Scrutinization of a Neutrosophic Fuzzy Erlangian Queuing Model Using a Parametric Programming Technique

P. Yasodai *, W. Ritha

Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University,
Tiruchirappalli, Tamil Nadu, India.
Emails: yasodairajendran@gmail.com; ritha_prakash@yahoo.co.in

Abstract
This article examines an Erlangian queuing model in a neutrosophic fuzzy environment. The inter-arrival rates and service rates are trapezoidal neutrosophic fuzzy numbers integrated into the Erlangian queuing model. The membership functions of the performance metrics of the corresponding queuing model have been outlined using parametric programming techniques in accordance with the (σ, β, γ)- cuts and Zadeh's extension principle. The neutrosophic fuzzy queues are converted into a family of crisp queues using this principle. The applicability of the provided approach for various cutting possibilities is highlighted by concrete examples.

Keywords: Fuzzy Sets; Membership Functions, Neutrosophic Fuzzy Erlangian Queuing Model; Trapezoidal Neutrosophic fuzzy number; Parametric Programming, (σ, β, γ)-cuts, Zadeh's Extension Principle; Queue length; Waiting time.

1. Introduction

Queuing Theory is an important facet of operational research. Applications of waiting line is very essential in day-to-day life but also in sequence of networks, medical diagnosis, computer programming, banking sectors, telecommunications, manufacturing firms and production systems. Many researchers have investigated the optimization problems of queuing systems, as well as their fine structure and techniques in several Markovian lines. Chen has discussed the queuing system in a fuzzy environment using Zadeh's extension principle. Kao et al derived the membership functions of the system measures for fuzzy queues using parametric programming technique. There is a diversity of methods in the literature of recent years.

At first, the intuitionistic fuzzy set was created by Atanassov. The generalisation of intuitionistic fuzzy sets is the neutrosophic set, which was introduced by Florentin Smarandache. Compared to the fuzzy set put forth by Zadeh, it is a powerful tool for dealing with ambiguity because it simultaneously takes into account an object's membership, indeterminacy, and non-membership degree. Additionally, Florentin Smarandache has studied a number of ideas, including neutrosophic measure, neutrosophic logic, neutrosophic probability, etc. The functions and characteristics of a single-valued Neutrosophic set were studied by Wang et al.

This paper is structured as follows: second segment provides some basic definitions of this research work. Third segment details a mathematical model which figures queue length and waiting time. Fourth segment describes the parametric programming technique of a neutrosophic fuzzy Erlangian queuing model with (α, β, γ)-cuts and Zadeh's extension principle. Fifth segment illustrates a numerical example. Sixth segment concludes the paper.

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2. Preliminaries

2.1 Fuzzy Set

C is a fuzzy set defined on U and can be written as a collection of ordered pairs if U is a universe of discourse and x is a particular element of U and $\phi_{\tilde{C}}(x)):U\to[0,1]$

$$\tilde{C}=\{(x,\ \phi_{\tilde{C}}(x))\ /\ x\in U\}$$

2.2 α -cut

The α -cut of a fuzzy number $\tilde{C}$ is defined as $\tilde{C}=\{\ x:\phi_{\tilde{C}}(x)\geq\alpha,\ \alpha\in[0,1]\}$.

2.3 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{c}$ is said to be trapezoidal fuzzy number if and only if there exists real numbers $c_1\leq c_2\leq c_3\leq c_4$, Such that:

$$\phi_{\tilde{c}}(x)=\begin{cases}\dfrac{x-c_1}{c_2-c_1}, & c_1\leq x\leq c_2\\ 1, & c_2\leq x\leq c_3\\ \dfrac{x-c_4}{c_3-c_4}, & c_3\leq x\leq c_4\\ 0, & otherwise\end{cases}$$

It is denoted by $\tilde{c}=(c_1,c_2,c_3,c_4)$ or $\tilde{c}=(c_1/c_2/c_3/c_4)$.

