A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations

Nader Mahmoud Taffach[1], **Ahmed Hatip**[2]

[1.]Faculty Of Science, Department Of Mathematics, Idleb University, Syria
[2.]Gaziantep University, Department Of Mathematics, Gaziantep, Turkey

Emails: ntaffash77@windowslive.com; Kollnaar5@gmail.com

Abstract

The main goal of this paper is to review the concepts of symbolic 2-plithogenic number theoretical concepts and algebraic equations, where many foundational concepts such as congruencies and linear equations and Diophantine linear equations.

Keywords: symbolic 2-plithogenic number theory; symbolic 2-plithogenic linear equation; symbolic 2-plithogenic Diophantine equation

1. Introduction

The concept of symbolic n-plithogenic sets was defined by Smarandache. This concept has made a good generalization of classical algebraic structures. Also, these structures have similar structures of neutrosophic and n-refined neutrosophic algebraic structures [10-40].

For n=2, we get symbolic 2-plithogenic algebraic structures, where we find symbolic 2-plithogenic equations, rings, spaces, and modules [1-10].

In this paper, we give the interested reader a good review about symbolic 2-plithogenic number theory and algebraic equations in many different types.

Symbolic 2-plithogenic algebraic equations

Definition.

Let $2-SP_Z=\{a+bP_1+cP_2;\ a,b,c\in Z\}$ be the symbolic 2-plithogenic ring of integers, the Diophantine equation with two variables is defined as follows:

$AX+BY=C; A=a_0+a_1P_1+a_2P_2, B=b_0+b_1P_1+b_2P_2, C=c_0+c_1P_1+c_2P_2,$

$X=x_0+x_1P_1+x_2P_2, Y=y_0+y_1P_1+y_2P_2, a_i,b_i,c_i,x_i,y_i\in 2-SP_Z.$

The following theorem describes an algorithm to solve the symbolic 2-plithogenic linear Diophantine equation with two variables.

Theorem.

Let $AX+BY=C$ be the symbolic 2-plithogenic linear Diophantine equation with two variables, it is solvable if and only if the following linear Diophantine equations are solvable.

$$\begin{cases} a_0x_0+b_0y_0=c_0 \\ (a_0+a_1)(x_0+x_1)+(b_0+b_1)(y_0+y_1)=c_0+c_1 \\ (a_0+a_1+a_2)(x_0+x_1+x_2)+(b_0+b_1+b_2)(y_0+y_1+y_2)=c_0+c_1+c_2 \end{cases}$$

The description of the algorithm.

To solve $AX+BY=C$ in $2-SP_Z$, we must follow these steps.

Step1.

We compute $gcd(a_0,b_0), gcd(a_0+a_1,b_0+b_1), gcd(a_0+a_1+a_2,b_0+b_1+b_2)$.

If $gcd(a_0,b_0)/c_0$, $gcd(a_0+a_1,b_0+b_1)/c_0+c_1, gcd(a_0+a_1+a_2,b_0+b_1+b_2)/c_0+c_1+c_2$, then it is solvable.

Step2.

We solve the equivalent system and get the values of $x_i,y_i; 0\le i\le 2$.

Doi: https://doi.org/10.54216/GJMSA.050103

Received: November 25, 2022 Accepted: March 20, 2023

Example.

Consider the following symbolic 2-plithogenic linear Diophantine equation:

$(2 + P_1 + P_2)X + (3 + 2P_1 - P_2)Y = 8 + 5P_1 + 7P_2$.

$gcd(a_0, b_0) = gcd(2,3) = 1/8$.

$gcd(a_0 + a_1, b_0 + b_1) = gcd(3,5) = 1/c_0 + c_1 = 13$.

$gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = gcd(4,4) = 4/c_0 + c_1 + c_2 = 20$.

So that, the equation is solvable.

The equivalent system of linear Diophantine equations is:

$$\begin{cases} 2x_0 + 3y_0 = 8 \ldots (1) \\ 3(x_0 + x_1) + 5(y_0 + y_1) = 13 \ldots (2) \\ 4(x_0 + x_1 + x_2) + 4(y_0 + y_1 + y_2) = 20 \ldots (3) \end{cases}$$

The equation (1) has a solution $(x_0 = 1, y_0 = 2)$.

The equation (2) has a solution $(x_0 + x_1 = 1, y_0 + y_1 = 2)$, there for $(x_1 = 0, y_1 = 0)$.

The equation (3) has a solution $(x_0 + x_1 + x_2 = 2, y_0 + y_1 + y_2 = 3)$, there for $(x_2 = 1, y_2 = 1)$.

