Other Certain Classes of Generalized Neutrosophic Soft Separation Structures

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Abstract

The main target of introduce our manuscript is to study and investigate another new extension for the classical meanings of neutrosophic soft neighbourhood and soft separation structures namely, neutrosophic soft *δ-ß*-neighbourhood and neutrosophic soft *δ-ß*-separation structures based on new idea of neutrosophic soft open sets called, neutrosophic soft *δ-ß*-open sets which is more general than of the neutrosophic soft open sets. This manuscript is devoted to obtaining some characterizations and essential properties concerning of neutrosophic soft *δ-ß*- T_i-spaces (i=0, 1, 2, 3, 4) as well the relationships among them. In addition, some relationships of neutrosophic soft *δ-ß*-separation structures with regard to neutrosophic soft points have been discussed. In addition, diverse suitable examples to support of our main results have been supplied.

Keywords: N_S-*Sδ-ß*-open sets, N_S-*Sδ-ß*-Neighbourhood, neutrosophic soft point. N_S-*Sδ-ß*-separation structures, N_S-*S δß*-T_i-spaces(i =0,…,4).

1. Introduction

The neutrosophic soft set is one of the newly emerging concepts and plays a significant role in solving complex problems in engineering, environment and economics . In recent times, many authors have been developed the neutrosophic soft set theory and safely used to many troubles in mathematics and other sciences, as well the idea of fuzzy sets and fuzzy logic was introduced in the beginning via L. Zadeh [1] and later; their topological structures were studied via C. Chang [2]. After that, K. Atanassov in [3, 4] expanded the notion of fuzzy sets into idea intutionistic fuzzy sets, after this, their intutionistic fuzzy topological structures was developed via Coker in [5]. The concept of soft set theory was introduced via Molodtsov [6, 7], and he successfully applied the soft theory in various directions as, smoothness, operations research, probability and theory of measurement, and so on. Later, M. Shabir & M. Naz [8] studied the soft topological spaces.

On the other hand, the neutrosophy & neutrosophic set theory was suggested via F. Smarandache in [9], as well in 2012, A. A. Salama & S. A. Alblowi [10] offered the concept of neutrosophic topological spaces using the idea of neutrosophic sets. Afterward, the neutrosophic soft sets was defined via P. K. Maji, in [11] and modified via I. Deli & S.Broumi [12], as well their topological structures was developed via T. Bera in [13].

Al-Sharqi et al. [14, 15] applied the idea of neutrosophic with complex values and used it to solve various real-life implementations. Alaa Al-Jumaili [16] applied another extended form of neutrosophic open set and studied another new idea of generalized neutrosophic cont-maps .

A new class of generalized open-sets namely,*δ-ß*-open set was offered in a general topology along with δ-ß-continuity via E. Hatir & T. Noiri in [17].

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Al-Jumaili et al. In [18] discussed a new class of maps with strongly closed graphs. After that, S. H. Abdulwahid & AL. Jumaili[19] presented new notions of generalized cont-maps via new generalized open set.

The notion of separation structures of neutrosophic soft Topological-sps was studied and investigated via G. Aras et al in [20]. Later, the notion of separation axioms has been studied and discussed in several directions; we refer the reader to see [21-23]. In recent times, A. Vadivel et al in [24] developed the notion of δ-open set in neutrosophic Topological-sps and presented another class of generalized neutrosophic open sets namely, neutrosophicδ-ß-open set and also studied their vital properties.

The major goal of present our manuscript is to study and discuss another expansion of neutrosophic soft separation structures called, neutrosophic soft δ-ß-separation structures. Several characterizations and fundamental properties relating of neutrosophic soft δ-ß-$\mathcal{T}_i$- spaces (i =0, 1, 2, 3, 4) have been obtained.

2. Materials and Methods

We recall the following required results of generalized neutrosophic soft sets which play vital role throughout this manuscript .

Definition 2.1: [3] A neutrosophic set (Concisely, $\mathcal{N}_{\mathcal{S}}S$) $\mathbb{L}$ on universe set $\mathcal{X}$ is described as: $\mathbb{L} = \{\langle x, \Gamma_{\mathbb{L}}(x), \lambda_{\mathbb{L}}(x), q_{\mathbb{L}}(x)\rangle : x \in \mathcal{X}\}$ wherever $\Gamma, \lambda, q : \mathcal{X} \longrightarrow [0,1]$ & $0 \le \Gamma_{\mathbb{L}}(x) + \lambda_{\mathbb{L}}(x) + q_{\mathbb{L}}(x) \le 3, \forall\, x \in \mathcal{X}$.

Definition2.2: [6] "Let $\mathcal{X}$ be initial universe set, $(\mathcal{F}, \mathcal{Q})$ called soft set over $\mathcal{X}$ provided that $\mathcal{F}$ is a map of a parameters set $\mathcal{Q}$ in to power set $\mathcal{P}(\mathcal{X})$ given by $\mathcal{F}: \mathcal{Q} \longrightarrow \mathcal{P}(\mathcal{X})$". For brief $(\mathcal{F}, \mathcal{Q}) = \{(e, \mathcal{F}(e)) : \mathcal{F}(e) \in \mathcal{P}(\mathcal{X}) \forall e \in \mathcal{Q}\}$. (i. e) for $e \in \mathcal{Q}, \mathcal{F}(e)$considered as the set of ε-approximate elements of $(\mathcal{F}, \mathcal{Q})$ & if $e \notin \mathcal{Q} \Longrightarrow \mathcal{F}(e) = \emptyset. (i.e)\ (\mathcal{F}, \mathcal{Q}) = \{\mathcal{F}(e) : e \in \mathcal{Q}, \mathcal{F}: \mathcal{Q} \longrightarrow \mathcal{P}(\mathcal{X})\}$.

After the neutrosophic soft set was described via [11], this idea was modified Deli & Broumi [12].

