On the Fuzzy Semi-Unital Rings and Their Algebraic Properties

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Abstract

Fuzzy rings are considered as generalizations of classical rings, where they are defined by using the fuzzical membership function. This paper is dedicated to defining and study fuzzy semi-unital ring of order n in a similar way to the same classical type of rings, where many elementary properties will be discussed in terms of theorems with many related examples that clarify the validity of this work.

Keywords: Fuzzy Set; Fuzzy ring; Fuzzy ideal; fuzzy semi-unital element.

1. Introduction and basic definitions

The concept of fuzzy subset was defined to explain the degree of truth and falsity [11], as follows:
A fuzzy subset μ of $X \neq \emptyset$ is $\mu: X \to [0,1]$, where μ represents the degree of membership.
Fuzzy sets and their generalization were very helpful in generalizing algebraic structures such as groups and rings [12-16].
The concept of semi-unital rings is a central concept in algebra, and it has many interesting properties and applications in pure algebra.
This motivates us to generalize it into fuzzy systems, where we recall some basic definitions and concepts.

Definition: [11]
Let λ, μ be two fuzzy subset of X, then:
1). $\lambda \subseteq \mu \Leftrightarrow \lambda(x) \leq \mu(x); x \in X\ (subset)$.
2). $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x); x \in X\ (equal)$.
We denote by $F(X)$ to the family of all fuzzy subsets of X.
Definition.
Let $\mu \in F(X)$, we say that μ is positive if $\mu(x) > 0$ for all $x \in X$.
Definition. [3]
Let $\mu \in F(X), t \in [0,1]$, we define the t-level subset of μ by $\mu_t = \{x \in X;\ \mu(x) \geq t\}$.
Definition. [1]
Let $X \neq \emptyset$ by a set with binary operation $(.)$ and $\lambda, \mu \in F(X)$, the product of λ, μ is defined as follow:

$$(\lambda \circ \mu)(x) = \begin{cases} sup\{min\{\lambda(y), \mu(z)\}\}; y, z \in X, x = yz \\ 0 \quad ; y, z \in X, x \neq yz \end{cases}$$

Definition. [1]
Let $X \neq \emptyset$ be a set, $\alpha \in X, t \in]0,1]$, the fuzzy point x_α is defined a follows:

$$\forall y \in X;\ x_\alpha(y) = \begin{cases} \alpha; x = y \\ 0; x \neq y \end{cases}$$

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Definition. [3]

The fuzzy subset $\mu: R \to [0,1]$ is called fuzzy subring of the ring R if:
1). $\mu(x) \geq min\{\mu(x), \mu(y)\}$.
2). $\mu(x.y) \geq min\{\mu(x), \mu(y)\}\ for\ all\ x, y \in R$.

2. Main discussion.

Theorem.

Let R be a ring and $A \subseteq R$, then A is a subring of R if and only if f_A is a fuzzy subring of R.

Proof.

Assume that A is a subring of R, we have two cases:
1). If $f_A(x) = 0$ or $f_A(y) = 0$,
then $min\{f_A(x), f_A(y)\} = 0$, thus:
$f_A(x - y) \geq min\{f_A(x), f_A(y)\}$,
$f_A(x.y) \geq min\{f_A(x), f_A(y)\}$.
2). If $f_A(x) = 1$ and $f_A(y) = 1$,
then $min\{f_A(x), f_A(y)\} = 1$, since $x.y, x - y \in A$,
then $f_A(x - y) = f(xy) = 1 \geq min\{f_A(x), f_A(y)\}$.
For the converse, suppose that f_A is a fuzzy subring of R.
Let $x, y \in A$, then $f_A(x) = f_A(y) = 1$.
On the other hand, we have:
$f_A(x - y) \geq min\{f_A(x), f_A(y)\} = 1 \Longrightarrow f_A(x - y) = 1 \Longrightarrow x - y \in A$.
$f_A(x.y) \geq \Longrightarrow f_A(x.y) = 1 \Longrightarrow xy \in A$, thus A is a subring of R.

Definition.

Let μ be a fuzzy subring of R, then $a_\alpha \in \mu$ is called fuzzy idempotent element of order m if m is the smallest positive integer such that ${a_\alpha}^{m+1} = a_\alpha$; $\alpha \in]0,1]$.

Definition.

