



The Integrated Novel Framework: Linguistic Variables in Pythagorean Neutrosophic Set with DEMATEL for Enhanced Decision Support

Jamiatun Nadwa Ismail¹, Zahari Rodzi^{1*}, Faisal Al-Sharqi², Hazwani Hashim³, Nor Hashimah Sulaiman⁴

¹College of Computing, Informatics and Media, UiTM Cawangan Negeri Sembilan, Kampus Seremban, 73000 Negeri Sembilan, Malaysia

²Department of Mathematics, Faculty of Education for Pure Sciences, University Of Anbar, Ramadi, Anbar, Iraq

³College of Computing, Informatics and Media, UiTM Cawangan Kelantan, Kampus Machang, 18500, Kelantan, Malaysia

⁴College of Computing, Informatics and Media, UiTM Cawangan Selangor, Kampus Dengkil, 43800 Selangor, Malaysia

Emails: ismail.nadwa@gmail.com, zahari@uitm.edu.my, faisal.ghazi@uoanbar.edu.iq, hazwanishashim@uitm.edu.my, _nhashima@tmsk.uitm.edu.my

Abstract

This study proposes a methodology that integrates the Pythagorean Neutrosophic Set (PNS) with the Decision-Making Trial and Evaluation Laboratory (DEMATEL) approach. This integration is intended to effectively handle the challenges of uncertainty and linguistic vagueness that are commonly encountered in multi-criteria decision-making scenarios. The PNS-DEMATEL framework comprises eight distinct steps, wherein the fundamental divergence lies in the creation of a linguistic variable within the PNS-DEMATEL framework. The linguistic variable has been formulated utilizing the PNS methodology, thereby enabling a more all-encompassing depiction of the viewpoints of experts. The application of the 7-point linguistic scale is utilized to acquire evaluations from experts, resulting in heightened precision and discernment. The method that has been put forth aims to improve the representation and management of linguistic variables, thereby enhancing the usability and validity of the model. The verification of the linguistic variable's validity is achieved by ensuring that the conditions outlined for the PNS are met. The integration of PNS with DEMATEL can facilitate the exploration of causal relationships in the barriers to halal certification. This approach can provide decision-makers with a more comprehensive and accurate representation of their opinions and judgements, leading to improved effectiveness in decision-making across various application domains.

Keywords: Pythagorean Neutrosophic Set; DEMATEL, Uncertainty; Halal principles, MCDM; Halal certification.

1. Introduction

Multi-Criteria Decision Making (MCDM) has been a prominent topic in decision-making since its introduction in the mid-1960s. Its interdisciplinary research area has attracted researchers from almost all fields, from psychologists and economists to computer scientists [1]. It is widely used in a range of fields, including engineering, economics, management, and environmental studies, among others [2–5]. As an important research branch of decision-making theory, MCDM methods cover a wide range of distinct approaches. One popular MCDM method is the Decision Making Trial and Evaluation Laboratory (DEMATEL), which was developed in the 1970s to address complex problems involving cause-effect relationships [6] DEMATEL can provide causal

relationships and also can capture the inter-relationships among the dependent factors [7]. DEMATEL has been applied in various situations, ranging from manufacturing planning and control to multi-criteria decision-making and analyzing world challenges such as administrative control systems, marketing strategy and customer performance, safety, and security measurement, fuzzy approaches and expert systems, and industry material selection process [8]. Classical DEMATEL employs crisp numbers to represent scale, but it has been criticized for its inability to handle linguistic uncertainty [9]. To overcome this limitation, researchers have extended DEMATEL by integrating it with fuzzy set theory, resulting in Fuzzy DEMATEL [10]. Researchers have also explored other extensions of DEMATEL, such as intuitionistic fuzzy set theory and type-2 fuzzy set theory [8]. These extensions aim to address different aspects of uncertainty and incompleteness in decision-making problems [11]. As a generalization of fuzzy sets and intuitionistic fuzzy sets, Smarandache [12] proposed the concept of a neutrosophic set (NS) from a philosophical point of view, where the fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth but also a falsity degree and an indeterminacy degree. One of neutrosophic set theory's recent extensions is the Pythagorean Neutrosophic Set (PNS); however, the combination of PNS with DEMATEL remains relatively unexplored [13]. To address this gap, this study proposes to integrate PNS with DEMATEL approach in order to improve the effectiveness of DEMATEL analysis on decision-maker judgement. This will involve identifying the appropriate linguistic terms and corresponding values to represent the indeterminacy and uncertainty in the sets, as well as developing a method for calculating and applying these values. Incorporating linguistic assessments can provide a more practical approach, as experts often rely on linguistic expressions to represent their opinions [14]. The case study proposed in this research focuses on assessing the barriers to halal certificate accreditation and application among food and beverage firms in Nigeria Sembilan, Malaysia. The halal certification bodies accredit a production facility for the manufacturing of halal products and play a crucial role in providing a higher level of customer satisfaction, but its adoption is challenging. Much previous research focuses on the consumer side, certification methods, and the performance and requirements of certification rather than on the acceptance of halal certification from the entrepreneur's perspective [15]. A study conducted by Khan et al. [7] employed the fuzzy DEMATEL technique to evaluate barriers to halal certification. Later, Khan et al. [16] utilized the ISM with Matrice d'Impacts Croisés-Multiplication Appliquée à un Classement (MICMAC) methods to identify the barriers to adopting halal certification. Hence using integrated PNS-DEMATEL approach, this study aims to identify and analyze the barriers in entrepreneurs' decision-making for applying halal certification and accreditation.

