b}_1) = \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.5)\} \\ \bar{\bar{\Psi}}(\mathfrak{b}_2) = \{(\hat{z}_1, 0.6, -0.9), (\hat{z}_2, 0.2, -0.7)\} \\ \bar{\bar{\Psi}}(\mathfrak{b}_3) = \{(\hat{z}_1, 0.4, -0.2), (\hat{z}_2, 0.2, -0.8)\} \end{Bmatrix}$$

Definition 2.6. [20] A HSS $\left(\bar{\bar{\Omega}}, V_1 \times V_2 \times V_3 \times ... \times V_n\right)$ on the fix universe $\hat{\mathfrak{Z}}$ is portrayed as follows.

$$\{(\alpha, \bar{\bar{\Omega}}(\alpha)) : \bar{\bar{\Omega}}(\alpha) \subseteq \hat{\mathfrak{Z}}, \forall \alpha \in V_1 \times V_2 \times V_3 \times ... \times V_n \subseteq \mathfrak{B}_1 \times \mathfrak{B}_2 \times \mathfrak{B}_3 \times ... \times \mathfrak{B}_n\},$$

where $\mathfrak{B}_i : i = 1, 2, ..., n$ are the pairwise disjoint sets of parameters and $V_i \subseteq \mathfrak{B}_i, \forall i = 1, 2, ..., n$.

Definition 2.7. [22] A FHSS $\left(\bar{\bar{\Gamma}}, V_1 \times V_2 \times V_3 \times ... \times V_n\right)$ on the fix universe $\hat{\mathfrak{Z}}$ is specified as:
$\bar{\bar{\Gamma}} : V_1 \times V_3 \times V_3 \times ... \times V_n \to \widehat{\mathbb{P}}\left(\hat{Z}\right)$, where $\widehat{\mathbb{P}}\left(\hat{Z}\right)$ is a family of all FSs over $\hat{\mathfrak{Z}}$ such that
$\bar{\bar{\Gamma}}(\mathfrak{b}) = \left\{ \left(\hat{z}, \mathfrak{F}_{\bar{\bar{\Gamma}}_{(\mathfrak{b})}}(\hat{z}) \right) \middle| \hat{z} \in \hat{Z}, \mathfrak{b} \in V_1 \times V_2 \times V_3 \times ... \times V_n \subseteq \mathfrak{B}_1 \times \mathfrak{B}_2 \times \mathfrak{B}_3 \times ... \times \mathfrak{B}_n \right\}$,
where $\mathfrak{F}_{\bar{\bar{\Gamma}}_{(\mathfrak{b})}}(\hat{z})$ is the fuzzy TM and $\mathfrak{B}_i : i = 1, 2, ..., n$ are pairwise disjoint sets of attribute values.

Here we rewrite the symbols for the sake of simplicity as follows: the relation $\mathfrak{B}_1 \times \mathfrak{B}_2 \times \mathfrak{B}_3 \times ... \times \mathfrak{B}_n = \bar{\bar{\Delta}}$ and $V_1 \times V_2 \times V_3 \times ... \times V_n = \bar{\bar{\Lambda}}$. Then we characterize the FHSS by the structure $\left(\bar{\bar{\Gamma}}, \bar{\bar{\Lambda}}\right)$.

3 The Concept of Bipolar Fuzzy Hypersoft Sets (BFHSSs)

In this part of our work, we represent the main definition of BFHSS with some numerical examples and fundamental set operations, as well as some elementary properties.

Definition 3.1. Let $\hat{\mathcal{Z}}$ be non-empty fixed set, $\widehat{P}\left(\hat{\mathcal{Z}}\right)$ denotes to the power of $\hat{\mathcal{Z}}$. Assume $\bar{\bar{\nu}}_i : i = 1, 2, ..., n$ are n-well defined attributes whose corresponding attribute values are respectively, the pairwise disjoint sets $\bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_i}$: $i = 1, 2, ..., n$. Let $\bar{\bar{\mathcal{B}}}_{\bar{\bar{\nu}}_i}$ be the nonempty subset of $\bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_i}$ $\forall i = 1, 2, ..., n$.
A BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ is characterized by the mapping $\bar{\bar{\Pi}} : \bar{\bar{\Lambda}} \to \widehat{P}\left(\hat{\mathcal{Z}}\right)$ whose functional value is the BFS

$$\bar{\bar{\Pi}}(\alpha) = \left\{ \left(\hat{z}, \mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}), \mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}) \right) \middle| \forall \hat{z} \in \hat{\mathcal{Z}}, \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}} \right\},$$

where $\bar{\bar{\Lambda}} = \bar{\bar{\mathcal{B}}}_{\bar{\bar{\nu}}_1} \times \bar{\bar{\mathcal{B}}}_{\bar{\bar{\nu}}_2} \times ... \times \bar{\bar{\mathcal{B}}}_{\bar{\bar{\nu}}_n}$, $\bar{\bar{\Delta}} = \bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_1} \times \bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_2} \times ... \times \bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_n}$ and $\mathcal{F}^+ : \bar{\bar{\Lambda}} \to [0, 1]$, $\mathcal{F}^- : \bar{\bar{\Lambda}} \to [-1, 0]$ denote positive-TM and negative-TM, respectively, such that $\mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})$ indicates the admissibility of the attribute α^* with regard to object $\hat{z}^*$ for the property corresponding to a BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$, while $\mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})$ represents some implicit counter-property of the attribute α^* with regard to object $\hat{z}^*$ corresponding to a BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$. We can view the BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ as follows:
$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right) = \left\{ \left\langle \alpha, \left\{ \left(\hat{z}, \mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}), \mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}) \right) \middle| \forall \hat{z} \in \hat{\mathcal{Z}} \right\} \right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}} \right\}.$