Definition2.3: [12] If $\mathcal{X}$ be an initial universe, $\mathcal{Q}$ is set of parameters, and $\mathcal{P}(\mathcal{X})$ indicates the set of each neutrosophic sets of $\mathcal{X}$.Then, a neutrosophic soft $(\tilde{\mathcal{F}}, \mathcal{Q})$ over $\mathcal{X}$ (Concisely, $\mathcal{N}_{\mathcal{S}}Ss$) is described by $(\tilde{\mathcal{F}}, \mathcal{Q}) = \{(u, \langle x, \Gamma_{\tilde{\mathcal{F}}(u)}(x), \lambda_{\tilde{\mathcal{F}}(u)}(x),\ q_{\tilde{\mathcal{F}}(u)}(x)\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$, wherever $\Gamma_{\tilde{\mathcal{F}}(u)}(x), \lambda_{\tilde{\mathcal{F}}(u)}(x), q_{\tilde{\mathcal{F}}(u)}(x) \in [0,1]$ are called the degree of membership map respectively, the degree of indeterminacy map and the degree of non-membership map of $\tilde{\mathcal{F}}(u)$. because the supremum of every Γ, λ, q is 1, the inequality $0 \le \Gamma_{\tilde{\mathcal{F}}(u)}(x) + \lambda_{\tilde{\mathcal{F}}(u)}(x) + q_{\tilde{\mathcal{F}}(u)}(x) \le 3$ is apparent.

Definition 2.4: [11,13] Let $\mathcal{X}$ be an initial universe & $\mathcal{N}_{\mathcal{S}}SS's$ $(\tilde{\mathcal{F}}, \mathcal{Q})$& $(\tilde{\mathcal{G}}, \mathcal{Q})$ are in the form $(\tilde{\mathcal{F}}, \mathcal{Q}) = \{(u, \langle x, \Gamma_{\tilde{\mathcal{F}}(u)}(x), \lambda_{\tilde{\mathcal{F}}(u)}(x),\ q_{\tilde{\mathcal{F}}(u)}(x)\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$ and

$(\tilde{\mathcal{G}}, \mathcal{Q}) = \{(u, \langle x, \Gamma_{\tilde{\mathcal{G}}(u)}(x), \lambda_{\tilde{\mathcal{G}}(u)}(x), q_{\tilde{\mathcal{G}}(u)}(x)\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$, then

(***a***) $0_{(\mathcal{X},\mathcal{Q})} = \{(u, \langle x, 0,0,1\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$ & $1_{(\mathcal{X},\mathcal{Q})} = \{(u, \langle x, 1,1,0\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$.

(***b***) $(\tilde{\mathcal{F}}, \mathcal{Q}) \subseteq (\tilde{\mathcal{G}}, \mathcal{Q})$ *iff* $\Gamma_{\tilde{\mathcal{F}}(u)}(x) \le \Gamma_{\tilde{\mathcal{G}}(u)}(x), \lambda_{\tilde{\mathcal{F}}(u)}(x) \le \lambda_{\tilde{\mathcal{G}}(u)}(x)$ & $q_{\tilde{\mathcal{F}}(u)}(x) \ge q_{\tilde{\mathcal{G}}(u)}(x) : x \in \mathcal{X} : u \in \mathcal{Q}$.

(***c***) $(\tilde{\mathcal{F}}, \mathcal{Q}) = (\tilde{\mathcal{G}}, \mathcal{Q}) \Leftrightarrow (\tilde{\mathcal{F}}, \mathcal{Q}) \subseteq (\tilde{\mathcal{G}}, \mathcal{Q})$ *and* $(\tilde{\mathcal{G}}, \mathcal{Q}) \subseteq (\tilde{\mathcal{F}}, \mathcal{Q})$.

(***d***) $(\tilde{\mathcal{F}}, \mathcal{Q})^c = \{(u, \langle x, q_{\tilde{\mathcal{F}}(u)}(x), 1 - \lambda_{\tilde{\mathcal{F}}(u)}(x), \Gamma_{\tilde{\mathcal{F}}(u)}(x)\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$.

(**e**) $(\tilde{\mathcal{F}}, \mathcal{Q}) \cap (\tilde{\mathcal{G}}, \mathcal{Q}) =$

$\{(u, \langle x, min(\Gamma_{\tilde{\mathcal{F}}(u)}(x), \Gamma_{\tilde{\mathcal{G}}(u)}(x)), min(\lambda_{\tilde{\mathcal{F}}(u)}(x), \lambda_{\tilde{\mathcal{G}}(u)}(x)), max(q_{\tilde{\mathcal{F}}(u)}(x), q_{\tilde{\mathcal{G}}(u)}(x))\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$.

(***f***) $(\tilde{\mathcal{F}}, \mathcal{Q}) \cup (\tilde{\mathcal{G}}, \mathcal{Q}) =$

$\{(u, \langle x, max(\Gamma_{\tilde{\mathcal{F}}(u)}(x), \Gamma_{\tilde{\mathcal{G}}(u)}(x)), max(\lambda_{\tilde{\mathcal{F}}(u)}(x), \lambda_{\tilde{\mathcal{G}}(u)}(x)), min(q_{\tilde{\mathcal{F}}(u)}(x), q_{\tilde{\mathcal{G}}(u)}(x))\rangle : x \in \mathcal{X}) : u \in \mathcal{Q}\}$.

Definition2.5: [13] Let$\mathcal{N}_{\mathcal{S}}S's$ $(\tilde{\mathcal{F}}, \mathcal{Q})$ be collection of each neutrosophic soft subsets over the universe $\mathcal{X}$ & $\mathcal{T} \subset \mathcal{N}_{\mathcal{S}}S's(\tilde{\mathcal{F}}, \mathcal{Q})$. So, $\mathcal{T}$ is neutrosophic soft topology on$\mathcal{X}$ (Concisely,$\mathcal{N}_{\mathcal{S}}S\mathcal{T}$) wherever $\mathcal{Q}$ is the set of parameters, satisfying the next conditions:

(***i***) $0_{(\mathcal{X},\mathcal{Q})}, 1_{(\mathcal{X},\mathcal{Q})} \in \mathcal{T}$.

(***ii***)$(\tilde{\mathcal{F}}, \mathcal{Q}) \cap (\tilde{\mathcal{G}}, \mathcal{Q}) \in \mathcal{T}, \forall (\tilde{\mathcal{F}}, \mathcal{Q}), (\tilde{\mathcal{G}}, \mathcal{Q}) \in \mathcal{T}$.

