Neutrosophic (m, n)-ideals in Semigroups

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Abstract

Our main results, we give the concept neutrosophic (m, n)-ideals and investigate the properties of neutrosophic (m, n)-ideals in semigroups. Finally, we study the minimality and primality of neutrosophic (m, n)-ideals in semigroups.

Keywords: (m, n)-ideals; neutrosophic (m, n)-ideals; minimal neutrosophic (m, n)-ideals; prime of neutrosophic (m, n)-ideals

1 introduction

Since the classical set is invalid to handle the described uncertainties, Zadeh[20] first gave the definition of a fuzzy set. K. T. Atanassov[3] introduced the notion of an intuitionistic fuzzy set as a generalization of a fuzzy set. In fact from his point of view for each element of the universe there are two degrees, one a degree of membership to a vague subset and the other is a degree of non-membership to that given subset. The concept of neutrosophic set (NPS) developed by Smarandache,[17] is a more general concept which extends the concepts of the classic set and fuzzy set. Neutrosophic set is a generalization of the fuzzy set and intuitionistic fuzzy set, where the truth-membership, indeterminacy-membership, and falsity-membership are represented independently. Vasantha Kandasamy and Florentin Smarandache[6] multiset neutrosophic fields, neutrosophic rings,neutrosophic vector spaces,[9] neutrosophic groups,neutrosophic bigroups[6] and neutrosophic N-groups,[4] neutrosophic semigroups,[6] neutrosophic bisemigroups, and neutrosophic, etc. The concept of fuzzy sets in semigroup was introduced by Kuroki,[11] introduced and studied fuzzy (left, right) ideals and fuzzy bi-ideals in semigroups. The theory of (m, n)-ideals in semigroups was studied by Lajos in 1963.[12] The notion of (m, n)-ideals of semigroups generalized the notion of one sided ideals of semigroups. The study of (m, n)-ideals in various algebraic structures had also been studied by many authors, for instance, Akram et al.,[2] Yaqoob and Chinram,[19] and many others. In 2019, Ahsan et al.[13] extended the notion of (m, n)-ideals in semigroups to the notion of fuzzy (m, n)-ideals in semigroups and they characterized (m, n)-regular semigroups by using fuzzy (m, n)-ideals. In 2022, T. Gaketem[5] discussed interval valued fuzzy almost (m, n)-ideals in semigroups. Recently, W. Nakkhasen[15] studied concept picture fuzzy (m, n)-ideals of semigroups and investigated some basic properties of picture fuzzy (m, n)-ideals of semigroups.

Our aim of this paper is to extend neutrosophic ideals to (m, n)-ideals in semigroups. Our main results is divided in the following sections, in section 2, we shall extend concepts of neutrosophic ideals to (m, n)-ideals in semigroups. In section 3, we give the concept neutrosophic (m, n)-ideals and investigated propertis of neutrosophic (m, n)-ideals insemigroups. Finally, we study minimality of neutrosophic (m, n)-ideals and prime fuzzy (m, n)-ideals in semigroups.

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2 Preliminaries

Before we discussed the concept of ideals in semigroups, fuzzy semigroups and neutrosophic ideal in semigrops and basic properties for the study of next sections.

A *subsemigroup* of a semigroup E is a non-empty set K of E such that $KK \subseteq K$. A *left (right) ideal* of a semigroup E is a non-empty set K of E such that $EK \subseteq K$ $(KE \subseteq K)$. By an *ideal* of a semigroup T, we mean a non-empty set of E which is both a left and a right ideal of E. A *regular* semigroup of E if for each $e \in E$ there exists $h \in E$ such that $e = ehe$.

Next, we reviews the smallest (m, n)-ideals and principal (m, n)-ideals in semigroups.

$$[K](m,n) = \bigcup_{r=1}^{m+n} K^r \cap K^m S K^n \text{ is the principal } (m,n)\text{-ideal by } K,$$

$$[K](m,0) = \bigcup_{r=1}^{m} K^r \cap K^m S \text{ is the principal } (m,0)\text{-ideal by } K \text{ and}$$

$$[K](0,n) = \bigcup_{r=1}^{n} K^r \cap S K^n \text{ is the principal } (0,n)\text{-ideal by } K,$$

i.e., the smallest (m, n)-ideal, the smallest $(m, 0)$-ideal and the smallest $(0, n)$-ideal of E containing K, respectively.

Lemma 2.1. [10] *Let E be a semigroup and m, n positive integers, $[u]_{(m,n)}$ the principal (m, n)-ideal generated by the element u. Then*

(1) $([u]_{(m,0)})^m E = u^m E$.

