On Neutrosophic filter and fantastic filter of BL-algebras

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Abstract

The main aim of this paper is to present a few characteristics of the neutrosophic filter of BL-algebras. Also, we introduce the notion of the neutrosophic fantastic filter of BL-algebras with an illustration and discuss a few of its properties. Moreover, we prove every neutrosophic fantastic filter is a neutrosophic filter in BL-algebras. Finally, we acquire an extension property and equivalent condition of the neurosophic fantastic filter of BL-algebras.

Keywords: BL-algebra; filter; neutrosophic filter; fantastic filter, neutrosophic fantastic filter.

1. Introduction

In 1998, Smarandache[9] introduced the ideology of neutrosophy to characterise the neutralites. The notion of fuzzy sets introduced by Zadeh[13] and intuitionistic fuzzy sets by K.T. Atanassov[2,3] laid the foundation for neutrosophic sets. Neutrosophy further became a stepping stone for many mathematical theories like neutrosophic set theory, neutrosophic statistics, neutrosophic probability, and neutrosophic logic. They are widely used in medical diagnosis, image segmentation, decision-making, layout planning and design, security, robotics, and many other fields. Hajek[4] introduced the notion of BL-algebras (basic logic). The unit interval [0, 1] furnished with the structure produced by a continuous t-norm is the primary illustration of a BL-algebra. The three most well-known classes of BL-algebras are MV-algebras, Godel algebras, and product algebras. The concept of filters was first initiated by Hajek[4] in BL algebras. Some types of filters, including fantastic filters, were defined by him in BL-algebras. But Turunen[10] was the first to study the theory of filters in BL-algebras well.

M. Haveshki, A. Borumand Saied and E. Eslami[5] have investigated some types of filters in BL algebras. S. Yahya Mohamed and P. Umamaheshwari[12] have investigated vague filters in BL-algebras. X. Zhang, X. Mao, Y. Wu, and X. Zhai[14] have defined neutrosophic filters in pseudo-BCI algebras. The authors[6] introduced the notion of a neutrosophic filter for BL-algebras and investigated some of its features with illustrations.

In this paper, we concentrate on neutrosophic filters and fantastic filters of BL-algebras. In Section 2, we explain some of the primary definitions and findings from the literature. In Section 3, we illustrate some features of neutrosophic filters. Also, we prove that the union of two neutrosophic filters need not be a neutrosophic filter. In Section 4, we introduce the notion of neutrosophic fantastic filters of BL-algebras along with some of their related features. We further prove that every fantastic neutrosophic filter is a neutrosophic filter.

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2. Preliminaries

In this section, some of the definitions and results from the literature are recalled to develop the primary findings.

Definition 2.1[4] A BL-algebra is an algebra $(\mathcal{B}, \vee, \wedge, \circ, \rightarrow, 0, 1)$ of type $(2,2,2,2,0,0)$ such that the following are satisfied for all $j_1, k_1, l_1 \in \mathcal{B}$,

(i) $(\mathcal{B}, \vee, \wedge, 0,1)$ is a bounded lattice,
(ii) $(\mathcal{B}, \circ, 1)$ is a commutative monoid,
(iii) $'\circ'$ and $'\rightarrow'$ form an adjoint pair, that is, $l_1 \leq j_1 \rightarrow k_1$ if and only if $j_1 \circ l_1 \leq k_1$ for all $j_1, k_1, l_1 \in \mathcal{B}$,
(iv) $j_1 \wedge k_1 = j_1 \circ (j_1 \rightarrow k_1)$,
(v) $(j_1 \rightarrow k_1) \vee (k_1 \rightarrow j_1) = 1$.

