n-refined Neutrosophic Fuzzy of Some Topological Concepts

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Abstract

In this paper, we present and study some concepts in topological space by using two tools in fuzzy theory which namely fuzzy and neutrosophic concepts. n-refined neutrosophic fuzzy topological space have been studied by n-refined neighborhood, n-refined compact set and other notions. Finally, some definitions, examples and new results have been presented in this paper.

Keywords: Topological space; fuzzy set; open set; neutrosophic set; compact set.

1. Introduction

A topological on a set is a collection τ of subsets of X, called the open sets, if $\emptyset$ and X belong to τ, any union of elements of τ belongs to τ, and any finite intersection of elements of τ belongs to τ. We say (X, τ) is a topological space. If $A \subset X$ then the subspace topology on A is the collection of intersections of A with open sets of (X, τ). With this topology, A is called a subspace of X. Interior of a set A ($int(A)$, or A^o) is the union of all open subsets of A. The set X is said to be a neighborhood of the point x, if $x \in Int(x)$ [2]. An element $x \in A$ is said to be a limit point of A if each neighborhood of x contains an element of A other than x. Closed set is one which contains all of its limit points. In another word, a closed set is one whose complement is open. Closure of a set A contained in a topological space (X, τ) is the intersection of all closed subsets of X containing A. a topological space X is called connected if the only subsets that are both open and closed are $\emptyset$ and X.

The neutrosophic [1] notion was founded by Smarandache in 1998 as an extension of fuzzy and intuitionistic fuzzy notions to deal with indeterminacy in real-life issues. This idea has been expanded and applied to various fields of mathematics, such as complex space [2-4], topology space [5-7], statistics, probability [8-10]. In neutrosophic algebra, several useful studies have emerged that have worked to highlight the role of indeterminacy in numerous algebraic structures. The neutrosophic triplet group was introduced by Smarandache and Ali [11]. Jun et al. [12] constructed neutrosophic quadruple BCK/BCI-numbers. Abobala [13] devoted some linear equations over the neutrosophic field. Abed et al. [14] introduced some new results of the neutrosophic multiplication module. Zail et al. [15] studied Some new results of a generalization of BCK-algebra (Ω-BCK-algebra). and other contributions, see [16-18]. Fuzzy topology concept presented by Chang [19]. Fuzzy and n-refined neutrosophic concept in a structure [21] was proposed by dividing the indeterminacy part into two levels of subindeterminacies. Recently, the idea of n-refined neutrosophic structures was defined as a generalization of Refined neutrosophic, and it was used by [22]. In this paper, we give some new results on n-Refined Indeterminacy of some Modules based on n-refined neutrosophic idea, where we present new relations of n-refined neutrosophic Modules.

2. Preliminaries

In this section, we shall present the basic definitions about fuzzy set, fuzzy topology, fuzzy compact and other concepts.

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Definition 2.1. [2]

Let $X \neq \emptyset$ be a set. A collection τ of subsets of X with $\tau = \{A, A \subseteq X\}$ is called topology if the following conditions are true:

1- $X \in \tau, \emptyset \in \tau$.
2- If $A_i \in \tau$, so $\cup A_i \in \tau, i \in I$.
3- If $A_i \in \tau$, so $\cap A_i \in \tau$.

Definition 2.2.

Suppose $(X,\ \tau_1)$and $(Y,\ \tau_2)$ be two topological spaces with $f: X \to Y$. Hence f is continuous if and only if every open set $B \in\ \tau_2$ imply $f^{-1}(B)$ is also open set inside τ_1.

Definition 2.3.

Let $X \neq \emptyset$ be a set and suppose that I is an interval $(I = [0, 1])$. So fuzzy set $A \in X$ is a mapping (function) from X to $[0, 1]$.

$A = \{(x, \mu_A(x)): x\ in\ X\}$ is called fuzzy set.

Definition 2.4.

We refer to fuzzy topology on the set X by collection τ of fuzzy set inside X such that:

1- $0 \in \tau, 1 \in \tau$.
2- When A and B inside τ, so $A \wedge B \in \tau$.
3- When $A_i \in \tau$, so $\vee A_i \in \tau, i \in J$.

Remark 2.5.

