Bipolar neutrosophic soft continuity mappings

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Abstract

In this article, we have introduced a bipolar neutrosophic soft point and investigated some of the properties with appropriate examples. Further, we have defined bipolar neutrosophic soft continuous mapping through bipolar neutrosophic soft points. Some results have been produced as theorems and examples. Further, we have discussed the relationship between the proposed mapping with the various existing mappings.

Keywords: Neutrosophic set; Bipolar neutrosophic soft set; Continuous mappings; Bipolar neutrosophic soft point; Bipolar neutrosophic soft continuous mapping; Bipolar neutrosophic soft open and closed sets.

1 introduction

Most of the real life problems consist some uncertain information. Fuzzy theory introduced by Zadeh,[8] is a tool for solving uncertainty problems. Fuzzy sets are used in many engineering and scientific problems to deal with uncertainty. Many researchers were proposed different types of fuzzy sets for problems under various fields since the introduction of Fuzzy set theory. In 1999, Atanassov[9] introduced intiutionistic fuzzy set theory which is very useful among other fuzzy set kinds. Molodtsov[4] pointed out some difficulties such as lack of parametric classification, in conventional fuzzy sets and introduced Soft set theory as a tool to deal with uncertainty precisely. Since the introduction of soft sets, many researchers have been working on the development of existing set by using soft set theory.

On the other side, fuzzy sets are not capable to analyze indeterminate information efficiently. Florentin Smarandache[1,2] introduced a new concept namely, Neutrosophy theory which is a generalization of conventional sets used to solve uncertainty problems. Neutrosophic set is the base of neutrosophy to deal with uncertainty problems. Many researchers have been working on neutrosophic set theory and various kinds of neutrosophic sets were proposed.[3,5–7] In 2013, Maji[13] proposed neutrosophic soft set which is a fusion of neutrosophy and soft set. In 2017, Mumtaz Ali et al.[5] have proposed bipolar neutrosophic soft sets and their application in decision making problems. In 2019, Arulpandy et al.[6] proposed a different approach on bipolar neutrosophic soft sets.

Many researchers have proposed topologies of neutrosophic sets of various kinds.[10,11] In 2014, Alkhazaleh et al.[12] proposed the concept of neutrosophic soft mappings. In 2021, Taha Yasin et al.[14] proposed some concepts on neutrosophic soft continuous mappings.

This paper introduce the concept of bipolar neutrosophic soft points and their properties along with bipolar neutrosopihc soft continuity mappings; also some related theorems and examples are also discussed. This paper is organized as follows: Section 2 consists the required preliminary definitions and results for the proposed concept. Section 3 deals with bipolar neutrosophic soft point and their properties with suitable examples. Section 4 deals with bipolar neutrosophic soft continuity mapping through bipolar neutrosophic soft points and related theorems. Section 5 concludes the paper and includes outline of future work.

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2 Preliminaries

Definition 2.1. Let X be a universe set. For every $x \in X$, the components $u(x), v(x)$ and $w(x)$ are truth, indeterminate and false degrees of x. Then the Neutrosophic set (NS) over X be defined as follows

$$N = \{u(x), v(x), w(x) : x \in X\}.$$

Here, $u(x), v(x), w(x)$ ranges in the non-standard interval $]^{-}0, 1^{+}[$ and their sum $^{-}0 \leq u + v + w \leq 3^{+}$. For scientific problems, we prefer standard interval $[0, 1]$ instead of non-standard interval and it is called single-valued neutrosophic set.

Definition 2.2. A soft set is a function which maps a parameter set to the power set of X. It is denoted by (f, E) and is defined by

$$f : E \to P(x),$$

where each member of X is parameterized with the parameter set E by the function f.

Definition 2.3. For the universe set X and positive member values $u^{+}, v^{+}, w^{+} : E \to [0, 1]$, negative member values $u^{-}, v^{-}, w^{-} : E \to [-1, 0]$, A bipolar neutrosophic set (BNS) is defined by

$$B = \Big\{ \langle x, u^{+}(x), v^{+}(x), w^{+}(x), u^{-}(x), v^{-}(x), w^{-}(x) \rangle : x \in X \Big\}.$$

Definition 2.4. A bipolar neutrosophic soft set (BNSS) is the fusion of soft set and bipolar neutrosophic set and is defined as follows.

$$BNSS = (f_A, E) = \{\langle e, f_A(x) \rangle : e \in A \subset E, f_A(x) \in BNS(X)\}.$$

Here $f_A(x) = \Big\{ \Big\langle x, u^{+}_{f_A(e)}(x), v^{+}_{f_A(e)}(x), w^{+}_{f_A(e)}(x), u^{-}_{f_A(e)}(x), v^{-}_{f_A(e)}(x), w^{-}_{f_A(e)}(x) \Big\rangle : x \in X \Big\}$.