Proposition 2.2[7] The following axioms are satisfied in a BL- algebra $\mathcal{B}$ for all $j_1, k_1, l_1 \in \mathcal{B}$,

(i) $k_1 \rightarrow (j_1 \rightarrow l_1) = j_1 \rightarrow (k_1 \rightarrow l_1) = (j_1 \circ k_1) \rightarrow l_1$,
(ii) $1 \rightarrow j_1 = j_1$,
(iii) $j_1 \leq k_1$ if and only if $j_1 \rightarrow k_1 = 1$,
(iv) $j_1 \vee k_1 = ((j_1 \rightarrow k_1) \rightarrow k_1) \wedge ((k_1 \rightarrow j_1) \rightarrow j_1)$,
(v) $j_1 \leq k_1$ implies $k_1 \rightarrow l_1 \leq j_1 \rightarrow l_1$,
(vi) $j_1 \leq k_1$ implies $l_1 \rightarrow j_1 \leq l_1 \rightarrow k_1$,
(vii) $j_1 \rightarrow k_1 \leq (l_1 \rightarrow j_1) \rightarrow (l_1 \rightarrow k_1)$,
(viii) $j_1 \rightarrow k_1 \leq (k_1 \rightarrow l_1) \rightarrow (k_1 \rightarrow l_1)$,
(ix) $j_1 \leq (j_1 \rightarrow k_1) \rightarrow k_1$,
(x) $j_1 \circ (j_1 \rightarrow k_1) = j_1 \wedge k_1$,
(xi) $j_1 \circ k_1 \leq j_1 \wedge k_1$
(xii) $j_1 \rightarrow k_1 \leq (j_1 \circ l_1) \rightarrow (k_1 \circ l_1)$,
(xiii) $j_1 \circ (k_1 \rightarrow l_1) \leq k_1 \rightarrow (j_1 \circ l_1)$,
(xiv) $(j_1 \rightarrow k_1) \circ (k_1 \rightarrow l_1) \leq j_1 \rightarrow l_1$,
(xv) $(j_1 \circ j_1^*) = 0$.

Note. In the above sequence, $\mathcal{B}$ is used to intend the BL- algebras and the operations.

$'\vee'$, $'\wedge'$, $'\circ'$ have preference on the way to the operations $'\rightarrow'$.

Note. In a BL- algebra $\mathcal{B}$, $'*'$ is a complement defined as $j_1^* = j_1 \rightarrow 0$ for all $j_1 \in \mathcal{B}$.

Definition 2.3[11,15] A non-empty subset F of a BL- algebra $\mathcal{B}$ is a filter of $\mathcal{B}$ if the following axioms hold for all $j_1, k_1 \in \mathcal{B}$,

(i) If $j_1, k_1 \in F$, then $j_1 \circ k_1 \in F$,
(ii) If $j_1 \in F$ and $j_1 \leq k_1$, then $k_1 \in F$.

Proposition 2.4[15] A nonempty subset F of a BL- algebra $\mathcal{B}$ is a filter of $\mathcal{B}$ if and only if the following are satisfied for all $j_1, k_1 \in \mathcal{B}$,

(i) $1 \in F$,
(ii) $j_1, j_1 \rightarrow k_1 \in F$ implies $k_1 \in F$.

A filter F of a BL-algebra $\mathcal{B}$ is proper if $F \neq \mathcal{B}$.

Definition 2.5[5] A non-empty subset F of a BL-algebra $\mathcal{B}$ is called a fantastic filter if it satisfies the following axioms for all $j_1, k_1, l_1 \in \mathcal{B}$,

(i) $1 \in F$
(ii) $l_1 \rightarrow (k_1 \rightarrow j_1) \in F$ *and* $l_1 \in F$ imply $\big((j_1 \rightarrow k_1) \rightarrow k_1\big) \rightarrow j_1 \in F$.

Definition 2.6[9] Let X be a universe of discourse. A neutrosophic subset R of X is a triple (T_R, I_R, F_R) where $T_R: X \rightarrow [0,1]$ is truth membership function, $I_R: X \rightarrow [0,1]$ is indeterminacy function and $F_R: X \rightarrow [0,1]$ is false membership function and $0 \leq T_R(j_1) + I_R(j_1) + F_R(j_1) \leq 3$ for all $j_1 \in X$.

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Hence, for each $j_1 \in X, T_R(j_1), I_R(j_1)$ and $F_R(j_1)$ are all standard real numbers in $[0,1]$.