We define $A \wedge B$ and $\vee A_i$ by:

$$(A \wedge B)(x) = \inf\{A(x), B(x)\}$$

$$\vee A_i(x) = \sup\{A_i(x), i \in J\}, x \in X.$$

Hence (X, τ) is called fuzzy topology space when τ is a topological space.

Definition 2.6.

The basic fuzzy sets operations:

1- $f = g$ if $f(x) = g(x), x \in X$, f and g are fuzzy sets in X.
2- $f \subseteq g$ if $f(x) \leq g(x), x \in X$.
3- $(f \cup g)(x) = \max\{f(X), g(x)\}, x \in X$.
4- $(f \cap g)(x) = \min\{f(X), g(x)\}, x \in X$.

Definition 2.7.

Let $\Phi: I^A \to I^A$ be a mapping. The Φ is a fuzzy closure operator if and only if:

1- $\Phi(\mathrm{k}) = \mathrm{k}, \mathrm{k}$ constant.
2- $\Phi(\mathrm{t}) \geq \mathrm{t}, \mathrm{t} \in\ I^A$.
3- $\Phi(\mathrm{t}) \vee \Phi(\mathrm{s}) = \Phi(\mathrm{t} \vee \mathrm{s}), \mathrm{t}, \mathrm{s} \in I^A$.
4- $\Phi(\Phi(\mathrm{t})) = \Phi(\mathrm{t}), \mathrm{t} \in\ I^A$.

Remark 2.8.

Fuzzy interior operator is dual of definition 2.7. because:

$\acute{\Phi}: I^A \to I^A$ is a fuzzy interior operator if and only if

1- $\acute{\Phi}(\mathrm{k}) = \mathrm{k}, \mathrm{k}$ constant.
2- $\acute{\Phi}(\mathrm{t}) \leq \mathrm{t}, \mathrm{t} \in\ I^A$.
3- $\acute{\Phi}(\mathrm{t}) \vee \acute{\Phi}(\mathrm{s}) = \acute{\Phi}(\mathrm{t} \vee \mathrm{s}), \mathrm{t}, \mathrm{s} \in I^A$.

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4- $\acute{\Phi}\left(\acute{\Phi}(t)\right) = \acute{\Phi}(t), t \in I^A$.

Definition 2.9.

We say a fuzzy point x_t belong to fuzzy set A in a fuzzy topological space (X, τ) is a fuzzy Φ-cluster point of a fuzzy set A if and only if fuzzy closure is also lower bounded of A.

3. The main results

Definition 3.1.

let $A_m(I) = \{(XI, \mu A_n(I)(XI)): XI \in X_n(I)\}$ and $B_n(I) = \{(XI, \mu A_n(I)(XI)): XI \in X_n(I)\}$ be two n-refine neutrosophic fuzzy sets in $X_n(I)$. Then $\frac{A_n(I)\vee B_n(I)}{H}$, $\frac{A_n(I)\wedge B_n(I)}{K}$ and $\frac{(A_n(I))^c}{L}$ are also n-refined by:

1- $\mu_{\frac{A_n(I)\vee B_n(I)}{H}}(XI) = NE[max\{\mu_{A_n(I)}(XI), \mu_{B_n(I)}(XI)\}], \forall xI \in X_n(I).$

2- $\mu_K(XI) = NE[min\{\mu_{A_n(I)}(XI), \mu_{B_n(I)}(XI)\}], \forall xI \in X_n(I).$

3- $\mu_L(XI) = NE\left(1 - \mu_{A_n(I)}(XI)\right), \forall xI \in X_n(I).$

Remark 3.2.

1- $A_n(I) \subseteq B_n(I)$ if and only if $NE\left(\mu_{A_n(I)}(XI) \leq \mu_{B_n(I)}(XI)\right), \forall xI \in X_n(I).$

2- $A_n(I) = B_n(I)$ if and only if $NE\left(\mu_{A_n(I)}(XI) = \mu_{B_n(I)}(XI)\right), \forall xI \in X_n(I).$

Definition 3.3.

