



Solving Thermal System Problem Via Fuzzy Mohand Transform under Neutrosophic Initial Conditions

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Abstract

In this paper, a new fuzzy integral transform is introduced and called “*fuzzy Mohand transform*” which is based on a mathematical formula of the classical Mohand transform. The fuzzy technique that is well-established from fuzzy Laplace transform, agrees with the neutrosophic logic. In fact, in mathematical field, neutrosophic logic, which is based on the idea of indeterminacy, is introduced as a generalization of fuzzy logic. In Neutrosophic logic, there are three values, grade of membership of Truth values (T), Indeterminate values (I) and False values (F). To demonstrate this mixed technique, a realistic example about heating system is discussed as an illustrative example for solving an ordinary differential equation of first order using fuzzy Mohand transform with neutrosophic numbers as an initial condition.

Keywords: Fuzzy number; Neutrosophic number; Neutrosophic triangular number; strongly generalized differentiability; Mohand transform.

1. Introduction

In certain practical issues, not considering that the membership value is insufficient, it is also necessary to consider the non-member value. Fuzzy sets are clarified by considering elements as a function of membership (characteristic) which allocates to each element a level of membership arises between zero and one [1].

For more a viable kind of fuzzy set in the life usages, Atanassov [2] proposed a new one so called “intuitionistic fuzzy set (IFS)”. Consequently, the conjectural fuzzy set offers a popularization of fuzzy set through the truth which supplies both the information of set, and those which belongs not to set [3].

A new deduced concept from the uncertainty world under a term of “neutrosophic set theory” is obtained by Florentin Smarandache to deal several subjects. The underlined searchable fields are description, the cause, belongingness of fact values, false values, values of indeterminate [4]. As a result, neutrosophic is a new logic in the mathematical world, which relies on the principle of indeterminacy, and this logic is considered as a generalization of fuzzy logic [1].

In accordance with the philosophical perspective, a premium result can be obtained by Neutrosophic set theory with non - standard interval. However, within the bounds of the practical application for the scientific and engineering troubles, the included data in this non - standard interval are unidentifiable. Wang et al. could solve this issue through defining single valued Neutrosophic set. [5]. Via taking in to account the unit of interval [0,1] according to its own standard form. The limited values in this interval are named Neutrosophic numbers.

It is observable addressing of fuzzy differential equations with uncertainty by regarding just the both of membership, and non - membership values. These two latter terms are excluded to regard the indeterminacy. Hence, evolving of theory of Neutrosophic differential equations is requested to formulate the whole three values, membership, non-membership, indeterminacy.

Lately, the logic of neutrosophic has turn out as one of the most valuable and important modeling tools in approximately all the implementation fields of engineering and science. The styling can be used in vast

phenomena of real life by using describing differential equations with imprecise and uncertain parameters, see [6],[7],[8],[9],[10].

According to the fuzzy integral transform novel, Mohand and Mahgoub [11] introduced a new transformation in 2017, known as “Mohand transform”. Nowadays, it becomes a well instrument to solve the fuzzy differential and integral equations. These fuzzy integral transforms reduce the equation (with fuzzy initial / boundary conditions) to an algebraic system [12].

In fact, this process means that there is switching between the operations of calculus and algebra which is called “Operational calculus”.

The presented mathematical innovation in this investigation is named “fuzzy Mohand transform”. To clarify the approach of a neutrosophic environment, the problem of the thermal system has been solved for different levels of time values and cut - points.

2. Mohand Transform [11]

Mohand integral transform is similar to Laplace, Smudu, Elzaki integral transforms that are used to solve differentiable equations and those used to solve fuzzy differential equations [15]. Consider the function in the set \mathbb{A} defined by :

$$\mathbb{A} = \left\{ h(w) : \exists M, k_1, k_2 > 0, |h(w)| < M e^{\frac{|w|}{k_j}}, \text{ if } w \in (-1)^j \times [0, \infty) \right\}$$

M is constant that must be a finite number, k_1, k_2 may be finite or infinite

The Mohand transform denoted by the operator \mathcal{M} and define by the integral equation

$$\mathcal{M}[h(w)] = R[v] = v^2 \int_0^\infty h(w) e^{-vw} dw, w \geq 0, k_1 \leq v \leq k_2$$

The variable v in this transform is used to factor the variable w in the argument of the function h .