In conclusion, the integration of PNS with DEMATEL provides a promising avenue for addressing uncertainty and linguistic vagueness in MCDM. By incorporating the PNS environment, decision-makers can achieve a more comprehensive and accurate representation of their opinions and judgements. They can capture not only the degrees of truth and falsity but also the degrees of neutrality, providing a comprehensive framework to model complex decision-making problems. The proposed method has the potential to contribute to various application domains and enhance the effectiveness of decision-making. The proposed method allows for a broader range of values and considers the linguistic uncertainties inherent in decision-making. This approach enhances the representation and handling of linguistic variables, thereby improving the accuracy and practicality of the model.

2. Methodology

3.1. Preliminaries

In this section, we introduce the basic definitions related to PNS.

Definition 1 A PNS proposed by Radha and Stanis [17]. Let X be a non-empty set (universe). A Pythagorean Neutrosophic Set with T and F as dependent neutrosophic components A on X is an object of the form

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \mid x \in X \right\} \quad (1)$$

Where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth, indeterminacy and false membership respectively such that $T, I, F \in [0, 1]$. Here when T and F are dependent components, then for all x in X ;

$$T + F \leq 1 \quad (2)$$

$$0 \leq T^2 + F^2 \leq 1 \quad (3)$$

$$0 \leq T^2 + I^2 + F^2 \leq 2 \quad (4)$$

3.2. Proposed Methods

This section focuses on the development of the Pythagorean Neutrosophic Set (PNS) DEMATEL framework. The overall structure of the proposed method is illustrated in Figure 1.

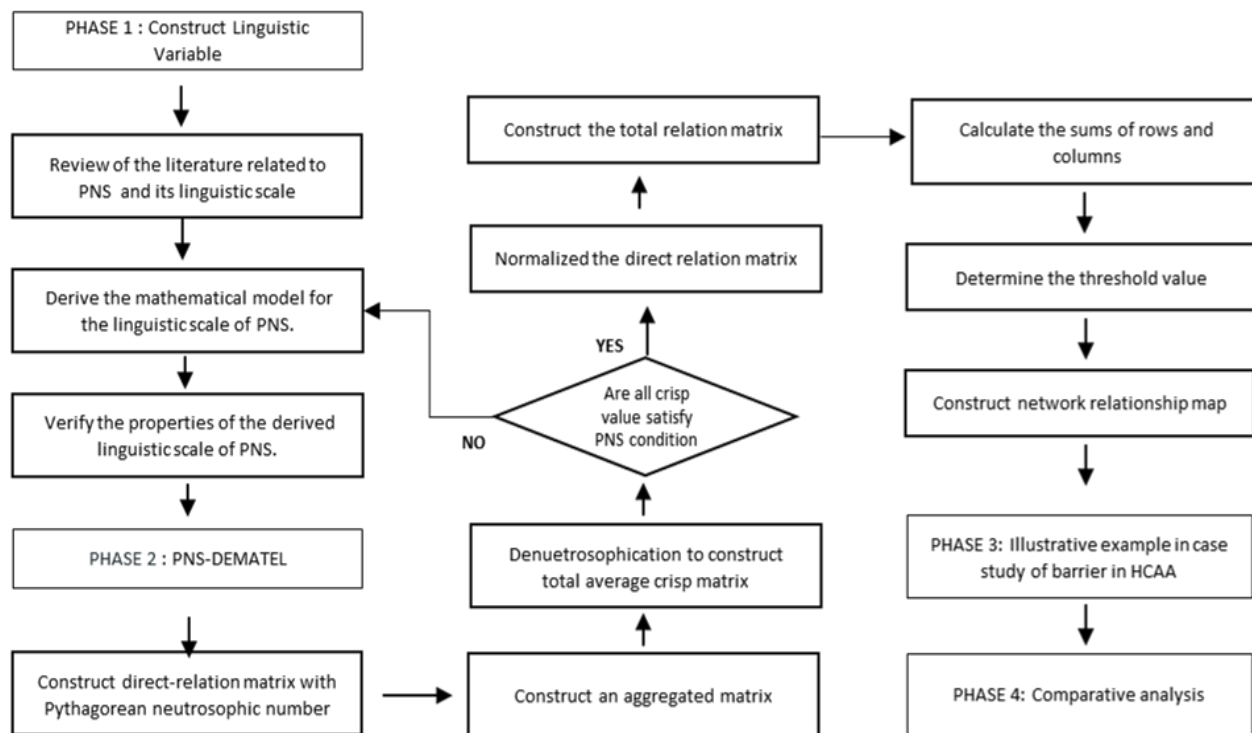


Figure 1: Proposed PNS-DEMATEL framework

The study introduces a new linguistic variable for PNS-DEMATEL and incorporates modifications into the original DEMATEL method while preserving its fundamental principles. The proposed method consists of eight steps and follows the basic concept of DEMATEL. However, the key difference lies in the development of the linguistic variable within the PNS-DEMATEL framework. In this approach, the linguistic variable is derived from the Single Values Neutrosophic Set (SVNS). The use of the PNS allows for a more comprehensive representation of experts' opinions.