The symbol I_o will denote the unit interval $[0, 1]$. Let $X_n(I)$ be a nonempty n-refined neutrosophic set. Let $A_n(I), B_n(I) \in I_o{}^{X_n(I)}$ and let $f: X_n(I) \to Y_n(I)$ be n-refined neutrosophic function. Then $f(A_n(I)) \in I_o{}^{Y_n(I)}$, i.e. $f(A_n(I))$ is n-refined neutrosophic fuzzy set in $Y_n(I)$ and defined by:

$$f\left(A_n(yI)\right) = \begin{cases} sup\{A_n(xI): XI \in f^{-1}(yI)\}; f^{-1}(yI) \neq \emptyset \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad ;\ f^{-1}(yI) = \emptyset \end{cases}$$

And $f^{-1}(B_n(I))$is n-refined neutrosophic set in $X_n(I)$, defined by $f^{-1}\left(B_n(xI)\right) = B_n(f(xI), xI \in I_n(I)$.

Definition 3.4.

The product $f_1 \times f_2: X_{1_n}(I) \times X_{2_n}(I) \to Y_{1_n}(I) \times Y_{2_n}(I)$ of n-refined neutrosophic mapping $f_1: X_{1_n}(I) \to Y_{1_n}(I)$ and $f_2: X_{2_n}(I) \to Y_{2_n}(I)$ is defined by : $(f_1 \times f_2)(x_1I, x_2I) = (f_1(x_1I),\ f_2(x_2I)\forall(x_1I, x_2I) \in X_{1_n}(I) \times X_{2_n}(I)$.

Remark 3.5.

For a mapping $f: X_n(I) \to Y_n(I)$, the graph $g: X_n(I) \to Y_n(I)$ of f is defined by:

$$g(xI) = \left(xI, f(xI)\right), \forall xI \in X_n(I).$$

Definition 3.6.

Let $A_n(I) \in I_o{}^{X_n(I)}$ and $B_n(I) \in I_o{}^{Y_n(I)}$. Then by $A_n(I) \times B_n(I)$, we denote n-refined neutrosophic fuzzy set in $X_n(I) \times Y_n(I)$ for which

$$\left(A_n(I) \times B_n(I)\right)(xI, yI) = NE[min\{(A_n(I))(xI), (B_n(I))(xI)\}], \forall(xI, yI) \in X_n(I) \times Y_n(I).$$

Definition 3.7.

A family $\tau_n(I) \subseteq I_o{}^{X_n(I)}$ of n-refined neutrosophic fuzzy sets is called n-refined neutrosophic fuzzy topology for $X_n(I)$ if:

1- $0I, 1I \in \tau_n(I)$.

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2- For all $(A_n(I), B_n(I) \in \tau_n(I), then\ A_n(I) \cap B_n(I) \in \tau_n(I)$.

3- $\forall \left(A_{j_n}(I)\right)_{j\in J} \in \tau_n(I), so\ \cup_{j\in J} A_{j_n}(I) \in \tau_n(I)$.

The pair $(X_n(I), \tau_n(I))$ is called n-refined neutrosophic fuzzy topological space.

Remark 3.8.

1- N-refined neutrosophic fuzzy $K_n(I)$ is called n-refined neutrosophic fuzzy closed if $(K_n(I))^{cl} \in \tau_n(I)$.

2- We denote by $(\tau_n(I))^{cl}$ of all n-refined neutrosophic fuzzy closed sets in n-refined neutrosophic fuzzy topological space.

Now, we present some properties of n-refined neutrosophic fuzzy closed sets.

1- $(\alpha I)^{cl} \in \tau_n(I)$.

2- If $K_n(I), H_n(I) \in \tau_n(I), so\ K_n(I) \cup H_n(I) \in (\tau_n(I))^{cl}$.

3- If $\{K_{j_n}(I): j \in J\} \in (\tau_n(I))^{cl}$, so $\cap\{K_{j_n}(I): j \in J\} \in (\tau_n(I))^{cl}$.

Example 3.9.

Let $X_n(I) = \{aI, bI\}$. Let $A_n(I)$ be n-refined neutrosophic fuzzy set on $X_n(I)$ defined by

$$\left(A_n(I)\right)(aI) = 0.5I, \left(A_n(I)\right)(bI) = 0.4I.$$

Then $\tau_n(I) = \{0I, A_n(I), 1I\}$ is n-refined neutrosophic fuzzy topological space.

$$0I(aI) = 0I, \forall\ aI \in xI, 1I(aI) = 1I, \forall\ aI \in xI.$$

Remark 3.10.