Definition 2.5. Let B be a $BNSS$. Then the complement of B is defined as

$$B^{c} = \{ \Big\langle e, w^{+}_{f}(e), 1 - v^{+}_{f}(e), u^{+}_{f}(e), w^{-}_{f}(e), -1 - v^{-}_{f}(e), u^{-}_{f}(e) \Big\rangle \}.$$

Definition 2.6. Let $\varphi_{\mathbb{B}}$ be a null $BNSS$ and is defined as

$$\varphi_{\mathbb{B}} = \{ \langle e_i, \{x_i, 0, 1, 1, 0, -1, -1\} \rangle : x \in X, e \in E \}.$$

Definition 2.7. Let $1_{\mathbb{B}}$ be a complete $BNSS$ and is defined as

$$1_{\mathbb{B}} = \{ \langle e_i, \{x_i, 1, 0, 0, -1, 0, 0\} \rangle : x \in X, e \in E \}.$$

Definition 2.8. Let B_1 and B_2 be two $BNSS$s. Then their union $B_1 \cup B_2$ is defined as

$$B_1 \cup B_2 = \Big\{ \Big\langle e, \cup_i f^{(i)}(e) \Big\rangle \Big\}.$$

Here,

$$\bigcup_i f^{(i)}(e) = \Big\{ \langle x, max\left[u^{+}_{fi}(e)(x)\right], min\left[v^{+}_{fi}(e)(x)\right], min\left[w^{+}_{fi}(e)(x)\right],$$
$$min\left[u^{-}_{fi}(e)(x)\right], max\left[v^{-}_{fi}(e)(x)\right], max\left[w^{-}_{fi}(e)(x)\right] \rangle \Big\}.$$

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Definition 2.9. Let B_1 and B_2 be two $BNSS$s. Then their intersection $B_1 \cap B_2$ is defined as

$$B_1 \cap B_2 = \left\{ \left\langle e, \cap_i f^{(i)}(e) \right\rangle \right\}.$$

Here,

$$\bigcap_i f^{(i)}(e) = \Bigg\{ \langle x, min\left[u^+_{f^i}(e)(x)\right], max\left[v^+_{f^i}(e)(x)\right], max\left[w^+_{f^i}(e)(x)\right],$$
$$max\left[u^-_{f^i}(e)(x)\right], min\left[v^-_{f^i}(e)(x)\right], min\left[w^-_{f^i}(e)(x)\right] \rangle \Bigg\}.$$

Definition 2.10. Let B_1 and B_2 be two $BNSS$s. Then B_1 is called subset of B_2 (i.e. $B_1 \subseteq B_2$) only if the following conditions are hold.
For every $x \in X$ and $e \in E$,

$$B_1 \subseteq B_2 = \Bigg\{ \left[u^+_{B_1}(x) \le u^+_{B_2}(x)\right], \left[v^+_{B_1}(x) \ge v^+_{B_2}(x)\right], \left[w^+_{B_1}(x) \ge w^+_{B_2}(x)\right],$$
$$\left[u^-_{B_1}(x) \ge u^-_{B_2}(x)\right], \left[v^-_{B_1}(x) \le v^-_{B_2}(x)\right], \left[w^-_{B_1}(x) \le w^-_{B_2}(x)\right] \Bigg\}.$$

Definition 2.11. Let $BNSS(X, E)$ be the family of all bipolar neutrosophic soft sets over X. Let $\tau_{\mathbb{B}} \subset BNSS(X, E)$. Then $\tau_{\mathbb{B}}$ is called a bipolar neutrosophic soft topology if the following conditions hold.

1. $0_{\mathbb{B}}$ and $1_{\mathbb{B}}$ are in $\tau_{\mathbb{B}}$,
2. Union of any number of $BNSS$ in $\tau_{\mathbb{B}}$ are also in τ,
3. Intersection of finite number of $BNSS$ in $\tau_{\mathbb{B}}$ are in τ.

Therefore, $(X, \tau_{\mathbb{B}}, E)$ be a bipolar neutrosophic soft topological space and every element of τ is a open set by default.

Definition 2.12. Let BNS be the set of all bipolar neutrosophic sets over the universe set X and every $x \in X$. Then the bipolar neutrosophic member $x_{(a,b,c,r,s,t)}$ is called bipolar neutrosophic point and is defined as follows

$$x_{(a,b,c,r,s,t)}(y) = \begin{cases} (u^+, v^+, w^+, u^-, v^-, w^-) & if \quad y = x \\ (0, 1, 1, 0, -1, -1) & if \quad y \ne x \end{cases}.$$

Here, $0 \le u^+, v^+, w^+ \le 1, -1 \le u^-, v^-, w^- \le 0$ and every BNS is the union of its neutrosophic points.