The fuzzy idempotent element $a_\alpha \in \mu$ I called fuzzy central idempotent if:
$\forall\, r_\alpha \in \mu$: $e_\alpha \circ r_\alpha =: \ r_\alpha \circ e_\alpha$; $\alpha \in]0,1]$.
μ is called fuzzy central ring if all elements $a_\alpha \in \mu$ are fuzzy central.

Definition.

The element $a_\alpha \in \mu$ is called fuzzy regular element if there exists $x_\alpha \subset \mu$ such that:
$a_\alpha = a_\alpha \circ x_\alpha \circ a_\alpha$; $\alpha \in]0,1]$.
μ is called fuzzy regular ring if all elements $a_\alpha \in \mu$ are fuzzy regular element.

Definition.

The element $a_\alpha \in \mu$ is called strong regular fuzzy element (right or left) if there exist $x_\alpha \in \mu$ such that:
$a_\alpha = {a_\alpha}^2 \circ x_\alpha\ (a_\alpha = x_\alpha \circ {a_\alpha}^2)$; $\alpha \in]0,1]$.
μ is called fuzzy strong regular ring if all elements $a_\alpha \in \mu$ are fuzzy strong regular.

Definition.

Let μ be a fuzzy subring of R, $a_\alpha \in \mu$ is called semi-unital (right or left) element if there exists $x_\alpha \in \mu$ such that:
$a_\alpha = a_\alpha \circ x_\alpha\ (a_\alpha = x_\alpha \circ a_\alpha)$; $\alpha \in]0,1]$.
μ is called semi-unital fuzzy subring if all elements $a_\alpha \in \mu$ are semi-unital.

Theorem.

Let μ be a fuzzy subring of R with $\alpha \in]0,1]$, then if $a_\alpha \in \mu$ is a semi-unital (right or left) element implies $(a_\alpha \subseteq \mu \circ a_\alpha)\ (a_\alpha \subseteq a_\alpha \circ \mu)$.

Proof.

We discuss the right case (the left case can be discussed by the same):
Since a_α is semi-unital right element, then:
$\exists x_\alpha \in \mu$: $a_\alpha = a_\alpha \circ x_\alpha \Longrightarrow a_\alpha = (a.x)_\alpha \Longrightarrow a = ax$.
Now, let's prove that $a_\alpha(t) \leq (a_\alpha \circ \mu)(t)$ for all $t \in R$.
1). For $t \neq \alpha$, we have $a_\alpha(t) = 0$, so that:
$a_\alpha(t) \leq (a_\alpha \circ \mu)(t)$, hence $a_\alpha \subseteq a_\alpha \circ \mu$ for all $t \neq \alpha$.
2). For $t = \alpha$, we get:

$$(a_\alpha \circ \mu)(a) = sup_{a=u.v}\left\{min\{a_\alpha(u), \mu(v)\}\right\} \geq min\{a_\alpha(u), \mu(x)\}; a = ax \geq min\{a, u\};\ x_\alpha \in \mu$$

$\Longrightarrow \mu(x) \geq a = \alpha = a_\alpha(a) \Longrightarrow a_\alpha \subseteq a_\alpha \circ \mu$.

Theorem.

Let μ be a fuzzy subring of the finite ring R, $a_\alpha \in A$; $\alpha \in]0,1]$, then if $a_\alpha \subseteq a_\alpha \circ \mu$ implies that a_α is semi-unital.

Proof.

If $a \neq x\acute{x}$; $\forall x, \acute{x} \in R$, then:
$(a_\alpha \circ \mu)(a) = 0 < (a_\alpha)(a) = \alpha$; $\alpha \in]0,1]$,

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which contradicts $a_\alpha \subseteq a_\alpha \circ \mu$, so there exists $x, \acute{x} \in R$ with $a = x\acute{x}$ and $(a_\alpha \circ \mu)(a) = sup_{a=x\acute{x}} \{min\{a_\alpha(\acute{x}), \mu(x)\}\}$.
According to the equation $a = x\acute{x}$, we get:
If $\acute{x} \neq a$; $\forall \acute{x} \in R$, then $a_\alpha(\acute{x}) = 0$, thus $(a_\alpha \circ \mu)(a) = 0 < (a_\alpha)(a) = \alpha$; $\alpha \in]0,1]$.
Which is a contradiction with $a_\alpha \subseteq a_\alpha \circ \mu$.
This means that $a = ax$; $x \in R$, then:
$(a_\alpha \circ \mu)(a) = sup_{a=ax} \{min\{\alpha, \mu(x)\}\} = max_{a=ax} \{min\{\alpha, \mu(x)\}\} \geq (a_\alpha)(a) = \alpha$.
So, there exists at least $x \in R$ with $a = ax$ such that $min\{\alpha, \mu(x)\} \geq \alpha$, hence $\mu(x) \geq \alpha$ and $x_\alpha \in \mu$.
This implies that $a_\alpha = (a.x)_\alpha = a_\alpha \circ x_\alpha$.

