



A Study on various Pentapartitioned Neutrosophic generalized closed sets

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Abstract

The purpose of this paper is to introduce the concept of various Pentapartitioned neutrosophic generalized closed sets such as PNg-closed set, PN_{ω} -closed set, PNgb-closed set in Pentapartitioned Neutrosophic Topological spaces. We also study some of their properties with counter examples.

Keywords: Pentapartitioned neutrosophic set; PNg-closed set; PN_{ω} -closed set; PNgb-closed set.

1. Introduction

Florentin Smarandache [12] in 1999, introduced the independent concept - Neutrality, the degree of indeterminacy. He also defined the neutrosophic set on three components (t, f, i) = (truth, falsehood, indeterminacy). Wang et al. [4] defined Single Valued Neutrosophic Set (SVNS) in 2010. In 2020, Rama Mallick[15] and Surpati Pramanik introduced Pentapartitioned Neutrosophic Set. In 2021, Suman Das[17] and Binod Chandra Tripathy introduced various sets namely pentapartitioned neutrosophic pre-open, semi-open, α -open and b-open sets and formulated basic results on pentapartitioned neutrosophic topological spaces.

The purpose of this paper is to introduce various Pentapartitioned neutrosophic generalized closed sets such as PNg-closed set, PN_{ω} -closed set, PNgb-closed set in Pentapartitioned neutrosophic topological spaces and to obtain its basic properties with counter examples.

2 Preliminaries

Definition 2.1. [6] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ where $\mu_A(x)$, $\sigma_A(x)$, $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A .

A Neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ can be identified as an ordered triple $(\mu_A(x), \sigma_A(x), \nu_A(x))$ in $]-0, 1+[$ on X .

Definition 2.2. [15] Let X be a universe. A Pentapartitioned neutrosophic set A on X is an object of the form $A = \{ \langle x, T_A, C_A, I_A, U_A, F_A \rangle : x \in X \}$ where $0 \leq T_A + C_A + I_A + U_A + F_A \leq 5$.

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $I_A(x)$ is an unknown membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

For simplicity, we denote Pentapartitioned neutrosophic set as PN set.

Definition 2.3. [15] Let X be a non-empty set, for any two pentapartitioned neutrosophic sets $A = \langle X, T_A, C_A, I_A, U_A, F_A \rangle$ and $B = \langle X, T_B, C_B, I_B, U_B, F_B \rangle$ are two PN sets. We have

- (i) $A \subseteq B$ iff $T_A \leq T_B, C_A \leq C_B, I_A \geq I_B, U_A \geq U_B$ and $F_A \geq F_B$
- (ii) $A \cup B = \langle X, \max(T_A, T_B), \max(C_A, C_B), \min(I_A, I_B), \min(U_A, U_B), \min(F_A, F_B) \rangle$
- (iii) $A \cap B = \langle X, \min(T_A, T_B), \min(C_A, C_B), \max(I_A, I_B), \max(U_A, U_B), \max(F_A, F_B) \rangle$

Definition 2.4. [6] The complement of a PN set (H, A) on X is defined as $H^c(x) = \langle x, F_A, U_A, 1 - I_A, C_A, T_A \rangle : x \in X$ and is denoted by $(H, A)^c$.

Definition 2.5. [15] A PN set (F, A) over the universe X is said to be a null PN set 0_X with respect to the parameter A if $0_X = \langle x, 0, 0, 1, 1, 1 \rangle \forall x \in X$.

Definition 2.6. [15] A PN set (F, A) over the universe X is said to be a absolute PN set 1_X with respect to the parameter A if $1_X = \langle x, 1, 1, 0, 0, 0 \rangle \forall x \in X$.