2.4 Neutrosophic fuzzy set

A neutrosophic fuzzy set $\tilde{N}$ be the subset of universe of discourse U is denoted by $\tilde{R}$ and defined by $\tilde{R}=\{x,M_{\tilde{R}}(x),I_{\tilde{R}}(x),N_{\tilde{R}}(x);\ x\in U\}$. Each element of R has a truth, indeterminacy, falsity functions and $M_{\tilde{R}}(x),I_{\tilde{R}}(x),N_{\tilde{R}}(x):U\to[0,1]$, Such that $0\leq M_{\tilde{R}}(x),I_{\tilde{R}}(x),N_{\tilde{R}}(x)\leq 3$.

2.5 Trapezoidal Neutrosophic Fuzzy Number

A trapezoidal neutrosophic fuzzy number $\tilde{r}$ with parameters $m_1\leq m_2\leq m_3\leq m_4$, $i_1\leq i_2\leq i_3\leq i_4$ & $n_1\leq n_2\leq n_3\leq n_4$ denoted as $\tilde{r}=\langle(m_1,m_2,m_3,m_4),(i_1,i_2,i_3,i_4),(n_1,n_2,n_3,n_4)\rangle$. Its truth-Membership function $M_{\tilde{R}}(x)$, Indeterminacy-membership function $I_{\tilde{R}}(x)$ and falsity membership function $Q_{\tilde{R}}(x)$ are defined as follows

$$M_{\tilde{R}}(x)=\begin{cases}\dfrac{x-m_1}{m_2-m_1}, & m_1\leq x\leq m_2\\ 1, & m_2\leq x\leq m_3\\ \dfrac{x-m_4}{m_3-m_4}, & m_3\leq x\leq m_4\\ 0, & otherwise\end{cases}$$

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$$I_{\tilde{R}}(x)=\begin{cases}\dfrac{i_2-x}{i_2-i_1} & , \; i_1\le x\le i_2\\ 0 & , \; i_2\le x\le i_3\\ \dfrac{x-i_3}{i_4-i_3} & , \; i_3\le x\le i_4\\ 1 & , \; otherwise\end{cases}$$

and

$$Q_{\tilde{R}}(x)=\begin{cases}\dfrac{n_2-x}{n_2-n_1} & , \; n_1\le x\le n_2\\ 0 & , \; n_2\le x\le n_3\\ \dfrac{x-n_3}{n_4-n_3} & , \; n_3\le x\le n_4\\ 1 & , \; otherwise\end{cases}$$

In the above definition, when $m_2=m_3$, $i_2=i_3$ and $n_2=n_3$ then we will get a neutrosophic triangular fuzzy number.

2.6 $(\alpha,\beta,\gamma)-$ Cut

The $(\alpha,\beta,\gamma)-$cut of a trapezoidal neutrosophic fuzzy number is defined as follows:

$N_{(\alpha,\beta,\gamma)}=\left[N_1(\alpha),N_2(\alpha)\right];\left[N_1^{'}(\beta),N_2^{'}(\beta)\right];\left[N_1^{''}(\gamma),N_2^{''}(\gamma)\right]$; Such that $0\le\alpha+\beta+\gamma\le 3$.

Where $\left[N_1(\alpha),N_2(\alpha)\right]=\left[\left(n_1^M+\alpha\left(n_2^M-n_1^M\right)\right),\left(n_4^M-\alpha\left(n_4^M-n_3^M\right)\right)\right]$

$\left[N_1^{'}(\beta),N_2^{'}(\beta)\right]=\left[\left(n_2^I-\beta\left(n_2^I-n_1^I\right)\right),\left(n_3^I+\beta\left(n_4^I-n_3^I\right)\right)\right]$

$\left[N_1^{''}(\gamma),N_2^{''}(\gamma)\right]=\left[\left(n_2^Q-\gamma\left(n_2^Q-n_1^Q\right)\right),\left(n_3^Q+\gamma\left(n_4^Q-n_3^Q\right)\right)\right]$

2.7 Interval Analysis Arithmetic

Let I_1 and I_2 be two interval numbers defined by ordered pair of real numbers with lower and upper bounds. Consider $I_1=[p,q],\ p<q$ and $I_1=[r,s],\ r<s$ and $I_1*I_2=[p,\,q]*[r,\,s]$, where $*=[+,\,-,\,\times,\,\div]$ symbolically.