In this study, the Likert scale used in the original DEMATEL is modified from a 5-point scale to a 7-point scale. This modification is made to provide a finer granularity in capturing the experts' assessments and to accommodate a wider range of opinions. The 7-point scale offers increased sensitivity and allows experts to express their viewpoints more accurately. This adjustment in the Likert scale enhances the precision and reliability of the assessment process within the PNS-DEMATEL framework.

3.3 The development of a linguistic variable.

The linguistic variable in the PNS-DEMATEL framework is constructed using the Pythagorean Neutrosophic Set (PNS), which enables the determination of the degree of membership, non-membership, and indeterminacy of values. In this step, the linguistic terms that accurately describe the PNS are identified, while ensuring the conditions specified in Definition 1 are met.

Previous research predominantly employed 3-scale and 4-scale linguistic variables Rahim et al. [9], exemplified by Gabus and Fontela's use of a 4-scale linguistic variable in the DEMATEL method. However, subsequent studies have explored different types of linguistic variables, including 5-scale and 8-scale, which have been widely adopted by researchers [11]. For instance, Biswas et al. [18] utilized a 5-scale linguistic variable when employing single-valued neutrosophic numbers (SVNN) in a TOPSIS-based study (Table 1).

Building upon these linguistic variables, the characteristics of Pythagorean neutrosophic sets are carefully considered to enhance their effectiveness. This results in the development of new and refined linguistic variables that align with the principles of the Pythagorean neutrosophic number system.

Table 1: Linguistic variable of SVNN

Score	Linguistic Variable	SVNNS
0	No Influence	<0.10, 0.80, 0.90>
1	Low Influence	<0.35, 0.60, 0.70>
2	Medium Influence	<0.50, 0.40, 0.45>
3	High Influence	<0.80,0.20,0.15>
4	Very High Influence	<0.90,0.10,0.10>

Define linguistic variable for PNS. This new linguistic variable "influence" is defined through seven linguistic terms, which are determined using the rating scales of DEMATEL. Pair-wise comparison scales are established within these seven terms as shown in Table 2 for proposed linguistic variable in PNS, with scores of 1 to 7 representing "Very low influence," "Low influence," "Medium low influence," "Medium influence," "Medium high influence," "High influence," and "Very high influence," respectively. These scales are now being introduced in PFS instead of crisp number.

By utilizing a 7-term linguistic scale, additional options are available compared to the 5-term linguistic scale, where the difference between terms is much greater. This is clearly shown in Table 1, where there is a big difference between scores 0 and 1. To bridge this large gap, we added additional scores, resulting in a new set of numbers shown in Table 2. A significant gap can also be observed between scores 2 and 3 in Table 1, indicating the need for another additional score. These supplementary scores are introduced to bridge the considerable difference and ensure a more precise representation of the underlying shades of variation. By incorporating these additional scores, we achieve a more balanced and finely graded set of measurements which offers increased sensitivity and allows experts to express their viewpoints more accurately.

Another notable enhancement we implemented when adapting from the 5-term linguistic scale to the new 7-term linguistic scale is the adjustment in scoring. In Table 2, we observed a change where the scoring now begins with 1 instead of 0. This modification stems from the integration of this PNS with DEMATEL, where employing 0 as a score would not be suitable due to the presence of diagonal elements of 0 in the decision matrix. Therefore, to align the scoring system appropriately and maintain consistency, we opted to start the scores from 1 in this context. The proposed Pythagorean neutrosophic linguistic variable is displayed in Table 2.

Table 2: The new Pythagorean neutrosophic linguistic variable

Score	Linguistic Variable	Rating Scale in Pythagorean Neutrosophic Set
1	No Influence	< 0.10, 0.80, 0.90 >
2	Low Influence	< 0.20, 0.70, 0.80 >
3	Medium Low Influence	< 0.35, 0.60, 0.60 >
4	Medium Influence	< 0.50, 0.40, 0.45 >
5	Medium High Influence	< 0.65, 0.30, 0.25 >
6	High Influence	< 0.80, 0.20, 0.15 >
7	Very High Influence	< 0.90, 0.10, 0.10 >

The newly added score of 2 in Table 2 has a corresponding truth membership value of 0.2. The reason for assigning a truth membership of 0.2 here is to maintain balance within the scale. The difference between the higher end of the scale, represented by score 6 with a truth membership of 0.8, and score 7 with a truth membership of 0.9, is 0.1. To ensure equilibrium, it is necessary for the difference between the truth membership values of the lower end of the scale, specifically score 1, and score 2 to be 0.1 as well. By assigning a truth membership of 0.2 to the new score of 2, the lower and upper ends of the scale are effectively balanced.

Hence, when assigning values for indeterminate and false membership, we derive them based on the corresponding truth value and ensure they satisfy the conditions for PNS as described in equations 2, 3, and 4. These equations provide a framework to maintain consistency and coherence within the neutrosophic set, ensuring that the memberships align with the intended meaning and logical relationships among the different scores. By adhering to these conditions, we can effectively capture the subtle difference and uncertainties inherent in the scale while maintaining a coherent and meaningful representation.

Similarly, the additional score 5 is derived using the same method and principle as score 2. It serves to bridge a considerable difference and provides a more precise representation of the underlying shades of variation within the scale. By introducing this intermediary score, we effectively ensure that the scale accurately reflects the subtle distinctions and variations that may exist, allowing for a more comprehensive understanding of the evaluated criteria.