Example 3.2. Assume one PhD student desires to buy a new computer among three computers $\hat{\mathcal{Z}} = \{\hat{z}_1, \hat{z}_2, \hat{z}_3\}$.
Let the attributes be $\bar{\bar{\nu}}_1$ = CPU class, $\bar{\bar{\nu}}_2$ = Size, $\bar{\bar{\nu}}_3$ = Hard drive Size, and their attribute's values be
$\bar{\bar{\mathcal{E}}}_{\bar{\nu}_1} = \{\mathfrak{b}_1 = Dual-core, \mathfrak{b}_2 = Quad-core\}, \bar{\bar{\mathcal{E}}}_{\bar{\nu}_2} = \{\mathfrak{b}_3 = Compact\ Case, \mathfrak{b}_4 = Full\ Tower, \mathfrak{b}_5 = Mid\ Tower\}$,
$\bar{\bar{\mathcal{E}}}_{\bar{\nu}_3} = \{\mathfrak{b}_6 = 256GB, \mathfrak{b}_7 = 512GB, \mathfrak{b}_8 = 1TB\}$.
Suppose
$\bar{\bar{\mathcal{B}}}_{\bar{\nu}_1} = \{\mathfrak{b}_1 = Dual-core\}$,
$\bar{\bar{\mathcal{B}}}_{\bar{\nu}_2} = \{\mathfrak{b}_3 = Compact\ Case, \mathfrak{b}_4 = Full\ Tower\}$, $\bar{\bar{\mathcal{B}}}_{\bar{\nu}_3} = \{\mathfrak{b}_7 = 512GB, \mathfrak{b}_8 = 1TB\}$ are subsets of $\bar{\bar{\mathcal{E}}}_{\bar{\nu}_i}$, $\forall i = 1, 2, 3$. Then the BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ is analyzed as follows

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right) =$$
$$\Big\{\{\langle((\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.5), (\hat{z}_3, 0.2, -0.4)\}),$$
$$((\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.9, -0.2), (\hat{z}_2, 0.8, -0.4), (\hat{z}_3, 0.7, -0.2)\}),$$
$$((\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.2, -1), (\hat{z}_3, 0.3, -0.9)\}),$$
$$((\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 1, -0.4)\})\rangle\}\Big\}.$$

Definition 3.3. A BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ over a fixed non empty set $\hat{\mathcal{Z}}$, is said to be empty BFHSS, denoted by $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_\Phi$ if both $\mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}) = \mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}) = 0, \forall \hat{z} \in \hat{\mathcal{Z}}, \forall \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}$ and defined as

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_\Phi = \left\{\left\langle \alpha, \left\{(\hat{z}, 0, 0) \,\middle|\, \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}.$$

Definition 3.4. A BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ over a fixed non empty set $\hat{\mathcal{Z}}$, is said to be absolute BFHSS, denoted by $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_{\hat{\mathcal{Z}}}$ if $\mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}) = 1$ and $\mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}) = -1, \forall \hat{z} \in \hat{\mathcal{Z}}, \forall \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}$ and defined as

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_{\hat{\mathcal{Z}}} = \left\{\left\langle \alpha, \left\{(\hat{z}, 1, -1) \,\middle|\, \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}.$$

Now, we clarify the idea of the complement operation of the BFHSS. Then, based on this definition, we also present a numerical example in order to put this definition more clearly into perspective, in addition to one of the important postulates in set theory.

Definition 3.5. Let $\hat{\mathcal{Z}}$ be a universe and $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ be a BFHSS over $\hat{\mathcal{Z}}$, which is defined as bellow:
$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right) = \left\{\left\langle \alpha, \left\{\left(\hat{z}, \mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}), \mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})\right) \,\middle|\, \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}$.
Then the complement of $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ is denoted by $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^c = \left(\bar{\bar{\Pi}}^c, \bar{\bar{\Lambda}}\right)$ and is defined as
$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^c = \left\{\left\langle \alpha, \left\{\left(\hat{z}, 1 - \mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}), -1 - \mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})\right) \,\middle|\, \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}$.

Example 3.6. Take Example 3.2, then the complement of the BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ calculate based on the definition 3.5 as:

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^c =$$
$$\Big\{\{\langle((\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7), \{(\hat{z}_1, 0.7, -0.9), (\hat{z}_2, 0.3, -0.5), (\hat{z}_3, 0.8, -0.6)\}),$$
$$((\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.1, -0.8), (\hat{z}_2, 0.2, -0.6), (\hat{z}_3, 0.3, -0.8)\}),$$
$$((\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7), \{(\hat{z}_1, 1, -0.5), (\hat{z}_2, 0.8, 0), (\hat{z}_3, 0.7, -0.1)\}),$$
$$((\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8), \{(\hat{z}_1, 0.3, -0.4), (\hat{z}_2, 0.6, -0.9), (\hat{z}_3, 0, -0.6)\})\rangle\}\Big\}.$$

Proposition 3.7. *If* $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ *is a BFHSS over the universe* $\hat{\mathcal{Z}}$. *Then* $\left(\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^c\right)^c = \left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$.

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Proof. From Definition 3.5, we have

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{c} = \left\{\left\langle \alpha, \left\{\left(\hat{z}, 1-\mathcal{F}^{+}_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}), -1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})\right) \middle| \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}.$$

Thus,

$$\left(\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{c}\right)^{c} = \left\{\left\langle \alpha, \left\{\left(\hat{z}, 1-\left(1-\mathcal{F}^{+}_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})\right), -1-\left(-1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})\right)\right) \middle| \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}.$$

$$= \left\{\left\langle \alpha, \left\{\left(\hat{z}, \mathcal{F}^{+}_{\bar{\bar{\Pi}}(\alpha)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}(\alpha)}(\hat{z})\right) \middle| \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}.$$

$$= \left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right).$$

□

Now, we proceed to discuss the idea of the subset of two BFHSSs.