Suppose that $\tau_{1_n}(I), \tau_{2_n}(I)$ are two n-refined neutrosophic fuzzy topology for $X_n(I)$. If $\tau_1(I) \subset \tau_2(I)$, so $\tau_2(I)$ is finer than $\tau_1(I)$ and $\tau_1(I)$ is coarser than $\tau_2(I)$.

Definition 3.11.

A base of n-refined neutrosophic fuzzy topological space $(X_n(I), \tau_n(I))$ is n-refined neutrosophic sub collection $B_n(I)$ of $\tau_n(I)$ such that every number $A_n(I)$ of $\tau_n(I)$ can be written by

$$A_n(I) = V_{j_n}(I)\left(A_{j_n}(I)\right), A_{j_n}(I) \in B_n(I)$$

Definition 3.12.

N-refined neutrosophic fuzzy point in $X_n(I)$ is a special n-refined neutrosophic fuzzy set with membership function defined by:

$$(P_n I)(xI) = \begin{cases} \lambda I & if\ xI = yI \\ 0 & if\ xI \neq yI \end{cases}$$

Where $0 < \lambda I \leq 1$. PI is called support yI, value λI and denoted by $(P_n(I))_{yI}^{\lambda I}$ or $P_n(I)(yI, \lambda I)$.

Remark 3.13.

1. n-refined neutrosophic fuzzy point $(P_n(I))_{yI}^{\lambda I} \in A_n(yI)$ if and only if $\propto I \leq A_n(I)((yI))$.
2. n-refined neutrosophic complement of the fuzzy point $(P_n(I))_{xI}^{\lambda I}$ is denoted by $\left(P_n(I)\right)_{xI}^{1-\lambda I}((P_n(I)_{xI}^{\lambda I})^c$.

Definition 3.14.

We say n-refined neutrosophic fuzzy point $(P_n)_{xI}^{\lambda I}$ is contained in n-refined neutrosophic fuzzy set $A_n(xI)$ or $(P_n)_{xI}^{\lambda I} \in A_n(xI)$ if and only if $\lambda I < A_n(xI)$.

Definition// let $(X_n(I), \tau_n(I))$ be n-refined neutrosophic fuzzy topological space and let $A_n(I) \subseteq X_n(I)$. We define n-refined neutrosophic closure of $A_n(I)$ by $A_n(I) \cup \acute{A}_n(I)$ and denoted by $\overline{A_n(I)}$ or $cl(A_n(I))$.

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Example 3.15.

Let $X_n(I) = \{aI, bI, cI\}, \tau_n(I) = \{X_n(I), \emptyset_n(I), \{aI, bI\}\}, A_n(I) = \{aI, cI\}$ then

$$\overline{A_n(I)} = A_n(I) \cup A_n^{\prime}(I)$$
$$= \{aI, cI\} \cup \{bI, cI\} = X_n(I)$$

Definition 3.16.

The n-refined neutrosophic $A_n^{\prime}(I)$ of fuzzy set A of X is define by

$$A_n^o(I) = max\{sup\{O: O \leq A_n(I), O \in \tau_n(I)\}\}$$

Remark 2.17.

N-refined neutrosophic closure of $A_n(I)$ is define by

$$\overline{A_n}(I) = min\{inf\{KI: A_n(I) \leq KI, K_n^c(I) \in \tau_n(I)\}\}$$

Example 3.18.

let $X_n(I) = \{aI, bI, cI\}, \tau_n(I) = \{X_n(I), \emptyset_n(I), \{aI\}, \{bI\}, \{aI, bI\}\}, A_n(I) = \{bI\}, B_n(I) = \{aI, cI\}$ then

$$A_n^o(I) = \{bI\}, \text{ because } bI \in \mathcal{M}_n(I) = \{bI\} \subseteq A_n(I) = \{bI\}.$$

Also,

$$B_n^o(I) = \{aI\}, \text{ because } aI \in \mathcal{U}_n(I) = \{aI\} \subseteq B_n(I) = \{aI\}.$$

Note that if $(X_n(I), \tau_n(I))$ be n-refined neutrosophic fuzzy topological space and $A_n(I) \subseteq X_n(I)$. We say $xI \in A_n(I)$ is n-refined neutrosophic interior point of $A_n(I)$ if and only if there exists n-refined neutrosophic open set $\mathcal{U}_n(I) \in \tau_n(I) \supset xI \ni xI \in \mathcal{M}_n(I) \subseteq A_n(I)$.