(2) $E([u]_{(0,n)})^n = Eu^n$.

(3) $([u]_{(m,0)})^m E([u]_{(0,n)})^n = u^m E u^n$.

Definition 2.2. [10] A subsemigroup K of a semigroup T is called an

(1) (m, n)-*ideal* of E if $K^m E K^n \subseteq K$for any $m, n \in \mathbb{N}$,

(2) $(m, 0)$-*ideal* of E if $K^m E \subseteq K$ for any $m \in \mathbb{N}$,

(3) $(0, n)$-*ideal* of E if $EK^n \subseteq K$ for any $n \in \mathbb{N}$.

For any $h_i \in [0, 1]$, $i \in \mathcal{F}$, define

$$\underset{i\in\mathcal{J}}{\vee} h_i := \sup_{i\in\mathcal{J}}\{h_i\} \quad \text{and} \quad \underset{i\in\mathcal{J}}{\wedge} h_i := \inf_{i\in\mathcal{J}}\{h_i\}.$$

We see that for any $h, r \in [0, 1]$, we have

$$h \vee r = \max\{h, r\} \quad \text{and} \quad h \wedge r = \min\{h, r\}.$$

The next, we review the definition of fuzzy sets, intuitionsic fuzzy sets and neutrosophic sets.

A **fuzzy set (fuzzy subset)** of a non-empty set E is a function $\vartheta : E \to [0, 1]$.

For any two fuzzy sets ϑ and ξ of a non-empty set E, define the symbol as follows:

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(1) $\vartheta \geq \xi \Leftrightarrow \vartheta(h) \geq \xi(h)$ for all $h \in E$,

(2) $\vartheta = \xi \Leftrightarrow \vartheta \geq \xi$ and $\xi \geq \vartheta$,

(3) $(\vartheta \wedge \xi)(h) = \min\{\vartheta(h), \xi(h)\} = \vartheta(h) \wedge \xi(h)$ for all $h \in E$,

(4) $(\vartheta \vee \xi)(h) = \max\{\vartheta(h), \xi(h)\} = \vartheta(h) \vee \xi(h)$ for all $h \in E$,

(5) $\vartheta \subseteq \xi$ if $\vartheta(h) \leq \xi(h)$ for all $h \in E$.

Definition 2.3. [13] A fuzzy set ϑ of a semigroup E is called a *fuzzy* (m, n)*-ideal* of E if $\vartheta(e_1 e_2 \cdots e_m k r_1 r_2 \cdots r_n) \geq \vartheta(e_1) \wedge \vartheta(e_2) \wedge \cdots \wedge \vartheta(e_m), \vartheta(r_1) \wedge \vartheta(r_2) \wedge \ldots \vartheta(r_n)$ for all $e_1, e_2, \ldots, e_m, k, r_1, r_2, \ldots r_n$ of E and $m, n \in \mathbb{N}$.

Definition 2.4. [17] A *neutrosophic set* (NPS) of a non-empty set E is a structure of the form $\vartheta := \{(e, \vartheta_T(e), \vartheta_I(e), \vartheta_F(e)) \mid e \in E\}$ where $\vartheta_T : E \to [0, 1]$ is called a *truth membership function*, $\vartheta_I : E \to [0, 1]$ is called a *indetreminate membership function*, and $\vartheta_F : E \to [0, 1]$ is called a *false membership function*

Definition 2.5. [6] For any non-empty subset K of set E. The characteristic neutrosophic function λ_K of K in E defined by

$$\lambda_K = \{(e, \lambda_{T_K}, \lambda_{I_K}, \lambda_{F_K}) \mid e \in E\},$$

where

$$\lambda_{T_K}(e) := \begin{cases} 1 & e \in K, \\ 0 & e \notin K, \end{cases}$$

$$\lambda_{I_K}(e) := \begin{cases} 0 & e \in K, \\ 1 & e \notin K, \end{cases}$$

$$\lambda_{F_K}(e) := \begin{cases} 1 & e \in K, \\ 0 & e \notin K. \end{cases}$$

The whole NPS λ_E in a semigroup E is defined $\lambda_E = \{1_{T_E}, 0_{I_E}, 1_{F_E}\}$, where λ_E is an NPS subset of E mapping every element of E to $\{1, 0, 1\}$.