This implies a solution $X = 1 + P_2, Y = 2 + P_2$.

2-symbolic plithogenic Quadratic equation.

Let $2 - SP_F$ be a symbolic 2-plithogenic field, the formula

$AX^2 + BY^2 + C = 0; A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2,$

$C = c_0 + c_1P_1 + c_2P_2, X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2, a_i, b_i, c_i, x_i, y_i \in 2 - SP_F$.

Is called the symbolic 2-plithogenic quadratic equation.

Theorem.

Let $AX^2 + BY^2 + C = 0$ be a symbolic 2-plithogenic quadratic equation over $2 - SP_F$, then it is solvable if and only if the following system is solvable:

$$\begin{cases} a_0{x_0}^2 + b_0{y_0}^2 + c_0 = 0 \ldots (1) \\ (a_0 + a_1)(x_0 + x_1)^2 + (b_0 + b_1)(y_0 + y_1)^2 + (c_0 + c_1) = 0 \ldots (2) \\ (a_0 + a_1 + a_2)(x_0 + x_1 + x_2)^2 + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2)^2 + (c_0 + c_1 + c_2) = 0 \ldots (3) \end{cases}$$

The description of algorithm.

To solve $AX^2 + BY^2 + C = 0$ in $2 - SP_F$, follow these steps:

Step1.

Solve the equivalent classical system of quadratic equations. If $(1), (2)$, and (3) are solvable in the field F,then the symbolic 2-plithogenic quadratic equation is solvable.

Step2.

Discuss all possible cases of x_0, x_1, x_2.

Remark.

If $AX^2 + BY^2 + C = 0$ is solvable in $2 - SP_F$, then it has at most 8 solutions.

Example.

Consider the following:

$(1 + P_1 + P_2)X^2 + (3 - P_1)X - 4 - 12P_2 = 0$

We have:

$$\begin{cases} a_0 = 1, a_1 = 1, a_2 = 1 \\ b_0 = 3, b_1 = -1, b_2 = 0 \\ c_0 = -4, c_1 = 0, c_2 = -12 \end{cases}$$

The equivalent system is:

$$\begin{cases} {x_0}^2 + 3x_0 - 4 = 0 \ldots (1) \\ 2(x_0 + x_1)^2 + 2(x_0 + x_1) - 4 = 0 \ldots (2) \\ 3(x_0 + x_1 + x_2)^2 + 2(x_0 + x_1 + x_2) - 16 = 0 \ldots (3) \end{cases}$$

The solutions of (1): $x_0 = 1, x_0 = -4$.

The solutions of (2): $x_0 + x_1 = 1, x_0 + x_1 = -2$.

The solutions of (3): $x_0 + x_1 + x_2 = 2, x_0 + x_1 + x_2 = -\frac{8}{3}$.

Case1.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 2$, then $x_1 = 0, x_2 = 1$, and $X = 1 + P_2$.

Case2.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = 0, x_2 = -\frac{11}{3}$, and $X = 1 - \frac{11}{3}P_2$.

Case3.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 2$, then $x_1 = -3, x_2 = 4$, and $X = 1 - 3P_1 + 4P_2$.

Case4.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = -3, x_2 = -\frac{2}{3}$, and $X = 1 - 3P_1 - \frac{2}{3}P_2$.

Case5.

Doi: https://doi.org/10.54216/GJMSA.050103

Received: November 25, 2022 Accepted: March 20, 2023

If $x_0 = -4, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 2$, then $x_1 = 5, x_2 = 1$, and $X = -4 + 5P_1 + P_2$.

Case6.

If $x_0 = -4, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = 5, x_2 = -\frac{11}{3}$, and $X = -4 + 5P_1 - \frac{11}{3}P_2$.

Case7.

If $x_0 = -4, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 2$, then $x_1 = 2, x_2 = 4$, and $X = -4 + 2P_1 + 4P_2$.

Case8.

If $x_0 = -4, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{8}{3}$, then $x_1 = 2, x_2 = -\frac{2}{3}$, and $X = -4 + 2P_1 - \frac{2}{3}P_2$.

So that, the solutions of the original symbolic 2-plithogenic quadratic equation are:

$$X \in \left\{-4 + 2P_1 - \frac{2}{3}P_2, -4 + 2P_1 + 4P_2, -4 + 5P_1 - \frac{11}{3}P_2, -4 + 5P_1 + P_2, 1 - 3P_1 - \frac{2}{3}P_2, 1 - 3P_1 + 4P_2, 1 - \frac{11}{3}P_2, 1 + P_2\right\}$$

Example.

