



Solving shortest path problems using an ant colony algorithm with triangular neutrosophic arc weights

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Abstract

Indeed, one of the most well-known topics in the area of graph theory is the shortest path (SP) problem, which has practical applications in various areas of research, including transportation, communication via networks, life-saving services, fire department services, etc. The edges of the connected SP problems are typically characterized by various numbers in practical applications. In this research paper, we calculate the shortest path using an ant colony optimization (ACO) algorithm with single value triangular neutrosophic numbers as arc weights. The method is used to estimate the shortest path of a neutrosophic network. One numerical example is used to test the suggested method, and outcomes are provided.

Keywords: Ant colony optimization; Neutrosophic shortest path problem; Neutrosophic directed graph; Single value triangular neutrosophic numbers; Neutrosophic network.

1. Introduction

One of the most significant and challenging mathematical problems is the SPP, which focuses on finding the shortest path between specific starting and ending nodes. Graph theory's essential mathematical network optimization problems [1,2] have been identified as sub-problems in a number of practical applications. Fields like wireless networking, logistics, and transportation. The SPP has been thoroughly studied in terms of effectiveness, efficacy, and logical algorithmic methods across a number of academic fields, including business operations research and the field of computer engineering. SPPs come in a variety of forms, and many of them are currently being studied. For instance, SPP for (n, k) -star graphs has been presented by Cheng et al. [3].

Generally, in telecommunication networks, due to a lack of data on traffic frequency, it is often very critical to find the actual arc length and weights of respective nodes. The random distribution in fuzzy networks is helpful in producing the exact arc length and weights, but it is uncertain in finding the SPP. For the above reason, different fundamentally significant methods are developed to solve the SPP. The addition and ranking methods are in the pictures and widely accepted in between fuzzy numbers in terms of their edge weights and nomenclature. In the last two or three decades, researchers have been doing prominent work on the fuzzy criteria and have done an effective job of describing fuzzy numbers by their arc lengths and weights; these types of things are called fuzzy SPPs (FSPPs) [4,5,6,7,8,9,10,11,12,13,14].

SPP was used by the decision maker to simulate a variety of real-world issues, including scheduling [15, 16], telecommunications [17, 18], etc. The SPP on fuzzy environments was first described by Prade and Dubois [19]. They presented a modified version of the traditional Ford Moore-Bellman method for the fuzzy shortest path problem. They employed the idea of a path's criticality in their essay. Kamburowski and Chanas[20] suggested the notion of a fuzzy inclination relationship for the purpose of determining the shortest path [21]. They used the

dynamic programming approach in their newly introduced algorithm to choose the one that was settled by the expert.

The Floyd-Warshall algorithm was created to calculate the SP between all the vertices of a given graph in a fuzzy network [22]. This algorithm incorporates both a dynamic programming approach and a ranking technique. Tajdin et al. [23] used Floyd's algorithm to find the shortest path in a wireless sensor network. Dou et al. [24] used a method of multi-criteria decision-making based on an ambiguous similarity measure to find the shortest route in a network with multiple constraints. Deng et al. [25] expanded the Dijkstra algorithm to address the shortest path problem with fuzzy arc weights.

The fuzzy physarum technique, constructed around a path-finding model and physiologically accepted, was proposed by Zhang et al. [26] as a solution to fuzzy shortest route problems. An evolutionary technique was employed by Hassanzadeh et al. [27] to identify the shortest possible route in a directed network with ambiguous weights. By turning the challenge of determining the shortest path into an optimization issue, they were able to determine the best route. When compared to the outcomes of the GA suggested, Ebrahimnejad et al. [28] used an approach based on PSO to reduce the run time and speed up convergence. The same ABC Network Model technique was used by Ebrahimnejad et al. [29] to expedite the process of locating the shortest path in complex illustrations with fuzzy edges.