Definition 2.6. [6] For two NPS set $\mathcal{A} = \{\vartheta_T, \vartheta_I, \vartheta_F\}$ and $\mathcal{B} = \{\xi_T, \xi_I, \xi_F\}$ in semigroup E, we define

(1) $\mathcal{A} \subseteq \mathcal{B} \Leftrightarrow \vartheta_T \leq \xi_T,\ \vartheta_I \geq \xi_I,\ \vartheta_F \leq \xi_F$,

(2) $\mathcal{A} \circ \mathcal{B} \Leftrightarrow \{(e, \vartheta_T \circ \xi_T,\ \vartheta_I \circ \xi_I,\ \vartheta_F \circ \xi_F) \mid e \in E\}$,
where

$$(\vartheta_T \circ \xi_T)(e) = \begin{cases} \bigvee\limits_{e=ph} \{\vartheta_T(p) \wedge \xi_T(h)\} & \text{if} \quad e = ph, \text{ for some } p, h \in E, \\ 0 & \text{otherwise,} \end{cases}$$

$$(\vartheta_I \circ \xi_I)(e) = \begin{cases} \bigwedge\limits_{e=ph} \{\vartheta_T(p) \vee \xi_T(h)\} & \text{if} \quad e = ph, \text{ for some } p, h \in E, \\ 1 & \text{otherwise,} \end{cases}$$

$$(\vartheta_F \circ \xi_F)(e) = \begin{cases} \bigvee\limits_{e=ph} \{\vartheta_I(p) \wedge \xi_I(h)\} & \text{if} \quad e = ph, \text{ for some } p, h \in E, \\ 0 & \text{otherwise,} \end{cases}$$

for all $e \in E$.

Definition 2.7. [6] Let ϑ be a NPS of a semigroup E is called

(1) a *neutrosophic subsemigroup* (NPSG) if $\vartheta_T(eh) \geq \vartheta_T(e) \wedge \vartheta_T(h),\ \vartheta_I(eh) \leq \vartheta_I(e) \vee \vartheta_I(h),\ \vartheta_F(eh) \geq \vartheta_F(e) \wedge \vartheta_F(h)$, for all $e, h \in E$,

(2) a *neutrosophic left ideal* (NPLI) if $\vartheta_T(eh) \geq \vartheta_T(h),\ \vartheta_I(eh) \leq \vartheta_I(h),\ \vartheta_F(eh) \geq \vartheta_F(h)$, for all $e, h \in E$,

(3) a *neutrosophic right ideal* (NPRI) if $\vartheta_T(eh) \geq \vartheta_T(e),\ \vartheta_I(eh) \leq \vartheta_I(e),\ \vartheta_F(eh) \geq \vartheta_F(e)$, for all $e, h \in E$,

(4) a *neutrosophic bi-ideal* (NPBI) if $\vartheta_T(ehk) \geq \vartheta_T(e) \wedge \vartheta_T(k),\ \vartheta_I(eh) \leq \vartheta_I(e) \vee \vartheta_I(k),\ \vartheta_F(eh) \geq \vartheta_F(e) \wedge \vartheta_F(k)$, for all $e, h, k \in E$.

3 Netrosophic (m,n)-ideals in Semigroups

Definition 3.1. An NPS ϑ of a semigroup E is called a *netrosophic* (m,n)*-ideal* (NP (m,n)-ideal) of E if $\vartheta_T(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \geq \vartheta_T(e_1)\wedge\vartheta_T(e_2)\wedge\cdots\wedge\vartheta_T(e_m),\vartheta_T(r_1)\wedge\vartheta_T(r_2)\wedge\cdots\wedge\vartheta_T(r_n)$, $\vartheta_I(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \leq \vartheta_I(e_1)\vee\vartheta_I(e_2)\vee\cdots\wedge\vartheta_I(e_m),\vartheta_I(r_1)\vee\vartheta_I(r_2)\vee\cdots\vee\vartheta_I(r_n)$ and $\vartheta_F(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \geq \vartheta_F(e_1)\wedge\vartheta_F(e_2)\wedge\cdots\wedge\vartheta_F(e_m),\vartheta_F(r_1)\wedge\vartheta_F(r_2)\wedge\cdots\wedge\vartheta_F(r_n)$ for all $e_1,e_2,\ldots,e_m,k,r_1,r_2,\ldots r_n$ of E and $m,n\in\mathbb{N}$.