Step 2: Verify the linguistic variable of PNS. In this step, the new linguistic variable for PNS as proposed in Table 2 is verified using all the three conditions stated in (2), (3) and (4). Using the definition stated in (1), let

$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \text{ then}$$

$$A = \left\{ \begin{array}{l} \langle x, 0.10, 0.80, 0.90 \rangle \\ \langle x, 0.20, 0.70, 0.80 \rangle \\ \langle x, 0.35, 0.60, 0.60 \rangle \\ \langle x, 0.50, 0.40, 0.45 \rangle \\ \langle x, 0.65, 0.30, 0.25 \rangle \\ \langle x, 0.80, 0.20, 0.15 \rangle \\ \langle x, 0.90, 0.10, 0.10 \rangle \end{array} \right\}$$

is a PN-subset of X where $T, I, F \in [0, 1]$. These numbers comply the conditions $0.10, 0.80, 0.90 \in [0, 1]$. To provide evidence and demonstrate the validity of the proposed number, conditions in (2), (3) and (4) are examined as follows:

Let $A_1 = \{\langle 0.1, 0.8, 0.9 \rangle\}$ and $A_2 = \{\langle 0.2, 0.7, 0.8 \rangle\}$ is the PN numbers. Conditions (2), (3) and (4) are verified as follows:

$$(i) \quad T + F = 0.1 + 0.9 = 1.0 \leq 1$$

$$T + F = 0.2 + 0.7 = 0.9 \leq 1$$

Then the inequality property is defined for condition in (2), $0 \leq T + F \leq 1$.

$$(ii) \quad T^2 + F^2 = (0.1)^2 + (0.9)^2 = 0.82 \leq 1$$

$$T^2 + F^2 = (0.2)^2 + (0.7)^2 = 0.53 \leq 1$$

Then the inequality property is defined for condition in (3) $0 \leq T^2 + F^2 \leq 1$.

$$(iii) \quad T^2 + I^2 + F^2 = (0.1)^2 + (0.8)^2 + (0.9)^2 = 1.46 \leq 2$$

$$T^2 + I^2 + F^2 = (0.2)^2 + (0.7)^2 + (0.8)^2 = 1.17 \leq 2$$

Then the inequality property is defined for condition in (4) $0 \leq T^2 + I^2 + F^2 \leq 2$.

Table 3 below shows the verification for each defined Pythagorean neutrosophic number where each set satisfies all the three conditions stated in (2), (3) and (4).

T	I	F	$T + F \leq 1$	$T^2 + F^2 \leq 1$	$T^2 + I^2 + F^2 \leq 2$
0.1	0.8	0.9	1	0.82	1.46
0.2	0.7	0.8	1	0.68	1.17

0.35	0.6	0.6	0.95	0.4825	0.8425
0.5	0.4	0.45	0.95	0.4525	0.6125
0.65	0.3	0.25	0.9	0.485	0.575
0.8	0.2	0.15	0.95	0.6625	0.7025
0.9	0.1	0.1	1	0.82	0.83

3.4 The PNS DEMATEL Procedures

The present discourse outlines the methodology of PNS DEMATEL incorporating the novel linguistic variable.

Step 1: Construct the initial direct influence matrix. A direct-relation matrix with Pythagorean neutrosophic number were constructed based on decision makers' (DM) preference. The score is determined using seven linguistic scales that ranged from no influence to very high influence based on Pythagorean neutrosophic linguistic variable. The k th DM gave the PNS score x_{ij}^k and the notation of x_{ij} shows the degree to which DM believes criteria i affects criteria j and the diagonal components are set to zero for decision making, where he matrix contains PNS in the form $x_{ij}^k = \langle T_{ij}, I_{ij}, F_{ij} \rangle$:

$$X^k = \begin{bmatrix} 0 & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & 0 & \cdots & x_{2n}^k \\ \vdots & \vdots & 0 & \vdots \\ x_{n1}^k & x_{n2}^k & \cdots & 0 \end{bmatrix}$$

Step 2: Compute the average matrix A_{ij} . By combining the l experts' opinions, the average direct-influence matrix $A = [a_{ij}]_{n \times n}$ can be obtained by

$$A_{ij}^k = \frac{1}{l} \sum_{k=1}^l x_{ij}^k, \quad i, j = 1, 2, \dots, n \quad (5)$$

Step 3: Deneutrosophication to obtain the total average crisp matrix B_{ij} . To deneutrosophicate the Pythagorean neutrosophic number $(T_A(x), I_A(x), F_A(x))$ to a crisp value, the deneutrosophication formula is used as follows:

$$B_{ij} = \frac{T_A(x) + 0.5I_A(x)}{1.5} \quad (6)$$

Step 4: Normalizing the direct- influence matrix. The normalized direct-influence matrix $Z = [z_{ij}]_{n \times n}$ can be achieved by using

$$Z = \frac{B}{s} \quad (7)$$

where $s = \max_{1 \leq i \leq n} \sum_{j=1}^n b_{ij}$. All elements in the matrix Z are complying with $0 \leq z_{ij} < 1$.

Step 5: Construct the total-influence matrix T_{ij} . From the normalized matrix Z , the total relation matrix T is calculated using

$$T = Z(I - Z)^{-1} \quad (8)$$

where I is the identity matrix.

Step 6: Calculating the sums of the rows and columns.