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$(\boldsymbol{iii})\ \bigcup_{\gamma \in \mathcal{A}} (\tilde{\mathcal{F}},\mathcal{Q})_\gamma \in \mathcal{T}, \forall\, (\tilde{\mathcal{F}},\mathcal{Q})_\gamma \in \mathcal{T} : \gamma \in \mathcal{A} \subseteq \mathcal{T}.$

In that case $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is neutrosophic soft topological space (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{STS}$) in $\mathcal{X}$. Each element of $\mathcal{T}$ called neutrosophic soft open sets(Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SOs}$) in $\mathcal{X}$. A $\mathcal{N}_\mathcal{S}\mathcal{S}s\ (\tilde{\mathcal{F}},\mathcal{Q})$ is called a netrosophic soft closed (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SC}s$) in $\mathcal{X}$ if $(\tilde{\mathcal{F}},\mathcal{Q})^c$ is $\mathcal{N}_\mathcal{S}\mathcal{SO}s$.

Definition 2.6: A subset$(\tilde{\mathcal{F}},\mathcal{Q})$is called:

$(\boldsymbol{a})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ Regular open set(Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SRO}s$) [13] $if\ (\tilde{\mathcal{F}},\mathcal{Q}) = \mathcal{N}_\mathcal{S}\mathcal{S}Int\left(\mathcal{N}_\mathcal{S}\mathcal{S}Cl(\tilde{\mathcal{F}},\mathcal{Q})\right)$

$(\boldsymbol{b})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ δ-open set (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{S}\delta\mathcal{O}s$) [25] $if\ (\tilde{\mathcal{F}},\mathcal{Q}) = \mathcal{N}_\mathcal{S}\mathcal{S}\delta Int(\tilde{\mathcal{F}},\mathcal{Q})$

$(\boldsymbol{c})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ α-open set (Briefly, $\mathcal{N}_\mathcal{S}\mathcal{S}\alpha\mathcal{O}s$) [26] $if\ (\tilde{\mathcal{F}},\mathcal{Q}) \subseteq \mathcal{N}_\mathcal{S}\mathcal{S}Int(\mathcal{N}_\mathcal{S}\mathcal{S}Cl(\mathcal{N}_\mathcal{S}\mathcal{S}Int((\tilde{\mathcal{F}},\mathcal{Q}))))$

$(\boldsymbol{d})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ *Pre*-open set (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SPO}s$) [22] $if\ (\tilde{\mathcal{F}},\mathcal{Q}) \subseteq \mathcal{N}_\mathcal{S}\mathcal{S}Int\left(\mathcal{N}_\mathcal{S}\mathcal{S}Cl(\tilde{\mathcal{F}},\mathcal{Q})\right)$

$(\boldsymbol{e})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ *Semi*-open set (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SSO}s$) [27] $if\ (\tilde{\mathcal{F}},\mathcal{Q}) \subseteq \mathcal{N}_\mathcal{S}\mathcal{S}Cl\left((\mathcal{N}_\mathcal{S}\mathcal{S}Int(\tilde{\mathcal{F}},\mathcal{Q})\right)$

$(\boldsymbol{f})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ *b*-open set (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SBO}s$) [21] *if*

$(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq \mathcal{N}_\mathcal{S}\mathcal{S}Cl\left(\mathcal{N}_\mathcal{S}\mathcal{S}Int(\tilde{\mathcal{F}},\mathcal{Q})\right) \cup \mathcal{N}_\mathcal{S}\mathcal{S}Int\left(\mathcal{N}_\mathcal{S}\mathcal{S}Cl(\tilde{\mathcal{F}},\mathcal{Q})\right)$

$(\boldsymbol{g})$ $\mathcal{N}_\mathcal{S}\mathcal{S}$ *E*-open set (Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SEO}s$) [23] *if*

$(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq \mathcal{N}_\mathcal{S}\mathcal{S}Cl\left(\mathcal{N}_\mathcal{S}\mathcal{S}\delta Int(\tilde{\mathcal{F}},\mathcal{Q})\right) \cup \mathcal{N}_\mathcal{S}\mathcal{S}Int\left(\mathcal{N}_\mathcal{S}\mathcal{S}\delta Cl(\tilde{\mathcal{F}},\mathcal{Q})\right).$

The complement of an $\mathcal{N}_\mathcal{S}\mathcal{SRO}s$

$(resp.\mathcal{N}_\mathcal{S}\mathcal{S}\delta\mathcal{O}s, \mathcal{N}_\mathcal{S}\mathcal{S}\alpha\mathcal{O}s, \mathcal{N}_\mathcal{S}\mathcal{SPO}s, \mathcal{N}_\mathcal{S}\mathcal{SSO}s, \mathcal{N}_\mathcal{S}\mathcal{SBO}s, \mathcal{N}_\mathcal{S}\mathcal{SEO}s)$ is neutrosophic soft closed sets, and denoted by $\mathcal{N}_\mathcal{S}\mathcal{SRC}s$ $(resp.\mathcal{N}_\mathcal{S}\mathcal{S}\delta\mathcal{C}s, \mathcal{N}_\mathcal{S}\mathcal{S}\alpha\mathcal{C}s, \mathcal{N}_\mathcal{S}\mathcal{SPC}s, \mathcal{N}_\mathcal{S}\mathcal{SSC}s, \mathcal{N}_\mathcal{S}\mathcal{SBC}s, \mathcal{N}_\mathcal{S}\mathcal{SEC}s)$ in $\mathcal{X}$.

Definition 2.7: [20] Let $\mathcal{N}_\mathcal{S}\mathcal{S}'s\ (\tilde{\mathcal{F}},\mathcal{Q})$ be the collection of each $\mathcal{N}_\mathcal{S}\mathcal{S}'s$ over the universe $\mathcal{X}$ & $x \in \mathcal{X}$, $0 \le \vartheta, \xi, \mu \le 1$, $u \in \mathcal{Q}$. Then, $\mathcal{N}_\mathcal{S}\mathcal{S}'s\ x^{u}_{(\vartheta,\xi,\mu)}$ is said to a neutrosophic soft point(Concisely, $\mathcal{N}_\mathcal{S}\mathcal{SP}$) and described as follows: $\forall\, z \in \mathcal{X}$,

$$x^{u}_{(\vartheta,\xi,\mu)}(u')(z) = \begin{cases} (\vartheta,\xi,\mu) & if\ u^* = u\ and\ z = x \\ (0,0,1) & if\ u^* \ne u\ or\ z \ne x \end{cases}$$