Theorem 3.2. *Let $\{\vartheta_i \mid i\in\mathcal{J}\}$ be a family of NPS (m,n)-ideals of a semigroup E. Then $\underset{i\in\mathcal{F}}{\wedge}\vartheta_i$ is an IVF (m,n)-ideal of E, where $\vartheta_i=\{(e,\vartheta_{i_T},\vartheta_{i_I},\vartheta_{i_F})\mid e\in E\}$.*

Proof. Let $e,h\in E$. Then,

$$\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(eh)\geq\underset{i\in\mathcal{J}}{\wedge}\{\vartheta_{i_T}(e)\wedge\vartheta_{i_T}(h)\}=\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(e)\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(e),$$

$$\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(eh)\leq\underset{i\in\mathcal{J}}{\vee}\{\vartheta_{i_I}(e)\vee\vartheta_{i_I}(h)\}=\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(e)\vee\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(e),$$

and

$$\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(eh)\geq\underset{i\in\mathcal{J}}{\wedge}\{\vartheta_{i_F}(e)\wedge\vartheta_{i_F}(h)\}=\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(e)\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(e).$$

Thus, $\underset{i\in\mathcal{J}}{\wedge}\vartheta_i$ is an NPSG of E. Let $e_1,e_2,\ldots,e_m,k,r_1,r_2,\ldots r_n\in E$. Then,

$$\begin{array}{rcl}\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) & \geq & \underset{i\in\mathcal{J}}{\wedge}\{\vartheta_{i_T}(e_1)\wedge\vartheta_{i_T}(e_2)\cdots\wedge\vartheta_{i_T}(e_n)\\ & & \wedge\vartheta_{i_T}(r_1)\wedge\vartheta_{i_T}(r_2)\ldots\vartheta_{i_T}(r_n)\}\\ & = & \underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(e_1)\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(e_2)\cdots\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(e_n)\\ & & \wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(r_1)\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(r_2)\ldots\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_T}(r_n),\end{array}$$

$$\begin{array}{rcl}\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) & \leq & \underset{i\in\mathcal{J}}{\vee}\{\vartheta_{i_I}(e_1)\vee\vartheta_{i_I}(e_2)\cdots\vee\vartheta_{i_I}(e_n)\\ & & \wedge\vartheta_{i_I}(r_1)\vee\vartheta_{i_I}(r_2)\ldots\vartheta_{i_I}(r_n)\}\\ & = & \underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(e_1)\vee\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(e_2)\cdots\vee\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(e_n)\\ & & \vee\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_I}(r_1)\vee\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(r_2)\ldots\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_I}(r_n)\end{array}$$

and

$$\begin{array}{rcl}\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) & \geq & \underset{i\in\mathcal{J}}{\wedge}\{\vartheta_{i_F}(e_1)\wedge\vartheta_{i_F}(e_2)\cdots\wedge\vartheta_{i_F}(e_n)\\ & & \wedge\vartheta_{i_F}(r_1)\wedge\vartheta_{i_F}(r_2)\ldots\vartheta_{i_F}(r_n)\}\\ & = & \underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(e_1)\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(e_2)\cdots\wedge\underset{i\in\mathcal{J}}{\vee}\vartheta_{i_F}(e_n)\\ & & \vee\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(r_1)\wedge\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(r_2)\ldots\underset{i\in\mathcal{J}}{\wedge}\vartheta_{i_F}(r_n).\end{array}$$

Thus, $\underset{i\in\mathcal{J}}{\wedge}\vartheta_i$ is an NP (m,n)-ideal of E. ☐

Theorem 3.3. *Let K is an ideal of a semigroup E and m,n are positive integers. Then K is an (m,n)-ideal of E if and only if the characteristic neutrosophic function λ_K is an NP (m,n)-ideal of E.*

Proof. Suppose that K is an (m,n)-ideal of E. Then, $\lambda_{T_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)=1$, $\lambda_{I_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)=0$ and $\lambda_{F_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)=1$.

Let $e_1,e_2,\ldots e_m,k,r_1,r_2,\ldots,r_n\in E$. Then the following cases:

Case 1 If $e_i\notin K$ for some $i\in\{1,2,\ldots,m\}$, then $\lambda_{T_K}(e_i)=0$, $\lambda_{I_K}(e_i)=1$ and $\lambda_{F_K}(e_i)=0$ for some $i\in\{1,2,\ldots,m\}$. Thus, $\lambda_{T_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)\geq\lambda_{T_K}(e_1)\wedge\lambda_{T_K}(e_2)\wedge\cdots\wedge\lambda_{T_K}(e_m)\wedge\cdots\wedge\lambda_{T_K}(r_1)\wedge\lambda_{T_K}(r_2)\wedge\cdots\wedge\lambda_{T_K}(r_n)$, $\lambda_{I_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)\leq\lambda_{I_K}(e_1)\vee\lambda_{I_K}(e_2)\vee\cdots\vee\lambda_{I_K}(e_m)\vee\cdots\vee\lambda_{I_K}(r_1)\vee\lambda_{I_K}(r_2)\vee\cdots\vee\lambda_{I_K}(r_n)$ and $\lambda_{F_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)\geq\lambda_{F_K}(e_1)\wedge\lambda_{T_K}(e_2)\wedge\cdots\wedge\lambda_{F_K}(e_m)\wedge\cdots\wedge\lambda_{F_K}(r_1)\wedge\lambda_{F_K}(r_2)\wedge\cdots\wedge\lambda_{F_K}(r_n)$.