Definition 3.8. For two BFHSSs $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ over $\hat{\mathcal{Z}}$. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ is a subset of $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$, denoted as $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)\subseteq\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ if

1. $\bar{\bar{\Lambda}}_1 \subseteq \bar{\bar{\Lambda}}_2$.

2. $\mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\alpha)}(\hat{z}) \leq \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\alpha)}(\hat{z})$ and $\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\alpha)}(\hat{z}) \geq \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\alpha)}(\hat{z}); \forall \alpha \in \bar{\bar{\Lambda}}_1$ and $\forall \hat{z} \in \hat{\mathcal{Z}}$.

Definition 3.9. For two BFHSSs $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ over $\hat{\mathcal{Z}}$. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ is a equal to $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$, denoted as $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)=\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ if

1. $\bar{\bar{\Lambda}}_1 = \bar{\bar{\Lambda}}_2$.

2. $\mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\alpha)}(\hat{z}) = \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\alpha)}(\hat{z})$ and $\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\alpha)}(\hat{z}) = \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\alpha)}(\hat{z}); \forall \alpha \in \bar{\bar{\Lambda}}_1$ and $\forall \hat{z} \in \hat{\mathcal{Z}}$.

Now, we move to clarify the ideas of union, intersection, and the difference between two BFHSSs. Then, based on these definitions, we also present a numerical example in order to put these definitions more clearly into perspective, in addition to one of the important postulates in set theory.

Definition 3.10. Let $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ be two BFHSSs over $\hat{\mathcal{Z}}$. The union of these two BFHSSs is denoted by $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$ where $\bar{\bar{\Lambda}}_3 = \bar{\bar{\Lambda}}_1 \cup \bar{\bar{\Lambda}}_2$ and $\forall \epsilon \in \bar{\bar{\Lambda}}_3, \forall \hat{z} \in \hat{\mathcal{Z}}$.Then,

$$\mathcal{F}^{+}_{\bar{\bar{\Pi}}_3(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \max\left(\bar{\bar{\mathcal{F}}}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

and for negative TM function,

$$\mathcal{F}^{-}_{\bar{\bar{\Pi}}_3(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \min\left(\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

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Definition 3.11. Let $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ be two BFHSSs over $\hat{\mathcal{Z}}$. The intersection of these two BFHSSs is denoted by $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$ where $\bar{\bar{\Lambda}}_3 = \bar{\bar{\Lambda}}_1 \cup \bar{\bar{\Lambda}}_2$ and $\forall \epsilon \in \bar{\bar{\Lambda}}_3, \forall \hat{z} \in \hat{\mathcal{Z}}$.Then,

$$\mathcal{F}^{+}_{\bar{\bar{\Pi}}_3(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \min\left(\mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

and for negative TM function,

$$\mathcal{F}^{-}_{\bar{\bar{\Pi}}_3(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \max\left(\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

Definition 3.12. Let $\hat{\mathcal{Z}}$ be a fixed non empty set and $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ be two BFHSSs. The difference of $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ is denoted by $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) - \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$ where $\left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right) = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c$.

Example 3.13. Consider the BFHSS $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ that is assigned in Example 3.2, as follows:

$\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) =$

$\Big\{\{\langle((\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.5), (\hat{z}_3, 0.2, -0.4)\})$,

$((\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.9, -0.2), (\hat{z}_2, 0.8, -0.4), (\hat{z}_3, 0.7, -0.2)\})$,

$((\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.2, -1), (\hat{z}_3, 0.3, -0.9)\})$,

$((\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 1, -0.4)\})\rangle\}\Big\}$.

Suppose that

$\bar{\bar{\mathcal{W}}}_{\bar{\nu}_1} = \{\mathfrak{b}_1 = Dual\ Core, b_2 = Quad\ Core\}$, $\bar{\bar{\mathcal{W}}}_{\bar{\nu}_2} = \{\mathfrak{b}_3 = Compact\ case, \mathfrak{b}_5 = Mid\ Tower\}$,
$\bar{\bar{\mathcal{W}}}_{\bar{\nu}_3} = \{\mathfrak{b}_8 = 1TB\}$ be another subsets of $\bar{\bar{\mathcal{E}}}_{\bar{\nu}_i}, i = 1, 2, 3$. Then the generated BFHSS $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ is as follows

$\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) =$

$\Big\{\langle(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.5, -0.1), (\hat{z}_2, 0.6, -0.8), (\hat{z}_3, 0.3, -0.9)\}$,

$(\mathfrak{b}_1, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.1, -0.3), (\hat{z}_2, 0.4, -0.7), (\hat{z}_3, 0.2, -0.1)\}$,

$(\mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.3, -0.8), (\hat{z}_2, 0.1, -0.9), (\hat{z}_3, 0.2, -0.6)\}$,

$(\mathfrak{b}_2, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.6, -0.8), (\hat{z}_2, 0.8, -0.2), (\hat{z}_3, 0.5, -0.5)\}\rangle\Big\}$.