In this step, the vectors R and C representing the sum of the rows and the sum of columns from the total-influence matrix T are defined by the following formulas:

$$R = [r_i]_{n \times 1} = \left[\sum_{j=1}^n t_{ij} \right]_{n \times 1} \quad (9)$$

$$C = [c_i]_{1 \times n} = \left[\sum_{i=1}^n t_{ij} \right]_{1 \times n}^T \quad (10)$$

The $R + C$ and $R - C$ values are calculated in which these values reflect the importance and relation values, respectively.

Step 7: Determine the threshold value. In this step, the threshold value referred as Θ is calculated by measuring the average of the element in matrix T . This value then become the basis of determining the new matrix T in terms of 0 and 1. Matrix T elements are considered to be one if they are more than the threshold value Θ and zero otherwise.

Step 8: Construct network relationship map (NRM). The graph is plotted using $(R + C, R - C)$ data set. The $R + C$ is labelled on the horizontal axis and $R - C$ is on vertical axis. The network relationship map's advantages can reflect the MCDM flow. Each graph node represents the object examined, while the arc between two nodes shows the direction and strength of the influence relationship.

4. Illustrative Example: Barriers of Halal accreditation among SMEs

In this section, an example is illustrated to show the applicability of the linguistic variable in decision making under PNS environment in accordance to the eight-step proposed algorithm as above. The case study involved is on assessing halal accreditation barriers among SMEs in Malaysia. Seven barriers towards the adoption of halal certification through assessment accreditation (HCAA) are adopted from a study by Khan et al. [7] where five members of expert's panel were involved to get their feedback. The barriers and their descriptions were presented in the Table 4 below.

Table 4: Barriers towards the adoption of HCAA

Barrier	Description
B1 - Lack of top management support	Lack of management support in terms of dedication and involvement to adopt the HCAA
B2 - Weak enforcement and compliance of halal logo	Less effective enforcement policies for the compliance of the halal standard provided by the credible halal certification body
B3 - Lack of consumer awareness	Lack of organisation's interest to educate the consumers regarding the halal product and services
B4 -Scarcity of halal certifying professionals	The number of available halal certifying professional is not enough to fulfil the demand of HCAA
B5 - Lack of infrastructure facilities for the halal Product	Lack of infrastructure to develop the required workforce skills and competencies and effective production facilities to produce the halal product
B6 - Lack of clear understanding of halal and additive issues for halal products	Lack of the clear understanding of halal additive related issues due to the less research in this area
B7 - Lack of research and development (R&D) in halal for commercialisation	Lack of research and development to understand the halal, educate the customers, and provide support for the development of halal business

Linguistic judgements from each expert are transformed into a non-negative 7×7 matrix $X^k = [x_{ij}^k]$ where k is the number of experts. Table 5 below shows the initial direct-relation matrix obtained from E_1 . Four other similar

results were also obtained from E_2 , E_3 , E_4 and E_5 as shown in Appendix. The aggregated matrix A_{ij} was calculated using Eq. (5) to represent the assessment made by six experts as shown in Table 6. Table 7 presents the denotroscopic crisp matrix B_{ij} which was obtained using Eq. (6). A normalized matrix Z_{ij} was then calculated using Eq. (7) as presented in Table 8. Table 9 presents the total relation matrix T_{ij} calculated using Eq. (8).

The sum of rows and columns are calculated using Eq. (9) and (10) as shown in Table 10. Based on the $R-C$ values, the barriers were separated into 'cause group' and 'effect group'. The barriers in the cause group were lack of top management support, lack of consumer awareness, scarcity of halal certifying professionals and lack of infrastructure facilities for the halal product. On the other hand, the barrier in the effect group were weak enforcement and compliance of halal logo, lack of clear understanding of halal and additive issues for halal products and lack of research and development (R&D) in halal for commercialisation. The importance of barrier was then determined based on the $R+C$ value where lack of clear understanding of halal and additive issues for halal products become the most important barrier in effect group.

Table 5: Initial direct-relation matrix, X_{ij} ¹

Barriers	B1	B2	B3	B4	B5	B6	B7
B1	0	1	2	7	4	6	6
B2	1	0	4	1	2	1	3
B3	6	2	0	4	1	6	1
B4	4	5	2	0	5	5	4
B5	1	6	1	4	0	2	4
B6	2	4	5	2	1	0	2
B7	1	1	1	4	3	4	0

Table 6: Average matrix, A_{ij}

Barriers	B1	B2	B3	B4	B5	B6	B7
B1	0	$\langle 0.19, 0.72, 0.80 \rangle$	$\langle 0.22, 0.70, 0.76 \rangle$	$\langle 0.84, 0.16, 0.13 \rangle$	$\langle 0.44, 0.48, 0.51 \rangle$	$\langle 0.76, 0.22, 0.18 \rangle$	$\langle 0.81, 0.18, 0.15 \rangle$
B2	$\langle 0.19, 0.72, 0.80 \rangle$	0	$\langle 0.47, 0.46, 0.47 \rangle$	$\langle 0.19, 0.72, 0.80 \rangle$	$\langle 0.19, 0.72, 0.80 \rangle$	$\langle 0.17, 0.74, 0.82 \rangle$	$\langle 0.27, 0.66, 0.70 \rangle$
B3	$\langle 0.86, 0.14, 0.12 \rangle$	$\langle 0.14, 0.76, 0.86 \rangle$	0	$\langle 0.50, 0.42, 0.44 \rangle$	$\langle 0.19, 0.72, 0.80 \rangle$	$\langle 0.81, 0.18, 0.15 \rangle$	$\langle 0.19, 0.72, 0.80 \rangle$
B4	$\langle 0.56, 0.36, 0.37 \rangle$	$\langle 0.56, 0.36, 0.37 \rangle$	$\langle 0.19, 0.72, 0.80 \rangle$	0	$\langle 0.68, 0.28, 0.25 \rangle$	$\langle 0.59, 0.34, 0.33 \rangle$	$\langle 0.53, 0.40, 0.40 \rangle$
B5	$\langle 0.22, 0.70, 0.76 \rangle$	$\langle 0.76, 0.22, 0.18 \rangle$	$\langle 0.17, 0.74, 0.82 \rangle$	$\langle 0.53, 0.40, 0.40 \rangle$	0	$\langle 0.24, 0.68, 0.74 \rangle$	$\langle 0.41, 0.52, 0.54 \rangle$