3 Bipolar neutrosophic soft point and their properties

Definition 3.1. Let $BNSS(X, E)$ be set of all bipolar neutrosophic soft sets over X. Then the term $x^e_{(a,b,c,r,s,t)}$ for all $x \in X, e \in E$ and $0 \le a, b, c \le 1, -1 \le r, s, t \le 0$, is called bipolar neutrosophic soft point and defined as follows

$$x^e_{(a,b,c,r,s,t)}(e')(y) = \begin{cases} (u^+, v^+, w^+, u^-, v^-, w^-) & if \quad e' = e \, and \, y = x \\ (0, 1, 1, 0, -1, -1) & if \quad e' \ne e \, and \, y \ne x \end{cases}.$$

Definition 3.2. Let (f_A, E) be bipolar neutrosophic soft set over X. Suppose $x^e_{(a,b,c,r,s,t)} \in (f_A, E)$ ($\in$ read as 'belongs to') only if $a \le u^+(x), b \ge v^+(x), c \ge w^+(x), r \ge u^-(x), s \le v^-(x) \, and \, t \le w^-(x)$.

Definition 3.3. Let $x^e_{(a,b,c,r,s,t)}$ and $x^{e'}_{(a',b',c',r',s',t')}$ be two bipolar neutrosophic soft points over the same universe X. If $x^e_{(a,b,c,r,s,t)} \cap x^{e'}_{(a',b',c',r',s',t')} = \varphi_B$(empty intersection), then $x^e_{(a,b,c,r,s,t)}$ and $x^{e'}_{(a',b',c',r',s',t')}$ are called distinct points. Further, $x^e_{(a,b,c,r,s,t)}$ and $x^{e'}_{(a',b',c',r',s',t')}$ are distinct points if and only if $x \ne y \, and \, e \ne e'$.

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Definition 3.4. Let $BNSS(X, E)$ and $BNSS(Y, E')$ be the collections of two bipolar neutrosophic soft sets and $m : X \to Y \; and \; n : E \to E'$ be two mappings between corresponding universe sets and parameter sets. Then the mapping $\psi = (m, n) : (X, E) \to (Y, E')$ is defined as follows.
The function value $\psi(f_A)$ is a bipolar neutrosophic soft set in $BNSS(Y, E')$, for any $f_A \in BNSS(X, E)$, $A \subseteq E$ and is obtained by

$$u^{+}_{m(f)(e')}(y) = \begin{cases} \sup\limits_{e \in n^{-1}(e') \cap A, x \in m^{-1}(y)} u^{+}_{f(e)}(x) & if \quad m^{-1}(y) \neq \varphi \\ 0 & otherwise \end{cases}$$

$$v^{+}_{m(f)(e')}(y) = \begin{cases} \inf\limits_{e \in n^{-1}(e') \cap A, x \in m^{-1}(y)} v^{+}_{f(e)}(x) & if \quad m^{-1}(y) \neq \varphi \\ 1 & otherwise \end{cases}$$

$$w^{+}_{m(f)(e')}(y) = \begin{cases} \inf\limits_{e \in n^{-1}(e') \cap A, x \in m^{-1}(y)} w^{+}_{f(e)}(x) & if \quad m^{-1}(y) \neq \varphi \\ 1 & otherwise \end{cases}$$

$$u^{-}_{m(f)(e')}(y) = \begin{cases} \inf\limits_{e \in n^{-1}(e') \cap A, x \in m^{-1}(y)} u^{-}_{f(e)}(x) & if \quad m^{-1}(y) \neq \varphi \\ 0 & otherwise \end{cases}$$

$$v^{-}_{m(f)(e')}(y) = \begin{cases} \sup\limits_{e \in n^{-1}(e') \cap A, x \in m^{-1}(y)} v^{-}_{f(e)}(x) & if \quad m^{-1}(y) \neq \varphi \\ -1 & otherwise \end{cases}$$

$$w^{-}_{m(f)(e')}(y) = \begin{cases} \sup\limits_{e \in n^{-1}(e') \cap A, x \in m^{-1}(y)} w^{-}_{f(e)}(x) & if \quad m^{-1}(y) \neq \varphi \\ -1 & otherwise \end{cases}$$

for every $n(A) \subseteq E', y \in Y$.

Definition 3.5. Let $BNSS(X, E)$ and $BNSS(Y, E')$ be the collections of two bipolar neutrosophic soft sets and $m : X \to Y \; and \; n : E \to E'$ be two mappings between corresponding universe sets and parameter sets. Then the inverse mapping $\psi^{-1} : (Y, E') \to (X, E)$ is defined as follows.
The function value $\psi^{-1}(g_B)$ is a bipolar neutrosophic soft set in $BNSS(X, E)$, for any $g_B \in BNSS(Y, E')$, $B \subseteq E'$ and is obtained by

$$u^{+}_{m^{-1}(f)(e)}(y) = \begin{cases} u^{+}_{g(n(e))}(m(x)) & if \quad n^{-1}(e) \in B \\ 0 & otherwise \end{cases}$$

$$v^{+}_{m^{-1}(f)(e)}(y) = \begin{cases} v^{+}_{g(n(e))}(m(x)) & if \quad n^{-1}(e) \in B \\ 1 & otherwise \end{cases}$$

$$w^{+}_{m^{-1}(f)(e)}(y) = \begin{cases} w^{+}_{g(n(e))}(m(x)) & if \quad n^{-1}(e) \in B \\ 1 & otherwise \end{cases}$$