The union, intersection and difference of $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ given as follows:

1. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) =$

$\Big\{\langle(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.5), (\hat{z}_3, 0.2, -0.4)\}$,

$(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.9, -0.2), (\hat{z}_2, 0.8, -0.8), (\hat{z}_3, 0.7, -0.9)\}$,

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$(\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.2, -0.1), (\hat{z}_3, 0.3, -0.9)\},$

$(\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 0.1, -0.4)\},$

$(\mathfrak{b}_1, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.1, -0.3), (\hat{z}_2, 0.4, -0.7), (\hat{z}_3, 0.2, -0.1)\},$

$(\mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.3, -0.8), (\hat{z}_2, 0.1, -0.9), (\hat{z}_3, 0.2, -0.6)\},$

$(\mathfrak{b}_2, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.6, -0.8), (\hat{z}_2, 0.8, -0.2), (\hat{z}_3, 0.5, -0.5)\}\rangle \Big\}$

2. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) =$
$\Big\{\langle(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.5), (\hat{z}_3, 0.2, -0.4)\},$

$(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.5, -0.1), (\hat{z}_2, 0.6, -0.4), (\hat{z}_3, 0.3, -0.2)\},$

$(\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.2, -0.1), (\hat{z}_3, 0.3, -0.9)\},$

$(\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 0.1, -0.4)\},$

$(\mathfrak{b}_1, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.1, -0.3), (\hat{z}_2, 0.4, -0.7), (\hat{z}_3, 0.2, -0.1)\},$

$(\mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.3, -0.8), (\hat{z}_2, 0.1, -0.9), (\hat{z}_3, 0.2, -0.6)\},$

$(\mathfrak{b}_2, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.6, -0.8), (\hat{z}_2, 0.8, -0.2), (\hat{z}_3, 0.5, -0.5)\}\rangle \Big\}$

3. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) - \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) =$
$\Big\{\langle(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.5), (\hat{z}_3, 0.2, -0.4)\},$

$(\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.5, -0.2), (\hat{z}_2, 0.4, -0.2), (\hat{z}_3, 0.7, -0.1)\},$

$(\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.2, -0.1), (\hat{z}_3, 0.3, -0.9)\},$

$(\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 0.1, -0.4)\},$

$(\mathfrak{b}_1, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.1, -0.3), (\hat{z}_2, 0.4, -0.7), (\hat{z}_3, 0.2, -0.1)\},$

$(\mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_8), \{(\hat{z}_1, 0.3, -0.8), (\hat{z}_2, 0.1, -0.9), (\hat{z}_3, 0.2, -0.6)\},$

$(\mathfrak{b}_2, \mathfrak{b}_5, \mathfrak{b}_8), \{(\hat{z}_1, 0.6, -0.8), (\hat{z}_2, 0.8, -0.2), (\hat{z}_3, 0.5, -0.5)\}\rangle \Big\}$

Proposition 3.14. *The BFHSSs* $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$, $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ *and* $\left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$ *over* $\hat{\mathcal{Z}}$ *fulfill the following properties.*

1. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$, $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$.

2. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_\Phi = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$, $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_\Phi = \left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)_\Phi$.

3. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) \hat{\cup} \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$, $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) \hat{\cap} \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$.

4. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) \hat{\cup} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)\right) = \left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right) \hat{\cup} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$,

$\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) \hat{\cap} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)\right) = \left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right) \hat{\cap} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$.

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5. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) \hat{\cup} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)\right) = \left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right) \hat{\cup} \left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)\right)$,

$\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) \hat{\cap} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)\right) = \left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right) \hat{\cap} \left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)\right)$.

Proof. The proof for these axioms is clear and depends on the definition 3.10 and definition 3.11. □

Proposition 3.15. *The BFHSSs* $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ *and* $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ *over* $\hat{\mathcal{Z}}$ *fulfills the following De Morgans Laws.*

1. $\left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right)^c = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)^c \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c$.

2. $\left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right)^c = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)^c \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c$.

Proof. To proof (1), we assume that $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$ where $\bar{\bar{\Lambda}}_3 = \bar{\bar{\Lambda}}_1 \cup \bar{\bar{\Lambda}}_2$ and $\forall \epsilon \in \bar{\bar{\Lambda}}_3$ then,

$$\mathcal{F}^{+}_{\bar{\bar{\Pi}}_3(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \max\left(\bar{\bar{\mathcal{F}}}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

Since $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)$. Then

$$\left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \hat{\cup} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right)^c = \left(\bar{\bar{\Pi}}_3, \bar{\bar{\Lambda}}_3\right)^c = \left(\bar{\bar{\Pi}}^c_3, \bar{\bar{\Lambda}}_3\right).$$

Hence $\forall \epsilon \in \bar{\bar{\Lambda}}_3$,

$$\mathcal{F}^{+}_{\bar{\bar{\Pi}}^c_3(\varepsilon)}(\hat{z}) = \begin{cases} 1 - \mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ 1 - \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ 1 - \max\left(\bar{\bar{\mathcal{F}}}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

Now since $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)^c = \left(\bar{\bar{\Pi}}^c_1, \bar{\bar{\Lambda}}_1\right)$. and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c = \left(\bar{\bar{\Pi}}^c_2, \bar{\bar{\Lambda}}_2\right)$. Then
$\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)^c \hat{\cap} \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c = \left(\bar{\bar{\Pi}}^c_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}^c_2, \bar{\bar{\Lambda}}_2\right)$

Suppose that $\left(\bar{\bar{\Pi}}^c_1, \bar{\bar{\Lambda}}_1\right) \hat{\cap} \left(\bar{\bar{\Pi}}^c_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\gamma}}, \bar{\bar{\lambda}}\right)$ where $\bar{\bar{\lambda}} = \bar{\bar{\Lambda}}_1 \cup \bar{\bar{\Lambda}}_2$
Hence $\forall \epsilon \in \bar{\bar{\lambda}}$ then

$$\mathcal{F}^{+}_{\bar{\bar{\gamma}}(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{+}_{\bar{\bar{\Pi}}^c_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{+}_{\bar{\bar{\Pi}}^c_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \min\left(\bar{\bar{\mathcal{F}}}^{+}_{\bar{\bar{\Pi}}^c_1(\varepsilon)}(\hat{z}), \mathcal{F}^{+}_{\bar{\bar{\Pi}}^c_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

$$= \begin{cases} 1 - \mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_1 - \bar{\bar{\Lambda}}_2 \\ 1 - \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if \varepsilon \in \bar{\bar{\Lambda}}_2 - \bar{\bar{\Lambda}}_1 \\ \min\left(1 - \bar{\bar{\mathcal{F}}}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), 1 - \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if \varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

$$= \begin{cases} 1-\mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_1-\bar{\bar{\Lambda}}_2 \\ 1-\mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_2-\bar{\bar{\Lambda}}_1 \\ 1-\max\left(\mathcal{F}^{+}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{+}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if\varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