Definition 2.7. [17]. Let X be a fixed set. Then, a set \mathfrak{T} of PN sets over X is called a Pentapartitioned Neutrosophic Topological Space (in short PNTS) on X , if the following three conditions hold:

- (i) $0_X, 1_X \in \mathfrak{T}$;
- (ii) $Y_1, Y_2 \in \mathfrak{T} \Rightarrow Y_1 \cap Y_2 \in \mathfrak{T}$
- (iii) $\{Y_i : i \in \Delta\} \subseteq \mathfrak{T} \Rightarrow \cup Y_i \in \mathfrak{T}$

Then the pair (X, \mathfrak{T}) is called a Pentapartitioned Neutrosophic Topological Space (in short PNTS). Each element of \mathfrak{T} is called a Pentapartitioned Neutrosophic open sets (in short PNOS). If $Y \in \mathfrak{T}$, then Y^c is said to be Pentapartitioned Neutrosophic closed sets (in short PNCS).

Definition 2.8. [15] Let A be a PNS in PNTS X . Then

- (i) $PNint(A) = \cup \{G : G \text{ is an PNOS in } X \text{ and } G \subseteq A\}$ is called a Pentapartitioned neutrosophic interior of A .
- (ii) $PNcl(A) = \cap \{K : K \text{ is a PNCS in } X \text{ and } A \subseteq K\}$ is called a Pentapartitioned neutrosophic closure of A .

Definition 2.9. [17] A PNS A of a PNTS X is said to be

- (1) a Pentapartitioned neutrosophic semi-open set (PNSOS)[17] if and only if $A \subseteq PNcl(PNint(A))$
- (2) a Pentapartitioned neutrosophic α -open set (PN α OS)[17] if and only if $A \subseteq PNint(PNcl(PNcl(A)))$
- (3) a Pentapartitioned neutrosophic b-open set (PNbOS)[17] if and only if $A \subseteq PNint(PNcl(A)) \cup PNcl(PNint(A))$.

The complements of the above sets are called Pentapartitioned neutrosophic semi-closed, α -closed and b-closed sets respectively and is denoted by PNSCS, PN α CS and PNbCS.

Remark 2.10. Throughout this paper, the following notations are used for simplicity. PNTS means Pentapartitioned Neutrosophic topological spaces and PNS means Pentapartitioned Neutrosophic set

3 Pentapartitioned Neutrosophic generalized - closed set

Definition 3.1: A subset A of a pentapartitioned neutrosophic topological space (X, τ) is called a pentapartitioned neutrosophic generalised-closed (PNg-closed) set if $\text{PNcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a pentapartitioned neutrosophic open set in (X, τ) .

The complement of a pentapartitioned neutrosophic generalised-closed set is called a pentapartitioned neutrosophic generalised-open set.

The family of pentapartitioned neutrosophic generalised-closed sets are denoted by $\text{PNgC}(X)$.

Example 3.2. Let $X = \{p, q, r\}$ with $\tau_{\text{PN}} = \{0_{\text{PN}}, A, B, C, 1_{\text{PN}}\}$ where

$A = \langle (p, 0.9, 0.5, 0.3, 0.6, 0.7), (q, 0.5, 0.6, 0.4, 0.5, 0.1), (r, 0.7, 0.4, 0.6, 0.7, 0.5) \rangle$

$B = \langle (p, 0.4, 0.4, 0.7, 0.7, 0.8), (q, 0.5, 0.4, 0.6, 0.8, 0.3), (r, 0.6, 0.4, 0.7, 0.8, 0.9) \rangle$

$C = \langle (p, 0.3, 0.4, 0.8, 0.7, 1.0), (q, 0.4, 0.3, 0.8, 0.9, 0.4), (r, 0.5, 0.3, 0.8, 0.8, 1.0) \rangle$

Consider the pentapartitioned neutrosophic sets

$D = \langle (p, 0.8, 0.4, 0.4, 0.7, 0.8), (q, 0.4, 0.5, 0.5, 0.5, 0.2), (r, 0.5, 0.3, 0.7, 0.7, 0.6) \rangle$

$E = \langle (p, 0.5, 0.5, 0.5, 0.5, 0.5), (q, 0.5, 0.5, 0.5, 0.5, 0.5), (r, 0.5, 0.5, 0.5, 0.5, 0.5) \rangle$

Then $\text{PNgC}(X)$ are A^c, B^c, C^c, D^c and E .