Consider the following:

$(2 + 3P_1 - P_2)X^2 + (4 + P_1 + P_2)X - 6 - 4P_1 = 0$

We have:

$$\begin{cases} a_0 = 2, a_1 = 3, a_2 = -1 \\ b_0 = 4, b_1 = 1, b_2 = 1 \\ c_0 = -6, c_1 = -4, c_2 = 0 \end{cases}$$

The equivalent system is:

$$\begin{cases} 2x_0^2 + 4x_0 - 6 = 0 \dots (1) \\ 5(x_0 + x_1)^2 + 5(x_0 + x_1) - 10 = 0 \dots (2) \\ 4(x_0 + x_1 + x_2)^2 + 6(x_0 + x_1 + x_2) - 10 = 0 \dots (3) \end{cases}$$

The solutions of (1): $x_0 = 1, x_0 = -3$.

The solutions of (2): $x_0 + x_1 = 1, x_0 + x_1 = -2$.

The solutions of (3): $x_0 + x_1 + x_2 = 1, x_0 + x_1 + x_2 = -\frac{5}{2}$.

Case1.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 1$, then $x_1 = x_2 = 0$, and $X = 1$.

Case2.

If $x_0 = 1, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = 0, x_2 = -\frac{7}{2}$, and $X = 1 - \frac{5}{2}P_2$.

Case3.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 1$, then $x_1 = -3, x_2 = 3$, and $X = 1 - 3P_1 + 3P_2$.

Case4.

If $x_0 = 1, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = -3, x_2 = -\frac{1}{2}$, and $X = 1 - 3P_1 - \frac{1}{2}P_2$.

Case5.

If $x_0 = -3, x_0 + x_1 = 1, x_0 + x_1 + x_2 = 1$, then $x_1 = 4, x_2 = 0$, and $X = -3 + 4P_1$.

Case6.

If $x_0 = -3, x_0 + x_1 = 1, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = 4, x_2 = -\frac{7}{2}$, and $X = -3 + 4P_1 - \frac{7}{2}P_2$.

Case7.

If $x_0 = -3, x_0 + x_1 = -2, x_0 + x_1 + x_2 = 1$, then $x_1 = 1, x_2 = 3$, and $X = -3 + P_1 + 3P_2$.

Case8.

If $x_0 = -3, x_0 + x_1 = -2, x_0 + x_1 + x_2 = -\frac{5}{2}$, then $x_1 = 1, x_2 = -\frac{1}{2}$, and $X = -3 + P_1 - \frac{1}{2}P_2$.

So that, the solutions of the original symbolic 2-plithogenic quadratic equation are:

$$X \in \left\{1, 1 - \frac{5}{2}P_2, 1 - 3P_1 + 3P_2, 1 - 3P_1 - \frac{1}{2}P_2, -3 + 4P_1, -3 + 4P_1 - \frac{7}{2}P_2, -3 + P_1 + 3P_2, -3 + P_1 - \frac{1}{2}P_2\right\}$$

2-plithogenic Linear equations.

We begin the simplest case, a symbolic 2-plithogenic linear equation with one variable $A.X = B$.

This equation is solvable uniquely if and only if A is invertible and $X = A^{-1}B$.

According to [31], $A^{-1} = a_0^{-1} + P_1[(a_0 + a_1)^{-1} - a_0^{-1}] + P_2[(a_0 + a_1 + a_2)^{-1} - (a_0 + a_1)^{-1}]$.

Example.

Consider the equation $(2 + P_1 + P_2)X = 3 - P_1$ over $2 - SP_R$.

$a_0 = 2, a_0^{-1} = \frac{1}{2}, a_0 + a_1 = 3, (a_0 + a_1)^{-1} = \frac{1}{3}, a_0 + a_1 + a_2 = 4, (a_0 + a_1 + a_2)^{-1} = \frac{1}{4}$, thus:

$A^{-1} = \frac{1}{2} - \frac{1}{6}P_1 - \frac{1}{12}P_2$, there for:

$$X = \left(\frac{1}{2} - \frac{1}{6}P_1 - \frac{1}{12}P_2\right)(3 - P_1) = \frac{3}{2} - \frac{1}{2}P_1 - \frac{1}{2}P_1 + \frac{1}{6}P_1 - \frac{1}{4}P_2 + \frac{1}{12}P_2 = \frac{3}{2} - \frac{5}{6}P_1 - \frac{1}{6}P_2$$

The general case is about a linear system of n symbolic 2-plithogenic equations $A_i.X_i = B_i; 1 \le i \le n$.