Therefor $\bar{\bar{\Pi}}_3^c$ and $\bar{\bar{\gamma}}$ are same operators and $\bar{\bar{\Lambda}}_3 = \bar{\bar{\lambda}}$ which implies

$$\mathcal{F}^{+}_{\left(\bar{\bar{\Pi}}_1(\varepsilon)\hat{\cup}\bar{\bar{\Pi}}_2(\varepsilon)\right)^c}(\hat{z}) = \mathcal{F}^{+}_{\left(\bar{\bar{\Pi}}_1^c(\varepsilon)\hat{\cap}\bar{\bar{\Pi}}_2^c(\varepsilon)\right)}(\hat{z}), \forall \hat{z} \in \hat{\mathcal{Z}}$$

Similarly, we can show it also holds for the negative-TM function as follows:

$$\mathfrak{F}^{-}_{\bar{\bar{\Pi}}_3(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_1-\bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_2-\bar{\bar{\Lambda}}_1 \\ \max\left(\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if\varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

$$\mathcal{F}^{-}_{\bar{\bar{\Pi}}_3^c(\varepsilon)}(\hat{z}) = \begin{cases} -1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_1-\bar{\bar{\Lambda}}_2 \\ -1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_2-\bar{\bar{\Lambda}}_1 \\ -1-\max\left(\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if\varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

Now,

$$\mathcal{F}^{-}_{\bar{\bar{\gamma}}(\varepsilon)}(\hat{z}) = \begin{cases} \mathcal{F}^{-}_{\bar{\bar{\Pi}}_1^c(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_1-\bar{\bar{\Lambda}}_2 \\ \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2^c(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_2-\bar{\bar{\Lambda}}_1 \\ \min\left(\bar{\bar{\mathcal{F}}}^{-}_{\bar{\bar{\Pi}}_1^c(\varepsilon)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2^c(\varepsilon)}(\hat{z})\right), & if\varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

$$= \begin{cases} 1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_1-\bar{\bar{\Lambda}}_2 \\ 1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_2-\bar{\bar{\Lambda}}_1 \\ \min\left(-1-\bar{\bar{\mathcal{F}}}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), -1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if\varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

$$= \begin{cases} -1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_1-\bar{\bar{\Lambda}}_2 \\ -1-\mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z}), & if\varepsilon \in \bar{\bar{\Lambda}}_2-\bar{\bar{\Lambda}}_1 \\ -1-\max\left(\mathcal{F}^{-}_{\bar{\bar{\Pi}}_1(\varepsilon)}(\hat{z}), \mathcal{F}^{-}_{\bar{\bar{\Pi}}_2(\varepsilon)}(\hat{z})\right), & if\varepsilon \in \bar{\bar{\Lambda}}_1 \cap \bar{\bar{\Lambda}}_2 \end{cases}$$

Therefor $\bar{\bar{\Pi}}_3^c$ and $\bar{\bar{\gamma}}$ are same operators and $\bar{\bar{\Lambda}}_3 = \bar{\bar{\lambda}}$ which implies

$$\mathcal{F}^{-}_{\left(\bar{\bar{\Pi}}_1(\varepsilon)\hat{\cup}\bar{\bar{\Pi}}_2(\varepsilon)\right)^c}(\hat{z}) = \mathcal{F}^{-}_{\left(\bar{\bar{\Pi}}_1^c(\varepsilon)\hat{\cap}\bar{\bar{\Pi}}_2^c(\varepsilon)\right)}(\hat{z}), \forall \hat{z} \in \hat{\mathcal{Z}}$$

This completes the proof.

The first proof and the second proof are alike.

□

The ideas of OR and AND operations are most important in set theory; therefore, now we move to clarify their definitions on our model BFHSSs, Then, based on these definitions, we give a numerical example and one of the most important set theory postulates to show how these definitions fit together.

Definition 3.16. Let $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ are two BFHSSs over $\hat{\mathcal{Z}}$, then the *OR* operation is a BFHSS over $\hat{\mathcal{Z}}$ and represented by $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \vee \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Xi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2\right)$, where $\bar{\bar{\Xi}}(\alpha_i, \beta_j) = \bar{\bar{\Pi}}_1(\alpha_i)\hat{\cup}\bar{\bar{\Pi}}_2(\beta_j)$, $\forall (\alpha_i, \beta_j) \in \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2$, where $\hat{\cup}$ is a BF-union.

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Definition 3.17. Let $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ and $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ are two BFHSSs over $\hat{\mathcal{Z}}$, then the *AND* operation is a BFHSS over $\hat{\mathcal{Z}}$ and represented by $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \wedge \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right) = \left(\bar{\bar{\Psi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2\right)$, where $\bar{\bar{\Psi}}\left(\alpha_i, \beta_j\right) = \bar{\bar{\Pi}}_1\left(\alpha_i\right) \hat{\cap} \bar{\bar{\Pi}}_2\left(\beta_j\right)$, $\forall(\alpha_i, \beta_j) \in \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2$, where $\hat{\cap}$ is a BF-intersection.