Theorem 1 [11]

Let $h(w)$ is a continuous function in $[0, r]$ and $\mathcal{M}[h(w)] = R(v)$, then

- For any functions $h(w)$ and $\psi(w)$ and any constants a, b then :

$$\mathcal{M}[ah(w) + b\psi(w)] = a(\mathcal{M}[h(w)]) + b(\mathcal{M}[\psi(w)])$$

- $\mathcal{M}[h^{(n)}(w)] = v^{(n)}R(v) - \sum_{i=0}^{n-1} v^{n-i+1} h^{(i)}(0)$

- If $\mathcal{M}[h(w)] = R(v)$ and $L[h(w)] = F(v)$, then $R(v) = v^2 F(v)$ and $F(v) = \frac{1}{v^2} R(v)$ where $F(v)$ is Laplace transformation of $h(w)$.

Example 2.1

Let the 1st - order ordinary differential equation

$$\frac{dx}{dw} + x = 0, x(0) = a$$

Apply Mohand transform for this equation to get:

$$vR(v) - v^2 h(0) + R(v) = 0$$

$$(v + 1)R(v) - av^2 = 0$$

$$R(v) = \frac{av^2}{v+1}$$

Now applying the inverse of Mohand transform, to get : $x(w) = e^{-aw}$

3. Fuzzy Mohand Transform ($\widehat{\mathcal{M}}$ -Transform)

In this section, we introduce fuzzy Mohand transform and denote by $\widehat{\mathcal{M}}$. Any fuzzy integral transform reduces fuzzy differential equation (FDE) to an algebraic system. This switching from operations of calculus to algebraic operations on transformations is called operational calculus which plays a significant role in applied mathematics.

Definition 3.1

Let $h(w)$ be a continuous fuzzy – valued function supposes that $v^2 h(w) e^{-vw}$ is an improper fuzzy Riemann – integrable on $[0, \infty)$, then

$v^2 \int_0^\infty h(w) e^{-vw} dw$ is called fuzzy Mohand transform and it is denoted by:

$$\widehat{\mathcal{M}}[h(w)] = v^2 \int_0^\infty h(w) e^{-vw} dw, (v > 0 \text{ and integer})$$

From theorem (2), we have

$$v^2 \int_0^\infty h(w) e^{-vw} dw = (v^2 \int_0^\infty \underline{h}(w, r) e^{-vw} dw, v^2 \int_0^\infty \bar{h}(w, r) e^{-vw} dw)$$

Using the definition of classical Mohand transform:

$\mathcal{M}[\underline{h}(w,r)] = v^2 \int_0^\infty \underline{h}(w,r) e^{-vw} dw$ and $\mathcal{M}[\overline{h}(w,r)] = v^2 \int_0^\infty \overline{h}(w,r) e^{-vw} dw$ then $\widehat{\mathcal{M}}[h(w,r)] = (\mathcal{M}[\underline{h}(w,r)], \mathcal{M}[\overline{h}(w,r)])$.

Definition 3.2

The integral transform

$\widehat{\mathcal{M}}[h(w)] = v^2 \int_0^\infty h(w) e^{-vw} dw$ is said to be absolutely convergent integral if

$v^2 \lim_{w \rightarrow \infty} \int_0^\infty |h(w) e^{-vw} dw|$ exist, that is mean :

$v^2 \lim_{w \rightarrow \infty} \int_0^\infty |\underline{h}(w,r) e^{-vw} dw|$ and $v^2 \lim_{w \rightarrow \infty} \int_0^\infty |\overline{h}(w,r) e^{-vw} dw|$ are exist.

Theorem 2

Let $h(w), \psi(w)$ be continuous fuzzy – valued functions, assume that c_1 and c_2 are constants, then:

- a) $\widehat{\mathcal{M}}[c_1 h(w)] = c_1 \widehat{\mathcal{M}}[h(w)]$.
- b) $\widehat{\mathcal{M}}[c_1(h(w)) \oplus c_2(\psi(w))] = c_1 \widehat{\mathcal{M}}[h(w)] \oplus c_2 \widehat{\mathcal{M}}[\psi(w)]$.