(i) $I_1+I_2=\left[p+r,\ q+s\right]$

(ii) $I_1-I_2=[p-s,\ q-r]$

(iii) $I_1\bullet I_2=[\min(pr,ps,qr,qs),\max(pr,ps,qr,qs)]$

(iv) $I_1\div I_2=[p,q]\bullet\left[\dfrac{1}{s},\dfrac{1}{r}\right]$, provided $s,r\neq 0$

(v) $\alpha[p,q]=\begin{cases}[\alpha p,\alpha q], & \alpha>0\\ [\alpha q,\alpha p], & \alpha<0\end{cases}$

2.8 Zadeh's Extension Principle

Let the inter arrival rate $\tilde{A}$ and service rate $\tilde{S}$ are all fuzzy numbers. The system performance measure $P(x,y)$ is defined as

$$\phi_{p(\tilde{A},\tilde{S})}(Z)=\underset{\substack{x\in X\\ y\in Y}}{Sup}\left\{\min(\phi_{\tilde{A}}(x),\phi_{\tilde{S}}(y))/Z=p(x,y)\right\}$$

3. Model Description: Single Server Erlangian Queuing Model

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Consider an Erlangian queuing system in k-phases single server facility, denoted by $M / ME_k / 1$; in which arrival occurs as exponential distribution with rate λ and service time according to Erlang's k distribution with rate μ. It consists of single service channels and a chain of 'k' similar stages gaining an average service time individually. The service discipline is first come first served and the system capacity are infinite. The Execution proportions of this queuing model are

(i) Expected queue length $\left(N_q\right)$

$$N_q = \frac{k+1}{2k}\left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$$

(ii) Expected waiting time $\left(T_q\right)$

$$T_q = \frac{k+1}{2k}\left(\frac{\lambda}{\mu(\mu-\lambda)}\right)$$

4. Parametric Programming Approach of a Neutrosophic Fuzzy Erlangian Queuing Model

In this section, we scrutinize a single server Erlangian queuing model in neutrosophic fuzzy environment. Consider a single server neutrosophic fuzzy Erlangian queuing model with four phases. The inter arrival time $\tilde{A}$ and service time $\tilde{S}$ are denoted by the following sets:

$$\tilde{A} = \left\{(a, M_{\tilde{A}}(a), I_{\tilde{A}}(a), Q_{\tilde{A}}(a)) / a \in X\right\}$$

$$\tilde{S} = \left\{(s, M_{\tilde{S}}(s), I_{\tilde{S}}(s), Q_{\tilde{S}}(s)) / s \in Y\right\}$$

Where X and Y are crisp sets of neutrosophic fuzzy inter arrival time and neutrosophic fuzzy service time and the corresponding membership functions are $\phi_{\tilde{A}}(a)$ and $\phi_{\tilde{S}}(s)$. (α,β,γ) -cut of inter arrival time $\tilde{A}$ and service time $\tilde{S}$ are defined by

$$\tilde{A}(\alpha,\beta,\gamma) = \left\{a \in X / M_{\tilde{A}}(a) \geq \alpha, I_{\tilde{A}}(a) \leq \beta, Q_{\tilde{A}}(a) \leq \gamma\right\}$$

$$\tilde{S}(\alpha,\beta,\gamma) = \left\{s \in Y / M_{\tilde{S}}(s) \geq \alpha, I_{\tilde{S}}(s) \leq \beta, Q_{\tilde{S}}(s) \leq \gamma\right\}$$

The crisp subsets of X and Y are $\tilde{A}(\alpha,\beta,\gamma)$ and $\tilde{S}(\alpha,\beta,\gamma)$. Using (α,β,γ) -cuts, the arrival rate and service rate can be represented by various levels of confidence intervals [0, 1]. The neutrosophic fuzzy queue can be reduced to a family of crisp queues with (α,β,γ) -cuts

$\left\{\tilde{A}(\alpha,\beta,\gamma); 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1\right\}$ and $\left\{\tilde{S}(\alpha,\beta,\gamma); 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1\right\}$.