Case 2 If $r_j \notin K$ for some $j \in \{1, 2, \dots, n\}$, then $\lambda_{T_K}(r_j) = 0$, $\lambda_{I_K}(r_i) = 1$ and $\lambda_{F_K}(r_i) = 0$ for some $i \in \{1, 2, \dots, m\}$. Thus, $\lambda_{T_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \geq \lambda_{T_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{T_K}(e_m) \wedge \cdots \wedge \lambda_{T_K}(r_1) \wedge \lambda_{T_K}(r_2) \wedge \cdots \wedge \lambda_{T_K}(r_n)$, $\lambda_{I_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \leq \lambda_{I_K}(e_1) \vee \lambda_{I_K}(e_2) \vee \cdots \wedge \lambda_{I_K}(e_m) \vee \cdots \vee \lambda_{I_K}(r_1) \vee \lambda_{I_K}(r_2) \vee \cdots \vee \lambda_{I_K}(r_n)$ and $\lambda_{F_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \geq \lambda_{F_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{F_K}(e_m) \wedge \cdots \wedge \lambda_{F_K}(r_1) \wedge \lambda_{F_K}(r_2) \wedge \cdots \wedge \lambda_{F_K}(r_n)$.

Case 3 If $e_i, r_j \in K$ for each $i \in \{1, 2, \dots, m\}$ and $j \in \{1, 2, ..., n\}$, then $e_1e_2\cdots e_mzr_1r_2\cdots r_n \in K^mEK^n \subseteq K$. Thus $\lambda_{T_K}(e_i) = 1 = \lambda_{T_K}(r_j)$, $\lambda_{I_K}(e_i) = 0 = \lambda_{I_K}(r_j)$ and $\lambda_{F_K}(e_i) = 1 = \lambda_{F_K}(r_j)$ for each $i \in \{1, 2, \dots, m\}$ and $j \in \{1, 2, ..., n\}$. Hence, $\lambda_{T_K}(e_1e_2\cdots e_mkr_1, r_2\cdots r_n) = 1 \geq \lambda_{T_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{T_K}(e_m) \wedge \cdots \wedge \lambda_{T_K}(r_1) \wedge \lambda_{T_K}(r_2) \wedge \cdots \wedge \lambda_{T_K}(r_n)$, $\lambda_{I_K}(e_1e_2\cdots e_mkr_1, r_2\cdots r_n) = 0 \leq \lambda_{I_K}(e_1) \vee \lambda_{I_K}(e_2) \vee \cdots \vee \lambda_{I_K}(e_m) \vee \cdots \vee \lambda_{I_K}(r_1) \vee \lambda_{I_K}(r_2) \vee \cdots \vee \lambda_{I_K}(r_n)$ and $\lambda_{F_K}(e_1e_2\cdots e_mkr_1, r_2\cdots r_n) = 1 \geq \lambda_{F_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{F_K}(e_m) \wedge \cdots \wedge \lambda_{F_K}(r_1) \wedge \lambda_{F_K}(r_2) \wedge \cdots \wedge \lambda_{F_K}(r_n)$.

Therefore, λ_K is an NP (m, n)-ideal of E.

Conversely, suppose that λ_K is an NP (m, n)-ideal of E and let $e_1, e_2, \dots e_m, k, r_1, r_2, \dots, r_n \in E$. Then, $\lambda_{T_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \geq \lambda_{T_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{T_K}(e_m) \wedge \cdots \wedge \lambda_{T_K}(r_1) \wedge \lambda_{T_K}(r_2) \wedge \cdots \wedge \lambda_{T_K}(r_n)$, $\lambda_{I_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) \leq \lambda_{I_K}(e_1) \vee \lambda_{I_K}(e_2) \vee \cdots \vee \lambda_{I_K}(e_m) \vee \cdots \vee \lambda_{I_K}(r_1) \vee \lambda_{I_K}(r_2) \vee \cdots \vee \lambda_{I_K}(r_n)$ and $\lambda_{F_K}(e_1e_2\cdots e_mkr_1, r_2\cdots r_n) \geq \lambda_{F_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{F_K}(e_m) \wedge \cdots \wedge \lambda_{F_K}(r_1) \wedge \lambda_{F_K}(r_2) \wedge \cdots \vee \lambda_{F_K}(r_n)$. Thus, $\lambda_{T_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) = 1 = \lambda_{F_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n)$ and $\lambda_{I_K}(e_1e_2\cdots e_mkr_1r_2\cdots r_n) = 0$. It impiles that, $e_1e_2\cdots e_mkr_1r_2\cdots r_n \in K$. Hence, $K^mEK^n \subseteq K$. Therefore, K is an (m, n)-ideal of E. □