Note. The values of $T_R(j_1), I_R(j_1)$ and $F_R(j_1)$ have no limitations and we have the obvious condition $0 \leq T_R(j_1) + I_R(j_1) + F_R(j_1) \leq 3$.

Definition 2.7[9] Let R and S be two neutrosophic sets on $\mathcal{B}$. Define $R \leq S$ if and only if $T_R(j_1) \leq T_S(j_1)$, $I_R(j_1) \geq I_S(j_1), F_R(j_1) \geq F_S(j_1)$ for all $j_1 \in \mathcal{B}$.

Definition 2.8[9] Let R and S be two neutrosophic sets on $\mathcal{B}$. Define

$R \wedge S = (T_R \wedge T_{S,} I_R \vee I_S, F_R \vee F_S)$

$R \vee S = (T_R \vee T_{S,} I_R \wedge I_S, F_R \wedge F_S)$

where $'\wedge'$ is the minimum and $'\vee'$ is the maximum between real numbers.

Definition 2.9[6] A neutrosophic set R of a BL- algebra $\mathcal{B}$ is called a neutrosophic filter if it satisfies the subsequent conditions:

(i) $T_R(j_1) \leq T_R(1), I_R(j_1) \geq I_R(1)$ and $F_R(j_1) \geq F_R(1)$,

(ii) $\min\{T_R(j_1 \rightarrow k_1), T_R(j_1)\} \leq T_R(k_1)$, $\min\{I_R(j_1 \rightarrow k_1), I_R(j_1)\} \geq I_R(k_1)$ and

$\min\{F_R(j_1 \rightarrow k_1), F_R(j_1)\} \geq F_R(k_1)\}$ for all $j_1, k_1 \in \mathcal{B}$.

Proposition 2.10[6] Let R be a neutrosophic set of a BL-algebra $\mathcal{B}$. $R$is a neutrosophic filter of $\mathcal{B}$ if and only if

(i) If $j_1 \leq k_1$then $T_R(j_1) \leq T_R(k_1), I_R(j_1) \geq I_R(k_1)$ and $F_R(j_1) \geq F_R(k_1)$,

(ii) $T_R(j_1 \circ k_1) \geq \min\{T_R(j_1), T_R(k_1)\}, I_R(j_1 \circ k_1) \leq \min\{I_R(j_1), I_R(k_1)\}$

and $F_R(j_1 \circ k_1) \leq \min\{F_R(j_1), F_R(k_1)\}$ for all $j_1, k_1 \in \mathcal{B}$.

Proposition 2.11[6] Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$. If $j_1 \leq k_1$then $T_R(j_1)$is order preserving and $I_R(j_1), F_R(j_1)$are order reversing.

3. Neutrosophic filter

In this section, several key characteristics of neutrosophic filter are emphasized.

Proposition 3.1 Let F be a neutrosophic filter in a BL-algebra $\mathcal{B}$. Then the sets

(i) $F_{T_R} = \{j_1 \in \mathcal{B}; T_R(j_1) = T_R(1)\}$
(ii) $F_{I_R} = \{j_1 \in \mathcal{B}; I_R(j_1) = I_R(1)\}$
(iii) $F_{F_R} = \{j_1 \in \mathcal{B}; F_R(j_1) = F_R(1)\}$ are filters of $\mathcal{B}$.

Proof: Let F be a neutrosophic filter in $\mathcal{B}$.

Obviously, $1 \in F_{T_R}$.

If $j_1, j_1 \rightarrow k_1 \in F_{T_R}$. Then, $T_R(j_1) = T_R(j_1 \rightarrow k_1) = T_R(1)$.

From the definition 2.9, we have
$$\begin{aligned} T_R(1) &= \min\{T_R(j_1), T_R(j_1 \rightarrow k_1)\} \\ &\leq T_R(k_1) \\ &\leq T_R(1) \end{aligned}$$

Hence, $T_R(k_1) = T_R(1)$. Therefore, $k_1 \in F_{T_R}$.