Example 3.18. Consider two BFHSSs given above in example 3.13.

Suppose that $\mathfrak{m}_1 = (\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_7)$, $\mathfrak{m}_2 = (\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8)$,$\mathfrak{m}_3 = (\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_7)$,$\mathfrak{m}_4 = (\mathfrak{b}_1, \mathfrak{b}_4, \mathfrak{b}_8)$ for $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$, $\mathfrak{n}_1 = (\mathfrak{b}_1, \mathfrak{b}_3, \mathfrak{b}_8)$, $\mathfrak{n}_2 = (\mathfrak{b}_1, \mathfrak{b}_5, \mathfrak{b}_8)$,$\mathfrak{n}_3 = (\mathfrak{b}_2, \mathfrak{b}_3, \mathfrak{b}_8)$,$\mathfrak{n}_4 = (\mathfrak{b}_2, \mathfrak{b}_5, \mathfrak{b}_8)$ for $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$, and

$\bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2 = \{(\mathfrak{m}_1 \times \mathfrak{n}_1), (\mathfrak{m}_1 \times \mathfrak{n}_2), (\mathfrak{m}_1 \times \mathfrak{n}_3), (\mathfrak{m}_1 \times \mathfrak{n}_4), (\mathfrak{m}_2 \times \mathfrak{n}_1), (\mathfrak{m}_2 \times \mathfrak{n}_2), (\mathfrak{m}_2 \times \mathfrak{n}_3),$
$(\mathfrak{m}_2 \times \mathfrak{n}_4), (\mathfrak{m}_3 \times \mathfrak{n}_1), (\mathfrak{m}_3 \times \mathfrak{n}_2), (\mathfrak{m}_3 \times \mathfrak{n}_3) (\mathfrak{m}_3 \times \mathfrak{n}_4), (\mathfrak{m}_4 \times \mathfrak{n}_1), (\mathfrak{m}_4 \times \mathfrak{n}_2),$
$(\mathfrak{m}_4 \times \mathfrak{n}_3), (\mathfrak{m}_4 \times \mathfrak{n}_4)\}$

Then the values of $\left(\bar{\bar{\Xi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2\right)$ and $\left(\bar{\bar{\Psi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2\right)$ can be represented by the following:

$$
\begin{aligned}
&\left(\bar{\bar{\Xi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2\right) = \\
&\{\langle(\mathfrak{m}_1 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.5, -0.1), (\hat{z}_2, 0.7, -0.8), (\hat{z}_3, 0.3, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_1 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.3, -0.3), (\hat{z}_2, 0.7, -0.7), (\hat{z}_3, 0.2, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_1 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.3, -0.8), (\hat{z}_2, 0.7, -0.9), (\hat{z}_3, 0.2, -0.6)\}\rangle, \\
&\langle(\mathfrak{m}_1 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.6, -0.8), (\hat{z}_2, 0.8, -0.5), (\hat{z}_3, 0.5, -0.5)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.9, -0.2), (\hat{z}_2, 0.7, -0.7), (\hat{z}_3, 0.2, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.9, -0.3), (\hat{z}_2, 0.8, -0.7), (\hat{z}_3, 0.7, -0.2)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.9, -0.8), (\hat{z}_2, 0.8, -0.9), (\hat{z}_3, 0.7, -0.6)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.9, -0.8), (\hat{z}_2, 0.8, -0.4), (\hat{z}_3, 0.7, -0.6)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.5, -0.5), (\hat{z}_2, 0.6, -1), (\hat{z}_3, 0.3, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.1, -0.5), (\hat{z}_2, 0.4, -1), (\hat{z}_3, 0.3, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.3, -0.8), (\hat{z}_2, 0.2, -1), (\hat{z}_3, 0.3, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.6, -0.8), (\hat{z}_2, 0.8, -0.1), (\hat{z}_3, 0.5, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.6, -0.8), (\hat{z}_3, 1, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_3, 0.4, -0.7), (\hat{z}_3, 1, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.7, -0.8), (\hat{z}_2, 0.4, -0.9), (\hat{z}_3, 1, -0.6)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.7, -0.8), (\hat{z}_2, 0.8, -0.2), (\hat{z}_3, 1, -0.5)\}\rangle\}.
\end{aligned}
$$

and

$$
\begin{aligned}
&\left(\bar{\bar{\Psi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_2\right) = \\
&\{\langle(\mathfrak{m}_1 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.6, -0.5), (\hat{z}_3, 0.2, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_1 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.1, -0.1), (\hat{z}_2, 0.4, -0.5), (\hat{z}_3, 0.2, -0.1)\}\rangle, \\
&\langle(\mathfrak{m}_1 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.1, -0.5), (\hat{z}_3, 0.2, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_1 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.3, -0.1), (\hat{z}_2, 0.7, -0.2), (\hat{z}_3, 0.2, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.5, -0.1), (\hat{z}_2, 0.6, -0.4), (\hat{z}_3, 0.3, -0.2)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.1, -0.2), (\hat{z}_2, 0.4, -0.4), (\hat{z}_3, 0.2, -0.1)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.3, -0.2), (\hat{z}_2, 0.1, -0.4), (\hat{z}_3, 0.2, -0.2)\}\rangle, \\
&\langle(\mathfrak{m}_2 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.6, -0.2), (\hat{z}_2, 0.8, -0.2), (\hat{z}_3, 0.5, -0.2)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_1), \{(\hat{z}_1, 0, -0.1), (\hat{z}_2, 0.2, -1), (\hat{z}_3, 0.3, -0.9)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_2), \{(\hat{z}_1, 0, -0.3), (\hat{z}_2, 0.2, -0.7), (\hat{z}_3, 0.2, -0.1)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_3), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.1, -0.9), (\hat{z}_3, 0.2, -0.6)\}\rangle, \\
&\langle(\mathfrak{m}_3 \times \mathfrak{n}_4), \{(\hat{z}_1, 0, -0.5), (\hat{z}_2, 0.2, -0.2), (\hat{z}_3, 0.3, -0.5)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_1), \{(\hat{z}_1, 0.5, -0.1), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 0.3, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_2), \{(\hat{z}_1, 0.1, -0.3), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 0.2, -0.1)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_3), \{(\hat{z}_1, 0.3, -0.6), (\hat{z}_2, 0.1, -0.1), (\hat{z}_3, 0.2, -0.4)\}\rangle, \\
&\langle(\mathfrak{m}_4 \times \mathfrak{n}_4), \{(\hat{z}_1, 0.6, -0.6), (\hat{z}_2, 0.4, -0.1), (\hat{z}_3, 0.5, -0.4)\}\rangle\}.
\end{aligned}
$$