Thus the crisp queue can be got from the neutrosophic fuzzy queue for different value of (α,β,γ) -cuts. The membership function of the performance measures $p(\tilde{A},\tilde{S})$ and the truth membership function, indeterminacy membership function and falsity membership function defined by using zadeh's extension principle, which are denoted by

$$M_{p(a,s)}(Z) = Sup\left\{\min(\phi_{\tilde{A}}(a), \phi_{\tilde{S}}(s)) / Z = p(a,s) / a \in X, s \in Y\right\}$$

$$I_{p(a,s)}(Z) = Inf\left\{\min(\phi_{\tilde{A}}(a), \phi_{\tilde{S}}(s)) / Z = p(a,s) / a \in X, s \in Y\right\}$$

$$Q_{p(a,s)}(Z) = Inf\left\{\min(\phi_{\tilde{A}}(a), \phi_{\tilde{S}}(s)) / Z = p(a,s) / a \in X, s \in Y\right\}$$

The parametric programming technique is used for finding upper and lower bounds of (α,β,γ) -cuts of $\phi_{p(a,s)}(Z)$ which are as follows:

$l_{p(\alpha,\beta,\gamma)} = \min p(a,s)$; Such that $l_{\tilde{A}(\alpha,\beta,\gamma)} \leq a \leq u_{\tilde{A}(\alpha,\beta,\gamma)}$ and $l_{\tilde{S}(\alpha,\beta,\gamma)} \leq s \leq u_{\tilde{S}(\alpha,\beta,\gamma)}$

$u_{p(\alpha,\beta,\gamma)} = \max p(a,s)$; Such that $l_{A(\alpha,\beta,\gamma)} \leq a \leq u_{A(\alpha,\beta,\gamma)}$ and $l_{S(\alpha,\beta,\gamma)} \leq s \leq u_{S(\alpha,\beta,\gamma)}$

Where $a \in \tilde{A}(\alpha,\beta,\gamma)$ and $s \in \tilde{S}(\alpha,\beta,\gamma)$.

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Both the upper and lower bounds $l_{p(\alpha,\beta,\gamma)}$ and $u_{p(\alpha,\beta,\gamma)}$ are iinvertible with respect to (α,β,γ). The left shape Function L(z) and right shape function R(z) can be obtained from $\left(l_{p(\alpha,\beta,\gamma)}\right)^{-1}$ and $\left(u_{p(\alpha,\beta,\gamma)}\right)^{-1}$.

The truth membership function is defined by

$$M_{P(\tilde{A},\tilde{S})}(z)=\begin{cases} L_M(z), & z_1^M \le z \le z_2^M \\ R_M(z), & z_3^M \le z \le z_4^M \\ 0 & , \ otherwise \end{cases}$$

Where $z_1^M \le z \le z_4^M$ and $L_M(z_1^M)=R_M(z_4^M)=0$.

The indeterminacy membership function is defined by

$$I_{P(\tilde{A},\tilde{S})}(z)=\begin{cases} L_I(z), & z_1^I \le z \le z_2^I \\ R_I(z), & z_3^I \le z \le z_4^I \\ 0 & , \ otherwise \end{cases}$$

Where $z_1^I \le z \le z_4^I$ and $L_I(z_1^I)=R_I(z_4^I)=0$.

The falsity membership function is defined by

$$Q_{P(\tilde{A},\tilde{S})}(z)=\begin{cases} L_Q(z), & z_1^Q \le z \le z_2^Q \\ R_Q(z), & z_3^Q \le z \le z_4^Q \\ 0 & , \ otherwise \end{cases}$$

Where $z_1^Q \le z \le z_4^Q$ and $L_Q(z_1^Q)=R_Q(z_4^Q)=0$.