$$u^{-}_{m^{-1}(f)(e)}(y) = \begin{cases} u^{-}_{g(n(e))}(m(x)) & if \quad n^{-1}(e) \in B \\ 0 & otherwise \end{cases}$$

$$v^{-}_{m^{-1}(f)(e)}(y) = \begin{cases} v^{-}_{g(n(e))}(m(x)) & if \quad n^{-1}(e) \in B \\ -1 & otherwise \end{cases}$$

$$w^{-}_{m^{-1}(f)(e)}(y) = \begin{cases} w^{-}_{g(n(e))}(m(x)) & if \quad n^{-1}(e) \in B \\ -1 & otherwise \end{cases}$$

for every $e \in n^{-1}(B) \subseteq E$ and $x \in X, \psi^{-1}(g_B)$ is called a bipolar neutrosopihc soft inverse image of BNSS g_B .

Definition 3.6. Let $BNSS(X, E)$ and $BNSS(Y, E')$ be the collections of two bipolar neutrosophic softs and $f_A \in BNSS(X, E), g_B \in BNSS(Y, E')$. Then $\psi = (m, n) : BNSS(X, E) \to BNSS(Y, E')$ be a bipolar neutrosophic soft mapping such that $m : X \to Y \; and \; n : E \to E'$.

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1. The bipolar neutrosopihc soft mapping $\psi = (m, n)$ is called a bipolar neutrosophic soft injective mapping if for every $(x_1)^{(e_1)}_{(a_1,b_1,c_1,r_1,s_1,t_1)}, (x_2)^(e_2)_{(a_2,b_2,c_2,r_2,s_2,t_2)} \in f_A$, $(x_1)^(e_1)_{(a_1,b_1,c_1,r_1,s_1,t_1)} \neq (x_2)^{(e_2)}_{(a_2,b_2,c_2,r_2,s_2,t_2)}$ implies $\psi((x_1)^{(e_1)}_{(a_1,b_1,c_1,r_1,s_1,t_1)}) = (m(x_1), n(e_1)) \neq \psi((x_2)^{(e_2)}_{(a_2,b_2,c_2,r_2,s_2,t_2)}) = (m(x_2), n(e_2))$.

2. The bipolar neutrosophic soft mapping $\psi = (m, n)$ is called a bipolar neutrosophic soft surjective mapping if there exist a bipolar neutrosophic soft point $x^e_{(a,b,c,r,s,t)} \in f_A$, such that $\psi(x^e_{(a,b,c,r,s,t)}) = y^{e'}_{(a',b',c',r',s',t')}$ for every $y^{e'}_{(a',b',c',r',s',t')} \in g_B$.

3. The bipolar neutrosophic soft mapping $\psi = (m, n)$ is called a bipolar neutrosophic soft bijective mapping if ψ is both injective and surjective.

4. The bipolar neutrosophic soft mapping $\psi = (m, n)$ is called a bipolar neutrosophic soft constant mapping if $\psi(x^e_{(a,b,c,r,s,t)}) = y^{e'}_{(a',b',c',r',s',t')}$ is provided for every $x^e_{(a,b,c,r,s,t)} \in f_A$, there exist $y^{e'}_{(a',b',c',r',s',t')} \in g_B$.

Definition 3.7. Let $BNSS(X, E)$ be the family of all bipolar neutrosohic soft set over X and $\tau_{\mathbb{B}}$ be the bipolar neutrosophic soft topology on X.

1. If $\tau_{\mathbb{B}} = \{0_{\mathbb{B}}, 1_{\mathbb{B}}\}$, then $\tau_{\mathbb{B}}$ is called a bipolar neutrosophic soft indiscrete topology and $(X, \tau_{\mathbb{B}}, E)$ is called bipolar neutrosophic soft indiscrete topological space over X.

2. If $\tau_{\mathbb{B}} = BNSS(X, E)$, then $\tau_{\mathbb{B}}$ is called a bipolar neutrosophic soft discrete topology and $(X, \tau_{\mathbb{B}}, E)$ is called bipolar neutrosophic soft discrete topological space over X.

Definition 3.8. Let $(X, \tau_{\mathbb{B}}, E)$ be a bipolar neutrosophic soft topological space over X and f_E be a bipolar neutrosophic soft set over X. Then f_E is called bipolar neutrosophic soft closed set if and only if its complement is a bipolar neutrosophic soft open set.

Definition 3.9. Let $(X, \tau_{\mathbb{B}}, E)$ be a bipolar neutrosophic soft topological space over X. A bipolar neutrosophic soft set $f_E \in (X, \tau_{\mathbb{B}}, E)$ is called a bipolar neutrosophic soft neighborhood of the bipolar neutrosophic soft point $x^e_{(a,b,c,r,s,t)} \in f_E$, if there exists a bipolar neutrosophic soft open set g_E such that $x^e_{(a,b,c,r,s,t)} \in g_E \subseteq f_E$.

Definition 3.10. Let $(X, \tau_{\mathbb{B}}, E)$ be a bipolar neutrosophic soft topological space over X and $f_E \in BNSS(X, E)$ be a bipolar neutrosophic soft set over X.