Doi: https://doi.org/10.54216/GJMSA.050103

Received: November 25, 2022 Accepted: March 20, 2023

To solve a system like that, we must transform it to an equivalent classical system. We present the following algorithm.

To solve the symbolic 2-plithogenic linear system:

$$\begin{cases} A_{11}.X_1 + A_{12}.X_2 + \cdots + A_{1n}.X_n = B_{1n} \\ A_{21}.X_1 + A_{22}.X_2 + \cdots + A_{2n}.X_n = B_{2n} \\ \vdots \\ A_{n1}.X_1 + A_{n2}.X_2 + \cdots + A_{nn}.X_n = B_{nn} \end{cases}$$

Where: $A_{ij} = a_{ij}^{(0)} + a_{ij}^{(1)}P_1 + a_{ij}^{(2)}P_2, X_i = X_i^{(0)} + X_i^{(1)}P_1 + X_i^{(2)}P_2, B_{ij} = b_{ij}^{(0)} + b_{ij}^{(1)}P_1 + b_{ij}^{(2)}P_2 \in 2 - SP_F$.

Follow these steps:

Step1.

Find the classical equivalent system as follows:

$$\begin{cases} \sum_{i,j=1}^{n} a_{ij}^{(0)} X_i^{(0)} = \sum_{i,j=1}^{n} b_{ij}^{(0)} \\ \sum_{i,j=1}^{n} (a_{ij}^{(0)} + a_{ij}^{(1)})(X_i^{(0)} + X_i^{(1)}) = \sum_{i,j=1}^{n} (b_{ij}^{(0)} + b_{ij}^{(1)}) \\ \sum_{i,j=1}^{n} (a_{ij}^{(0)} + a_{ij}^{(1)} + a_{ij}^{(2)})(X_i^{(0)} + X_i^{(1)} + X_i^{(2)}) = \sum_{i,j=1}^{n} (b_{ij}^{(0)} + b_{ij}^{(1)} + b_{ij}^{(2)}) \end{cases}$$

step2.

Solve each system and remark that:

The first system gives the values of $X_i^{(0)}; 1 \le i \le n$.

The second one gives the values of $X_i^{(0)} + X_i^{(1)}; 1 \le i \le n$.

The third one gives values of $X_i^{(0)} + X_i^{(1)} + X_i^{(2)}; 1 \le i \le n$.

Step3.

If each system is solvable, then the original 2-plithogenic system is solvable, and if the number of solutions of every classical system is k, then the number of solutions for the 2-plithogenic system is k^3.

Example.

Consider the following symbolic 2-plithogenic system of three linear equations with three variables:

$$\begin{cases} (1 + P_2)X_1 + (3 - P_1)X_2 + (1 + P_1 - P_2)X_3 = 5 \\ P_2X_1 + P_1X_2 + (P_1 - P_2)X_3 = 2P_1 + 2P_2 \\ (1 + P_1 - P_2)X_1 + (4 + 3P_1 - P_2)X_2 + (5 + 2P_2)X_3 = 11 + 4P_2 \end{cases}$$

the equivalent classical systems are:

$$\begin{cases} X_1^{(0)} + 3X_2^{(0)} + X_3^{(0)} = 5 \\ 0X_1^{(0)} + 0X_2^{(0)} + 0X_3^{(0)} = 0 \\ 2X_1^{(0)} + 4X_2^{(0)} + 5X_3^{(0)} = 11 \end{cases} \quad \dots \quad \text{system(1)}$$

$$\begin{cases} (X_1^{(0)} + X_1^{(1)}) + 2(X_2^{(0)} + X_2^{(1)}) + 2(X_3^{(0)} + X_3^{(1)}) = 5 \\ 0(X_1^{(0)} + X_1^{(1)}) + (X_2^{(0)} + X_2^{(1)}) + (X_3^{(0)} + X_3^{(1)}) = 2 \\ 3(X_1^{(0)} + X_1^{(1)}) + 7(X_2^{(0)} + X_2^{(1)}) + 5(X_3^{(0)} + X_3^{(1)}) = 15 \end{cases} \quad \dots \quad \text{system(2)}$$

$$\begin{cases} 2(X_1^{(0)} + X_1^{(1)} + +X_1^{(2)}) + 2(X_2^{(0)} + X_2^{(1)} + X_2^{(2)}) + (X_3^{(0)} + X_3^{(1)} + X_3^{(2)}) = 5 \\ (X_1^{(0)} + X_1^{(1)} + +X_1^{(2)}) + (X_2^{(0)} + X_2^{(1)} + X_2^{(2)}) + 2(X_3^{(0)} + X_3^{(1)} + X_3^{(2)}) = 4 \\ 2(X_1^{(0)} + X_1^{(1)} + +X_1^{(2)}) + 6(X_2^{(0)} + X_2^{(1)} + X_2^{(2)}) + 7(X_3^{(0)} + X_3^{(1)} + X_3^{(2)}) = 15 \end{cases} \quad \dots \text{system(3)}$$

The system(1) has infinite solutions, thus the 2-plithogenic system has infinite solutions.