5. Numerical Illustration

Let us contemplate the arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ are trapezoidal neutrosophic fuzzy numbers with Phase k=4.

$$\tilde{\lambda}=[(2,3,4,5)(3,4,5,6)(5,6,7,8)]$$

$$\tilde{\mu}=[(12,13,14,15)(13,14,15,16)(15,16,17,18)]$$

The (α,β,γ) - cut of $\tilde{\lambda}$ and $\tilde{\mu}$ are

$$\tilde{\lambda}=[(2+\alpha,5-\alpha)(4-\beta,5+\beta)(6-\gamma,7+\gamma)]$$

$$\tilde{\mu}=[(12+\alpha,15-\alpha)(14-\beta,15+\beta)(16-\gamma,17+\gamma)]$$

5.1 Expected Queue Length $\left(\tilde{N}_q\right)$

$$\tilde{N}_q=\frac{k+1}{2k}\left(\frac{\tilde{\lambda}^2}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})}\right)$$

The parametric programming for the queue length is

$$l_{N_q(\alpha)}=Min\left\{\frac{5}{8}\left(\frac{x^2}{y(y-x)}\right)\right\}, \text{ such that } 2+\alpha \prec x \prec 5-\alpha \ and \ 12+\alpha \prec y \prec 15-\alpha$$

$$u_{N_q(\alpha)}=Max\left\{\frac{5}{8}\left(\frac{x^2}{y(y-x)}\right)\right\}, \text{ such that } 2+\alpha \prec x \prec 5-\alpha \ and \ 12+\alpha \prec y \prec 15-\alpha$$

$$For \ \ l_{N_q(\alpha)}, x \to 2+\alpha \ and \ y \to 15-\alpha$$

$$l_{N_q(\alpha)}=\frac{5\alpha^2+20\alpha+20}{16\alpha^2-344\alpha+1560}$$

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$$For\ \ u_{N_q(\alpha)}, x \to 5-\alpha \ and\ y \to 12+\alpha$$

$$u_{N_q(\alpha)} = \frac{5\alpha^2 - 50\alpha + 125}{16\alpha^2 + 248\alpha + 672}$$

The truth membership function is

$$M_{\tilde{N}_q}(Z) = \begin{cases} L_M(z), & \left[l_{N_q(\alpha)}\right]_{\alpha=0} \le z \le \left[l_{N_q(\alpha)}\right]_{\alpha=1} \\ R_M(z), & \left[u_{N_q(\alpha)}\right]_{\alpha=1} \le z \le \left[u_{N_q(\alpha)}\right]_{\alpha=0} \\ 0 & ,\ otherwise \end{cases}$$

which is defined as

$$M_{\tilde{N}_q}(Z) = \begin{cases} \dfrac{(344z+20) \pm \sqrt{18496z^2 + 46240z}}{32z-10} & , 0.0128 \le z \le 0.0365 \\ \dfrac{-(248z+50) \pm \sqrt{18496z^2 + 46240z}}{32z-10} & , 0.0855 \le z \le 0.1860 \\ 0 & ,\ otherwise \end{cases}$$

$$l_{N_q(\beta)} = Min\left\{\frac{5}{8}\left(\frac{x^2}{y(y-x)}\right)\right\}, \text{ such that } 4-\beta \prec x \prec 5+\beta \ and\ 14-\beta \prec y \prec 15+\beta$$

$$u_{N_q(\beta)} = Max\left\{\frac{5}{8}\left(\frac{x^2}{y(y-x)}\right)\right\}, \text{ such that } 4-\beta \prec x \prec 5+\beta \ and\ 14-\beta \prec y \prec 15+\beta$$

$$For\ \ l_{N_q(\beta)}, x \to 4-\beta \ and\ y \to 15+\beta$$

$$l_{N_q(\beta)} = \frac{5\beta^2 - 40\beta + 80}{16\beta^2 + 328\beta + 1320}$$

$$For\ \ u_{N_q(\beta)}, x \to 5+\beta \ and\ y \to 14-\beta$$

$$u_{N_q(\beta)} = \frac{5\beta^2 + 50\beta + 125}{16\beta^2 - 296\beta + 1008}$$

The indeterminacy membership function is

$$I_{\tilde{N}_q}(Z) = \begin{cases} L_I(z), & \left[l_{N_q(\beta)}\right]_{\beta=1} \le z \le \left[l_{N_q(\alpha)}\right]_{\beta=0} \\ R_I(z), & \left[u_{N_q(\beta)}\right]_{\beta=0} \le z \le \left[u_{N_q(\alpha)}\right]_{\beta=1} \\ 0 & ,\ otherwise \end{cases}$$

which is estimated as

$$I_{\tilde{N}_q}(Z) = \begin{cases} \dfrac{-(328z+40) \pm \sqrt{23104z^2 + 57760z}}{32z-10} & , 0.0270 \le z \le 0.0606 \\ \dfrac{(296z+50) \pm \sqrt{23104z^2 + 57760z}}{32z-10} & , 0.1240 \le z \le 0.2473 \\ 0 & ,\ otherwise \end{cases}$$