The ACO algorithm was expanded by Calle et al. [30] by giving ants the ability to smell. This method rapidly locates a path between two nodes through a dynamic graph, even though it might not find the shortest path, which makes it useful in situations where optimality is not required. A brand-new algorithm to address the Traveling Salesman Problem was presented by Ashour et al. [31]. These authors used adaptive affinity propagation to cluster vertices, and then ACO was used to determine the best path for each cluster separately. A genetic-ACO algorithm was created by Changdar et al. [32] to evaluate solid multiple-travelling salesman problems in an uncertain setting.

In a neutrosophic environment, many scientists do prominent work to get the exact solution using algorithmic methods, mathematical methods, the exact algorithm method, heuristic methods, and metaheuristic methods. But in this paper, we calculate SP by using the social behaviours of ants in a neutrosophic environment. The analysis of the literature has been summarized in Section-1. Section-2 gives a general description of a neutrosophic set and trapezoidal neutrosophic sets and modified. Section-3 shows proposed ant colony optimization algorithm. Section-4 which contains an Implementation of algorithm for computing the shortest path (SP) in a neutrosophic network with respect to a single value triangular neutrosophic number. The conclusion of the proposed methodology is described in Section-5.

2. Preliminaries

Definition 2.2

Assume that X is a universe set and A neutrosophic set A over X is defined by

$$\hat{N} = \{[\hat{x}, T_{\hat{N}}(\hat{x}), I_{\hat{N}}(\hat{x}), F_{\hat{N}}(\hat{x})] | \hat{x} \in \hat{X}\} \text{----- (1)}$$

Where $T_{\hat{N}}(\hat{x}) \in [0,1]$, $I_{\hat{N}}(\hat{x}) \in [0,1]$ and $F_{\hat{N}}(\hat{x}) \in [0,1]$ are called truth, indeterminacy and falsity membership function.

$$T_{\hat{N}}(\hat{x}): X \rightarrow]0, 1[, I_{\hat{N}}(\hat{x}): X \rightarrow]0, 1[, F_{\hat{N}}(\hat{x}): X \rightarrow]0, 1[$$

Such that $0 \leq T_{\hat{N}}(\hat{x}) + I_{\hat{N}}(\hat{x}) + F_{\hat{N}}(\hat{x}) \leq 3, \hat{x} \in \hat{X}$.

Definition 2.3

Assume that single valued triangular neutrosophic number (e_1, e_2, e_3) . Then its three membership function is defined as.

$$T_{\hat{e}}(\hat{x}) = \begin{cases} \frac{(\hat{x} - e_1)T_{\hat{e}}}{(e_2 - e_1)}, & e_1 \leq \hat{x} \leq e_2 \\ \frac{(e_3 - \hat{x})T_{\hat{e}}}{(e_3 - e_2)}, & e_2 \leq \hat{x} \leq e_3 \\ 0, & \text{otherwise} \end{cases} \text{----- (2.0)}$$

$$I_{\hat{e}}(\hat{x}) = \begin{cases} \frac{(e_2 - \hat{x}) + I_{\hat{e}}(\hat{x} - e_1)}{(e_2 - e_1)}, & e_1 \leq \hat{x} \leq e_2, \\ \frac{(\hat{x} - e_2) + I_{\hat{e}}(e_3 - \hat{x})}{(e_3 - e_2)}, & e_2 \leq \hat{x} \leq e_3 \\ 1, & \text{otherwise} \end{cases} \quad \text{----- (2.1)}$$

$$F_{\hat{e}}(\hat{x}) = \begin{cases} \frac{(e_2 - \hat{x}) + F_{\hat{e}}(\hat{x} - e_1)}{(e_2 - e_1)}, & e_1 \leq \hat{x} \leq e_2, \\ \frac{(\hat{x} - e_2) + F_{\hat{e}}(e_3 - \hat{x})}{(e_3 - e_2)}, & e_2 \leq \hat{x} \leq e_3 \\ 1, & \text{otherwise} \end{cases} \quad \text{----- (2.3)}$$

Definition 2.3

Assume that single valued trapezoidal neutrosophic number (e_1, e_2, e_3, e_4) . Then its three membership function is defined as.