Theorem 3.4. *Let E be a semigroup and m, n be positive integers. Then K is an $(m, 0)$-ideal ($(0, n)$-ideal) of E if and only if the characteristic neutrosophic function λ_K is an NP an $(m, 0)$-ideal ($(0, n)$-ideal) of E.*

Proof. Suppose that K is an $(m, 0)$-ideal of E and let $e_1, e_2, \dots e_m, k \in E$. Then the following cases:

Case 1 If $e_i \notin K$ for some $i \in \{1, 2, \dots, m\}$, then $\lambda_{T_K}(e_i) = 0 = \lambda_{F_K}(e_i)$ and $\lambda_{I_K}(e_i) = 1$ for some $i \in \{1, 2, \dots, m\}$. Thus, $\lambda_{T_K}(e_1e_2\cdots e_mk) \geq \lambda_{T_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{T_K}(e_m)$, $\lambda_{I_K}(e_1e_2\cdots e_mk) \leq \lambda_{I_K}(e_1) \vee \lambda_{I_K}(e_2) \vee \cdots \vee \lambda_{I_K}(e_m)$ and $\lambda_{F_K}(e_1e_2\cdots e_mk) \geq \lambda_{F_K}(e_1) \wedge \lambda_{F_K}(e_2) \wedge \cdots \wedge \lambda_{F_K}(e_m)$.

Case 2 If $u_i \in K$ for each $i \in \{1, 2, \dots, m\}$, then $\lambda_{T_K}(e_i) = 1 = \lambda_{I_K}(e_i)$ and $\lambda_{F_K}(e_i) = 0$ for each $i \in \{1, 2, \dots, m\}$. Thus, $\lambda_{T_K}(e_1e_2\cdots e_mk) \geq \lambda_{T_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{T_K}(e_m)$, $\lambda_{I_K}(e_1e_2\cdots e_mk) \leq \lambda_{I_K}(e_1) \vee \lambda_{I_K}(e_2) \vee \cdots \vee \lambda_{I_K}(e_m)$ and $\lambda_{F_K}(e_1e_2\cdots e_mk) \geq \lambda_{F_K}(e_1) \wedge \lambda_{F_K}(e_2) \wedge \cdots \wedge \lambda_{F_K}(e_m)$.

Therefore, λ_K is an NP $(m, 0)$-ideal of E.

Conversely, suppose that λ_K is an NP $(m, 0)$-ideal of E. Then, $\lambda_{T_K}(e_1e_2\cdots e_mk) \geq \lambda_{T_K}(e_1) \wedge \lambda_{T_K}(e_2) \wedge \cdots \wedge \lambda_{T_K}(e_m)$, $\lambda_{I_K}(e_1e_2\cdots e_mk) \leq \lambda_{I_K}(e_1) \vee \lambda_{I_K}(e_2) \vee \cdots \vee \lambda_{I_K}(e_m)$ and $\lambda_{F_K}(e_1e_2\cdots e_mk) \geq \lambda_{F_K}(e_1) \wedge \lambda_{F_K}(e_2) \wedge \cdots \wedge \lambda_{F_K}(e_m)$. Thus, $\lambda_{T_K}(e_1e_2\cdots e_mk) = 1 = \lambda_{F_K}(e_1e_2\cdots e_mk)$ and $\lambda_{I_K}(e_1e_2\cdots e_mk) = 0$. It implies that, $e_1e_2\cdots e_mk \in K$. Hence, $K^mE \subseteq K$. Therefore K is an $(m, 0)$-ideal of E □

Definition 3.5. Let E be a semigroup and m, n be positive intergers. Then E is called (m, n)*-regular* if for each $e \in E$ there exists $h \in E$ such that $e = e^mze^n$ equivalently for each subset K of E if $K \subseteq K^mEK^n$ or for each element e of E, $e \in e^mEe^n$.