1. The bipolar neutrosophic soft interior of f_E, denoted by f^o_E, is defined as the union of all bipolar neutrosophic soft open subsets of f_E. It is clear that f^o_E is the biggest bipolar neutrosophic soft open set that is contained in f_E.

2. The bipolar neutrosophic soft closure of f_E, denoted by $\overline{f_E}$, is defined as the intersection of all bipolar neutrosophic soft open subsets of f_E. It is clear that $\overline{f_E}$ is the smallest bipolar neutrosophic soft closed set that is containing f_E.

4 Bipolar neutrosophic soft continuity

Definition 4.1. Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi = (m, n) : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. For every bipolar neutrosophic soft neighborhood $g_{E'}$ of $\psi(x^e_{(a,b,c,r,s,t)})$, if there exist a bipolar neutrosophic soft neighborhood f_E of bipolar neutrosophic soft point $x^e_{(a,b,c,r,s,t)} \in BNSS(X, E)$ such that $\psi(f_E) \subseteq g_{E'}$, then ψ is called bipolar neutrosophic soft continuous mapping at $x^e_{(a,b,c,r,s,t)}$.
If ψ is a bipolar neutrosophic soft continuous mapping for all $x^e_{(a,b,c,r,s,t)} \in BNSS(X, E)$, then ψ is called a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$.

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Theorem 4.2. *Let* $(X, \tau^1_{\mathbb{B}}, E)$ *and* $(Y, \tau^2_{\mathbb{B}}, E')$ *be two bipolar neutrosophic soft topological spaces and* $\psi : (X, E) \to (Y, E')$ *be a bipolar neutrosophic soft mapping. Then* ψ *is a bipolar neutrosophic soft continuous mapping on* $(X, \tau^1_{\mathbb{B}}, E)$ *if and only if* $\psi^{-1}(g'_E)$ *is a bipolar neutrosophic soft open set for every bipolar neutrosophic soft open set* $g_E \in \tau^2_{\mathbb{B}}$.

Proof. Suppose that ψ is a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$ and $g'_E \in \tau^2_{\mathbb{B}}$. To prove $\psi^{-1}(g'_E) \in \tau^1_{\mathbb{B}}$. For every $x^e_{(a,b,c,r,s,t)} \in \psi^{-1}(g'_E)$, since $\psi(x^e_{(a,b,c,r,s,t)}) \in g'_E$ and ψ is bipolar neutrosophic soft continuous mapping, then there exists a neighborhood f_E of $x^e_{(a,b,c,r,s,t)}$ such that $\psi(f_E) \subseteq g'_E$. Hence $x^e_{(a,b,c,r,s,t)} \in f_E \subseteq \psi^{-1}(g'_E)$. i.e. $\psi^{-1}(g'_E)$ is a bipolar neutrosophic soft open set.

On the other hand, let $x^e_{(a,b,c,r,s,t)}$ be a bipolar neutrosophic soft point of $BNSS(X, E)$ and $\psi(x^e_{(a,b,c,r,s,t)}) \in g'_E$ be a bipolar neutrosophic soft open set in $BNSS(Y, E')$. Then $x^e_{(a,b,c,r,s,t)} \in \psi^{-1}(g'_E)$ is a bipolar neutrosophic soft open set in (X, E) and $\psi(\psi^{-1}(g'_E) \subseteq g'_E$. Hence ψ is a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$. □

Theorem 4.3. *Let* $(X, \tau^1_{\mathbb{B}}, E)$ *and* $(Y, \tau^2_{\mathbb{B}}, E')$ *be two bipolar neutrosophic soft topological spaces and* $\psi : (X, E) \to (Y, E')$ *be a bipolar neutrosophic soft mapping. Then* ψ *is a bipolar neutrosophic soft continuous mapping on* $(X, \tau^1_{\mathbb{B}}, E)$ *if and only if* $\psi^{-1}(g'_E)$ *is a bipolar neutrosophic soft closed set for every bipolar neutrosophic soft closed set* $g_E \in \tau^2_{\mathbb{B}}$.

Proof. Let $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft continuous mapping and let g'_E be a bipolar neutrosophic soft closed set in Y. Since, $\psi^{-1}((g'_E)^c) = (\psi^{-1}(g'_E))^c$ and $(g'_E)^c$ is bipolar neutrosophic soft open set, $(\psi^{-1}(g'_E))^c$ is bipolar neutrosophic soft open set in X. Hence $\psi^{-1}(g'_E)$ is a bipolar neutrosophic soft closed set in X.

On the other hand, let $\psi^{-1}(g'_E)$ be bipolar neutrosophic soft closed set in X whenever (g'_E) is a bipolar neutrosophic soft closed set in Y. For any $h'_E \in Y$, $\psi^{-1}((h'_E)^c) = (\psi^{-1}(h'_E))^c$. From the hypothesis, $\psi^{-1}((h'_E)^c))$ is a bipolar neutrosophic soft closed set in X. So $\psi^{-1}(h'_E)$ is a bipolar neutrosophic soft open set in X. Hence, ψ is a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$. □

Example 4.4. Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping.