$$T_{\hat{e}}(\hat{x}) = \begin{cases} \frac{(\hat{x} - e_1)T_{\hat{e}}}{(e_2 - e_1)}, & e_1 \leq \hat{x} \leq e_2 \\ T_{\hat{e}}, & e_2 \leq \hat{x} \leq e_3 \\ \frac{(e_4 - \hat{x})T_{\hat{e}}}{(e_4 - e_3)}, & e_3 \leq \hat{x} \leq e_4 \\ 0, & \text{otherwise} \end{cases} \quad \text{----- (2.4)}$$

$$I_{\hat{e}}(\hat{x}) = \begin{cases} \frac{(e_2 - \hat{x}) + I_{\hat{e}}(\hat{x} - e_1)}{(e_2 - e_1)}, & e_1 \leq \hat{x} \leq e_2, \\ I_{\hat{e}}, & e_2 \leq \hat{x} \leq e_3 \\ \frac{(\hat{x} - e_3) + I_{\hat{e}}(e_4 - \hat{x})}{(e_2 - e_1)}, & e_3 \leq \hat{x} \leq e_4 \\ 1, & \text{otherwise} \end{cases} \quad \text{----- (2.5)}$$

$$F_{\hat{e}}(\hat{x}) = \begin{cases} \frac{(e_2 - \hat{x}) + F_{\hat{e}}(\hat{x} - e_1)}{(e_2 - e_1)}, & e_1 \leq \hat{x} \leq e_2 \\ F_{\hat{e}}, & e_2 \leq \hat{x} \leq e_3 \\ \frac{(\hat{x} - e_3) + F_{\hat{e}}(e_4 - \hat{x})}{(e_2 - e_1)}, & e_3 \leq \hat{x} \leq e_4 \\ 1, & \text{otherwise} \end{cases} \quad \text{----- (2.6)}$$

Statement-1:

The α – cut of a single value neutrosophic number is given by the real interval $\check{E} = (e_1, e_2, e_3, e_4)$, $[\check{E}]_{\alpha} = [\check{E}_{\alpha}^{-}, \check{E}_{\alpha}^{+}] = (e_2 - e_1)\alpha + e_1, e_4 - (e_4 - e_3)\alpha$.

Statement-2:

The α – cut of a normal single value neutrosophic number $\check{E} = (m, \sigma)$ is given by the real interval $[\check{E}]_{\alpha} = [\check{E}_{\alpha}^{-}, \check{E}_{\alpha}^{+}] = m - \sigma\sqrt{-\ln(\alpha)}, m + \sigma\sqrt{-\ln(\alpha)}$.

α – cut methods adding two trapezoidal neutrosophic numbers in neutrosophic environment

Consider A single value trapezoidal neutrosophic number $\check{E}_1 = (e_1, e_2, e_3, e_4)$ and a normal neutrosophic number $\check{E}_2 = (m, \sigma)$.

The related membership function and its sum are estimated by dividing the α – cut from

$[0, 1]$, and $\alpha_0 = 0, \alpha_i = \alpha_{i-1} + \Delta\alpha_i$

Here, $\Delta\alpha_i = \frac{1}{n}$ and $n = 1, 2, \dots, n$.

Given $\alpha_i \in (0, 1], 1 \leq i \leq n$, the α – cut of sum of this neutrosophic numbers is derived using statement-1 and statement-2.

$$[\check{E}]_{\alpha} = [\check{E}_{\alpha}^{-}, \check{E}_{\alpha}^{+}] = [\check{F}_{\alpha}^{L} + \check{G}_{\alpha}^{L}, \check{F}_{\alpha}^{R} + \check{G}_{\alpha}^{R}]$$

$$= (e_2 - e_1)\alpha + e_1 + m - \sigma\sqrt{-\ln(\alpha)}, e_4 - (e_4 - e_3)\alpha + m + \sigma\sqrt{-\ln(\alpha)} \dots (5)$$