Remark 3.19.

We denote to n-refined neutrosophic interior points of $A_n(I)$ by $A_n^o(I)$ or $Int(A_n(I))$.

Example 3.20.

Let $A_n(I)$, $B_n(I)$ and $C_n(I)$ be n-refined neutrosophic fuzzy sets of $\mathcal{S}_n(I)$ and define by:

$$A_n(xI) = \begin{cases} 0I & ; \quad 0I \leq xI \leq \frac{1}{2}I \\ (2x)I & ; \quad \frac{1}{2}I \leq xI \leq 1I \end{cases}$$

$$B_n(xI) = \begin{cases} 1I & ; \quad 0I \leq xI \leq \frac{1}{4}I \\ (-4x)I + 2I & ; \quad \frac{1}{4}I \leq xI \leq \frac{1}{2}I \\ 0 & ; \quad \frac{1}{2}I \leq xI \leq 1I \end{cases}$$

$$C_n(xI) = \begin{cases} 0 & ; \quad 0I \leq xI \leq \frac{1}{4}I \\ \frac{(4x)I - 1I}{3I} & ; \quad \frac{1}{2}I \leq xI \leq 1I \end{cases}$$

Then $\tau_n(I) = \{oI, A_n(I), B_n(I), A_n(I) \vee B_n(I), 1I\}$ is n-refined neutrosophic fuzzy topology on $\mathcal{S}_n(I)$. Hence $\overline{A_n}(I) = B_n^c(I)$ and $\overline{B_n}(I) = A_n^c(I)$ and $\overline{(A_n(I) \vee B_n(I))} = 1I$, and $A_n^o(I) = B_n(I)$ and $B_n^o(I) = A_n(I)$ and so $(A_n(I) \vee B_n(I))^o = 0I$.

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Definition 3.21.

let $(X_n(I), \tau_n(I))$ be n-refined neutrosophic topological space and let $xI \in X_n(I)$ and $A_n(I) \subseteq X_n(I)$. We say $A_n(I)$ is n-refined neutrosophic neighborhood of n-refined neutrosophic point xI when there exists n-refined neutrosophic open set $\mathcal{U}_n(I) \supset xI$ and contain in $A_n(I)$.

Remark 3.22.

If $A_n(I)$ is n-refined neutrosophic open set and contains xI, so $A_n(I)$ is n-refined neutrosophic open neighborhood of xI.

Now we present a definition of n-refined neutrosophic neighborhood of $(P_n)_{xI}^{\lambda I}$ by other words:

N-refined neutrosophic set $A_n(I)$ in $(X_n(I), \tau_n(I))$ is called n-refined neutrosophic fuzzy neighborhood of $(P_n(I))_{xI}^{\lambda I}$ if and only if

$$\exists\, B_n(I) \in \tau_n(I) \ni (P_n(I))_{xI}^{\lambda I} \in B_n(I) \leq A_n(I)$$

Note that n-refined neutrosophic neighborhood $A_n(I)$ is n-refined neutrosophic open if and only if $A_n(I)$ is open.

Definition 3.23.

let $(X_n(I), \tau_n(I))$ be n-refined neutrosophic topological space. $A_n(I) \subseteq X_n(I)$ is called n-refined neutrosophic compact if every n-refined neutrosophic open cover of $X_n(I)$ has a finite subcover.

Example 3.24.

Let $X_n(I) = \mathbb{R}_n(I)$ and let $\tau_n(I) = \{\emptyset_n(I), \mathbb{R}_n(I), Q_n(I), \hat{Q}_n(I)\}$ where $Q_n(I)$ is n-refined neutrosophic rational numbers, $\mathbb{R}_n(I)$ is n-refined neutrosophic real numbers and $\hat{Q}_n(I)$ is n-refined neutrosophic irrational numbers. Then $Q_n(I)$ is n-refined neutrosophic compact in $\mathbb{R}_n(I)$, because

$Q_n(I) \subseteq \cup\{\emptyset_n(I), \mathbb{R}_n(I), Q_n(I)\}$, also $\hat{Q}_n(I)$ is n-refined neutrosophic compact in $\mathbb{R}_n(I)$, because $\hat{Q}_n(I) \subseteq \cup\{\hat{Q}_n(I),, Q_n(I)\}$.