1. If $\tau^1_{\mathbb{B}}$ is the bipolar neutrosophic soft discrete topology on X, then ψ is a bipolar neutrosophic soft continuous mapping.
2. It $\tau^2_{\mathbb{B}}$ is the bipolar neutrosophic soft indiscrete topology on Y, then ψ is a bipolar neutrosophic soft continuous mapping.
3. Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}, E = \{e_1, e_2\}, E' = \{e'_1, e'_2\}$ and $\tau^1_{\mathbb{B}}, \tau^2_{\mathbb{B}}$ be two bipolar neutrosophic soft topologies, defined as follows:

$$\tau^1_{\mathbb{B}} = \left\{0_{X(\mathbb{B})}, 1_{X(\mathbb{B})}, f^1_E, f^2_E, f^3_E\right\}$$
$$\tau^2_{\mathbb{B}} = \left\{0_{Y(\mathbb{B})}, 1_{Y(\mathbb{B})}, g^1_{E'}, g^2_{E'}, g^3_{E'}\right\}$$

where

$$f^1_E = \left\{ \begin{array}{c} (e_1, \langle x_1, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle), \\ (e_2, \langle x_1, 0, 1, 1, 0, -1, -1\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle) \end{array} \right\}$$

$$f^2_E = \left\{ \begin{array}{c} (e_1, \langle x_1, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle x_2, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle), \\ (e_2, \langle x_1, 0, 1, 1, 0, -1, -1\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle) \end{array} \right\}$$

$$f^3_E = \left\{ \begin{array}{c} (e_1, \langle x_1, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle x_2, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle), \\ (e_2, \langle x_1, 0, 1, 1, 0, -1, -1\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle) \end{array} \right\}$$

$$g^1_{E'} = \left\{ \begin{array}{c} (e'_1, \langle y_1, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle y_2, 0, 1, 1, 0, -1, -1\rangle, \langle y_3, 0, 1, 1, 0, -1, -1\rangle), \\ (e'_2, \langle y_1, 0, 1, 1, 0, -1, -1\rangle, \langle y_2, 0, 1, 1, 0, -1, -1\rangle, \langle y_3, 0, 1, 1, 0, -1, -1\rangle) \end{array} \right\}$$

Received: Month Day, Year Accepted: Month Day, Year

$$g^2_{E'} = \left\{ \begin{array}{c} (e'_1, \langle y_1, 0, 1, 1, 0, -1, -1\rangle, \langle y_2, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle y_3, 0, 1, 1, 0, -1, -1\rangle), \\ (e'_2, \langle y_1, 0, 1, 1, 0, -1, -1\rangle, \langle y_2, 0, 1, 1, 0, -1, -1\rangle, \langle y_3, 0, 1, 1, 0, -1, -1\rangle) \end{array} \right\}$$

$$g^3_{E'} = \left\{ \begin{array}{c} (e'_1, \langle y_1, 0, 1, 1, 0, -1, -1\rangle, \langle y_2, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle y_3, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle), \\ (e'_2, \langle y_1, 0, 1, 1, 0, -1, -1\rangle, \langle y_2, 0, 1, 1, 0, -1, -1\rangle, \langle y_3, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle) \end{array} \right\}$$

The bipolar neutrosophic soft points of the above sets are given below.

$$f^1_E = \left\{ x^{e_1}_{1(0.5,0.7,0.4,-0.4,-0.4,-0.2)} \right\}$$
$$f^2_E = \left\{ x^{e_1}_{1(0.5,0.7,0.4,-0.4,-0.4,-0.2)}, x^{e_1}_{2(0.4,0.5,0.2,-0.6,-0.4,-0.8)} \right\}$$
$$f^3_E = \left\{ x^{e_1}_{1(0.5,0.7,0.4,-0.4,-0.4,-0.2)}, x^{e_1}_{2(0.4,0.5,0.2,-0.6,-0.4,-0.8)}, x^{e_2}_{3(0.4,0.5,0.2,-0.6,-0.4,-0.8)} \right\}$$
$$g^1_{E'} = \left\{ y^{e'_1}_{1(0.5,0.7,0.4,-0.4,-0.4,-0.2)} \right\}$$
$$g^2_{E'} = \left\{ y^{e'_1}_{2(0.5,0.7,0.4,-0.4,-0.4,-0.2)} \right\}$$
$$g^3_{E} = \left\{ y^{e'_1}_{2(0.5,0.7,0.4,-0.4,-0.4,-0.2)}, y^{e'_1}_{3(0.4,0.5,0.2,-0.6,-0.4,-0.8)}, y^{e'_2}_{3(0.4,0.5,0.2,-0.6,-0.4,-0.8)} \right\}.$$

The mapping $\psi = (m, n) : (X, E) \to (Y, E')$ defined as

$$m(x_1) = y_2 \quad n(e_1) = e'_1$$
$$m(x_2) = y_3 \quad n(e_2) = e'_2$$
$$m(x_3) = y_3$$

. Then $\psi = (m, n)$ is bipolar neutrosophic soft continuous mapping at $x^{e_1}_{1(0.5,0.7,0.4,-0.4,-0.4,-0.2)}$.