Then, from the proposition 2.4, we get $F_{T_R}$is a filter of $\mathcal{B}$.

Similarly, we can prove (ii) and (iii).

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Proposition 3.2 Let R be a neutrosophic set of a BL-algebra $\mathcal{B}$. Let R be a neutrosophic filter of $\mathcal{B}$. Then the following hold for all $j_1, k_1, l_1 \in \mathcal{B}$.

(i) $T_R(j_1 \wedge k_1) = \min\{T_R(j_1), T_R(k_1)\}$, $I_R(j_1 \wedge k_1) = \min\{I_R(j_1), I_R(k_1)\}$, $F_R(j_1 \wedge k_1) = \min\{F_R(j_1), F_R(k_1)\}$

(ii) $T_R(j_1 \circ k_1) = \min\{T_R(j_1), T_R(k_1)\}$, $I_R(j_1 \circ k_1) = \min\{I_R(j_1), I_R(k_1)\}$, $F_R(j_1 \circ k_1) = \min\{F_R(j_1), F_R(k_1)\}$

(iii) $T_R(0) = \min\{T_R(j_1), T_R(j_1{}^*)\}$, $I_R(0) = \min\{I_R(j_1), I_R(j_1{}^*)\}$, $F_R(0) = \min\{F_R(j_1), F_R(j_1{}^*)\}$

Proof: (i) Let R be a neutrosophic set of $\mathcal{B}$. Let R be a neutrosophic filter of $\mathcal{B}$.

Since $j_1 \wedge k_1 \le j_1, j_1 \wedge k_1 \le k_1$,

From the proposition 2.11, we get $T_R(j_1 \wedge k_1) \le \min\{T_R(j_1), T_R(k_1)\}$

From the definition 2.10, we have

$$\begin{aligned} T_R(j_1 \wedge k_1) &\ge \min\{T_R(j_1 \to (j_1 \wedge k_1)), T_A(j_1)\} \\ &= \min\{T_R((j_1 \to j_1) \wedge (j_1 \to k_1)), T_R(j_1)\} \\ &= \min\{T_R(j_1 \to k_1), T_R(j_1)\} \\ &\ge \min\{\min\{T_R(k_1 \to (j_1 \to k_1)), T_R(k_1), T_R(j_1)\} \\ &= \min\{\min\{T_R(1), T_R(k_1)\}, T_R(j_1)\} \\ &= \min\{T_R(k_1), T_R(j_1)\} \\ &= \min\{T_R(j_1), T_R(k_1)\} \end{aligned}$$

Hence, $T_R(j_1 \wedge k_1) = \min\{T_R(j_1), T_R(k_1)\}$.

Similarly, we can prove for I_R , F_R.

(ii) Let R be a neutrosophic set of $\mathcal{B}$. Let R be a neutrosophic filter of $\mathcal{B}$.

From (ii) of the proposition 2.10, we have $T_R(j_1 \circ k_1) \ge \min\{T_R(j_1), T_R(k_1)\}$

Since , $j_1 \circ k_1 \le j_1 \wedge k_1$, From proposition 2.11 and (i),
we have $T_R(j_1 \circ k_1) \le T_R(j_1 \wedge k_1)$

$$= \min\{T_R(j_1), T_R(k_1)\}$$

Thus, $T_R(j_1 \circ k_1) = \min\{T_R(j_1), T_R(k_1)\}$.

Similarly, we can prove for I_R , F_R.

(iii) Let R be a neutrosophic set of $\mathcal{B}$. Let R be a neutrosophic filter of $\mathcal{B}$.

From (ii), we have $\min\{T_R(j_1), T_R(j_1{}^*)\} = T_R(j_1 \circ j_1{}^*) = T_R(0)$

Therefore, $T_R(0) = \min\{T_R(j_1), T_R(j_1{}^*)\}$.

Similarly, we can prove for I_R , F_R.

Proposition 3.3 Union of two neutrosophic filters of $\mathcal{B}$ need not be a neutrosophic filter of $\mathcal{B}$.