Definition 2.8: [20] Presume that the universe set $\mathcal{X}$ is given by $\mathcal{X} = \{x_1, x_2\}$ with the parameters $\mathcal{Q} = \{u_1, u_2\}$. Regard as $\mathcal{N}_\mathcal{S}\mathcal{S}'s\ (\tilde{\mathcal{F}},\mathcal{Q})$ on $\mathcal{X}$ as:

$$(\tilde{\mathcal{F}},\mathcal{Q}) = \begin{Bmatrix} u_1 = \{\langle x_1, (0.3, 0.7, 0.6)\rangle, \langle x_2, (0.4, 0.3, 0.8)\rangle\ \} \\ u_2 = \{\langle x_1, (0.4, 0.6, 0.8)\rangle, \langle x_2, (0.3, 0.7, 0.2)\rangle\ \} \end{Bmatrix}$$

It's obvious $(\tilde{\mathcal{F}},\mathcal{Q})$the union of It's $\mathcal{N}_\mathcal{S}\mathcal{SP}s$

$x^{u_1}_{1(0.3,0.7,0.6)}, x^{u_2}_{1(0.4,0.6,0.8)}, x^{u_1}_{2(0.4,0.3,0.8)}, x^{u_2}_{2(0.3,0.7,0.2)}$. Now,

$$x^{u_1}_{1\,(0.3\,,0.7\,,0.6)} = \begin{Bmatrix} u_1 = \langle x_1, (0\,.3, 0\,.7, 0.\,6)\rangle, \langle x_2, (0, 0\,,1)\rangle \\ u_2 = \langle x_1, (0, 0, 1)\rangle, \langle x_2,\ (0,0,1)\rangle \end{Bmatrix},$$

$$x^{u_2}_{1\,(0.4\,,0.6\,,0.8)} = \begin{Bmatrix} u_1 = \langle x_1, (0,0,1)\rangle, \langle x_2,\ (0,0,1)\rangle \\ u_2 = \langle x_1, (0.4, 0.6, 0.8)\rangle, \langle x_2,\ (0, 0, 1)\rangle \end{Bmatrix},$$

$$x^{u_1}_{2\,(0.4\,,0.3\,,0.8)} = \begin{Bmatrix} u_1 = \langle x_1, (0\,, 0, 1)\rangle, \langle x_2, (0.4, 0.3, 0.8)\rangle \\ u_2 = \langle x_1, (0, 0, 1)\rangle, \langle x_2, (0,0,1)\rangle \end{Bmatrix},$$

$$x^{u_2}_{2\,(0.3\,,0.7,0.2)} = \begin{Bmatrix} u_1 = \langle x_1, (0,0,1)\rangle, \langle x_2, (0,0,1)\rangle \\ u_2 = \langle x_1, (0\,,0,1)\rangle, \langle x_2, (0\,.3,0\,.7,0\,.2)\rangle \end{Bmatrix}.$$

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Definition 2.9: [20] Let $(\tilde{\mathcal{F}},\mathcal{Q})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{S}'s$ over the universe set $\mathcal{X}$, we say that $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})$ read as belonging to the $\mathcal{N}_{\mathcal{S}}\mathcal{S}'s$ $(\tilde{\mathcal{F}},\mathcal{Q})$, whenever $\vartheta \leqq \Gamma_{\tilde{\mathcal{F}}(u)}(x), \xi \leqq \lambda_{\tilde{\mathcal{F}}(u)}(x)$ & $\mu \geqq q_{\tilde{\mathcal{F}}(u)}(x)$.

Definition 2.10: [20] Let $x^{u}_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ be two $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$. For the $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$ $x^{u}_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ over common universe$\mathcal{X}$, say $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$ are different points, if $x^{u}_{(\vartheta,\xi,\mu)} \cap z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} = 0_{(\mathcal{X},\mathcal{Q})}$. It is obvious that $x^{u}_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ are distinct $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$ *iff* $x \neq z$ and $u^* \neq u$.

Definition 2.11: [20] Let $(\tilde{\mathcal{F}},\mathcal{Q})$ be an arbitrary $\mathcal{N}_{\mathcal{S}}\mathcal{S}'s$ and $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. In that case$\mathcal{T}_{(\tilde{\mathcal{F}},\mathcal{Q})} = \{(\tilde{\mathcal{F}},\mathcal{Q})\cap(\tilde{\mathcal{G}},\mathcal{Q}) : (\tilde{\mathcal{G}},\mathcal{Q}) \in \mathcal{T}\}$ is said to $\mathcal{N}_{\mathcal{S}}\mathcal{ST}$ on $(\tilde{\mathcal{F}},\mathcal{Q})$ and $\left((\tilde{\mathcal{F}},\mathcal{Q}),\mathcal{T}_{(\tilde{\mathcal{F}},\mathcal{Q})},\mathcal{Q}\right)$ is said to neutrosophic soft topological sub-space (Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{STsp}$) of$(\mathcal{X},\mathcal{T},\mathcal{Q})$.

3. Results and discussion

This phase is devoted to display the idea of neutrosophic soft δ-ß-open sets to obtain various properties and characterizations essential concerning of neutrosophic softδ-ß-$\mathcal{T}_i$-spaces(i =0, 1, 2, 3, 4) . Also, the relationships of neutrosophic soft δ-ß-separation axioms with respect to neutrosophic soft points have been investigated.

Definition 3.1: A subset $(\tilde{\mathcal{F}},\mathcal{Q})$ of $\mathcal{N}_{\mathcal{S}}\mathcal{STS}(\mathcal{X},\mathcal{T},\mathcal{Q})$ is called neutrosophic softδ-ß-open set (Concisely, $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{O}s$) *if* $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq \mathcal{N}_{\mathcal{S}}\mathcal{SCl}\left(\mathcal{N}_{\mathcal{S}}\mathcal{SInt}\left(\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta\mathcal{Cl}(\tilde{\mathcal{F}},\mathcal{Q})\right)\right)$. The complement of an $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{OS}$ is neutrosophic soft closed sets($\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{C}s$).