We will find some solutions to clarify the algorithm.

For example system(1) has a solution $X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 1$.

The system (2) has a solution $X_1^{(0)} + X_1^{(1)} = X_2^{(0)} + X_2^{(1)} = X_3^{(0)} + X_3^{(1)} = 1$, thus $X_1^{(1)} = X_2^{(1)} = X_3^{(1)} = 0$.

The system (3) has a solution $X_1^{(0)} + X_1^{(1)} + X_1^{(2)} = X_2^{(0)} + X_2^{(1)} + X_2^{(2)} = X_3^{(0)} + X_3^{(1)} + X_3^{(2)} = 1$, there for $X_1^{(2)} = X_2^{(2)} = X_3^{(2)} = 0$, and $X_1 = X_1^{(0)} + X_1^{(1)}P_1 + X_1^{(2)}P_2 = 1$, $X_2 = 1, X_3 = 1$ is a solution for the 2-plithogenic system.

Also, the system (1) has a solution $X_1^{(0)} = \frac{13}{2}, X_2^{(0)} = -\frac{1}{2}, X_3^{(0)} = 0$.

The system (2) has a solution $X_1^{(0)} + X_1^{(1)} = X_2^{(0)} + X_2^{(1)} = X_3^{(0)} + X_3^{(1)} = 1$.

The system (3) has a solution $X_1^{(0)} + X_1^{(1)} + X_1^{(2)} = X_2^{(0)} + X_2^{(1)} + X_2^{(2)} = X_3^{(0)} + X_3^{(1)} + X_3^{(2)} = 1$.

Doi: https://doi.org/10.54216/GJMSA.050103
Received: November 25, 2022 Accepted: March 20, 2023

There for $X_1^{(1)} = 1 - \frac{13}{2} = -\frac{11}{2}, X_2^{(1)} = 1 + \frac{1}{2} = \frac{3}{2}, X_3^{(1)} = 1, X_1^{(2)} = X_2^{(2)} = X_3^{(2)} = 0$.
This implies that:
$X_1 = \frac{13}{2} - \frac{11}{2}P_1, X_2 = -\frac{1}{2} + \frac{3}{2}P_1, X_3 = P_1$ is a solution of the 2-plithogenic system.

2-Plithogenic Number Theory

Definition.
Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_Z$, we say that $A \setminus B$ if and only if there exists $C \in 2 - SP_Z$ such that $A \times B = C$.
Definition.
Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2$ be three symbolic 2-plithogenic integers, then $A \equiv B(mod\ C)$ if and only if $C \setminus A - B$.
Also, $C = gcd(A, B)$ if and only if $C \setminus A$ and $C \setminus B$ and for any $D \setminus A, D \setminus B$, then $D \setminus C$.
Definition.
We say that $A \leq B$ if $a_0 \leq b_0, a_0 + a_1 \leq b_0 + b_1, a_0 + a_1 + a_2 \leq b_0 + b_1 + b_2$.
Theorem.
Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2 \in 2 - SP_Z$, then:
1). $(\leq)$ is a partial order relation.
2). $A \setminus B$ if and only if $a_0 \setminus b_0, a_0 + a_1 \setminus b_0 + b_1, a_0 + a_1 + a_2 \setminus b_0 + b_1 + b_2$.
3). $gcd(A, B) = C$ if and only if $gcd(a_0, b_0) = c_0, gcd(a_0 + a_1, b_0 + b_1) = c_0 + c_1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = c_0 + c_1 + c_2$.
4). $A \equiv B(mod\ C)$ if and only if:

$$\begin{cases} a_0 \equiv b_0(mod\ c_0) \\ a_0 + a_1 \equiv b_0 + b_1(mod\ c_0 + c_1) \\ a_0 + a_1 + a_2 \equiv b_0 + b_1 + b_2(mod\ c_0 + c_1 + c_2) \end{cases}$$