Proposition 3.19. *The BFHSSs* $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$ *and* $\left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)$ *over* $\hat{\mathcal{Z}}$ *fulfill the following properties.*

1. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \vee \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$.

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2. $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \wedge \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$

3. $\left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \vee \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right)^c = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)^c \wedge \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c.$

4. $\left(\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \wedge \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)\right)^c = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)^c \vee \left(\bar{\bar{\Pi}}_2, \bar{\bar{\Lambda}}_2\right)^c.$

Proof. To prove (1), let $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \vee \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) = \left(\bar{\bar{\Xi}}, \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_1\right)$, and $\alpha_i \in \bar{\bar{\Lambda}}_1$. Then based on the definition 3.16, we have $\bar{\bar{\Xi}}\left(\alpha_i, \alpha_i\right) = \bar{\bar{\Pi}}_1\left(\alpha_i\right) \hat{\cup} \bar{\bar{\Pi}}_1\left(\alpha_i\right), \forall\left(\alpha_i, \alpha_i\right) \in \bar{\bar{\Lambda}}_1 \times \bar{\bar{\Lambda}}_1$
$\Rightarrow \bar{\bar{\Pi}}_1\left(\alpha_i\right) \hat{\cup} \bar{\bar{\Pi}}_1\left(\alpha_i\right) = \bar{\bar{\Pi}}_1\left(\alpha_i\right) \Rightarrow \bar{\bar{\Xi}}\left(\alpha_i, \alpha_i\right) = \bar{\bar{\Pi}}_1\left(\alpha_i\right) \Rightarrow \left(\bar{\bar{\Xi}}, \Lambda_1 \times \Lambda_1\right) = \bar{\bar{\Pi}}_1\left(\alpha_i\right).$
Hence $\left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) \vee \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right) = \left(\bar{\bar{\Pi}}_1, \bar{\bar{\Lambda}}_1\right)$.

The proof of first item and the proof of second item are alike. Proofs of third item and fourth item are similar to that of proposition 3.15.

□

4 Decision-making under BFHSSs

In this section, we will develop an algorithm based on the aggregation BFHSS operator in order to show the feasibility and effectiveness of this concept in modeling some real-life problems of decision-making.

Definition 4.1. Let $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right) = \left\{\left\langle \alpha, \left\{\left(\hat{z}, \mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}\left(\hat{z}\right), \mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}\left(\hat{z}\right)\right) \middle| \forall \hat{z} \in \hat{\mathcal{Z}}\right\}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\}$ be a BFHSS on $\hat{\mathcal{Z}}$. Then the aggregation BFHSS operator indicate by $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{agg}$ is given by

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{agg} = \left\{\alpha, \left\langle\left(\hat{z}_i, \hat{\Theta}_{\bar{\bar{\Pi}}(\alpha)}\left(\hat{z}_i\right)\right) \middle| \hat{z}_i \in \hat{\mathcal{Z}}\right\rangle : \alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}\right\},$$

where,

$$\hat{\Theta}_{\bar{\bar{\Pi}}(\alpha)}\left(\hat{z}_i\right) = \frac{1}{\left|\hat{\mathcal{Z}} \times \bar{\bar{\Delta}}\right|} \sum_{\alpha \in \bar{\bar{\Lambda}} \subseteq \bar{\bar{\Delta}}} \left(\mathcal{F}^+_{\bar{\bar{\Pi}}(\alpha)}\left(\hat{z}_i\right) + \left|\mathcal{F}^-_{\bar{\bar{\Pi}}(\alpha)}\left(\hat{z}_i\right)\right|\right).$$

Here $\left|\hat{\mathcal{Z}} \times \bar{\bar{\Delta}}\right|$ denote to the cardinality of $\hat{\mathcal{Z}} \times \bar{\bar{\Delta}}$.

Now we will build an algorithm based on the definition 4.1

Algorithm
1. Build the BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ on $\hat{\mathcal{Z}}$.
2. Calculate the aggregation $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{agg}$ of the BFHSS operator.
3. Discover an optimum alternative set on $\hat{\mathcal{Z}}$, by taking the maximum degree of the alternative in $\hat{\mathcal{Z}}$.