Remark 3.2: The following diagram describes the relationships among some well-known generalized neutrosophic soft open sets in neutrosophic soft Topological spaces . None of these implications and some neutrosophic properties are reversible we refer the readers to see the following references [25-30]

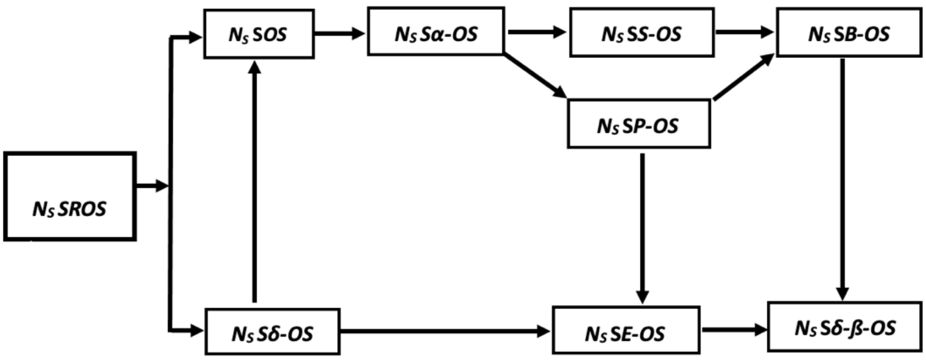

Figure 1: The relationships among some well-known generalized neutrosophc soft-open-sets

Definition3.3: A$\mathcal{N}_{\mathcal{S}}\mathcal{S}'s(\tilde{\mathcal{F}},\mathcal{Q})$ of $\mathcal{N}_{\mathcal{S}}\mathcal{STS}(\mathcal{X},\mathcal{T},\mathcal{Q})$ over $\mathcal{X}$ called neutrosophic soft δ-ß-neighborhood (Concisely,$\mathcal{N}_{\mathcal{S}}\mathrm{S}\delta-$ß$-$Nbd) of the $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})$if there exists a $\mathcal{N}_{\mathcal{S}}\mathrm{S}\delta-$ß$\mathcal{OS}$ $(\tilde{\mathcal{G}},\mathcal{Q})$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{G}},\mathcal{Q}) \subseteq (\tilde{\mathcal{F}},\mathcal{Q})$.

Theorem 3.4: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$ and $(\tilde{\mathcal{F}},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{S}'s$ on$\mathcal{X}$. Then, $(\tilde{\mathcal{F}},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{OS}$ *iff* $(\tilde{\mathcal{F}},\mathcal{Q})$ $\mathcal{N}_{\mathcal{S}}\mathrm{S}\delta-$ß$-$Nbd of it's $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$.

Proof: Presume, $(\tilde{\mathcal{F}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{OS}$ and $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})$. In that case, $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\tilde{\mathcal{F}},\mathcal{Q})$. Therefore, $(\tilde{\mathcal{F}},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathrm{S}\delta-$ß$-$Nbd of $x^{u}_{(\vartheta,\xi,\mu)}$.

Conversely: Assume, $(\tilde{\mathcal{F}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathrm{S}\delta-$ß$-$Nbd of its $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$, and $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})$. Since $(\tilde{\mathcal{F}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathrm{S}\delta-$ß$-$Nbd of the $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ $x^{u}_{(\vartheta,\xi,\mu)}$, so $\exists$ $(\tilde{\mathcal{G}},\mathcal{Q}) \in \mathcal{T}$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{G}},\mathcal{Q}) \subseteq (\tilde{\mathcal{F}},\mathcal{Q})$. Since$(\tilde{\mathcal{F}},\mathcal{Q}) = \cup\{x^{u}_{(\vartheta,\xi,\mu)}: x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})\}$, consequently$(\tilde{\mathcal{F}},\mathcal{Q})$ is union of $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{O}$ sets. Then, $(\tilde{\mathcal{F}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ß$\mathcal{O}s$.

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Remark 3.5: The $\mathcal{N}_{\mathcal{S}}S\delta - ß - \text{Nbd}$ system of $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ $x^{u}_{(\vartheta,\xi,\mu)}$ denoted via $\cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$, is the family of each its $\mathcal{N}_{\mathcal{S}}S\delta - ß - \text{Nbds}$.

Theorem 3.6: Let $\cup(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$, be a $\mathcal{N}_{\mathcal{S}}S\delta - ß -$ Nbd system at $x^{u}_{(\vartheta,\xi,\mu)}$ in a $\mathcal{N}_{\mathcal{S}}\mathcal{STS}(\mathcal{X},\mathcal{T},\mathcal{Q})$. So, the next statements hold:

(***a***) *If* $(\tilde{\mathcal{F}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$, then $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})$.

(***b***) *If* $(\tilde{\mathcal{F}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$ & $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\tilde{\mathcal{K}},\mathcal{Q})$, *then* $(\tilde{\mathcal{K}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$.

(***c***) If $(\tilde{\mathcal{F}},\mathcal{Q})$ and $(\tilde{\mathcal{G}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$, then $(\tilde{\mathcal{F}},\mathcal{Q}) \cap (\tilde{\mathcal{G}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$.

(***d***) If $(\tilde{\mathcal{F}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$, then $\exists\,(\tilde{\mathcal{G}},\mathcal{Q}) \in \cup\,(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$ (s. t) $\forall\, z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{G}},\mathcal{Q}), (\tilde{\mathcal{G}},\mathcal{Q}) \in \cup(z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)},\mathcal{Q})$,

Proof: The proofs of phrases (a), (b) & (c) are evident and consequence from definition(2.8).

(***d***) Presume $(\tilde{\mathcal{F}},\mathcal{Q}) \in \cup(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$. in that case $\exists$ $\mathcal{N}_{\mathcal{S}}S\delta - ß\mathcal{O}s$ $(\tilde{\mathcal{G}},\mathcal{Q})$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{G}},\mathcal{Q}) \subseteq (\tilde{\mathcal{F}},\mathcal{Q})$. Consequently by Theorem (3.4), $(\tilde{\mathcal{G}},\mathcal{Q}) \in \cup(x^{u}_{(\vartheta,\xi,\mu)},\mathcal{Q})$. So $\forall\, z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{G}},\mathcal{Q}), (\tilde{\mathcal{G}},\mathcal{Q}) \in \cup(z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)},\mathcal{Q})$.

Definition 3.7: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$, and $x^{u}_{(\vartheta,\xi,\mu)}$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ be distinct $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$. If $\exists$ $\mathcal{N}_{\mathcal{S}}S\delta - ß\mathcal{S}'s$ $(\tilde{\mathcal{F}},\mathcal{Q})$ & $(\tilde{\mathcal{G}},\mathcal{Q})$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q})$ and $x^{u}_{(\vartheta,\xi,\mu)} \cap (\tilde{\mathcal{G}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$ or $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{G}},\mathcal{Q})$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \cap (\tilde{\mathcal{F}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$, so $(\mathcal{X},\mathcal{T},\mathcal{Q})$ called a neutrosophic soft $\delta - ß - \mathcal{T}_0 -$ space (Concisely, $\mathcal{N}_{\mathcal{S}}S\delta - ß - \mathcal{T}_0 -$ space).