Lemma 3.6. *Let E be an (m, n)-regular of semigroup and m, n be positive intergers. Then every NP (m, n)-ideal of E is an NPBI of E.*

Proof. Suppose that ϑ is an NP (m, n)-ideal of E and $i, j, k \in E$. By assumption there exist $x, y \in E$ such that $ijk = i^mxi^njk^myk^n$. Thus,

$$\vartheta_T(ijk) = \vartheta_T(i^mxi^njk^myk^n) = \vartheta_T(i^m(xi^njk^my)k^n) \geq \vartheta_T(i^m) \wedge \vartheta_T(k^n) \geq \vartheta_T(i) \wedge \vartheta_T(k),$$

$$\vartheta_I(ijk) = \vartheta_I(i^mxi^njk^myk^n) = \vartheta_I(i^m(xi^njk^my)k^n) \leq \vartheta_I(i^m) \vee \vartheta_I(k^n) \leq \vartheta_I(i) \vee \vartheta_I(k),$$

and

$$\vartheta_F(ijk) = \vartheta_F(i^mxi^njk^myk^n) = \vartheta_F(i^m(xi^njk^my)k^n) \geq \vartheta_F(i^m) \wedge \vartheta_F(k^n) \leq \vartheta_F(i) \wedge \vartheta_F(k).$$

Hence, ϑ is an NPBI of E. □

Definition 3.7. An (m,n)-ideal K of a semigroup E is called a *minimal* if for every (m,n)-ideal of J of E such that $J \subseteq K$, we have $J = K$.

Definition 3.8. An NP (m,n)-ideal ϑ of a semigroup E is a *minimal* if for all NP (m,n)-ideal ξ of E such that $\xi \leq \vartheta$, then $\xi = \vartheta$.

Lemma 3.9. *For any non-empty subsets I and K of a semigroup E, we have $I \subseteq K$ if and only if $\lambda_I \leq \lambda_K$, where λ_I and λ_K are characteristic neutrosophic functions I and K respective.*

Theorem 3.10. *A non-empty subset K of a semigroup E is a minimal (m,n)-ideal if and only if λ_K is a minimal NP (m,n)-ideal.*

Proof. Let K be a minimal (m,n)-ideal of E. Then K is an (m,n)-ideal. Thus, by Theorem 3.3, λ_K is a NP (m,n)-ideal of E. Let I be an (m,n)-ideal of E such that $I \subseteq K$. Then by Theorem 3.3, λ_I is a NP (m,n)-ideal of E and $\lambda_I \leq \lambda_K$. Since K is a minimal (m,n)-ideal of E we have $J = K$. Thus, $\lambda_I = \lambda_K$. Hence, λ_K is minimal NP (m,n)-ideal of E.

Conversely, λ_K is minimal NP (m,n)-ideal of E. Then λ_K is an NP (m,n)-ideal of E. Thus, by Theorem 3.3, K is an (m,n)-ideal of E. Let λ_J be an NP (m,n)-ideal of E such that $\lambda_J \leq \lambda_K$. Then by Theorem 3.3, J is an (m,n)-ideal of E such that $J \subseteq K$. Since λ_K is minimal NP (m,n)-ideal of E. we have $\lambda_J = \lambda_K$. Thus, $J = K$. Hence, K is a minimal (m,n)-ideal of E. □

Definition 3.11. An (m,n)-ideal K of a semigroup E is called a *maximal* if for every (m,n)-ideal of J of E such that $K \subseteq J$, we have $J = K$.

Definition 3.12. An NP (m,n)-ideal ϑ of a semigroup E is a *maximal* if for all NP (m,n)-ideal ξ of E such that $\vartheta \leq \xi$, then $\xi = \vartheta$.

Theorem 3.13. *A non-empty subset K of a semigroup E is a maximal (m,n)-ideal if and only if λ_K is a maximal NP (m,n)-ideal.*

Proof. It follows form Theorem 3.10. □

4 Prime and Semiprime Neutrosophic (m,n)-ideals

In this section, we give relationship between prime, semiprime (m,n)-ideals and prime, semiprime NP (m,n)-ideals.

Definition 4.1. Let P be an (m,n)-ideal of a semigroup E is called

(1) *prime* if $eh \in P$ implies $e \in P$ or $h \in P$ for all $e,h \in E$,

(2) *semiprime* if $e^2 \in P$ implies $e \in P$ for all $e \in E$.

Definition 4.2. Let ϑ be a NP (m,n)-ideal of a semigroup is called

(1) *prime* if $\vartheta_T(eh) \leq \vartheta_T(e) \vee \vartheta_T(h)$, $\vartheta_I(eh) \geq \vartheta_I(e) \wedge \vartheta_I(h)$ and $\vartheta_F(eh) \leq \vartheta_F(e) \vee \vartheta_F(h)$ for all $e,h \in E$,

(2) *semiprime* if $\vartheta_T(e^2) \leq \vartheta_T(e)$, $\vartheta_I(e^2) \geq \vartheta_I(e)$ and $\vartheta_F(e^2) \leq \vartheta_F(e)$ for all $e \in E$.