Theorem.
Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_Z$, then $gcd(A, B) = 1$ if and only if $gcd(a_0, b_0) = 1, gcd(a_0 + a_1, b_0 + b_1) = 1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = 1$.
The proof is clear.
Theorem.
Let $A, B, C, D, E \in 2 - SP_Z$, where:
$A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2, D = d_0 + d_1P_1 + d_2P_2, E = e_0 + e_1P_1 + e_2P_2$; $c_i, a_i, b_i, e_i, d_i \in Z$, then:
1). If $A \equiv B(mod\ C), D \equiv E(mod\ C)$, then $A + D \equiv B + E(mod\ C), A - D \equiv B - E(mod\ C)$.
2). $A.D \equiv B.E(mod\ C)$.
3). If $gcd(A, B) = 1$, then:

$$A^{-1}(mod\ B) = a_0^{-1}(mod\ b_0) + P_1[(a_0 + a_1)^{-1}(mod\ b_0 + b_1) - a_0^{-1}(mod\ b_0)] + P_2[(a_0 + a_1 + a_2)^{-1}(mod\ b_0 + b_1 + b_2) - (a_0 + a_1)^{-1}(mod\ b_0 + b_1)]$$

Example:
Consider $A = 5 + 4P_1 + 2P_2, B = 2 + P_1 + P_2, C = 3 + 4P_2$, we have:
$5 \equiv 2(mod\ 3), 5 + 4 = 9 \equiv (2 + 1)(mod\ 3 + 0), 5 + 4 + 2 = 11 \equiv (2 + 1 + 1)(mod\ 3 + 0 + 4)$, thus $A \equiv B(mod\ C)$.
$gcd(A, B) = gcd(5,2) + P_1[gcd(9,3) - gcd(5,2)] + P_2[gcd(11,4) - gcd(9,3)] = 1 + P_1(3 - 1) + P_2(1 - 3) = 1 + 2P_1 - 2P_2$.
Example.
Consider $A = 2 + P_1 + P_2, B = 3 + P_1 + P_2$, it is clear that $gcd(A, B) = 1$.
$A^{-1}(mod\ B) = 2^{-1}(mod\ 3) + P_1[3^{-1}(mod\ 4) - 2^{-1}(mod\ 3)] + P_2[4^{-1}(mod\ 5) - 3^{-1}(mod\ 4)] = 2 + P_1(3 - 2) + P_2(4 - 3) = 2 + P_1 + P_2$.
Definition.
Let $A = a_0 + a_1P_1 + a_2P_2 > 0$ be a symbolic 2-plithogenic integer, we define $\varphi_S: 2 - SP_Z \to 2 - SP_Z$ such that:
$\varphi_S(A) = \varphi(a_0) + P_1[\varphi(a_0 + a_1) - \varphi(a_0)] + P_2[\varphi(a_0 + a_1 + a_2) - \varphi(a_0 + a_1)]$.
Where φ is the classical phi-Euler's function.
Example.
Take $A = 3 + 5P_1 - P_2, a_0 = 3, a_1 = 5, a_2 = -1$. We have:
$a_0 = 3 > 0, a_0 + a_1 = 8 > 0, a_0 + a_1 + a_2 = 7 > 0$, so that $A > 0$.
$\varphi(a_0) = 2, \varphi(a_0 + a_1) = 4, \varphi(a_0 + a_1 + a_2) = 6$, hence:
$\varphi_S(A) = 2 + P_1[4 - 2] + P_2[6 - 4] = 2 + 2P_1 + 2P_2$.
Theorem.
Let $A = a_0 + a_1P_1 + a_2P_2, M = m_0 + m_1P_1 + m_2P_2 \in 2 - SP_Z$ such that $gcd(A, M) = 1$, then

Doi: https://doi.org/10.54216/GJMSA.050103
Received: November 25, 2022 Accepted: March 20, 2023

$A^{\varphi_S(M)} \equiv 1(mod\ M)$.

Example.

Take $A = 2 + 3P_1 - 2P_2, M = 3 + 4P_1 + 4P_2$**,** we have $gcd(A, M) = 1$.

$\varphi_S(M) = 2 + P_1(6 - 2) + P_2(10 - 6) = 2 + 4P_1 + 4P_2$

$A^{\varphi_S(M)} = 2^2 + P_1[5^6 - 2^2] + P_2[3^{10} - 5^6]$

$2^2 \equiv 1(\ mod\ 3), 5^6 \equiv 1(\ mod\ 7), 3^{10} \equiv 1(\ mod\ 11)$, thus $A^{\varphi_S(M)} \equiv 1(mod\ M)$

Theorem.

Let $C = gcd(A, B) \in 2 - SP_Z$, then there exists $M, N \in 2 - SP_Z$ such that $C = MA + NB$.