It can be proved by an example.

Let $\mathcal{B}$ $=\{0, u, v, w, 1\}$. The binary operations $\circ$ and $\to$ are given by the subsequent tables (3.1) and (3.2).

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Table 3.1 :'∘' Operation

∘	0	u	v	w	1
0	1	1	1	1	1
u	0	1	1	1	1
v	0	u	1	w	1
w	0	u	v	1	1
1	0	u	v	w	1

Table 3.2 :' → ' Operation

→	0	u	v	w	1
0	1	0	0	0	0
u	0	1	u	u	u
v	0	u	1	u	v
w	0	0	u	1	w
1	0	u	v	w	1

Then, $(\mathcal{B}, \vee, \wedge, \circ, \rightarrow, 0, 1)$ is a BL- algebra. Define neutrosophic sets R and S in $\mathcal{B}$ as follows:

$$T_R(0) = T_R(v) = T_R(w) = T_R(1) = 0.7, T_R(u) = 0.5,$$

$$I_R(0) = I_R(u) = I_R(v) = I_R(w) = 0.4, I_R(1) = 0.3,$$

$$F_R(0) = F_R(u) = F_R(v) = F_R(w) = 0.2, F_R(1) = 0.1;$$

$$T_S(0) = T_S(u) = T_S(w) = T_S(1) = 0.8, T_S(v) = 0.6,$$

$$I_S(0) = I_S(u) = I_S(v) = I_S(w) = 0.5, I_S(1) = 0.1,$$

$$F_S(0) = F_S(u) = F_S(v) = F_S(w) = 0.4, F_S(1) = 0.1;$$

Then, R and S are neutrosophic filters in $\mathcal{B}$. Let $D= R \cup S$, then

$$T_D(0) = T_D(u) = T_D(w) = T_D(1) = 0.8, T_D(v) = 0.7,$$

$$I_D(0) = I_D(u) = I_D(v) = I_D(w) = 0.5, I_D(1) = 0.3,$$

$$F_D(0) = F_D(u) = F_D(v) = F_D(w) = 0.4, F_D(1) = 0.1.$$

Here, $D = R \cup S$ is not a neutrosophic filters in $\mathcal{B}$.

Since, $T_D(v) = 0.7 \ngeq 0.8 = \min\{T_D(u \circ v), T_D(u)\}$, $I_D(u) = 0.5 \nleq 0.3 = \min\{I_D(0 \circ u), I_D(0)\}$,

$F_D(u) = 0.4 \nleq 0.1 = \min\{F_D(0 \circ u), F_D(0)\}$.

4. Neutrosophic fantastic filter

In this section, we introduce a notion of neutrosophic fantastic filter and investigate some related properties.

Definition 4.1 Let R be a neutrosophic subset of a BL-algebra $\mathcal{B}$. Then R is called a neutrosophic fantastic filter of $\mathcal{B}$, if it satisfies the following axioms for all $j_1, k_1, l_1 \in \mathcal{B}$,

(i) $T_R(1) \geq T_R(j_1), I_R(1) \leq I_R(j_1), F_R(1) \leq F_R(j_1)$.

(ii) $T_R\big((j_1 \rightarrow k_1) \rightarrow k_1\big) \rightarrow j_1) \geq \min\{T_R\big(l_1 \rightarrow (k_1 \rightarrow j_1)\big), T_R(l_1)\}$,
$I_R\big((j_1 \rightarrow k_1) \rightarrow k_1\big) \rightarrow j_1) \leq \min\{I_R\big(l_1 \rightarrow (k_1 \rightarrow j_1)\big), I_R(l_1)\}$,
$F_R((j_1 \rightarrow k_1) \rightarrow k_1) \rightarrow j_1) \leq \min\{F_R(l_1 \rightarrow (k_1 \rightarrow j_1)), F_R(l_1)\}$.

Example 4.2 Let $\mathcal{B} =\{0, r, s, 1\}$. The binary operations ∘ and → are given by the subsequent tables (4.1) and (4.2).