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$$l_{N_q(\gamma)} = Min\left\{\frac{5}{8}\left(\frac{x^2}{y(y-x)}\right)\right\}, \text{ such that } 6-\gamma \prec x \prec 7+\gamma \; and \; 16-\gamma \prec y \prec 17+\gamma$$

$$u_{N_q(\gamma)} = Max\left\{\frac{5}{8}\left(\frac{x^2}{y(y-x)}\right)\right\}, \text{ such that } 6-\gamma \prec x \prec 7+\gamma \; and \; 16-\gamma \prec y \prec 17+\gamma$$

$For \;\; l_{N_q(\gamma)}, x \rightarrow 6-\gamma \; and \; y \rightarrow 17+\gamma$

$$l_{N_q(\gamma)} = \frac{5\gamma^2-60\gamma+180}{16\gamma^2+360\gamma+1496}$$

$For \;\; u_{N_q(\gamma)}, x \rightarrow 7+\gamma \; and \; y \rightarrow 16-\gamma$

$$u_{N_q(\gamma)} = \frac{5\gamma^2+70\gamma+245}{16\gamma^2-328\gamma+1152}$$

The falsity membership function is

$$Q_{\tilde{N}_q}(Z) = \begin{cases} L_Q(z), & \left[l_{N_q(\gamma)}\right]_{\gamma=1} \le z \le \left[l_{N_q(\gamma)}\right]_{\gamma 0} \\ R_Q(z), & \left[u_{N_q(\gamma)}\right]_{\gamma=0} \le z \le \left[u_{N_q(\gamma)}\right]_{\gamma=1} \\ 0 & , \; otherwise \end{cases}$$

which is defined as

$$Q_{\tilde{N}_q}(Z) = \begin{cases} \dfrac{-(360z+60) \pm \sqrt{33856z^2+84640z}}{32z-10}, & 0.0668 \le z \le 0.1203 \\ \dfrac{(328z+70) \pm \sqrt{33856z^2+84640z}}{32z-10}, & 0.2127 \le z \le 0.3810 \\ 0 & , \; otherwise \end{cases}$$

For different values of $(\alpha, \beta, \gamma) \in [0,1]$, the lower and upper bounds of expected number of customers in the queue is tabulated as follows.

Table 1: Expected Queue Length

α	$l_{N_q(\alpha)}$	$u_{N_q(\alpha)}$	β, γ	$l_{N_q(\beta)}$	$u_{N_q(\beta)}$	$l_{N_q(\gamma)}$	$u_{N_q(\gamma)}$
0.0	0.0128	0.1860	0.0	0.0606	0.1240	0.1203	0.2127
0.1	0.0145	0.1722	0.1	0.0562	0.1329	0.1136	0.2252
0.2	0.0162	0.1595	0.2	0.0521	0.1424	0.1072	0.2384
0.3	0.0181	0.1477	0.3	0.0482	0.1526	0.1012	0.2525
0.4	0.0202	0.1367	0.4	0.0446	0.1634	0.0955	0.2676
0.5	0.0224	0.1266	0.5	0.0412	0.1751	0.0900	0.2835
0.6	0.0249	0.1171	0.6	0.0380	0.1875	0.0849	0.3005

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0.7	0.0275	0.1083	0.7	0.0350	0.2009	0.0800	0.3187
0.8	0.0303	0.1002	0.8	0.0321	0.2152	0.0754	0.3381
0.9	0.0333	0.0925	0.9	0.0295	0.2307	0.0710	0.3588
1	0.0365	0.0855	1	0.0270	0.2473	0.0668	0.3810

5.2 Expected Waiting Time $\left(\tilde{T}_q\right)$

$$\tilde{T}_q = \frac{k+1}{2k}\left(\frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})}\right)$$