We get equation (5) by using n points of $[\check{E}_{\alpha}^{-}, \check{E}_{\alpha}^{+}]$

Assume that $u_i = \check{E}_{\alpha}^{L}$ and $v_i = \vartheta \check{E}_{\alpha}^{L}$ and for n points u_i, v_i . Let us assume fitting model

$$Y = e^{-\frac{(u-\lambda_1)^2}{\beta_1}}$$

We proposed α – cut based addition [] []

$$\beta_1 = \frac{n \sum u_i \sqrt{-\ln(v_i)} - \sum \sqrt{-\ln(v_i)} \sum u_i}{-n \sum_i \sqrt{-\ln(v_i)} - \sum_i \sqrt{-\ln(v_i)} \times \sum_i \sqrt{-\ln(v_i)}} \dots (6)$$

$$\lambda_1 = \frac{\sum \ln(v_i) (-\sum u_i) \sum u_i \sqrt{-\ln(v_i)} - \sum \sqrt{-\ln(v_i)} \sum u_i \times \sum \sqrt{-\ln(v_i)}}{n \sum_i \sqrt{\ln(v_i)} + \sum_i \sqrt{-\ln(v_i)} \times \sum_i \sqrt{-\ln(v_i)}} \dots (7)$$

Assume that $u_i = \check{E}_{\alpha}^{L}$ and $v_i = \vartheta \check{E}_{\alpha}^{L}$ and for n points u_i, v_i . Let us assume fitting model $Y = e^{-\frac{(u-\lambda_2)^2}{\beta_2}}$.

$$\beta_2 = \frac{n \sum u_i \sqrt{-\ln(v_i)} - \sum \sqrt{-\ln(v_i)} \sum u_i}{n \sum_i \sqrt{\ln(v_i)} + \sum_i \sqrt{-\ln(v_i)} \times \sum_i \sqrt{-\ln(v_i)}} \dots (8)$$

$$\lambda_2 = \frac{\sum \ln(v_i) \times (\sum u_i) + \sum(u_i \times \sqrt{-\ln(v_i)}) \times \sum_i \sqrt{-\ln(v_i)}}{n \sum_i \sqrt{\ln(v_i)} + \sum_i \sqrt{-\ln(v_i)} \times \sum_i \sqrt{-\ln(v_i)}} \dots (9)$$

Therefore, the estimated membership sum of neutrosophic numbers is described below:

$$\vartheta_e(u) = \begin{cases} e^{-\frac{(u-\lambda_2)^2}{\beta_2}}, & x < \lambda_2 \\ 1, & \lambda_1 < x < \lambda_2 \\ e^{-\frac{(u-\lambda_1)^2}{\beta_1}}, & x > \lambda_1 \end{cases} \dots (10)$$

In neutrosophic number it describes α – cut process. Here in this paper we describe the distance between two neutrosophic numbers using the resulting points from the α – cut process.

The distance between two neutrosophic numbers \widehat{M} and \widehat{N} is defined as

$$\widehat{D}_{\hat{r}, \hat{s}}(\widehat{M}, \widehat{N}) = \begin{cases} (1 - \hat{s}) \int_0^1 (\widehat{M}^{-}_{\alpha_i}, \widehat{N}^{-}_{\alpha_i})^{\hat{r}} d\alpha + (\hat{s}) \int_0^1 (\widehat{M}^{+}_{\alpha_i}, \widehat{N}^{-}_{\alpha_i})^{\hat{r}}, \hat{r} < \infty \\ (\hat{s}) \sup(\widehat{M}^{-}_{\alpha_i}, \widehat{N}^{-}_{\alpha_i})^{\hat{r}} + \inf(\widehat{M}^{-}_{\alpha_i}, \widehat{N}^{-}_{\alpha_i}), \hat{r} = \infty \end{cases} \dots (11)$$

Here the parameter \hat{r} is defined as weight, \widehat{M}^{-} and \widehat{M}^{+} components of the neutrosophic numbers. \hat{s} denotes logical properties of $\widehat{D}_{\hat{r}, \hat{s}}$. and if possible then, $\widehat{D}_{\hat{r}, \frac{1}{2}}$ is used.