Example.

Consider $A = 3 + 2P_1 + P_2, B = 3 + P_1 + 3P_2$, we have:

$a_0 = 3, a_1 = 2, a_2 = 1, b_0 = 3, b_1 = 1, b_2 = 3$.

$gcd(a_0, b_0) = 3, gcd(a_0 + a_1, b_0 + b_1) = gcd(5,4) = 1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = gcd(6,7) = 1$

Thus $gcd(A, B) = 3 + (1 - 3)P_1 + (1 - 1)P_2 = 3 - 2P_1$.

On the other hand, we have:

$$\begin{cases} 3 = 1.3 + 0.3\ hence\ m_0 = 1, n_0 = 0 \\ 1 = 1.5 - 1.4\ hence\ m_1 = 1, n_1 = -1 \\ 1 = -1.6 + 1.7\ hence\ m_2 = -1, n_2 = 1 \end{cases}$$

Thus $M = 1 + (1 - 3)P_1 + (-1 - 1)P_2 = 1 - 2P_2, N = 0 + (-1 - 0)P_1 + (1 + 1)P_2 = -P_1 + 2P_2$

We can see that:

$$\begin{aligned} MA + NB &= (1 - 2P_2)(3 + 2P_1 + P_2) + (-P_1 + 2P_2)(3 + P_1 + 3P_2) \\ &= 3 + 2P_1 + P_2 - 6P_2 - 4P_2 - 2P_2 - 3P_1 - P_1 - 3P_1 + 6P_2 + 2P_2 + 6P_2 = 3 - 2P_1 = C \\ &= gcd(A, B) \end{aligned}$$

References

[1] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", International Journal of Neutrosophic Science, 2023.

[2] Taffach, N., " An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol 54, 2023.

[3] Taffach, N., and Ben Othman, K., " An Introduction to Symbolic 2-Plithogenic Modules

Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

[4] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", Neutrosophic Sets and Systems, vol. 53, 2023.

[5] Florentin Smarandache, Plithogenic Algebraic Structures. Chapter in “Nidus idearum Scilogs, V: joining the dots” (third version), Pons Publishing Brussels, pp. 123-125, 2019.

[6] Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.

[7] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and Their

Algebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.

[8] Khaldi, A., Ben Othman, K., Von Shtawzen, O., Ali, R., and Mosa, S., " On Some Algorithms for Solving Different Types of Symbolic 2-Plithogenic Algebraic Equations", Neutrosophic Sets and Systems, Vol 54, 2023.

[9] Merkepci, H., and Rawashdeh, A., " On The Symbolic 2-Plithogenic Number Theory and

Integers ", Neutrosophic Sets and Systems, Vol 54, 2023.

[10] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[11] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[12] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.

Doi: https://doi.org/10.54216/GJMSA.050103

Received: November 25, 2022 Accepted: March 20, 2023

[13] Kandasamy, V.W.B., and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phonex, Arizona, 2006.

[14] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011

[15] Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.

[16] Abobala, M., "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 72-75. 2020.

[17] Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.

[18] Kandasamy, I., Kandasamy, V., and Smarandache, F., "Algebraic structure of Neutrosophic Duplets in Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 18, pp. 85-95. 2018.

[19] Yingcang, Ma., Xiaohong Zhang ., Smarandache, F., and Juanjuan, Z., "The Structure of Idempotents in Neutrosophic Rings and Neutrosophic Quadruple Rings", Symmetry Journal (MDPI), Vol. 11. 2019.

[20] Kandasamy, V. W. B,. Ilanthenral, K., and Smarandache, F., "Semi-Idempotents in Neutrosophic Rings", Mathematics Journal (MDPI), Vol. 7. 2019.

[21] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.

[22] Smarandache, F., " An Introduction To neutrosophic Genetics", International Journal of neutrosophic Science, Vol.13, 2021.

[23] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86 . 2020.

[24] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.

[25] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[26] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", Neural. Compute. Appl. 2019.

[27] Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.

[28] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[29] Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

[30] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

[31] M. Ali, F. Smarandache, M. Shabir and L. Vladareanu., "Generalization of Neutrosophic Rings and Neutrosophic Fields", Neutrosophic Sets and Systems, vol. 5, pp. 9-14, 2014.

[32] Anuradha V. S., "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka", Neutrosophic Sets and Systems, vol. 31, pp. 179-199. 2020.

[33] Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.

[34] Abobala, M., Bal, M., and Hatip, A.," A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices**",** International Journal of Neutrosophic Science, Vol. 17, 2021.