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Table 4.1: $'\circ'$ Operation

$\circ$	0	r	s	1
0	0	r	s	1
r	0	0	0	1
s	0	0	0	1
1	1	1	1	0

Table 4.2: $' \rightarrow '$ Operation

$\rightarrow$	0	r	s	1
0	0	0	0	0
r	r	0	0	r
s	r	1	0	s
1	1	1	0	0

Then, $(\mathcal{B}, \vee, \wedge, \circ, \rightarrow, 0, 1)$ is a BL- algebra. Define a neutrosophic set R of $\mathcal{B}$ as follows:

$R = \{(0, [0.8,0.3,0.1]), (r, [0.6,0.4,0.2]), (s, [0.6,0.4,0.2]), (1, [0.8,0.3,0.1])\}$.

It is evident that R assures the conditions (i) and (ii) of the definition 4.1, and hence is a neutrosophic fantastic filter of $\mathcal{B}$.

Example 4.3 Let $\mathcal{B} = \{0, r, s, 1\}$. The binary operations are given by the subsequent tables (4.3) and (4.4).

Table 4.3: $'\circ'$ Operation

$\circ$	0	r	s	1
0	0	0	0	0
r	0	r	r	s
s	0	r	s	s
1	0	0	s	1

Table 4.4: $' \rightarrow '$ Operation

$\rightarrow$	0	r	s	1
0	1	1	1	1
r	r	1	1	1
s	0	r	1	1
1	0	r	s	1

Let S of $\mathcal{B}$ be a neutrosophic set as follows:

$S = \{(0, [0.4,0.2,0.1]), (r, [0.3,0.3,0.2]), (s, [0.5,0.3,0.2]), (1, [0.5,0.2,0.1])\}$.

Here, S is not a neutrosophic fantastic filter of $\mathcal{B}$.

Since $T_S(r) = 0.3 \ngeq 0.4 = \min\{T_S(1 \rightarrow (s \rightarrow r)), T_S(1)\}$.

Theorem 4.4 Every neutrosophic fantastic filter of a BL-algebra $\mathcal{B}$ is a neutrosophic filter of $\mathcal{B}$.

Proof: Let R be a neutrosophic fantastic filter of $\mathcal{B}$.

Then, from (i) of the definition 4.1, we have

$T_R(1) \geq T_R(j_1), I_R(1) \leq I_R(j_1), F_R(1) \leq F_R(j_1)$for all $j_1 \in \mathcal{B}$. (4.1)

From (ii) of the definition 4.1,

$T_R(((j_1 \rightarrow k_1) \rightarrow k_1) \rightarrow j_1) \geq \min\{T_R(l_1 \rightarrow (k_1 \rightarrow j_1)), T_R(l_1)\}$for all$j_1, k_1, l_1 \in \mathcal{B}$.

Similarly,$I_R\left(((j_1 \rightarrow k_1) \rightarrow k_1) \rightarrow j_1\right) \leq \min\{I_R(l_1 \rightarrow (k_1 \rightarrow j_1)), I_R(l_1)\}$,

$F_R(((j_1 \rightarrow k_1) \rightarrow k_1) \rightarrow j_1) \leq \min\{F_R(l_1 \rightarrow (k_1 \rightarrow j_1)), F_R(l_1)\}$.

Put k_1=1, we have $T_R((j_1 \rightarrow 1) \rightarrow 1) \rightarrow j_1) \geq \min\{T_R(l_1 \rightarrow (1 \rightarrow j_1)), T_R(l_1)\}$

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$$T_R((1 \to 1) \to j_1) \geq \min\{T_R(l_1 \to (1 \to j_1)), T_R(l_1)\}$$

$$T_R(j_1) \geq \min\{T_R(l_1 \to j_1), T_R(l_1)\} \ for\ all\ j_1, l_1 \in \mathcal{B}.$$

Similarly, $I_R(j_1) \leq \min\{I_R(l_1 \to j_1), I_R(l_1)\}$,

$$F_R(j_1) \leq \min\{F_R(l_1 \to j_1), F_R(l_1)\} for\ all\ j_1, l_1 \in \mathcal{B}. \quad (4.2)$$

Therefore, from (4.1) and (4.2), R is a neutrosophic filter of $\mathcal{B}$.