Definition 3.8: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$, and $x^{u}_{(\vartheta,\xi,\mu)}$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ be distinct $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$. If $\exists$ $\mathcal{N}_{\mathcal{S}}S\delta - ß\mathcal{S}'s$ $(\tilde{\mathcal{F}},\mathcal{Q})$ & $(\tilde{\mathcal{G}},\mathcal{Q})$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q}), and\ x^{u}_{(\vartheta,\xi,\mu)} \cap (\tilde{\mathcal{G}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{G}},\mathcal{Q}), z^{q'}_{(\varepsilon',\zeta',\eta')} \cap (\tilde{\mathcal{F}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$, so $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is said neutrosophic soft $\delta - ß - \mathcal{T}_1 -$ space (Concisely, $\mathcal{N}_{\mathcal{S}}S\delta - ß - \mathcal{T}_1 -$ space).

Definition 3.9: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$, and $x^{u}_{(\vartheta,\xi,\mu)}$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ be distinct $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$. If $\exists$ $\mathcal{N}_{\mathcal{S}}S\delta - ß\mathcal{S}'s$ $(\tilde{\mathcal{F}},\mathcal{Q})$ & $(\tilde{\mathcal{G}},\mathcal{Q})$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q}), z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{G}},\mathcal{Q})\&(\tilde{\mathcal{F}},\mathcal{Q}) \cap (\tilde{\mathcal{G}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$, so $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is said to neutrosophic soft$\delta - ß - \mathcal{T}_2 -$ space (Concisely, $\mathcal{N}_{\mathcal{S}}S\delta - ß - \mathcal{T}_2 -$ space).

Example 3.10: Assume, $\mathcal{X} = \{x_1, x_2\}$ is a universe set, and $\mathcal{Q} = \{u_1, u_2\}$ is a parameters set and $x^{u_1}_{1(0.1,0.4,0.7)}$, $x^{u_2}_{1(0.2,0.5,0.6)}$, $x^{u_1}_{2(0.3,0.3,0.5)}$ & $x^{u_2}_{2(0.4,0.4,0.4)}$ are $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$. In that case, the family $\mathcal{T} = \{0_{(\mathcal{X},\mathcal{Q})}, 1_{(\mathcal{X},\mathcal{Q})}, (\tilde{\mathcal{F}}_1,\mathcal{Q}), (\tilde{\mathcal{F}}_2,\mathcal{Q}), (\tilde{\mathcal{F}}_3,\mathcal{Q}), (\tilde{\mathcal{F}}_4,\mathcal{Q}), (\tilde{\mathcal{F}}_5,\mathcal{Q}), (\tilde{\mathcal{F}}_6,\mathcal{Q}), (\tilde{\mathcal{F}}_7,\mathcal{Q}), (\tilde{\mathcal{F}}_8,\mathcal{Q})\}$, wherever

$(\tilde{\mathcal{F}}_1,\mathcal{Q}) = \{x^{u_1}_{1(0.1,0.4,0.7)}\}, (\tilde{\mathcal{F}}_2,\mathcal{Q}) = \{x^{u_2}_{1(0.2,0.5,0.6)}\}, (\tilde{\mathcal{F}}_3,\mathcal{Q}) = \{x^{u_1}_{2(0.3,0.3,0.5)}\}$

$(\tilde{\mathcal{F}}_4,\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_2,\mathcal{Q}),\ (\tilde{\mathcal{F}}_5,\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q}), (\tilde{\mathcal{F}}_6,\mathcal{Q}) = (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q})$

$(\tilde{\mathcal{F}}_7,\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q})$

$(\tilde{\mathcal{F}}_8,\mathcal{Q}) = \{x^{u_1}_{1(0.1,0.4,0.7)}, x^{u_2}_{1(0.2,0.5,0.6)}, x^{u_1}_{2(0.3,0.3,0.5)}, x^{u_2}_{2(0.4,0.4,0.4)}\}$

Is a $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. Thus, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. As well, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}S\delta - ß - \mathcal{T}_0$ space but not a $\mathcal{N}_{\mathcal{S}}S\delta - ß - \mathcal{T}_1 -$ space since for $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s $x^{u_1}_{1(0.1,0.4,0.7)}$ & $x^{u_2}_{1(0.4,0.4,0.4)}$, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ isn't a $\mathcal{N}_{\mathcal{S}}S\delta - ß - \mathcal{T}_1 -$space.

Example 3.11: Assume, $\mathcal{X} = \{x_1, x_2\}$ is a universe set, and $\mathcal{Q} = \{u_1, u_2\}$ is a parameters set and $x^{u_1}_{1(0.1,0.4,0.7)}$, $x^{u_2}_{1(0.2,0.5,0.6)}$, $x^{u_1}_{2(0.3,0.3,0.5)}$ and $x^{u_1}_{2(0.4,0.4,0.4)}$ be $\mathcal{N}_{\mathcal{S}}\mathcal{SP}'s$. So, the collection $\mathcal{T} = \{0_{(\mathcal{X},\mathcal{Q})}, 1_{(\mathcal{X},\mathcal{Q})}, (\tilde{\mathcal{F}}_1,\mathcal{Q}), (\tilde{\mathcal{F}}_2,\mathcal{Q}), (\tilde{\mathcal{F}}_3,\mathcal{Q}), \ldots\ldots, (\tilde{\mathcal{F}}_{15},\mathcal{Q})\}$, wherever

$$(\tilde{\mathcal{F}}_1,\mathcal{Q}) = \{x^{u_1}_{1(0.1,0.4,0.7)}\}, (\tilde{\mathcal{F}}_2,\mathcal{Q}) = \{x^{u_2}_{1(0.2,0.5,0.6)}\}, (\tilde{\mathcal{F}}_3,\mathcal{Q}) = \{x^{u_1}_{2(0.3,0.3,0.5)}\}$$

$$(\tilde{\mathcal{F}}_4,\mathcal{Q}) = y^{u_2}_{2(0.4,0.4,0.4)}, (\tilde{\mathcal{F}}_5,\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_2,\mathcal{Q}), (\tilde{\mathcal{F}}_6,\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q})$$

$$(\tilde{\mathcal{F}}_7,\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_4,\mathcal{Q}), (\tilde{\mathcal{F}}_8,\mathcal{Q}) = (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q}), (\tilde{\mathcal{F}}_9,\mathcal{Q}) = (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_4,\mathcal{Q})$$

$$(\tilde{\mathcal{F}}_{10},\mathcal{Q}) = (\tilde{\mathcal{F}}_3,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_4,\mathcal{Q}), (\tilde{\mathcal{F}}_{11},\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q}), (\tilde{\mathcal{F}}_{12},\mathcal{Q}) = (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cup (\tilde{\psi}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_4,\mathcal{Q}), (\tilde{\mathcal{F}}_{13},\mathcal{Q}) = (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_3,\mathcal{Q}) \cup (\tilde{\mathcal{F}}_4,\mathcal{Q})$$