Remark 4.3. Every prime (m,n)-ideal is semiprime (m,n)-ideal in a semigroup.

Theorem 4.4. *Let P a non-empty subset of a semigroup E. Then P is a prime (m,n)-ideal of S if and only if λ_P is a prime NP (m,n)-ideal of E.*

Proof. Suppose that P is a prime (m,n)-ideal of S. Then P is a (m,n)-ideal of S. Thus by Theorem 3.3 λ_P is a NP (m,n)-ideal of S. Let $e,h \in E$.

Case 1: If $eh \in P$, then $e \in P$ or $h \in P$. Thus $\lambda_{T_P}(eh) = 1 = \lambda_{T_P}(e)$ $\lambda_{I_P}(eh) = 0 = \lambda_{I_P}(e)$ or $\lambda_{T_P}(eh) = 1 = \lambda_{F_P}(h)$, $\lambda_{F_P}(eh) = 1 = \lambda_{F_P}(h)$ and $\lambda_{I_P}(eh) = 0 = \lambda_{I_P}(e)$ or $\lambda_{I_P}(eh) = 0 = \lambda_{I_P}(h)$. Hence, $\lambda_{T_P}(eh) \leq \lambda_{T_P}(e) \vee \lambda_{T_P}(h)$, $\lambda_{I_P}(eh) \geq \lambda_{I_P}(e) \wedge \lambda_{I_P}(h)$ and $\lambda_{F_P}(eh) \leq \lambda_{T_P}(e) \vee \lambda_{T_P}(h)$.

Case 2: If $eh \notin P$, then $\lambda_{T_P}(eh) = 0 = \lambda_{F_P}(eh)$ and $\lambda_{I_P}(eh) = 1$. Thus, $\lambda_{T_P}(eh) \leq \lambda_{T_P}(e) \vee \lambda_{T_P}(h)$, $\lambda_{I_P}(eh) \geq \lambda_{I_P}(e) \wedge \lambda_{I_P}(h)$ and $\lambda_{F_P}(eh) \leq \lambda_{T_P}(e) \vee \lambda_{T_P}(h)$.

From 2 case, we have λ_P is a prime NP (m,n)-ideal of E.

Conversely, suppose that λ_P is a prime NP (m,n)-ideal of E. Then λ_P is a NP (m,n)-ideal of E. Thus, by Theorem 3.3, P is an (m,n)-ideal of E. Let $e,h \in E$. If $e \notin P$ and $h \notin P$, then $\lambda_{T_P}(e) = 0 = \lambda_{T_P}(h)$ $\lambda_{I_P}(e) = 1 = \lambda_{I_P}(h)$ and $\lambda_{F_P}(e) = 0 = \lambda_{F_P}(h)$. By assumption, $\lambda_{T_P}(eh) \leq \lambda_{T_P}(e) \vee \lambda_{T_P}(h)$, $\lambda_{I_P}(eh) \leq \lambda_{I_P}(e) \wedge \lambda_{I_P}(h)$ and $\lambda_{F_P}(eh) \geq \lambda_{T_P}(e) \vee \lambda_{T_P}(h)$. Thus, $\lambda_{T_P}(eh) = 1 = \lambda_{T_P}(e)$, $\lambda_{I_P}(eh) = 0 = \lambda_{I_P}(e), \lambda_{T_P}(eh) = 0 = \lambda_{I_P}(h)$, $\lambda_{I_P}(eh) = 0 = \lambda_{I_P}(h)$ and $\lambda_{F_P}(eh) = 1$, $\lambda_{F_P}(e) = 1 = \lambda_{F_P}(h)$. It is a contradiction, so $e \in P$ or $h \in P$. Hence, P is a prime (m,n)-ideal of E. □

Theorem 4.5. *Let P a non-empty subset of a semigroup E. Then P is a semiprime (m,n)-ideal of E if and only if λ_P is a semiprime NP (m,n)-ideal of E.*

Proof. It follows from Theorem 4.4. □

5 Conclusion

In this paper, we give the concept of neutrosophic (m,n)-ideals and prove the basic properties of neutrosophic (m,n)-ideals in semigroups. Additionally, we study minimal and prime neutrosophic (m,n)-ideals in semigroups. In the future we extend study intuitionsic fuzzy (m,n)-ideal and essential neutrosophic (m,n)-ideals in semigroups.

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