Proof (a):

$$\begin{aligned} \widehat{\mathcal{M}}[c_1 h(w)] &= (\mathcal{M}[c_1 \underline{h}(w,r)], \mathcal{M}[c_1 \overline{h}(w,r)]) \\ &= (v^2 \int_0^\infty c_1 \underline{h}(w,r) e^{-vw} dw, v^2 \int_0^\infty c_1 \overline{h}(w,r) e^{-vw} dw) \\ &= (c_1 v^2 \int_0^\infty \underline{h}(w,r) e^{-vw} dw, c_1 v^2 \int_0^\infty \overline{h}(w,r) e^{-vw} dw) \\ &= c_1 (v^2 \int_0^\infty \underline{h}(w,r) e^{-vw} dw, v^2 \int_0^\infty \overline{h}(w,r) e^{-vw} dw) = c_1 \widehat{\mathcal{M}}[h(w)]. \end{aligned}$$

(b) suppose $h(w) = (\underline{h}(w,r), \overline{h}(w,r)), \psi(w) = (\underline{\psi}(w,r), \overline{\psi}(w,r))$, then

$$\begin{aligned} \widehat{\mathcal{M}}[c_1(h(w)) \oplus c_2(\psi(w))] &= (\mathcal{M}[(c_1 \underline{h}(w,r)) + c_2(\underline{\psi}(w,r))], \mathcal{M}[c_1 \overline{h}(w,r) + c_2 \overline{\psi}(w,r)]) \\ &= (v \int_0^\infty e^{-vw} (c_1 \underline{h}(w,r) + c_2 \underline{\psi}(w,r)) dw, v \int_0^\infty e^{-vw} (c_1 \overline{h}(w,r) + c_2 \overline{\psi}(w,r)) dw) \\ &= (v \int_0^\infty e^{-vw} c_1 \underline{h}(w,r) dw + v \int_0^\infty e^{-vw} c_2 \underline{\psi}(w,r) dw, v \int_0^\infty e^{-vw} c_1 \overline{h}(w,r) dw \\ &\quad + v \int_0^\infty e^{-vw} c_2 \overline{\psi}(w,r) dw) \\ &= (v \int_0^\infty e^{-vw} c_1 \underline{h}(w,r) dw, v \int_0^\infty e^{-vw} c_1 \overline{h}(w,r) dw) + \\ &\quad (v \int_0^\infty e^{-vw} c_2 \underline{\psi}(w,r) dw, v \int_0^\infty e^{-vw} c_2 \overline{\psi}(w,r) dw) \\ &= c_1 (v \int_0^\infty e^{-vw} \underline{h}(w,r) dw, v \int_0^\infty e^{-vw} \overline{h}(w,r) dw) \\ &\quad + c_2 (v \int_0^\infty e^{-vw} \underline{\psi}(w,r) dw, v \int_0^\infty e^{-vw} \overline{\psi}(w,r) dw) \\ &= c_1 \mathcal{M}(\underline{h}(w,r), \overline{h}(w,r)) + c_2 \mathcal{M}(\underline{\psi}(w,r), \overline{\psi}(w,r)) = c_1 \widehat{\mathcal{M}}[h(w)] \oplus c_2 \widehat{\mathcal{M}}[\psi(w)]. \end{aligned}$$

Theorem 3

Let $h(w)$ be an integrable fuzzy – valued function, and $h(w)$ is primitive of $h'(w)$ on $[0, \infty]$, then:

- (a) $\widehat{\mathcal{M}}[h'(w)] = v \widehat{\mathcal{M}}[h(w)] \ominus v^2 h(0)$, where h is (i)- differentiable.
- (b) $\widehat{\mathcal{M}}[h'(w)] = -v^2 h(0) \ominus (-v) \widehat{\mathcal{M}}[h(w)]$, where h is (ii)- differentiable.

Proof : (a)

Since $h'(w)$ is continuous fuzzy – valued function and for any arbitrary fixed $r \in [0,1]$, then

$$\widehat{\mathcal{M}}[h'(w)] = (\mathcal{M}[\underline{h}'(w,r)], \mathcal{M}[\overline{h}'(w,r)]) \tag{1}$$

By definition of Mohand derivative of first order we get :

$$\widehat{\mathcal{M}}[\underline{h}'(w)] = v \mathcal{M}[\underline{h}(w,r)] - v^2 \underline{h}(0,r), \widehat{\mathcal{M}}[\overline{h}'(w)] = v \mathcal{M}[\overline{h}(w,r)] - v^2 \overline{h}(0,r) \tag{2}$$