Proposition 3.25.

let $(X_n(I), \tau)$ be n-refined neutrosophic fuzzy topological space. Then every xI in $X_n(I)$ has at least one n-refined neutrosophic fuzzy neighborhood point.

Proof.

suppose that $X_n(I)$ being n-refined neutrosophic fuzzy open set it is n-refined neutrosophic fuzzy neighborhood of all points. Hence $xI \in X_n(I)$ has at least one n-refined neutrosophic fuzzy points as a neighborhood.

Corollary 3.26.

Let $(X_n(I), \tau)$ be n-refined neutrosophic fuzzy topological space. Every n-refined neutrosophic fuzzy neighborhood of $xI \in X_n(I)$ contains xI.

Proof.

Let $A_n(I)$ n-refined neutrosophic fuzzy open set. So $\forall\, x \in A_n(I)\ \exists$ n-refined neutrosophic fuzzy open set in $A_n(I) \subseteq X_n(I), xI \in A_n(I) \subseteq A_n(I)$. Then $A_n(I)$ is a n-refined neutrosophic fuzzy neighborhood of every its n-refined neutrosophic fuzzy point.

Thus, each n-refined neutrosophic fuzzy neighborhood contains $xI \in X_n(I)$.

Proposition 3.27.

Let $(X_n(I), \tau)$ be n-refined neutrosophic fuzzy topological space and let $F_n(I)$ be n-refined neutrosophic fuzzy closed subset of $X_n(I)$ with $xI \in F_n(I)$. Then there exists n-refined neutrosophic fuzzy neighborhood $N_n(I)$ of xI such that $N_n(I) \cap F_n(I) = \emptyset$.

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Proof.

Since $\left(F_n(I)\right)^c$ is n-refined neutrosophic fuzzy open set containing xI, then $\left(F_n(I)\right)^c$ is n-refined neutrosophic fuzzy neighborhood of xI. Put $\left(F_n(I)\right)^c = N_n(I)$. Then it follows that $N_n(I)$ is n-refined neutrosophic fuzzy neighborhood of xI such that

$$N_n(I) \cap F_n(I) = \left(F_n(I)\right)^c \cap F_n(I) = \emptyset$$

Recall that when $(X_n(I), \tau)$ is n-refined neutrosophic fuzzy topological space and if $A_n(I) \subseteq X_n(I)$ is n-refined neutrosophic fuzzy subset of $X_n(I)$. Then a n-refined neutrosophic fuzzy point $xI \in A_n(I)$ is called n-refined neutrosophic fuzzy interior point of $A_n(I)$ if and only if $A_n(I)$ is n0refined neutrosophic fuzzy neighborhood of xI.

Example 3.28.

Let $X_n(I) = \{aI, bI, cI\}$ and let $\tau = \{X_n(I), \{bI\}, \{bI, cI\}, \{aI, bI\}, \emptyset\}$ be n-refined neutrosophic fuzzy topological space on $X_n(I)$. If $A_n(I) = \{aI, cI\}$, so to find $\left(A_n(I)\right)^o$, we need to check every n-refined neutrosophic fuzzy point of $A_n(I)$. Consider $aI \in A_n(I)$, ∃ no n-refined neutrosophic fuzzy open set containing aI in $A_n(I)$. Hence aI is not n-refined neutrosophic fuzzy interior point of $A_n(I)$. Also cI is not n-refined neutrosophic fuzzy interior point. Thus $\left(A_n(I)\right)^o = \emptyset$.

4. Conclusion

In this work, we have employed the idea of the refined neutrosophic set to produce some modules, such as cyclic, simple, and finitely generated modules. Also, we showed the new relations of n-refined neutrosophic modules as well as several examples and properties that have been studied about the relations. Finally, we hope that this study will show the importance of neutrosophic ideas in strengthening different algebraic structures.

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