Theorem 4.5 Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$. Then, R is a fantastic filter of $\mathcal{B}$ if and only if $T_R\left(\left((j_1 \to k_1) \to k_1\right) \to j_1\right) \geq T_R(k_1 \to j_1)$, $I_R(((j_1 \to k_1) \to k_1) \to j_1) \leq I_R(k_1 \to j_1)$ and $F_R(((j_1 \to k_1) \to k_1) \to j_1) \leq F_R(k_1 \to j_1)$.

Proof: Let R be a neutrosophic fantastic filter of a BL-algebra $\mathcal{B}$.

Then, from (ii) of the definition 4.1,

we have $T_R((j_1 \to k_1) \to k_1) \to j_1) \geq \min\{T_R(l_1 \to (k_1 \to j_1)), T_R(l_1)\}$ for all $j_1, k_1, l_1 \in \mathcal{B}$.

Similarly, $I_R((j_1 \to k_1) \to k_1) \to j_1) \leq \min\{I_R(l_1 \to (k_1 \to j_1)), I_R(l_1)\}$,

$$F_R((j_1 \to k_1) \to k_1) \to j_1) \leq \min\{F_R(l_1 \to (k_1 \to j_1)), F_R(l_1)\}.$$

Put l_1=1, we get, $T_R(((j_1 \to k_1) \to k_1) \to j_1) \geq \min\{T_R(1 \to (k_1 \to j_1)), T_R(1)\}$

$$= \min\{T_R(k_1 \to j_1), T_R(1)\} = T_R(k_1 \to j_1).$$

Thus, we have $T_R(((j_1 \to k_1) \to k_1) \to j_1) \geq T_R(k_1 \to j_1)$.

Similarly, $I_R(((j_1 \to k_1) \to k_1) \to j_1) \leq I_R(k_1 \to j_1)$, $F_R(((j_1 \to k_1) \to k_1) \to j_1) \leq F_R(k_1 \to j_1)$ for all $j_1, k_1 \in \mathcal{B}$.

Conversely, Let R be a neutrosophic filter of a BL-algebra $\mathcal{B}$.

It satisfies $T_R(((j_1 \to k_1) \to k_1) \to j_1) \geq T_R(k_1 \to j_1)$,

$I_R(((j_1 \to k_1) \to k_1) \to j_1) \leq I_R(k_1 \to j_1)$, $F_R(((j_1 \to k_1) \to k_1) \to j_1) \leq F_R(k_1 \to j_1)$ for all $j_1, k_1 \in \mathcal{B}$.

Then from (ii) of the definition 2.10, we have $T_R(((j_1 \to k_1) \to k_1) \to j_1)$

$$\geq T_R(k_1 \to j_1)$$

$$\geq \min\{T_R(l_1 \to (k_1 \to j_1)), T_R(l_1)\}. \quad (4.3)$$

Similarly, $I_R(((j_1 \to k_1) \to k_1) \to j_1) \leq \min\{I_R(l_1 \to (k_1 \to j_1)), I_R(l_1)\}$ and

$$F_R(((j_1 \to k_1) \to k_1) \to j_1) \leq \min\{F_R(l_1 \to (k_1 \to j_1)), F_R(l_1)\} \text{for all } j_1, k_1, l_1 \in \mathcal{B}.$$

Since, R is a neutrosophic filter of $\mathcal{B}$, we have

$$T_R(1) \geq T_R(j_1), I_R(1) \leq I_R(j_1) \text{ and } F_R(1) \leq F_R(j_1) \text{ for all } j_1 \in \mathcal{B}. \quad (4.4)$$

Therefore, from (4.3) and (4.4), R is a neutrosophic fantastic filter of $\mathcal{B}$.