If we substitute (2) in (1), we get :

$$\widehat{\mathcal{M}}[h'(w)] = (v \mathcal{M}[\underline{h}(w,r)] - v^2 \underline{h}(0,r), v \mathcal{M}[\overline{h}(w,r)] - v^2 \overline{h}(0,r))$$

$\widehat{\mathcal{M}}[h'(w)] = v \mathcal{M}[h(w, r), \bar{h}(w, r)] - v^2[h(0, r) - \bar{h}(0, r)]$
 By theorem 2 , we get : $\widehat{\mathcal{M}}[h'(w)] = v\widehat{\mathcal{M}}[h(w)] \ominus v^2h(0)$

(b) Now we suppose that h is the second form , for any arbitrary $r \in [0,1]$.
 $-v^2h(0) \ominus (-v)\widehat{\mathcal{M}}[h(w)] = (-v^2\bar{h}(0, r) + v \mathcal{M}[\bar{h}(w, r)], -v^2\underline{h}(0, r) + v\mathcal{M}[\underline{h}(w, r)])$
 Since $(-v^2\bar{h}(0, r) + v \mathcal{M}[\bar{h}(w, r)]) = \mathcal{M}[\bar{h}'(w, r)]$ and
 $(-v^2\underline{h}(0, r) + v\mathcal{M}[\underline{h}(w, r)]) = \mathcal{M}[\underline{h}'(w, r)]$
 $-v^2h(0) \ominus (-v)\widehat{\mathcal{M}}[h(w)] = (\mathcal{M}[\bar{h}'(w, r)], \mathcal{M}[\underline{h}'(w, r)])$
 $-v^2h(0) \ominus (-v)\widehat{\mathcal{M}}[h(w)] = \mathcal{M}[\bar{h}'(w, r), \underline{h}'(w, r)]$
 Since h is the second form , it follows that : $-v^2h(0) \ominus (-v)\widehat{\mathcal{M}}[h(w)] = \widehat{\mathcal{M}}[h'(w)]$.

4. Application of Fuzzy Mohand Transform of Ordinary Differential Equations

In this section a problem of heating system is considered. The differential equation formed for this problem is solved in Neutrosophic environment. It is shown that, Neutrosophic environment includes differential equations with initial conditions containing parameters of belongingness, non – belongingness and indeterminacy.

Example 4.1

Consider the Neutrosophic initial value problem

$$y'(\tau) = -\frac{1}{s\sigma} y(\tau), y_{T\alpha}(0) = [-\alpha, \alpha], y_{I\beta}(0) = [-\beta, \beta], y_{F\gamma}(0) = [-\gamma, \gamma].$$

Where $s = 0.5$, the thermal capacitance is $\sigma = 2$, y is the temperature

Using Neutrosophic Mohand transform, we get the expression in Truth, Indeterminacy and Falsity as:

$y'(\tau) + y(\tau) = 0$, so applying fuzzy Mohand transform gives:

$$\widehat{\mathcal{M}}[y'(\tau)] + \widehat{\mathcal{M}}[y(\tau)] = 0$$

Case 1: if $y(\tau)$ be (i) – differentiable

$$v\mathcal{M}[y_{T\alpha}(\tau)] - v^2y_{T\alpha}(0) + \mathcal{M}[y_{T\alpha}(\tau)] = 0$$

$$(v + 1)\mathcal{M}[y_{T\alpha}(\tau)] = -\alpha v^2$$

$$\mathcal{M}[y_{T\alpha}(\tau)] = \frac{-\alpha v^2}{v+1}, \mathcal{M}[\bar{y}_{T\alpha}(\tau)] = \frac{\alpha v^2}{v+1}.$$

Now we use inverse fuzzy Mohand Transform

$$\underline{y}_{T\alpha}(\tau) = -\alpha e^{-\tau}, \bar{y}_{T\alpha}(\tau) = \alpha e^{-\tau}$$

$$\Rightarrow v\mathcal{M}[y_{I\beta}(\tau)] - v^2y_{I\beta}(0) + \mathcal{M}[y_{I\beta}(\tau)] = 0$$

$$(v + 1)\mathcal{M}[y_{I\beta}(\tau)] = -\beta v^2$$

$$\mathcal{M}[y_{I\beta}(\tau)] = \frac{-\beta v^2}{(v+1)}, \mathcal{M}[\bar{y}_{I\beta}(\tau)] = \frac{\beta v^2}{(v+1)}$$

$$\underline{y}_{I\beta}(\tau) = -\beta e^{-\tau}, \bar{y}_{I\beta}(\tau) = \beta e^{-\tau}$$

$$\text{Also, } \underline{y}_{F\gamma}(\tau) = -\gamma e^{-\tau}, \bar{y}_{F\gamma}(\tau) = \gamma e^{-\tau}$$