[35] Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

Doi: https://doi.org/10.54216/GJMSA.050103
Received: November 25, 2022 Accepted: March 20, 2023

[36] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.

[37] Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

[38] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[39] Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 9, pp. 110-116 . 2020.

[40] Hatip, A., and Olgun, N., "On Refined Neutrosophic R-Module", International Journal of Neutrosophic Science, Vol. 7, pp.87-96. 2020.

[41] Bal, M., Abobala, M., "On The Representation Of Winning Strategies Of Finite Games By Groups and Neutrosophic Groups", Journal Of Neutrosophic and Fuzzy Systems, 2022.

[42] Smarandache F., and Abobala, M., *"n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

[43] Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.

[44] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.

[45] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

[46] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.

[47] Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.

[48] Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.

[49] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[50] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[51] Florentin Smarandache: Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited, *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166.

[52] Florentin Smarandache, Physical Plithogenic Set, 71st Annual Gaseous Electronics Conference, Session LW1, Oregon Convention Center Room, Portland, Oregon, USA, November 5–9, 2018.

[53] F. Smarandache, Plithogeny, Plithogenic Set, Logic, Probability, and Statistics, 141 pages, Pons Editions,Brussels, Belgium, 2017. arXiv.org (Cornell University), Computer Science - Artificial Intelligence, 03Bxx:

[54] F. Smarandache, Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers,

Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers. In *Symbolic NeutrosophicTheory*, Chapter 7, pages 186-193, Europa Nova, Brussels, Belgium, 2015.

[55] Khaldi, A**., "** A Study On Split-Complex Vector Spaces**",** Neoma Journal Of Mathematics and Computer Science, 2023.

[56] Ahmad, K., " On Some Split-Complex Diophantine Equations", Neoma Journal Of Mathematics and Computer Science, 2023.

[57] Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.

Doi: https://doi.org/10.54216/GJMSA.050103
Received: November 25, 2022 Accepted: March 20, 2023

[58] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[59] Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", Neutrosophic Knowledge, Vol. 1, 2020.

[60] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[61] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021

[62] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[63] Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces**",** Neoma Journal Of Mathematics and Computer Science, 2023.

[64] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.

[65] Celik, M., and Olgun, N., " An Introduction To Neutrosophic Real Banach And Hillbert Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022.

[66] Abobala, M., and Hatip, A., "An Algebraic Approch To Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[67] Sundar, J., Vadivel, A., " New operators Using Neutrosophic $\vartheta -$Open Sets", Journal Of Neutrosophic and Fuzzy Systems, 2022.

[68] Celik, M., and Olgun, N., " On The Classification Of Neutrosophic Complex Inner Product Spaces", Galoitica Journal Of Mathematical Structures And Applications, 2022**.**

[69] Hatip, A., " On Intuitionistic Fuzzy Subgroups of (M-N) Type and Their Algebraic Properties", Galoitica Journal Of Mathematical Structures and Applications, Vol.4, 2023.

[70] Abobala, M., and Ziena, M.B., (2023) . A Study Of Neutrosophic Real Analysis By Using One Dimensional Geometric AH-Isometry. Galoitica Journal Of Mathematical Structures and Applications, Vol 3.

[71] Merkepci, M., Abobala, M., and Allouf, A., " The Applications of Fusion Neutrosophic Number Theory in Public Key Cryptography and the Improvement of RSA Algorithm ", Fusion: Practice and Applications, 2023.

[72] Merkepci, M., and Abobala, M., " Security Model for Encrypting Uncertain Rational Data Units Based on Refined Neutrosophic Integers Fusion and El Gamal Algorithm ", Fusion: Practice and Applications, 2023.

[73] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.

[74] Ahmad, K., Bal, M., Hajjari, A., Ali, R.," On Imperfect Duplets In Some refined Neutrosophic Rings", Journal of Neutrosophic and Fuzzy Systems, 2022.

[75] Ahmad, K., Bal, M., and Aswad, M.," A Short Note on The Solution Of Fermat's Diophantine Equation In Some Neutrosophic Rings", Journal of Neutrosophic and Fuzzy Systems, Vol. 1, 2022.

[76] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 45, 2021.

[77] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

[78] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 16, pp. 72-79, 2021.

[79] Bal, M., Ahmad, K., Hajjari, A., Ali, R.," The Structure Of Imperfect Triplets In Several Refined Neutrosophic Rings" Journal of Neutrosophic and Fuzzy Systems, 2022.

Doi: https://doi.org/10.54216/GJMSA.050103
Received: November 25, 2022 Accepted: March 20, 2023