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$$(\tilde{\mathcal{F}}_{14},\mathcal{Q})=(\tilde{\mathcal{F}}_{1},\mathcal{Q})\cup(\tilde{\mathcal{F}}_{3},\mathcal{Q})\cup(\tilde{\mathcal{F}}_{4},\mathcal{Q}),$$

$$(\tilde{\mathcal{F}}_{15},\mathcal{Q})=\{x^{u_1}_{1\,(0.1,0.4,0.7)},x^{u_2}_{1\,(0.2,0.5,0.6)},x^{u_1}_{2\,(0.3,0.3,0.5)},x^{u_2}_{2\,(0.4,0.4,0.4)}\}$$

a $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. Thus, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. As well,$(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_2-$ space.

Example 3.12: Assume, $\mathcal{X}=\mathbb{N}$(Natural numbers) and $\mathcal{Q}=\{u\}$ is parameter set. Now, $n^{u}_{(\vartheta_n,\xi_n,\mu_n)}$ are$\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s. Now we can provide $(\vartheta_n,\xi_n,\mu_n)$ appropriate values and $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s $n^{u}_{(\vartheta_n,\xi_n,\mu_n)}$, $r^{u}_{(\vartheta_r,\xi_r,\mu_r)}$ are different $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s *iff* $n\neq r$. "Apparent that there is one-to-one compatibility between the set of natural numbers and the set of" $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s $\mathbb{N}^{u}=\{n^{u}_{(\vartheta_n,\xi_n,\mu_n)}\}$. in that case give Co-finite topology on this set. So, $\mathcal{N}_{\mathcal{S}}\mathcal{S}$'s $(\tilde{\mathcal{F}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$ *iff* the finite $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s are neglected from $\mathbb{N}^{u}$. Thus,$(\mathcal{X},\mathcal{T},\mathcal{Q})$is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1-$ space but not a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_2$ −space.

Theorem 3.13: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. Then, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1$-space *iff* every $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{C}s$.

Proof: Assume, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1-$ space and $x^{u}_{(\vartheta,\xi,\mu)}$ be an arbitrary all $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$. We demonstrate that $x^{u}_{(\vartheta,\xi,\mu)}$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{C}s$. Assume, $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in(x^{u}_{(\vartheta,\xi,\mu)})^c$. In that case $x^{u}_{(\vartheta,\xi,\mu)}$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ are different $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s. Hence, $x\neq z$ or $u'\neq u$. Because, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1-$ space. So, $\exists\,\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$ $(\tilde{\mathcal{G}},\mathcal{Q})$ (s.t) $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in(\tilde{\mathcal{G}},\mathcal{Q})$ & $x^{u}_{(\vartheta,\xi,\mu)}\cap(\tilde{\mathcal{G}},\mathcal{Q})=0_{(\mathcal{X},\mathcal{Q})}$. Because, $x^{u}_{(\vartheta,\xi,\mu)}\cap(\tilde{\mathcal{G}},\mathcal{Q})=0_{(\mathcal{X},\mathcal{Q})}$, we get $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in(\tilde{\mathcal{G}},\mathcal{Q})\subseteq(x^{u}_{(\vartheta,\xi,\mu)})^c$. Consequently, $(x^{u}_{(\vartheta,\xi,\mu)})^c$ is a$\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$ this mean $x^{u}_{(\vartheta,\xi,\mu)}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{C}s$.

Conversely: Presume that all $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ $x^{u}_{(\vartheta,\xi,\mu)}$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{C}s$. In that case $(x^{u}_{(\vartheta,\xi,\mu)})^c$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$. Presume, $x^{u}_{(\vartheta,\xi,\mu)}\cap z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}=0_{(\mathcal{X},\mathcal{Q})}$. Hence,$z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in(x^{u}_{(\vartheta,\xi,\mu)})^c$ and $x^{u}_{(\vartheta,\xi,\mu)}\cap(x^{u}_{(\vartheta,\xi,\mu)})^c=0_{(\mathcal{X},\mathcal{Q})}$. Consequently $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1-$ space on $\mathcal{X}$.

Theorem 3.14: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$.Then, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_2-$ space *iff* for distinct $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s $x^{u}_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$, there exists$\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$ $(\tilde{\mathcal{F}},\mathcal{Q})$ containing $x^{u}_{(\vartheta,\xi,\mu)}$ but not $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ (s. t) $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ does not belong to $\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{F}},\mathcal{Q})$.

Proof: Assume, $x^{u}_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ are two $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s in $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_2$- space$(\mathcal{X},\mathcal{T},\mathcal{Q})$.

In that case, there exists disjoint $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{OS}$'s $(\tilde{\mathcal{F}},\mathcal{Q})$ & $(\tilde{\mathcal{G}},\mathcal{Q})$ (s. t)

$x^{u}_{(\vartheta,\xi,\mu)}\in(\tilde{\mathcal{F}},\mathcal{Q})$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in(\tilde{\mathcal{G}},\mathcal{Q})$.

Since, $x^{u}_{(\vartheta,\xi,\mu)}\cap z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}=0_{(\mathcal{X},\mathcal{Q})}$ and $(\tilde{\mathcal{F}},\mathcal{Q})\cap(\tilde{\mathcal{G}},\mathcal{Q})=0_{(\mathcal{X},\mathcal{Q})}$, so, $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\notin(\tilde{\mathcal{F}},\mathcal{Q})$.

It implies that $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\notin\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{F}},\mathcal{Q})$.