Now to test the presented algorithm, we organize the following problem:

Mr. Brown started his job at a company that is 50 kilometers away from his home, and for the purpose of early arrival at the work site, he decided to buy a car from a car dealership. At the auto show, there were five cars for sale (options) that comprised the universal set $\hat{\mathcal{Z}} = \{\hat{z}_1, \hat{z}_2, \hat{z}_3, \hat{z}_4, \hat{z}_5\}$. The best pick may be assessed by scrutinizing the attributes $\bar{\bar{\nu}}_1$ =Brands, $\bar{\bar{\nu}}_2$ =Color, $\bar{\bar{\nu}}_3$ =Price, and $\bar{\bar{\nu}}_4$ =Engine power. The attribute-value sets blending to these attributes are: $\bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_1} = \{\mathfrak{b}_1 = Hyundai, \mathfrak{b}_2 = Toyota, \mathfrak{b}_3 = BMW\}$, $\bar{\bar{\mathcal{E}}}_{\bar{\bar{\nu}}_2} =$

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$\{\mathfrak{b}_4 = white, \mathfrak{b}_5 = red, \mathfrak{b}_6 = blue\}$,$\bar{\bar{\mathcal{E}}}_{\bar{\nu}_3} = \{\mathfrak{b}_7 = 2500\, dollar, \mathfrak{b}_8 = 3500\, dollar, \mathfrak{b}_9 = 4500\, dollar\}, \bar{\bar{\mathcal{E}}}_{\bar{\nu}_4} = \{\mathfrak{b}_{10} = 1600\ , \mathfrak{b}_{11} = 2000, \mathfrak{b}_{12} = 3000\}$.
Now, to help Mr. Brown in his choice we can apply the method as follows:

1. Build the BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ on $\hat{\mathcal{Z}}$ as follows:
Suppose
$\bar{\bar{\mathcal{B}}}_{\bar{\nu}_1} = \{\mathfrak{b}_1 = Hyundai, \mathfrak{b}_2 = Toyota, \mathfrak{b}_3 = BMW\}$, $\bar{\bar{\mathcal{B}}}_{\bar{\nu}_2} = \{\mathfrak{b}_4 = white, \mathfrak{b}_6 = blue\}$,
$\bar{\bar{\mathcal{B}}}_{\bar{\nu}_3} = \{\mathfrak{b}_7 = 2500\, dollar, \mathfrak{b}_8 = 3500\, dollar, \mathfrak{b}_9 = 4500\, dollar\}$,
$\bar{\bar{\mathcal{B}}}_{\bar{\nu}_4} = \{\mathfrak{b}_{10} = 1600\ , \mathfrak{b}_{11} = 2000\ \}$ are subsets of $\bar{\bar{\mathcal{E}}}_{\bar{\nu}_i}, \forall i = 1, 2, 3$. Then the BFHSS $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)$ is analyzed as follows

$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right) =$$
$$\{\langle (b_1, b_4, b_7, b_{10}), \{(\hat{z}_1, 0.5, -0.1), (\hat{z}_2, 0.7, -0.8)\ , (\hat{z}_3, 0.3, -0.9), (\hat{z}_4, 0.3, -0.3), (\hat{z}_5, 0.7, -0.7)\}\rangle,$$
$$\langle (b_1, b_6, b_8, b_{11}), \{(\hat{z}_1, 0.3, -0.2), (\hat{z}_2, 0.4, -0.7)\ , (\hat{z}_3, 0.2, -0.4), (\hat{z}_4, 0.9, -0.8), (\hat{z}_5, 0.8, -0.9)\}\rangle,$$
$$\langle (b_1, b_4, b_9, b_{11}), \{(\hat{z}_1, 0.7, -0.5), (\hat{z}_2, 0.1, -0.2)\ , (\hat{z}_3, 0.1, -0.4), (\hat{z}_4, 0.5, -0.7), (\hat{z}_5, 0.4, -0.8)\}\rangle,$$
$$\langle (b_2, b_6, b_8, b_{10}), \{(\hat{z}_1, 0.2, -0.2), (\hat{z}_2, 0.2, -0.8)\ , (\hat{z}_3, 0.2, -0.5), (\hat{z}_4, 0.3, -0.7), (\hat{z}_5, 0.2, -0.7)\}\rangle,$$
$$\langle (b_2, b_4, b_7, b_{11}), \{(\hat{z}_1, 0.7, -0.6), (\hat{z}_2, 0.3, -0.5)\ , (\hat{z}_3, 0.7, -0.2), (\hat{z}_4, 0.1, -0.1), (\hat{z}_5, 0.3, -0.9)\}\rangle,$$
$$\langle (b_3, b_6, b_8, b_{10}), \{(\hat{z}_1, 0.6, -0.7), (\hat{z}_2, 0.1, -0.4)\ , (\hat{z}_3, 0.1, -0.4), (\hat{z}_4, 0.6, -0.5), (\hat{z}_5, 0.6, -0.5)\}\rangle,$$
$$\langle (b_3, b_4, b_9, b_{11}), \{(\hat{z}_1, 0.9, -0.8), (\hat{z}_2, 0.5, -0.3)\ , (\hat{z}_3, 0.6, -0.8), (\hat{z}_4, 0.6, -0.4), (\hat{z}_5, 0.7, -0.1)\}\rangle\}.$$

2. Calculate the aggregation $\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{agg}$ of the BFHSS operator.
$$\left(\bar{\bar{\Pi}}, \bar{\bar{\Lambda}}\right)^{agg} = \{\langle (\hat{z}_1, 0.35), (\hat{z}_2, 0.3), (\hat{z}_3, 0.29), (\hat{z}_4, 0.34), (\hat{z}_5, 0.41)\rangle\}.$$

3. Finally, DM select $\hat{z}_5$ as the best car Mr. Brown could buy.

5 Conclusion

As a new extension of HSSs, in this work we demonstrated the notion of BFHSSs via a blend of the notions of BFS and HSS. The concept of BFHSS is modified to provided the user with more focus and accuracy in dealing with the parametric environment. Based on this concept, we defined the basic operations with some properties that clarify the relationship of this concept to the basic properties of set theory. In addition, we provided some numerical examples to illustrate the mechanism of action of these concepts. Finally, we developed an algorithm based on the aggregation BFHSS operator in order to show the feasibility and effectiveness of this concept in modeling some real-life problems of decision-making.

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