Consider two neutrosophic numbers \widehat{M} and \widehat{N} , $\widehat{D}_{\hat{r}, \hat{s}}$ is proportional to:

$$\widehat{D}_{\hat{r}, \hat{s}}(\widehat{M}, \widehat{N}) = [(1 - \hat{s}) \sum_{i=1}^n (\widehat{M}^{-}_{\alpha_i}, \widehat{N}^{-}_{\alpha_i})^{\hat{r}} + \hat{s} \sum_{i=1}^n (\widehat{M}^{+}_{\alpha_i}, \widehat{N}^{+}_{\alpha_i})^{\hat{r}}]^{\frac{1}{\hat{r}}} \dots (12)$$

If $\hat{s} = \frac{1}{2}$ and $\hat{r} = 2$, We get this

$$\widehat{D}_{2, \frac{1}{2}}(\widehat{M}, \widehat{N}) = \sqrt{[(1 - \frac{1}{2}) \sum_{i=1}^n (\widehat{M}^{-}_{\alpha_i}, \widehat{N}^{-}_{\alpha_i})^{\hat{r}} + \frac{1}{2} \sum_{i=1}^n (\widehat{M}^{+}_{\alpha_i}, \widehat{N}^{+}_{\alpha_i})^{\hat{r}}]^{\frac{1}{\hat{r}}}} \dots (13)$$

Here we compare two neutrosophic edge weights \widehat{M} and \widehat{N} to $\check{0} = (0, 0 \dots \dots, 0)$ using the α – cut methodology. Finally we conclude weights are positive and equation- (13) is used to calculate $\widehat{D}_{2, \frac{1}{2}}(\widehat{M}, 0)$ and $\widehat{D}_{2, \frac{1}{2}}(\widehat{M}, 0)$. Here $\widehat{M} \leq \widehat{N}$ iff $\widehat{D}_{2, \frac{1}{2}}(\widehat{M}, 0) \leq \widehat{D}_{2, \frac{1}{2}}(\widehat{N}, 0)$.

3. Proposed ant colony optimization algorithm-

In colonies especially large number of ants and some other species of bees are lived together. The large number of insects swarm they resolve their own problem by means of togetherness. When we talk about swarm intelligence the best example is ACO algorithm which shows great characters in colony optimization.

The large number of ants basically used the ACO algorithms to find the shortest path between the nest and source vertex i.e. The food points. The ultimate is to find the best route by utilizing the pheromone in advance.

Suppose $G = (V, E)$ by considering this we have to find the SP between vertex I and vertex J respectively. Any edge which starting from vertex I and end at j is denoted as l_{ij} . In each l_{ij} we have denote an amount of pheromone we given to each. The total process was calculated and study by the amount of pheromone the ants utilized. In every edge pheromone used much concordantly to measure the best path on accountant. Large number of pheromones present leads to a better path in absolute ways. In the beginning all nodes have equal amount of pheromone present i.e., τ_0 .

$\nabla\tau_{ij}$ is the amount of pheromone when the ants passing through it and add the adequate pheromone to each vertex. Starting from vertex I and j at time t if the ant passes then the expression is modified and described in the equation as follows:

$$\tau_{ij}(t) \leftarrow \tau_{ij}(t) + \nabla\tau_{ij} \text{ --- (14)}$$

The ants are searching food in different ways as they built the adequate graph by continuous evaporating of pheromones leads to a decreasing function. It stimulates their forging behaviour as previous so they are now more aggressive towards the food in terms of better paths, in the evaporation. Then the above expression is explained as follows:

$$\tau_{ij}(t) \leftarrow (1 - \rho)\tau_{ij}(t), \rho \in [0,1] \text{ --- (15)}$$