The parametric programming of an expected waiting time is

$$l_{T_q(\alpha)} = Min\left\{\frac{5}{8}\left(\frac{x}{y(y-x)}\right)\right\}, \text{ such that } 2+\alpha \prec x \prec 5-\alpha \text{ and } 12+\alpha \prec y \prec 15-\alpha$$

$$u_{T_q(\alpha)} = Max\left\{\frac{5}{8}\left(\frac{x}{y(y-x)}\right)\right\}, \text{ such that } 2+\alpha \prec x \prec 5-\alpha \text{ and } 12+\alpha \prec y \prec 15-\alpha$$

$For\ \ l_{T_q(\alpha)}, x \to 2+\alpha\ and\ y \to 15-\alpha$

$$l_{T_q(\alpha)} = \frac{10+5\alpha}{16\alpha^2-344\alpha+1560}$$

$For\ \ u_{T_q(\alpha)}, x \to 5-\alpha\ and\ y \to 12+\alpha$

$$u_{T_q(\alpha)} = \frac{25-5\alpha}{16\alpha^2+248\alpha+672}$$

The truth membership function is

$$M_{\tilde{T}_q}(Z) = \begin{cases} L_M(z), & \left[l_{T_q(\alpha)}\right]_{\alpha=0} \le z \le \left[l_{T_q(\alpha)}\right]_{\alpha=1} \\ R_M(z), & \left[u_{T_q(\alpha)}\right]_{\alpha=1} \le z \le \left[u_{T_q(\alpha)}\right]_{\alpha=0} \\ 0 & , otherwise \end{cases}$$

which is estimated as

$$M_{\tilde{T}_q}(Z) = \begin{cases} \dfrac{(344z+5) \pm \sqrt{18496z^2+4080z+25}}{32z} & , 0.0064 \le z \le 0.0122 \\ \dfrac{-(248z+5) \pm \sqrt{18496z^2+4080z+25}}{32z} & , 0.0214 \le z \le 0.0372 \\ 0 & , otherwise \end{cases}$$

$$l_{T_q(\beta)} = Min\left\{\frac{5}{8}\left(\frac{x}{y(y-x)}\right)\right\}, \text{ such that } 4-\beta \prec x \prec 5+\beta \text{ and } 14-\beta \prec y \prec 15+\beta$$

$$u_{T_q(\beta)} = Max\left\{\frac{5}{8}\left(\frac{x}{y(y-x)}\right)\right\}, \text{ such that } 4-\beta \prec x \prec 5+\beta \text{ and } 14-\beta \prec y \prec 15+\beta$$

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For $l_{T_q(\beta)}, x \to 4-\beta \textit{ and } y \to 15+\beta$

$$l_{T_q(\beta)} = \frac{20-5\beta}{16\beta^2+328\beta+1320}$$

For $u_{T_q(\beta)}, x \to 5+\beta \textit{ and } y \to 14-\beta$

$$u_{T_q(\beta)} = \frac{25+5\beta}{16\beta^2-296\beta+1008}$$

The indeterminacy membership function is

$$I_{\tilde{T}_q}(Z)=\begin{cases} L_I(z), & \left[l_{T_q(\beta)}\right]_{\beta=1} \le z \le \left[l_{T_q(\beta)}\right]_{\beta=0} \\ R_I(z), & \left[u_{T_q(\beta)}\right]_{\beta=0} \le z \le \left[u_{T_q(\beta)}\right]_{\beta=1} \\ 0 & , \ otherwise \end{cases}$$

which is defined as

$$I_{\tilde{T}_q}(Z)=\begin{cases} \dfrac{-(328z+5)\pm\sqrt{23104z^2+4560z+25}}{32z}, & 0.0090 \le z \le 0.0152 \\ \dfrac{(296z+5)\pm\sqrt{23104z^2+4560z+25}}{32z} & ,0.0248 \le z \le 0.0412 \\ 0 & , \ otherwise \end{cases}$$

$$l_{T_q(\gamma)} = Min\left\{\frac{5}{8}\left(\frac{x}{y(y-x)}\right)\right\}, \text{ such that } \ 6-\gamma \prec x \prec 7+\gamma \textit{ and } 16-\gamma \prec y \prec 17+\gamma$$

$$u_{T_q(\gamma)} = Max\left\{\frac{5}{8}\left(\frac{x}{y(y-x)}\right)\right\}, \text{ such that } \ 6-\gamma \prec x \prec 7+\gamma \textit{ and } 16-\gamma \prec y \prec 17+\gamma$$