4. The bipolar neutrosophic soft open sets in $\tau^2_{\mathbb{B}}$ are $g^1_{E'}, g^2_{E'}, g^3_{E'}, 0_{Y(\mathbb{B})}, 1_{Y(\mathbb{B})}$. So the inverse image of these open sets are as follows.

$$\psi^{-1}(g^1_{E'}) = 0_{X(\mathbb{B})} \in \tau^1_{\mathbb{B}}$$

$$\psi^{-1}(g^2_{E'}) = \left\{ \begin{array}{c} (e_1, \langle x_1, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle), \\ (e_2, \langle x_1, 0, 1, 1, 0, -1, -1\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0, 1, 1, 0, -1, -1\rangle) \end{array} \right\} = f^1_E \in \tau^1_{\mathbb{B}}$$

$$\psi^{-1}(g^3_{E'}) = \left\{ \begin{array}{c} (e_1, \langle x_1, 0.5, 0.7, 0.4, -0.4, -0.4, -0.2\rangle, \langle x_2, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle, \\ \langle x_3, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle), \\ (e_2, \langle x_1, 0, 1, 1, 0, -1, -1\rangle, \langle x_2, 0, 1, 1, 0, -1, -1\rangle, \langle x_3, 0.4, 0.5, 0.2, -0.6, -0.4, -0.8\rangle) \end{array} \right\} \notin \tau^1_{\mathbb{B}}$$

Hence ψ is not bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$.

Theorem 4.5. *Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. Then ψ is a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$ if and only if $\psi^{-1}((g'_E)^o) \subseteq \psi^{-1}((g'_E))^o$ for each $g_B \in (Y, E')$.*

Proof. Let ψ be a bipolar neutrosophic soft continuous mapping and $g_B \in (Y, E')$. Then $\psi^{-1}((g_B)^o) \in \tau^1_{\mathbb{B}}$ and $(g_B)^o \subseteq g_B$, so that $\psi^{-1}((g_B)^o) \subseteq \psi^{-1}(g_B)$.
Since $(\psi^{-1}((g_B))^o$ is the largest bipolar neutrosophic soft open set contained in $\psi^{-1}(g_B)$, $\psi^{-1}((g'_B)^o) \subseteq (\psi^{-1}((g_B)))^o$.

On the other hand, let $\psi^{-1}((g_B)^o) \subseteq (\psi^{-1}((g_B)))^o$, for every $g_B \in (Y, E')$.
If $g_B \in \tau^2_{\mathbb{B}}$, then
$\psi^{-1}(g_B) = \psi^{-1}((g_B)^o) \subseteq (\psi^{-1}((g_B)))^o \subseteq \psi^{-1}(g_B)$. Therefore, $\psi^{-1}(g_B) \in \tau^1_{\mathbb{B}}$. Hence ψ is a bipolar neutrosophic soft continuous mapping. □

Theorem 4.6. *Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. Then ψ is a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$ if and only if $\psi(\overline{f_E}) \subseteq \overline{\psi(f_E)}$ for $f_E \in (X, E)$.*

Received: Month Day, Year Accepted: Month Day, Year

Proof. Let ψ be bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$ and $f_E \in (X, E)$. Since $\overline{\psi(f_E)}$ is a bipolar neutrosophic soft closed set in Y, $\psi^{-1}(\overline{\psi(f_E)})$ is a bipolar neutrosophic soft closed set in X.
Then $\overline{\psi^{-1}(\overline{\psi(f_E)})} = \psi^{-1}(\overline{\psi(f_E)})$ and $\psi(f_E) \subseteq \overline{\psi(f_E)}$.
Thus $f_E \subseteq \psi^{-1}(\psi(f_E)) \subseteq \psi^{-1}(\overline{\psi(f_E)})$.
Also $f_E \subseteq \overline{\psi^{-1}(\psi(f_E))} = \psi^{-1}(\overline{\psi(f_E)})$. Hence $\psi(\overline{f_E}) \subseteq \overline{\psi(f_E)}$.