Theorem.
Let A be a subring of R, then A is a semi-unital if and only if f_A is a fuzzy semi-unital subring of R.

Proof.
Assume that A is a semi-unital subring (from right), then f_A is a fuzzy subring of R.
On the other hand, for any $x_\alpha \in f_A$; $\alpha \in]0,1]$, we have $f_A(x) \geq \alpha$ implie $f_A(x) = 1$, thus $x \in A$.
According to the assumption there exists $y \in A; x = x.y \Longrightarrow x_\alpha = (x.y)_\alpha = x_\alpha \circ y_\alpha$,
$f_A(y) = 1 \geq \alpha \in]0,1] \Longrightarrow y_\alpha \in f_A$.
Hence, $\forall x_\alpha \in f_A, \exists y_\alpha \in f_A$; $x_\alpha = x_\alpha \circ y_\alpha$.
For the converse, we assume that f_A is a fuzzy semi-unital subring of R, this means that A is a subring of R.
On the other hand, we have:
$\forall x \in A$; $f_A(x) = 1 \geq \alpha \in]0,1] \Longrightarrow x_\alpha \in f_A$.
According to the assumption, there exits $y_\alpha \in f_A; x_\alpha = x_\alpha \circ y_\alpha \Longrightarrow x_\alpha = (x.y)_\alpha \Longrightarrow x = x.y$.
Since $y_\alpha \in f_A$, we get $f_A(y) \geq \alpha \in]0,1]$, thus $f_A(y) = 1$ and $y \in A$.
This implies the proof.

Theorem.
Let R be a ring and μ is a fuzzy subring of R, and let $\mu_\alpha \neq \emptyset; 0 \leq \alpha \leq \mu(0)$,
then μ_α is semi-unital (right or left) fuzzy subring of R if and only if μ is a semi-unital of R.

Proof.
Suppose that μ_α is a semi-unital subring of R, then:
$\forall a_\alpha \in \mu \implies a \in \mu_\alpha$, there exists $x \in \mu_\alpha$ such that $a = ax \Longrightarrow a_\alpha = (a.x)_\alpha = a_\alpha \circ x_\alpha$.
This implies that $x_\alpha \in \mu$ and μ is semi-unital.
Now, suppose that μ is semi-unital, then:
$\forall a_\alpha \in \mu \neq \emptyset \implies a \in \mu_\alpha$, there exists $x_\alpha \in \mu$ such that $a_\alpha = a_\alpha \circ x_\alpha \Longrightarrow a_\alpha = (a.x)_\alpha \Longrightarrow a = ax$, this means that $x \in \mu_\alpha$ and the proof holds.

Theorem.
Let $a \in R$, μ be a fuzzy subring of R and positive, if μ is semi-unital right or left) then R is semi-unital.

Proof.
$\forall a_\alpha \in \mu, \exists x_\alpha \in \mu$; $a_\alpha = a_\alpha \circ x_\alpha, \alpha \in]0,1] \Longrightarrow a_\alpha = (a.x)_\alpha \Longrightarrow a = ax$.
Since $a_\alpha \in \mu$, then $a \in \mu_\alpha = \{x \in R;\ \mu(x) \geq \alpha\}$, thus $\forall a \in R, \exists x \in R$ such that $a = ax$.

Remark.
The converse of the previous theorem is not true in general.