Definition 3.3: A subset A of a pentapartitioned neutrosophic topological space (X, τ) is called a pentapartitioned neutrosophic ω -closed (PN ω -closed) set if $\text{PNcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a pentapartitioned neutrosophic semi-open set in (X, τ) .

The complement of a pentapartitioned neutrosophic ω -closed set is called a pentapartitioned neutrosophic ω -open set.

The family of pentapartitioned neutrosophic ω -closed sets are denoted by $\text{PN}\omega\text{C}(X)$.

Example 3.4. Let $X = \{p\}$ with $\tau_{\text{PN}} = \{0_{\text{PN}}, A, B, C, D, E, F, 1_{\text{PN}}\}$ where

$A = \langle (p, 0.1, 0.6, 0.5, 0.2, 0.6) \rangle$

$B = \langle (p, 0.4, 0.6, 0.1, 0.2, 0.4) \rangle$

$C = \langle (p, 0.4, 0.1, 0.5, 0.6, 0.4) \rangle$

$D = \langle (p, 0.1, 0.4, 0.7, 0.5, 0.6) \rangle$

$E = \langle (p, 0.4, 0.6, 0.5, 0.2, 0.4) \rangle$

$F = \langle (p, 0.4, 0.4, 0.5, 0.5, 0.4) \rangle$

Then $\text{PN}\omega\text{C}(X)$ are A^c, B^c, C^c, D^c, E^c and F^c .

Definition 3.5: A subset A of a pentapartitioned neutrosophic topological space (X, τ) is called a pentapartitioned neutrosophic generalized b-closed (PNgb-closed) set if $\text{PNb-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a pentapartitioned neutrosophic open set in (X, τ) .

The complement of a pentapartitioned neutrosophic generalized b-closed set is called a pentapartitioned neutrosophic generalized b-open set.

The family of pentapartitioned neutrosophic generalized b-closed sets are denoted by $\text{PNgb-C}(X)$.

Example 3.6. In the above example 3.2, $\text{PNgb-C}(X)$ are A^c, B^c, C^c, D^c, E and F .

Theorem 3.7. Each PN – closed set is a PNgb-closed set in X .

Proof. Let $A \subseteq U$ where U is a pentapartitioned neutrosophic open set in X . Since A is a PN-closed set $\text{PNcl}(A) = A$. We have $\text{PNb-cl}(A) \subseteq \text{PNcl}(A) = A \subseteq U$. Therefore, A is a PNgb-closed set in X .

The converse of the above theorem need not be true as shown in the following example.

Example 3.8. Let $X = \{p, q\}$ with $\tau_{\text{PN}} = \{0_{\text{PN}}, A, B, 1_{\text{PN}}\}$ where

$A = \langle (p, 0.5, 0.6, 0.5, 0.7, 0.8), (q, 0.5, 0.5, 0.5, 0.5, 0.6) \rangle$

$B = \langle (p, 0.4, 0.4, 0.8, 0.8, 0.8), (q, 0.5, 0.5, 0.8, 0.8, 0.8) \rangle$

Consider the pentapartitioned neutrosophic sets

$C = \langle (p,0.3,0.5,0.7,0.8,0.9), (q,0.4,0.4,0.6,0.5,0.7) \rangle$
 $D = \langle (p,0.2,0.4,0.9,0.8,0.9), (q,0.4,0.3,0.9,1.0,1.0) \rangle$
 $E = \langle (p,0.5,0.5,0.5,0.5,0.5), (q, 0.5,0.5,0.5,0.5,0.5) \rangle$
 Here C, D and E are PNgb-C(X), but not PN-C(X).