On the other hand, let $\psi(\overline{f_E}) \subseteq \overline{\psi(f_E)}$ for every $f_E \in (X, E)$. Let g'_E be any bipolar neutrosophic soft closed set in Y. Then $\overline{g_{E'}} = g_{E'}$.
From the hypothesis, $\psi(\overline{\psi^{-1}(g_{E'})} \subseteq \overline{\psi(\psi^{-1}(g_{E'})} \subseteq \overline{g_{E'}} = g_{E'}$ is obtained.
Hence $\overline{\psi^{-1}(g_{E'})} \subseteq \psi^{-1}(g_{E'})$ and $\psi^{-1}(g_{E'}) \subseteq \overline{\psi^{-1}(g_{E'})}$.
So $\overline{\psi^{-1}(g_{E'})} = \psi^{-1}(g_{E'})$. This means that $\psi^{-1}(g_{E'})$ is a bipolar neutrosophic soft closed set in X. Hence ψ is a bipolar neutrosophic soft continuous mapping on $(X, \tau^1_{\mathbb{B}}, E)$. □

Definition 4.7. Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. Then,

1. A bipolar neutrosophic soft mapping ψ is called a bipolar neutrosophic soft open mapping if $\psi(f_E)$ is a bipolar neutrosophic soft open set in (Y, E') for each bipolar neutrosophic soft open set f_E of $BNSS(X, E)$.
2. A bipolar neutrosophic soft mapping ψ is called a bipolar neutrosophic soft closed mapping if $\psi(g, E')$ is a bipolar neutrosophic soft closed set in (Y, E') for each bipolar neutrosophic soft closed set f_E of $BNSS(X, E)$.

Theorem 4.8. *Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. Then*

1. *ψ is a bipolar neutrosophic soft open mapping if and only if $\psi((f_E)^o) \subseteq (\psi(f_E))^o$ for each bipolar neutrosophic soft set f_E of (X, E).*
2. *ψ is a bipolar neutrosophic soft closed mapping if and only if $\overline{\psi(f_E)} \subseteq \psi(\overline{f_E})$ for each bipolar neutrosophic soft set f_E of (X, E).*

Proof. 1. Let ψ be a bipolar neutrosophic soft open mapping and $(f_E)^o \in (X, E)$. Then $(f_E)^o$ is a bipolar neutrosophic open mapping $\psi((f_E)^o) \subseteq f_E$. Since ψ is a neutrosophic open mapping $\psi((f_E)^o)$ is a bipolar neutrosophic open set in (Y, E') and $\psi((f_E)^o) \subseteq \psi(f_E)$. Thus $\psi((f_E)^o) \subseteq (\psi(f_E))^o$.

On the other hand, let $\psi((f_E)^o) \subseteq (\psi(f_E))^o$ for any bipolar neutrosophic soft open set $f_E \in (X, E)$. Since $f_E = (f_E)^o$, we have $\psi(f_E) = \psi((f_E)^o) \subseteq (\psi(f_E))^o \subseteq \psi(f_E)$. This gives $\psi(f_E) = (\psi(f_E))^o$. Hence ψ is a bipolar neutrosophic soft open mapping.

2. Let ψ be a bipolar neutrosophic soft open mapping and $(f_E)^o \in (X, E)$. Since ψ is a bipolar neutrosophic soft closed mapping, $\psi(\overline{f_E})$ is a bipolar neutrosophic soft closed set in (Y, E') and $\psi(f_E) \subseteq \psi(\overline{f_E})$. Hence $\overline{\psi(f_E)} \subseteq \psi(\overline{f_E})$.

On the other hand, let f_E is a bipolar neutrosophic soft closed set in (x, E). Then $f_E = \overline{f_E}$. From the given condition, $\overline{\psi(f_E)} \subseteq \psi(\overline{f_E}) = \psi(f_E) \subseteq \overline{\psi(f_E)}$. So $\overline{\psi(f_E)} = \psi(f_E)$. Hence ψ is a bipolar neutrosophic soft closed mapping. □

Proposition 4.9. *A bipolar neutrosophic soft mapping ψ is bipolar neutrosophic soft continuous and bipolar neutrosophic soft closed mapping if and only if $\overline{\psi(f_E)} = \psi(\overline{f_E})$.*

Definition 4.10. Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. Then ψ is called a bipolar neutrosophic soft homeomorphism if,

1. ψ is a bipolar neutrosophic soft bijection.

Received: Month Day, Year Accepted: Month Day, Year

2. ψ is a bipolar neutrosophic soft continuous mapping.
3. ψ^{-1} is a bipolar neutrosophic soft continuous mapping.

Theorem 4.11. *Let $(X, \tau^1_{\mathbb{B}}, E)$ and $(Y, \tau^2_{\mathbb{B}}, E')$ be two bipolar neutrosophic soft topological spaces and $\psi : (X, E) \to (Y, E')$ be a bipolar neutrosophic soft mapping. Then*

1. *ψ is a bipolar neutrosophic soft homeomorphism*
2. *ψ is a bipolar neutrosophic soft continuous and bipolar neutrosophic soft closed mapping*
3. *ψ is a bipolar neutrosophic soft continuous and bipolar neutrosophic soft open mapping.*

5 Conclusion

We have introduced bipolar neutrosophic soft point and bipolar neutrosophic soft continuity mappings and investigated through theorems with appropriate examples. Further, the connection between proposed mapping with other continuity mappings were discussed. The preservation of topological structures in continuity mapping, is explained via theorems.

In future, we will use bipolar neutrosophic soft points in various topological mappings to prove the invariability. Also, the proposed method will be used in decision making models. Further, we will try to incorporate the proposed bipolar neutrosophic soft point with some of the image processing techniques via bipolar neutrosophic image domain.

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