Examples.
Let $(Z_4, +, .)$ Be the ring of integer, module 4.
$\mu = \left\{(0,1), \left(1, \frac{1}{2}\right), \left(2, \frac{3}{4}\right), \left(3, \frac{1}{2}\right)\right\}$.
Z_4 is semi-unital ring, that is because:
$\forall x \in Z_4; x = \begin{cases} x.1 \in \acute{x}Z_4 \\ 1.x \in Z_4 x \end{cases}$
μ is not semi-unital fuzzy subring, that is because:
For $2 \in Z_4, \mu(2) = \frac{3}{4}, 2_\alpha \in \mu$; $\alpha \in \left]0, \frac{3}{4}\right]$.
$\begin{cases} 2 = 2.1 \Longrightarrow 2_\alpha = 2_\alpha \circ 1_\alpha \\ 2 = 2.3 \Longrightarrow 2_\alpha = 2_\alpha \circ 3_\alpha \end{cases}$
$\mu(3) = \mu(1) = \frac{1}{2}$, hence $3_\alpha \in \mu, 1_\alpha \in \mu$ for $\alpha \in \left]\frac{1}{2}, \frac{3}{4}\right]$, but $1_\alpha, 3_\alpha \notin \mu$.

Theorem.
Let R be a ring, μ is fuzzy subring of R, $\alpha \in]0,1]$.
If $a_\alpha \in \mu$ is fuzzy idempotent element of order m, then a_α is semi-unital if R.

Proof.
a_α is idempotent of order m implies that:
$a_\alpha = {a_\alpha}^{m+1} = \begin{cases} a_\alpha = {a_\alpha}^m \\ {a_\alpha}^m \circ a_\alpha \end{cases}$

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$\mu(a_\alpha{}^m) \geq \mu(a) \geq \alpha$, hene $a_\alpha{}^m \in \mu$ and a_α is semi-unital.

Theorem.

Let R be a ring, μ be a fuzzy subring of R, $\alpha \in]0,1]$, then if $a_\alpha \in \mu$ is fuzzy regular in R, then a_α is semi-unital in R.

Proof.

a_α is fuzzy regular element implies:

$\exists x_\alpha \in \mu;\ a_\alpha = a_\alpha \circ x_\alpha \circ a_\alpha = a_\alpha \circ (x_\alpha \circ a_\alpha) = a_\alpha \circ (x.a)_\alpha$.

Since $x.a \in R$, there exists $\acute{x} \in R$ such that $\acute{x} = x.a$ and $\acute{x}_\alpha = (x.a)_\alpha$, with:

$\mu(\acute{x}) = \mu(x.a) \geq min\{\mu(a), \mu(x)\} \geq min\{a, \alpha\} = \alpha$, thus $\acute{x}_\alpha \in \mu$ and a_α is semi-unital.

Theorem.

Let μ be a fuzzy subring of R, $\alpha \in]0,1]$, if $a_\alpha \in \mu$ is strong regular fuzzy element from (right or left) of $x_\alpha \in R$, then a_α semi-unital from (right or left).

Proof.

a_α is strong regular fuzzy element implies:

$\exists x_\alpha \in \mu;\ a_\alpha = a_\alpha{}^2 \circ x_\alpha = a_\alpha \circ x_\alpha \circ a_\alpha = a_\alpha \circ (a_\alpha \circ x_\alpha) = a_\alpha \circ (a.x)_\alpha$

We have $a.x \in R$, so there exists $\acute{x} \in R$ such that $\acute{x} = a.x$ and $\acute{x}_\alpha = (a.x)_\alpha \in \mu$.

Theorem.

Let μ be fuzzy subring of R, $\alpha \in]0,1]$, if $a_\alpha \in \mu$ is semi-unital fuzzy element and $x_\alpha \acute{\in} \mu$, then $r_\alpha \circ a_\alpha$ is semi-unital right element, $a_\alpha \circ r_\alpha$ is semi-unital left element, and $a_\alpha \circ r_\alpha \circ a_\alpha$ is semi-unital.

Proof.

We prove the right case. The other cases can be proved by the same.

$\exists x_\alpha \in \mu;\ a_\alpha = a_\alpha \circ x_\alpha \Longrightarrow a = a.x \Longrightarrow r_\alpha = r_\alpha.x \Longrightarrow (r.a)_\alpha = (r_\alpha.x)_\alpha \Longrightarrow r_\alpha \circ a_\alpha = r_\alpha \circ a_\alpha \circ x_\alpha$.

Thus, the proof is complete.

Definition.

Let R be a ring, μ be a fuzzy subring of R, we say that $a_\alpha \in \mu$ is semi-unital of order n from the right 9left) if there exists $x_\alpha \in \mu$ such that:

$a_\alpha = a_\alpha{}^n \circ x_\alpha\ (a_\alpha = x_\alpha \circ a_\alpha{}^n); \alpha \in]0,1], n \in N$.