Basically, in the evaporation process due to low evaporation it leads to the high convergence rate so that the ants have less interest in exploring edges. Sometimes when high evaporation occurs it creates a slow in the speed of the convergence rate which leads to a barrier in obtaining suitable graph. For this the initial parameter (ρ) have magnificent values with respect to every edge is called surjective values. It is denoted by η_{ij} . For the best shortest path assuming the surjective values as $\eta_{ij} = 1/d_{ij}$, where d_{ij} is the respective distance from I to j . In the existing colonies ants have different types of properties:

On each step when ants move from source vertex i and move towards vertex j then we used the probability formula with respect to the individual step. We have taken ant k is moving from vertex I and vertex j is the destination point got selected which is explain in the following equation.

$$p_{ij}^k = \begin{cases} \frac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{m \in N_i^k} (\tau_{im})^\alpha (\eta_{im})^\beta}, & j \in N_i^k \\ 0, & j \notin N_i^k \end{cases} \text{ --- (16) Where } N_i^k \text{ denotes the neighborhood of vertex } i.$$

- ❖ The pheromone and the surjective information are denoted by α and β respectively which is positive numbers. In equation (19) assuming the effectiveness of ants if $\alpha=0$ then the vertices got closure to the probabilistic value is higher. Then the ACO algorithm is absolute by randomness. Assuming $\beta=0$ then it leads to the information on about pheromones only. Which in result gives a rapid convergence and stagnation in the algorithm?
 - ❖ Ant continues to move since they have not got a suitable feasible path leads to the end of the condition.
 - ❖ The amount of pheromone τ_{ij} on each edge I_{ij} modified in well manner so that each vertex in the renewal process gives rise to the decrease in the pheromones.
 - ❖ Ants have ability to store the data by their personal memory say M . When they pass and obtained the feasible solution then ants shift alongside the path in opposite direction.
- The shortest path in ACO algorithm is considered by flow chart as follows:

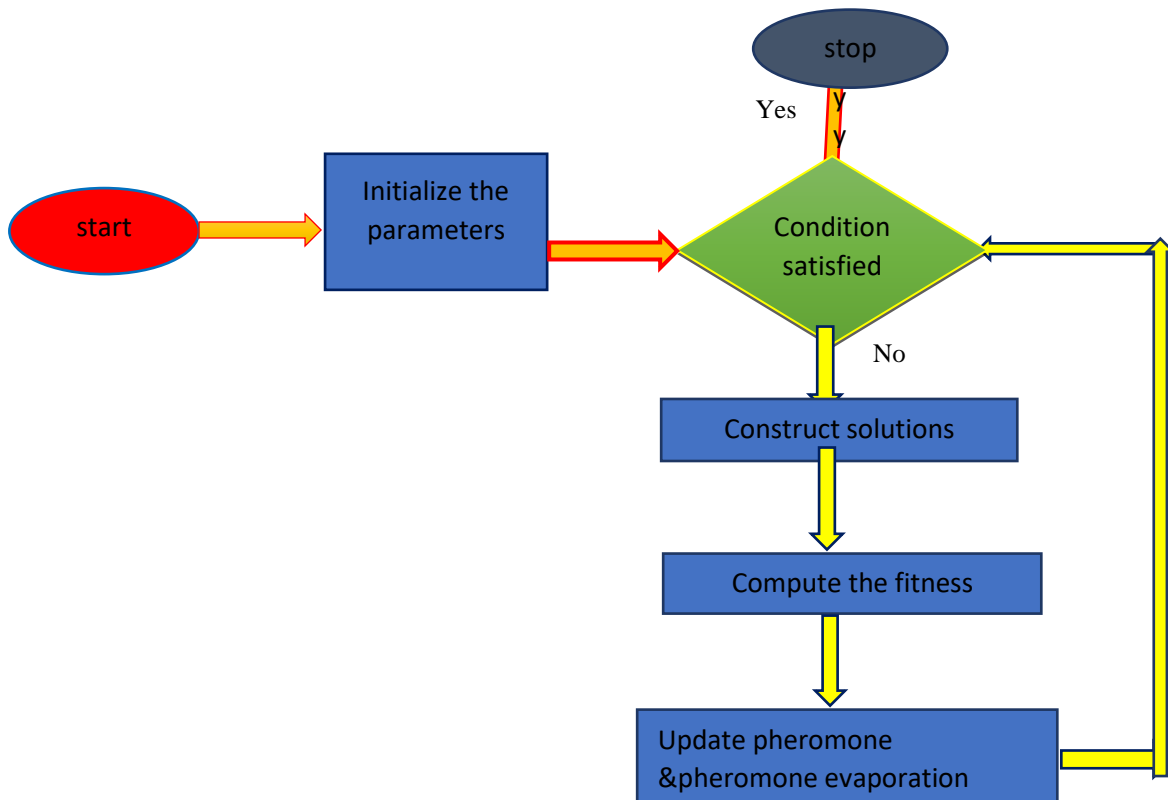


Figure 1: Flowchart of ACO algorithm

4. Implementation:

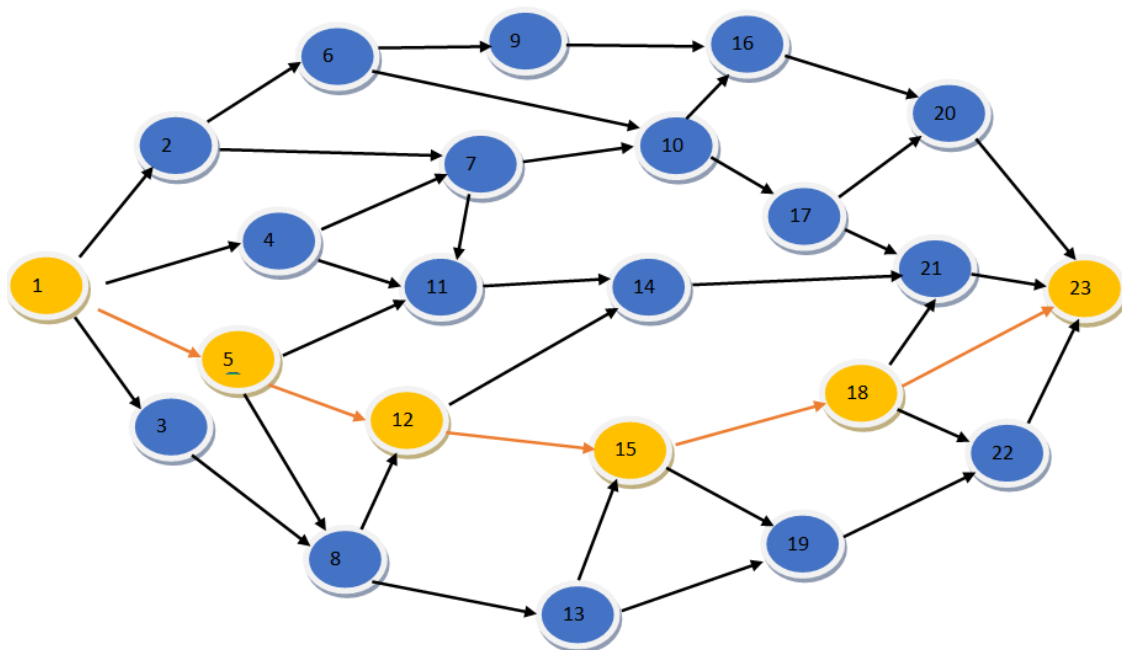


Figure 2: Network

Table 1: Arc weight in single value triangular Neutrosophic number.