Case 2 : if $y(\tau)$ be (ii) – differential

$$-v^2y(0) \ominus (-v)\mathcal{M}[y(\tau)] + \mathcal{M}[y(\tau)] = 0$$

$$-v^2\bar{y}_{T\alpha}(0) + v\mathcal{M}[\bar{y}_{T\alpha}(\tau)] + \mathcal{M}[y_{T\alpha}(\tau)] = 0$$

$$v\mathcal{M}[\bar{y}_{T\alpha}(\tau)] + \mathcal{M}[y_{T\alpha}(\tau)] = v^2\bar{y}_{T\alpha}(0) \tag{1}$$

$$v\mathcal{M}[y_{T\alpha}(\tau)] + \mathcal{M}[\bar{y}_{T\alpha}(\tau)] = v^2y_{T\alpha}(0) \tag{2}$$

Solve equations (1) & (2), we get:

$$\underline{y}_{T\alpha}(\tau) = \alpha \cosh \tau + \alpha \sinh \tau, \bar{y}_{T\alpha}(\tau) = -\alpha \cosh \tau - \alpha \sinh \tau$$

$$\text{Also, } -v^2\bar{y}_{I\beta}(0) + v\mathcal{M}[\bar{y}_{I\beta}(\tau)] + \mathcal{M}[y_{I\beta}(\tau)] = 0$$

$$v\mathcal{M}[\bar{y}_{I\beta}(\tau)] + \mathcal{M}[y_{I\beta}(\tau)] = v^2\bar{y}_{I\beta}(0) \tag{1}$$

$$v\mathcal{M}[y_{I\beta}(\tau)] + \mathcal{M}[\bar{y}_{I\beta}(\tau)] = v^2y_{I\beta}(0) \tag{2}$$

Solve equations (1) & (2) , we get :

$$\underline{y}_{I\beta}(\tau) = \beta \cosh \tau + \beta \sinh \tau, \bar{y}_{I\beta}(\tau) = -\beta \cosh \tau - \beta \sinh \tau$$

$$\text{Also, } -v^2 \bar{y}_{F\gamma}(0) + v\mathcal{M}[\bar{y}_{F\gamma}(\tau)] + \mathcal{M}[y_{F\gamma}(\tau)] = 0$$

$$v\mathcal{M}[\bar{y}_{F\gamma}(\tau)] + \mathcal{M}[y_{F\gamma}(\tau)] = v^2 \bar{y}_{F\gamma}(0) \quad (1)$$

$$-v^2 y_{F\gamma}(0) + v\mathcal{M}[y_{F\gamma}(\tau)] + \mathcal{M}[\bar{y}_{F\gamma}(\tau)] = 0$$

$$v\mathcal{M}[y_{F\gamma}(\tau)] + \mathcal{M}[\bar{y}_{F\gamma}(\tau)] = v^2 y_{F\gamma}(0) \quad (2)$$

Solve equations (1) & (2), we get :

$$\underline{y}_{F\gamma}(\tau) = \gamma \cosh \tau + \gamma \sinh \tau, \quad \bar{y}_{F\gamma}(\tau) = -\gamma \cosh \tau - \gamma \sinh \tau.$$

5. Conclusion

In fact, in mathematical field, neutrosophic logic, that is based on the idea of indeterminacy, is introduced as a generalization of fuzzy logic. This article discusses a practical example of a heating system to illustrate the mixed neutrosophic and fuzzy technique for solving first-order ordinary differential equations utilizing fuzzy Mohand transform with neutrosophic numbers as an initial condition. Such approach offers accuracy while cutting down on the time needed to address these problems. In the future, this mixed technique of fuzzy- neutrosophic logic will be generalized to solve second and third-order ordinary differential equations.as well as partial differential equations.

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