μ is called semi-unital fuzzy subring if all elements are semi-unital of order n.

Remark.

If R a finite ring, μ is semi-unital of order n if and only if:

$\forall a_\alpha \in \mu: a_\alpha \subseteq a_\alpha{}^n \circ \mu\ (a_\alpha \subseteq \mu \circ a_\alpha{}^n); \alpha \in]0,1]$.

Theorem.

Let μ be a fuzzy subring of R, if $a_\alpha \in \mu$ is semi-unital of order two, then a_α is semi-unital of order n for any $n \in N, \alpha \in]0,1]$.

Proof.

There exists $x_\alpha \in \mu$ such that $a_\alpha = a_\alpha{}^2 \circ x_\alpha$.

For $n = 1$, we have:

$a_\alpha = a_\alpha{}^2 \circ x_\alpha = a_\alpha \circ a_\alpha \circ x_\alpha = a_\alpha \circ (a.x)_\alpha; a.x \in R$.

So, there exists $\acute{x} = a.x \in R$ with $\acute{x}_\alpha = (a.x)_\alpha \in \mu$.

For $n \geq 3$, we have:

$a_\alpha = a_\alpha{}^2 \circ x_\alpha = a_\alpha \circ a_\alpha \circ x_\alpha = a_\alpha \circ a_\alpha{}^2 \circ x_\alpha \circ x_\alpha = a_\alpha \circ a_\alpha \circ a_\alpha{}^2 \circ x_\alpha \circ x_\alpha \circ x_\alpha = \cdots = a_\alpha{}^{n+1} \circ x_\alpha{}^n = a_\alpha{}^n \circ a_\alpha \circ x_\alpha{}^n = a_\alpha{}^n \circ (a_\alpha \circ x_\alpha{}^n) = a_\alpha{}^n \circ (a \circ x^n)_\alpha; a.x^n \in R\ and\ \acute{x}_\alpha = (a.x^n)_\alpha$, and

$\mu(\acute{x}) = \mu(a.x^n) \geq \min\{\mu(a), \mu(x^n)\} \geq \min\{\mu(a), \mu(x)\} \geq \alpha$, thus $\acute{x}_\alpha \in \mu$.

Theorem.

Let μ be a fuzzy subring of R, $\alpha \in]0,1]$.

If $a_\alpha \in \mu$ is semi-unital of order $n \geq 2$, then a_α is semi-unital of order $n = 2$.

Proof.

There exists $x_\alpha \in \mu$ uch that $a_\alpha = a_\alpha{}^n \circ x_\alpha$.

Assume that $n > 2$, then $n = 2 + m; m \in N$.

$a_\alpha = a_\alpha{}^n \circ x_\alpha = (a^n.x)_\alpha = (a^2 a^m.x)_\alpha = a_\alpha{}^2 \circ (a^m.x)_\alpha$, where $(a^m.x)_\alpha \in \mu$, thus a_αI semi-unital of order two.

Example.

Take $R = Z$, the ring of integers, with $\mu: Z \to [0,1]$:

$$\mu(x) = \begin{cases} 1\,; x \in 2Z \\ 0.5\,; x \notin 2Z \end{cases}$$

$3 \in Z, \mu(3) = 0.5$, so:

$3 \in \mu_{0.5} = \{xR; \mu(x) \geq 0.5\} \Longrightarrow 3_{0.5} \in \mu$.

$3_{0.5} \circ 1_{0.5} = (3.1)_{0.5} = 3_{0.5}$.

$1 \in Z, \mu(1) = 0.5, 1_{0.5} \in \mu$, thus $3_{0.5}$ is semi-unital fuzzy element.

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We cannot find $x_{0.5} \in \mu$ with $3_{0.5} = {3_{0.5}}^2 \circ x_{0.5}$, that is beause if we assumed the existence of such element, we would get:
$3_{0.5} = 9_{0.5} \circ x_{0.5} = (9x)_{0.5} \Longrightarrow 3 = 9x; x \in Z$, and this is a contradiction.

3. Conclusion

In this scientific paper, we have defined and studied semi-unital fuzzy rings, where we have presented many algebraic properties that these algebraic structures have through some theorems and examples related to them.
In the future, we suggest that the researchers continue to study this type of rings, and contribute to finding the combination of algebraic properties that they have in a similar way to the same corresponding type in classical rings.

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