Arc	membership value
1-2	(0.1,0.2,0.3)
1-3	(0.1,0.3,0.4)
1-4	(0.3,0.5,0.7)
1-5	(0.3,0.4,0.5)
2-6	(0.2,0.3,0.6)
2-7	(0.1,0.2,0.4)
3-8	(0.4,0.7,0.9)
4-7	(0.3,0.4,0.7)
5-8	(0.1,0.2,0.5)
5-11	(0.5,0.6,0.8)
5-12	(0.5,0.7,0.9)
6-9	(0.6,0.7,0.8)
6-10	(0.5,0.7,0.9)
7-10	(0.2,0.6,0.7)
7-11	(0.2,0.4,0.5)
8-12	(0.2,0.5,0.8)
8-13	(0.6,0.8,0.9)
9-16	(0.4,0.5,0.9)
10-16	(0.2,0.4,0.5)
10-17	(0.6,0.5,0.8)
11-14	(0.5,0.6,0.9)
12-14	(0.1,0.2,0.5)
12-15	(0.3,0.4,0.7)
13-15	(0.5,0.7,0.9)
14-21	(0.1,0.3,0.5)
15-18	(0.4,0.5,0.8)
15-19	(0.4,0.5,0.6)
16-20	(0.5,0.7,0.9)
17-20	(0.3,0.4,0.6)
17-21	(0.4,0.5,0.6)
18-21	(0.4,0.5,0.9)
18-22	(0.3,0.4,0.8)
18-23	(0.3,0.4,0.6)
19-22	(0.4,0.2,0.5)
20-23	(0.2,0.5,0.6)
21-23	(0.5,0.6,0.9)
22-23	(0.6,0.8,0.9)

In this part, the directed and weighted graphs of varying complexity are used to implement and analyze the ACO algorithm. and also used MATLAB Programming , a computer running Windows 11 with an Ryzen Core i5 1.6 GHz processor and 8GB of RAM was used.

The results of experiments using the ACO algorithm from overall 20 iterations are displayed on this graph. Let's also assume that there is a group of 10 ants.

Table 2: Final Results

SI No	Number of Iteration	Shortest Path	converge total Number of iterations	Time span(s)	Total time (s)
			ACO	ACO	ACO
1	20	1-5-12-15-18-23	04	0.93	4.23
2	20	1-5-12-15-18-23	03	0.39	4.22
3	20	1-5-12-15-18-23	03	0.89	4.16
4	20	1-5-12-15-18-23	01	0.29	4.08
5	20	1-5-12-15-18-23	01	0.20	4.09
6	20	1-5-12-15-18-23	06	0.99	4.21
7	20	1-5-12-15-18-23	03	0.54	4.23
8	20	1-5-12-15-18-23	01	0.18	4.15
9	20	1-5-11-5-12-15-18-23	03	0.69	4.24
10	20	1-5-12-15-18-23	01	0.32	4.23
Min	20	-	01	0.25	4.20
Max	20	-	06	1.39	4.21
Avg	20	-	04	0.59	4.21

The data in this table shows how the algorithms were able to identify the graph's shortest path, which is 1-5-12-15-18-23. The ACO method succeeded approximately in the fourth attempt in terms of the total number of rounds required for the process of convergence.

Therefore, in the overall setting of the considering Network, this is the shortest path (SP) from source vertex to destination vertex.

5. Conclusion

In conclusion, the challenge of identifying the shortest paths in a neutrosophic environment is one that must require careful consideration of ambiguous and inaccurate data. It is possible to create algorithms that can quickly find the best routes in these circumstances by utilizing neutrosophic sets and the associated techniques. However, more investigation is required to create more efficient algorithms that can take into account the intricate interactions between neutrosophic information, network structure, and other path selection-influencing variables. Ultimately, we will be better able to develop solutions that are both effective and dependable if we keep improving our comprehension of the distinct difficulties and chances that arise in neutrosophic shortest path problems.

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Compliance with ethical standards

Conflicts of Interest: The authors declare that none of them have any conflicts of interest. This is a combined effort of all the authors and none of them have received any financial grant from any organization for carrying out this work.

Ethical approval

This article is the outcome of exploration involving Graph Theory and Algorithm and as such it does not contain any studies with human participants or animals performed by any of the authors.

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