B6	$\langle 0.24, 0.68, 0.74 \rangle$	$\langle 0.47, 0.46, 0.47 \rangle$	$\langle 0.50, 0.44, 0.43 \rangle$	$\langle 0.24, 0.68, 0.74 \rangle$	$\langle 0.22, 0.70, 0.76 \rangle$	0	$\langle 0.21, 0.70, 0.78 \rangle$
B7	$\langle 0.19, 0.72, 0.80 \rangle$	$\langle 0.22, 0.70, 0.76 \rangle$	$\langle 0.17, 0.74, 0.82 \rangle$	$\langle 0.44, 0.50, 0.50 \rangle$	$\langle 0.24, 0.68, 0.74 \rangle$	$\langle 0.21, 0.70, 0.78 \rangle$	0

Table 7: The aggregated crisp matrix B_{ij}

Barriers	B1	B2	B3	B4	B5	B6	B7
B1	0.0000	0.3667	0.3800	0.6133	0.4533	0.5800	0.6000
B2	0.3667	0.0000	0.4667	0.3667	0.3667	0.3600	0.4000
B3	0.6200	0.3467	0.0000	0.4733	0.3667	0.6000	0.3667
B4	0.4933	0.4933	0.3667	0.0000	0.5467	0.5067	0.4867
B5	0.3800	0.5800	0.3600	0.4867	0.0000	0.3867	0.4467
B6	0.3867	0.4667	0.4800	0.3867	0.3800	0.0000	0.3733
B7	0.3667	0.3800	0.3600	0.4600	0.3867	0.3733	0.0000

Table 8: Normalized matrix, Z_{ij}

Barriers	B1	B2	B3	B4	B5	B6	B7
B1	0.0000	0.1225	0.1269	0.2049	0.1514	0.1938	0.2004
B2	0.1225	0.0000	0.1559	0.1225	0.1225	0.1203	0.1336
B3	0.2071	0.1158	0.0000	0.1581	0.1225	0.2004	0.1225
B4	0.1648	0.1648	0.1225	0.0000	0.1826	0.1693	0.1626
B5	0.1269	0.1938	0.1203	0.1626	0.0000	0.1292	0.1492
B6	0.1292	0.1559	0.1604	0.1292	0.1269	0.0000	0.1247
B7	0.1225	0.1269	0.1203	0.1537	0.1292	0.1247	0.0000

Table 9: Total influence matrix, T_{ij}

Barriers	B1	B2	B3	B4	B5	B6	B7
B1	1.0255	1.1490	1.0742	1.2586	1.1262	1.2567	1.2206
B2	0.9344	0.8327	0.9073	0.9818	0.9058	0.9864	0.9612
B3	1.1445	1.0847	0.9094	1.1652	1.0486	1.2054	1.1045
B4	1.1340	1.1490	1.0412	1.0529	1.1174	1.2030	1.1589
B5	1.0267	1.0905	0.9655	1.1077	0.8855	1.0878	1.0669
B6	0.9842	1.0131	0.9532	1.0338	0.9524	0.9263	0.9999
B7	0.9340	0.9477	0.8800	1.0065	0.9131	0.9898	0.8450

Table 10: Rankings of barriers

Barrier	R	C	R+C	R-C	Cause/Effect
B1	8.1108	7.1832	15.2941	0.9276	Cause
B2	6.5096	7.2667	13.7763	-0.7571	Effect
B3	7.6622	6.7309	14.3932	0.9313	Cause
B4	7.8564	7.6065	15.4629	0.2498	Cause
B5	7.2306	6.9490	14.1797	0.2816	Cause
B6	6.8630	7.6554	14.5184	-0.7923	Effect
B7	6.5160	7.3569	13.8730	-0.8409	Effect

Network relationship map as presented in Figure 3 below was constructed using R+C and R-C values. The importance order of barrier is obtained as B4 > B1 > B6 > B3 > B5 > B7 > B2 where “scarcity of halal certifying

professionals” become the most influential barrier and “weak enforcement and compliance of halal logo” become the least influential barrier.