Theorem 3.9. Each PN semi-closed set is a PNgb-closed set in X.

Proof. Let $A \subseteq U$ where U is a pentapartitioned neutrosophic open set in X. Since A is a PNsemi-closed set, $PNscl(A) = A$. We have $PNb-cl(A) \subseteq PNscl(A) = A \subseteq U$. Therefore, A is a PNgb-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.10. In the above example 3.6, C, D and E are PNgb-C(X), but not PNs-C(X).

Theorem 3.11. Each PNb-closed set is a PNgb-closed set in X.

Proof. Let $A \subseteq U$ where U is a pentapartitioned neutrosophic open set in X. Since A is a PNb-closed set, $PNbcl(A) = A$. We have $PNb-cl(A) = A \subseteq U$. Therefore, A is a PNgb-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.12. Let $X = \{p,q\}$ with $\tau_{PN} = \{0_{PN,A,B}, 1_{PN}\}$ where

$A = \langle (p,0.3,0.4,0.5,0.4,0.3), (q,0.4,0.3,0.7,0.3,0.4) \rangle$

$B = \langle (p,0.4,0.6,0.4,0.4,0.1), (q,0.5,0.4,0.5,0.1,0.3) \rangle$

Consider the pentapartitioned neutrosophic set $Q = \langle (p,0.6,0.6,0.3,0.4,0.1), (q,0.9,0.8,0.4,0.1,0.2) \rangle$

Here Q is a PNgb-C(X) but not PNb-C(X).

Theorem 3.13. Each $PN\alpha$ -closed set is a PNgb-closed set in X.

Proof. Let $A \subseteq U$ where U is a pentapartitioned neutrosophic open set in X. Since A is a $PN\alpha$ -closed set, $PN\alpha cl(A) = A$. We have $PNb-cl(A) \subseteq PN\alpha-cl(A) = A \subseteq U$. Therefore, A is a PNgb-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.14. Let $X = \{p,q,r\}$ with $\tau_{PN} = \{0_{PN,A,B,C}, 1_{PN}\}$ where

$A = \langle (p,0.9,0.5,0.3,0.6,0.7), (q,0.5,0.6,0.4,0.5,0.1), (r,0.7,0.4,0.6,0.7,0.5) \rangle$

$B = \langle (p,0.4,0.4,0.7,0.7,0.8), (q,0.5,0.4,0.6,0.8,0.3), (r,0.6,0.4,0.7,0.8,0.9) \rangle$

$C = \langle (p,0.3,0.4,0.8,0.7,1.0), (q,0.4,0.3,0.8,0.9,0.4), (r,0.5,0.3,0.8,0.8,1.0) \rangle$

Consider the pentapartitioned neutrosophic sets

$D = \langle (p,0.8,0.4,0.4,0.7,0.8), (q,0.4,0.5,0.5,0.5,0.2), (r,0.5,0.3,0.7,0.7,0.6) \rangle$

$E = \langle (p,0.5,0.5,0.5,0.5,0.5), (q, 0.5,0.5,0.5,0.5,0.5), (r, 0.5,0.5,0.5,0.5,0.5) \rangle$

Here C, E and D^c are PNgb-C(X), but not $PN\alpha$ -C(X).

Theorem 3.15. Each PNg-closed set is a PNgb-closed set in X.

Proof. Let $A \subseteq U$ where U is a pentapartitioned neutrosophic open set in X. Since A is a PNg-closed set, $PNcl(A) \subseteq U$. We have $PNb-cl(A) \subseteq PNcl(A) \subseteq U$. Therefore, A is a PNgb-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.16. Let $X = \{p,q\}$ with $\tau_{PN} = \{0_{PN,A,B}, 1_{PN}\}$ where