Next, we show the extension property neutrosophic filters

Theorem 4.6 Let R and S be two neutrosophic filters of a BL-algebra $\mathcal{B}$. Let $R \subseteq S$ and $T_R(1) = T_S(1)$, $I_R(1) = I_S(1), F_R(1) = F_S(1)$. If R is a neutrosophic filter, then so is S.

Proof: Let R and S be two neutrosophic filters of a BL-algebra $\mathcal{B}$.

Let $R \subseteq S$ and $T_R(1) = T_S(1)$.

Then, we have $T_R(((k_1 \to j_1) \to j_1) \to k_1) \to k_1) \to ((k_1 \to j_1) \to j_1))$

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$$\geq T_S\left(\left(\left((k_1 \to j_1) \to j_1\right) \to k_1\right) \to k_1\right) \to ((k_1 \to j_1) \to j_1)) \text{for all } j_1, k_1 \in \mathcal{B}.$$

$$\geq T_R(k_1 \to (k_1 \to j_1) \to j_1))$$

$$= T_R((k_1 \to j_1)(k_1 \to j_1))$$

$$= T_R(1)$$

$$= T_S(1)$$

Since, $(\left(\left(\left((k_1 \to j_1) \to j_1\right) \to k_1\right) \to k_1\right) \to j_1) \to (((j_1 \to k_1) \to k_1) \to j_1)$

$$\geq ((j_1 \to k_1) \to k_1) \to (\left((k_1 \to j_1) \to j_1) \to j_1\right) \to k_1) \to k_1)$$

$$\geq (\left((k_1 \to j_1) \to j_1\right) \to k_1) \to (j_1 \to k_1)$$

$$\geq j_1 \to \left((k_1 \to j_1) \to j_1\right)$$

$$= (k_1 \to j_1) \to (j_1 \to j_1)$$

$$= (k_1 \to j_1) \to 1$$

$$= 1$$

Consider $T_S\left(\left((j_1 \to k_1) \to k_1\right) \to j_1\right) \geq \min\left\{T_S(1), T_S\left(\left(\left(\left((k_1 \to j_1) \to j_1\right) \to k_1\right) \to k_1\right) \to j_1\right)\right\}$ for all $j_1, k_1 \in \mathcal{B}$.

$$= T_S(\left(\left(\left((k_1 \to j_1) \to j_1\right) \to k_1\right) \to k_1\right) \to j_1)$$

$$\geq \min\left\{T_S((k_1 \to j_1) \to \left(\left(\left(\left((k_1 \to j_1) \to j_1\right) \to k_1\right) \to k_1\right) \to j_1\right), T_S((k_1 \to j_1))\right\}$$

$$= \min\{T_S\left(\left(\left((k_1 \to j_1) \to j_1\right) \to k_1\right) \to y\right), T_S\left((k_1 \to j_1)\right)\}$$

$$\geq \min\{T_S(1), T_S(k_1 \to j_1)\}$$

$$= T_S(k_1 \to j_1).$$

Similarly, we can prove for I_R , I_S, and F_R , F_S.

Therefore, from the theorem 4.5, S is a neutrosophic fantastic filter of a BL-algebra $\mathcal{B}$.

6. Conclusion

In the present paper, several key characteristics of neutrosophic filters of BL-algebras are discussed. We have proven some equivalent conditions for a neutrosophic set to be a filter. Also, we have justified that the union of two neutrosophic filters of a BL-algebra need not be a neutrosophic filter with an example. Further, the notion of a neutrosophic fantastic filter is introduced with an illustration, and the related properties are discussed. Moreover, we have proved that every neutrosophic fantastic filter is a neutrosophic filter in BL-algebra. Finally, we have acquired an extension property and equivalent condition of the neutrosophic fantastic filter of BL-algebras. In the future, we can extend the above concepts to neutrosophic ultra filter, normal, implicative filters, and so on. Further, it will be useful to establish the relationship between the neutrosophic fantastic, implicative, and positive implicative filters in BL-algebras.

Conflicts of Interest: "The authors declare no conflict of interest."

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