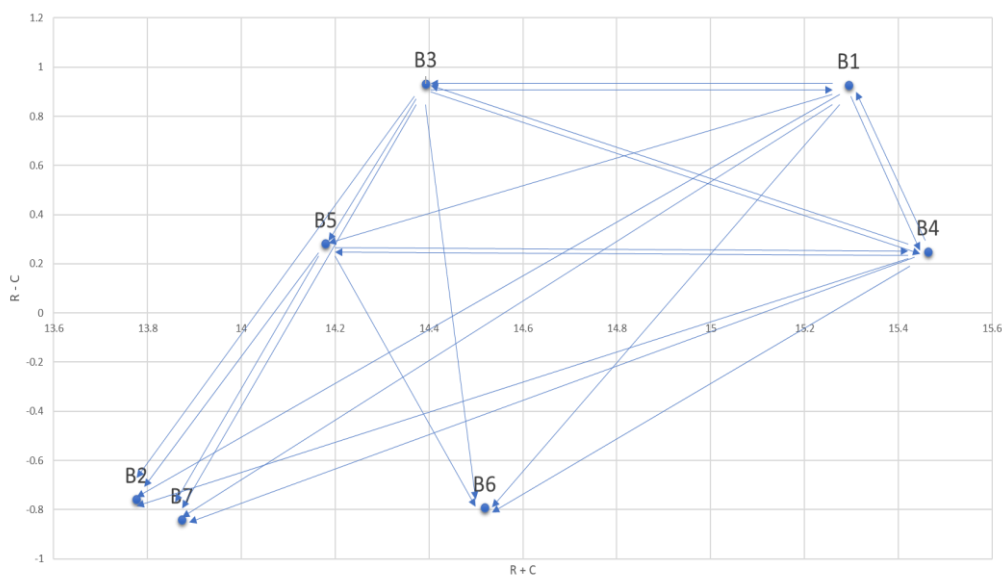


Figure 3: Network Relationship Map

5. Comparative analysis

The ranking obtained from the proposed PNS-DEMATEL approach is subjected to a comparative analysis to validate its feasibility. We compare the PNS-DEMATEL method with DEMATEL and Pythagorean fuzzy DEMATEL as shown in Table 10 below.

Table 11: Comparative Analysis

Method	Order of Barrier	Cause	Effect
DEMATEL	B4 > B1 > B6 > B3 > B5 > B7 > B2 $\theta = 0.426693$	B1, B3, B4, B5	B2, B6, B7
PF - DEMATEL	B4 > B1 > B6 > B3 > B5 > B7 > B2 $\theta = 0.343457$	B1, B3, B4, B5	B2, B6, B7
PNS - DEMATEL	B4 > B1 > B6 > B3 > B5 > B7 > B2 $\theta = 1.035688$	B1, B3, B4, B5	B2, B6, B7

6. Conclusion

The consistency of the barrier ranking is evident across all three comparative sets. Employing the proposed 7-scale linguistic variables and weighted average method appears to have no significant different on the outcome. In this research article, we have presented a method for developing linguistic variables for PNS. The lack of linguistic variables in PNS can make it difficult for users to express their opinions and judgments in a meaningful and consistent way. By developing linguistic variables for PNS, we can provide a more intuitive and expressive way for users to interact with the framework and make more informed decisions. Our proposed method involves defining the universe of discourse and the set to be evaluated, determining the linguistic terms and their associated membership functions, developing the Pythagorean neutrosophic linguistic variable, and evaluating the linguistic variable using PNS. By utilizing a 7-term linguistic scale, additional options are available compared to the 5-term linguistic scale, where the difference between set is much greater in 5-term linguistic scale. It can also potentially reduce response bias by providing respondents with more options to choose from by allowing a wider range of responses and reduces the likelihood of respondents feeling compelled to choose a middle or neutral option, which might occur more frequently with a 5-term scale. A 7-term scale provides more gradations or levels of measurement compared to a 5-term scale. This increased granularity allows for a more enhance understanding of respondents' attitudes, opinions, or perceptions. It can capture subtler differences and provide more detailed insights into the variable being measured. Furthermore, a 7-term scale provides more sensitivity in capturing variations and it can capture subtler differences and provide more detailed insights into the variable being measured. Moreover, this approach effectively addresses the uncertainty that commonly emerges in decision-

making problems thus improving the expressiveness and interpretability of the PNS framework and make it a more useful tool for decision-making in situations where the truth membership and false membership is dependent.

Acknowledgments

We would like to express our gratitude to UiTM Malaysia for MYRA GRANT - 600-RMC/GPM LPHD 5/3 (097/2022) for providing financial support for this research project.