For $l_{T_q(\gamma)}, x \to 6-\gamma \textit{ and } y \to 17+\gamma$

$$l_{T_q(\gamma)} = \frac{30-5\gamma}{16\gamma^2+360\gamma+1496}$$

For $u_{T_q(\gamma)}, x \to 7+\gamma \textit{ and } y \to 16-\gamma$

$$u_{T_q(\beta)} = \frac{35+5\gamma}{16\gamma^2-328\gamma+1152}$$

The falsity membership function is

$$Q_{\tilde{T}_q}(Z)=\begin{cases} L_Q(z), & \left[l_{T_q(\gamma)}\right]_{\gamma=1} \le z \le \left[l_{T_q(\gamma)}\right]_{\gamma=0} \\ R_Q(z), & \left[u_{T_q(\gamma)}\right]_{\gamma=0} \le z \le \left[u_{T_q(\gamma)}\right]_{\gamma=1} \\ 0 & , \ otherwise \end{cases}$$

which is defined as

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$$Q_{\tilde{T}_q}(Z)=\begin{cases} \dfrac{-(360z+5)\pm\sqrt{33856z^2+5520z+25}}{32z} & ,0.0134\le z\le 0.0201 \\ \dfrac{(328z+5)\pm\sqrt{33856z^2+5520z+25}}{32z} & ,0.0304\le z\le 0.0476 \\ 0 & ,\ otherwise \end{cases}$$

For different values of $(\alpha,\beta,\gamma)\in[0,1]$, the lower and upper bounds of expected waiting time of a customers in the queue are tabulated as follows.

Table 2: Expected Waiting Time

α	$l_{T_q(\alpha)}$	$u_{T_q(\alpha)}$	β,γ	$l_{T_q(\beta)}$	$u_{T_q(\beta)}$	$l_{T_q(\gamma)}$	$u_{T_q(\gamma)}$
0	0.0064	0.0372	0	0.0152	0.0248	0.0201	0.0304
0.1	0.0069	0.0352	0.1	0.0144	0.0261	0.0193	0.0317
0.2	0.0074	0.0332	0.2	0.0137	0.0274	0.0185	0.0331
0.3	0.0079	0.0314	0.3	0.0130	0.0288	0.0178	0.0346
0.4	0.0084	0.0297	0.4	0.0124	0.0303	0.0170	0.0362
0.5	0.0090	0.0281	0.5	0.0118	0.0318	0.0164	0.0378
0.6	0.0096	0.0266	0.6	0.0112	0.0335	0.0157	0.0395
0.7	0.0102	0.0252	0.7	0.0106	0.0352	0.0151	0.0414
0.8	0.0108	0.0238	0.8	0.0100	0.0371	0.0145	0.0433
0.9	0.0115	0.0226	0.9	0.0095	0.0391	0.0139	0.0454
1	0.0122	0.0214	1	0.0090	0.0412	0.0134	0.0476

5. Conclusion

This paper investigates the qualitative behavior of single server neutrosophic fuzzy Erlangian queuing model with phase service. Due to the uncertainty in the parameter estimates for queuing decision models, neutrosophic environments can be used to cope with system performance measures. Trapezoidal neutrosophic fuzzy numbers are used to express the service time and arrival time of the proposed model. Using the (α,β,γ) -cuts and Zadeh's extension principle, we determine the queue length and waiting time. Also, the parametric programming technique is utilized for finding upper and lower bounds of queue length and waiting time for different (α,β,γ) -cuts. The neutrosophic fuzzy queue can be converted into a family of crisp queues. The performance measurements of the suggested model, which are created using the truth, indeterminacy and falsity membership degrees of a trapezoidal neutrosophic fuzzy number are demonstrated by an example.

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