$A = \langle (p,0.5,0.6,0.5,0.7,0.8), (q,0.5,0.5,0.5,0.5,0.6) \rangle$

$B = \langle (p,0.4,0.4,0.8,0.8,0.8), (q,0.5,0.5,0.8,0.8,0.8) \rangle$

Consider the pentapartitioned neutrosophic sets

$C = \langle (p,0.3,0.5,0.7,0.8,0.9), (q,0.4,0.4,0.6,0.5,0.7) \rangle$

$D = \langle (p,0.2,0.4,0.9,0.8,0.9), (q,0.4,0.3,0.9,1.0,1.0) \rangle$

$E = \langle (p,0.5,0.5,0.5,0.5,0.5), (q, 0.5,0.5,0.5,0.5,0.5) \rangle$

Here A, C and D are PNgb-C(X), but not PNg-C(X).

Theorem 3.17. Each $PN\omega$ -closed set is a PNgb-closed set in X.

Proof. Let $A \subseteq U$ where U is a pentapartitioned neutrosophic open set in X. Since A is a $PN\omega$ -closed set, and also every PNO set is PNSO set, we have $PNcl(A) \subseteq U$. We know that, $PNb-cl(A) \subseteq PNcl(A) \subseteq U$. Therefore, A is a PNgb-closed set in X.

The converse of the above theorem need not be true as shown in the following example.

Example 3.18. Let $X = \{p,q\}$ with $\tau_{PN} = \{0_{PN}, A, B, 1_{PN}\}$ where

$A = \langle (p,0.5,0.6,0.5,0.7,0.8), (q,0.5,0.5,0.5,0.5,0.6) \rangle$

$B = \langle (p,0.4,0.4,0.8,0.8,0.8), (q,0.5,0.5,0.8,0.8,0.8) \rangle$

Consider the pentapartitioned neutrosophic sets

$C = \langle (p,0.3,0.5,0.7,0.8,0.9), (q,0.4,0.4,0.6,0.5,0.7) \rangle$

$D = \langle (p,0.2,0.4,0.9,0.8,0.9), (q,0.4,0.3,0.9,1.0,1.0) \rangle$

$E = \langle (p,0.5,0.5,0.5,0.5,0.5), (q, 0.5,0.5,0.5,0.5,0.5) \rangle$

Here A, C and D are PNgb-C(X), but not PN_{ω} -C(X).

Remark 3.19. figure 1 shows the relationship between PN ω -closed set , PNgb-closed and PN ω -closed sets with the known existing pentapartitioned neutrosophic sets. A \rightarrow B represents A implies B but not conversely.

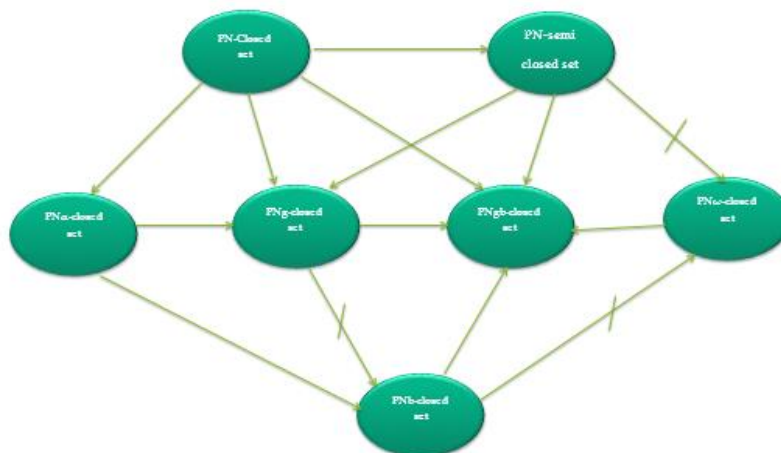


Figure 1: the relationship between PN ω -closed set , PNgb-closed and PN ω -closed sets

4. Conclusion and Future work

In this paper we introduced PN ω -closed set, PN ω -closed set, PNgb-closed set and some of its properties were discussed and derived some contradicting examples. This idea can be developed and extended in continuity, homeomorphisms, compactness, connectedness and so on.

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