References

- [1] Si, S. L., You, X. Y., Liu, H. C., & Zhang, P. (2018). DEMATEL Technique: A Systematic Review of the State-of-the-Art Literature on Methodologies and Applications. *Mathematical Problems in Engineering*, 2018(1). <https://doi.org/10.1155/2018/3696457>
- [2] F. Al-Sharqi, A. G. Ahmad, A. Al Quran, Mapping on interval complex neutrosophic soft sets, *International Journal of Neutrosophic Science*, vol.19(4), pp.77-85, 2022.
- [3] F. Al-Sharqi, A.G. Ahmad, A. Al-Quran, Fuzzy parameterized-interval complex neutrosophic soft sets and their applications under uncertainty, *Journal of Intelligent and Fuzzy Systems*, vol. 44, pp.1453–1477, 2023.
- [4] F. Al-Sharqi, A. G. Ahmad and A. Al-Quran, Similarity Measures on Interval-Complex Neutrosophic Soft Sets with Applications to Decision Making and Medical Diagnosis under Uncertainty. *Neutrosophic Sets and Systems*, 51(1) (2022), 495-515.
- [5] Mohammed Abed, M., Hassan, N., & Al-Sharq, F. (2022). On Neutrosophic Multiplication Module. *Neutrosophic Sets and Systems*, 49(1), 12
- [6] Mamites, I., Almerino, P., Sitoy, R., Atibing, N. M., Almerino, J. G., Cebe, D., Ybañez, R., Tandag, J., Villaganas, M. A., Lumayag, C., Plando, D., Añero, M., Acebes, H. M., Maturan, F., Evangelista, S. S., Aro, J. L., Himang, C., & Ocampo, L. (2022). Factors Influencing Teaching Quality in Universities: Analyzing Causal Relationships Based on Neutrosophic DEMATEL. *Education Research International*, 2022. <https://doi.org/10.1155/2022/9475254>
- [7] Khan, S., Khan, M. I., & Haleem, A. (2019). Evaluation of barriers in the adoption of halal certification: a fuzzy DEMATEL approach. *Journal of Modelling in Management*, 14(1), 153–174. <https://doi.org/10.1108/JM2-03-2018-0031>
- [8] Zulkifli, N., Abdullah, L., & Garg, H. (2021). An Integrated Interval-Valued Intuitionistic Fuzzy Vague Set and Their Linguistic Variables. *International Journal of Fuzzy Systems*, 23(1), 182–193. <https://doi.org/10.1007/s40815-020-01011-8>
- [9] Rahim, N., Abdullah, L., & Yusoff, B. (2020). A border approximation area approach considering bipolar neutrosophic linguistic variable for sustainable energy selection. *Sustainability (Switzerland)*, 12(10). <https://doi.org/10.3390/SU12103971>
- [10] Hajiyan, H., & Badakhshan, P. (2020). A DEMATEL-Fuzzy ANP Hybrid Method for Supplier Selection Based on IT Security Factors A DEMATEL-Fuzzy ANP Hybrid Method for Supplier Selection Based on IT Security Factors. *December*.
- [11] Abdullah, L., & Goh, P. (2019). Decision making method based on Pythagorean fuzzy sets and its application to solid waste management. *Complex and Intelligent Systems*, 5(2), 185–198. <https://doi.org/10.1007/s40747-019-0100-9>
- [12] Smarandache, F. (2006). Neutrosophic set - A generalization of the intuitionistic fuzzy set. *2006 IEEE International Conference on Granular Computing*, 1(1), 38–42. <https://doi.org/10.1109/grc.2006.1635754>
- [13] Prema, R., & Radha, R. (2022). Generalized Neutrosophic Pythagorean Set. *International Research Journal of Modernization in Engineering Technology and Science*, 11, 1571–1575. <https://doi.org/10.56726/irjmets31596>
- [14] Al-Quran, A., Hashim, H., & Abdullah, L. (2020). A hybrid approach of interval neutrosophic vague sets and DEMATEL with new linguistic variable. *Symmetry*, 12(2), 1–15. <https://doi.org/10.3390/sym12020275>
- [15] Haleem, A., Khan, M. I., & Khan, S. (2020). Halal certification, the inadequacy of its adoption, modelling and strategising the efforts. *Journal of Islamic Marketing*, 11(2), 393–413. <https://doi.org/10.1108/JIMA-05-2017-0062>
- [16] Khan, S., Haleem, A., & Khan, M. I. (2021). Risk management in Halal supply chain: an integrated fuzzy Delphi and DEMATEL approach. *Journal of Modelling in Management*, 16(1), 172–214. <https://doi.org/10.1108/JM2-09-2019-0228>
- [17] Radha, R., & Stanis Arul Mary, A. (2021). Neutrosophic Pythagorean Soft Set With T and F as Dependent Neutrosophic Components. *Neutrosophic Sets and Systems*, 42(Dm), 65–78. <https://doi.org/10.5281/zenodo.4711505>
- [18] Biswas, P., Pramanik, S., & Giri, B. C. (2016). TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Computing and Applications*, 27(3), 727–737. <https://doi.org/10.1007/s00521-015-1891-2>

Appendix

Expert 2-5 Direct Influential Matrix

E2	B1	B2	B3	B4	B5	B6	B7	
B1		0	2	3	7	3	5	6
B2		3	0	3	1	2	1	3
B3		7	1	0	4	2	7	1
B4		5	5	2	0	4	4	5
B5		3	5	1	5	0	1	3
B6		3	4	4	3	3	0	1
B7		2	3	2	5	1	2	0

E3	B1	B2	B3	B4	B5	B6	B7	
B1		0	3	1	6	4	7	7
B2		2	0	4	3	1	3	1
B3		6	2	0	3	1	5	2
B4		4	4	1	0	5	5	4
B5		1	7	3	4	0	2	4
B6		1	3	5	1	1	0	2
B7		3	1	3	3	3	1	0

E4	B1	B2	B3	B4	B5	B6	B7	
B1		0	1	1	6	3	5	5
B2		2	0	3	2	3	1	2
B3		7	1	0	4	2	7	3
B4		4	4	1	0	6	4	5
B5		3	5	2	5	0	3	3
B6		3	3	3	2	3	0	3
B7		1	3	1	3	2	2	0

E5	B1	B2	B3	B4	B5	B6	B7	
B1		0	2	3	6	4	4	7
B2		1	0	5	2	1	2	3
B3		7	1	0	5	3	6	2
B4		5	4	3	0	6	5	3
B5		2	6	1	3	0	3	3

B6	2	5	3	3	2	0	2
B7	2	2	1	3	2	2	0