Conversely: Presume, for different $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s $x^{u}_{(\vartheta,\xi,\mu)}$& $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$, there exists $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$ $(\tilde{\mathcal{F}},\mathcal{Q})$ containing $x^{u}_{(\vartheta,\xi,\mu)}$ but not $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ (s. t) $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\notin\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{F}},\mathcal{Q})$. In that case $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{F}},\mathcal{Q}))^c$,(i. e), $(\tilde{\mathcal{F}},\mathcal{Q})$ & $(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{F}},\mathcal{Q}))^c$ disjoint$\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{OS}$'s containing $x^{u}_{(\vartheta,\xi,\mu)}$& $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ respectively.

Theorem 3.15: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1$- space for every $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ $x^{u}_{(\vartheta,\xi,\mu)}\in(\tilde{\mathcal{F}},\mathcal{Q})\in\mathcal{T}$. If there exists a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{O}s$ $(\tilde{\mathcal{G}},\mathcal{Q})$ (s.t) $x^{u}_{(\vartheta,\xi,\mu)}\in(\tilde{\mathcal{G}},\mathcal{Q})\subseteq\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{G}},\mathcal{Q})\subseteq(\tilde{\mathcal{F}},\mathcal{Q})$, then $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_2$- space.

Proof: Assume, $x^{u}_{(\vartheta,\xi,\mu)}\cap z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}=0_{(\mathcal{X},\mathcal{Q})}$. Since, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_1$- space, $x^{u}_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ are $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{CS}$'s in $\mathcal{T}$. So, $x^{u}_{(\vartheta,\xi,\mu)}\in\left(z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\right)^c\in\mathcal{T}$. Consequently,

$\exists\,\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß\mathcal{OS}$ $(\tilde{\mathcal{G}},\mathcal{Q})$ in $\mathcal{T}$ (s. t) $x^{u}_{(\vartheta,\xi,\mu)}\in(\tilde{\mathcal{G}},\mathcal{Q})\subseteq\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{G}},\mathcal{Q})\subseteq\left(z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\right)^c$. As a result, obtain $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}\in\left(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{G}},\mathcal{Q})\right)^c$ & $x^{u}_{(\vartheta,\xi,\mu)}\in(\tilde{\mathcal{G}},\mathcal{Q})$ and $(\tilde{\mathcal{G}},\mathcal{Q})\cap(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\tilde{\mathcal{G}},\mathcal{Q}))^c=0_{(\mathcal{X},\mathcal{Q})}$, i. e., $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-ß-\mathcal{T}_2$- space.

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Remark 3.16: Suppose $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta$-ß-$\mathcal{T}_i$- spaces($i$ =0,1,2) $\forall\, x \neq z$, and $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s $x_{(\vartheta,\xi,\mu)}$ & $z_{(\vartheta^*,\xi^*,\mu^*)}$ have neighbourhoods satisfying conditions of δ-ß-$\mathcal{T}_i$- spaces in $\mathcal{N}_{\mathcal{S}}\mathcal{STS}(\mathcal{X},\mathcal{T}^u)$ $\forall\, u \in \mathcal{Q}$ since $x^u_{(\vartheta,\xi,\mu)}$ and $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)}$ are different $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$'s.

Definition 3.17: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$, and let $(\tilde{\mathcal{F}},\mathcal{Q})$ be a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$ and $x^u_{(\vartheta,\xi,\mu)}\cap(\tilde{\mathcal{F}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$. If $\exists$ $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{O}s$'s $(\widetilde{\mathcal{W}}_1,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_2,\mathcal{Q})$ (s. t) $x^u_{(\vartheta,\xi,\mu)} \in (\widetilde{\mathcal{W}}_1,\mathcal{Q}), (\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_1,\mathcal{Q})\cap(\widetilde{\mathcal{W}}_2,\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$, then $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is said to neutrosophic soft$\delta-$ ß – regular (Concisely, $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß $-\mathcal{R}$eg) space. Also, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is called neutrosophic soft$\delta-$ ß – $\mathcal{T}_3-$ space (Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_3$)-space if it's both $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – regular & $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_1$-space.

Theorem3.18: Let$(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$, and $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_3$ space *iff* for each $x^u_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q}) \in \mathcal{T}$, there exists $(\widetilde{\mathcal{W}},\mathcal{Q}) \in \mathcal{T}$ (s. t) $x^u_{(\vartheta,\xi,\mu)} \in (\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq \mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq (\tilde{\mathcal{F}},\mathcal{Q})$.

Proof: Assume, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_3$ space and $x^u_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}},\mathcal{Q}) \in \mathcal{T}$. Since, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_3$ space for the $\mathcal{N}_{\mathcal{S}}\mathcal{SP}$ $x^u_{(\vartheta,\xi,\mu)}$ and $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$ $(\tilde{\mathcal{F}},\mathcal{Q})^c$, there exists $(\widetilde{\mathcal{W}}_1,\mathcal{Q})$ & $(\widetilde{\mathcal{W}}_2,\mathcal{Q}) \in \mathcal{T}$ (s.t) $x^u_{(\vartheta,\xi,\mu)} \in (\widetilde{\mathcal{W}}_1,\mathcal{Q}), (\tilde{\mathcal{F}},\mathcal{Q})^c \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_1,\mathcal{Q}) \cap (\widetilde{\mathcal{W}}_2,\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$. In that case obtain $x^u_{(\vartheta,\xi,\mu)} \in (\widetilde{\mathcal{W}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})^c \subseteq (\tilde{\mathcal{F}},\mathcal{Q})$. Since, $(\widetilde{\mathcal{W}}_2,\mathcal{Q})^c$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$, $\mathcal{N}_{\mathcal{S}}\mathcal{SCl} \in (\widetilde{\mathcal{W}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})^c$.

Conversely: Assume, $x^u_{(\vartheta,\xi,\mu)} \cap (\tilde{\mathcal{K}},\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$ and $(\tilde{\mathcal{K}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$. Then, $x^u_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{K}},\mathcal{Q})^c$ "and from the condition of the theorem", we get $x^u_{(\vartheta,\xi,\mu)} \in (\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq \mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq (\tilde{\mathcal{K}},\mathcal{Q})^c$. As a result,$x^u_{(\vartheta,\xi,\mu)} \in (\widetilde{\mathcal{W}},\mathcal{Q}), (\tilde{\mathcal{K}},\mathcal{Q}) \subseteq (\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q}))^c$

and $(\widetilde{\mathcal{W}},\mathcal{Q}) \cap (\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q}))^c = 0_{(\mathcal{X},\mathcal{Q})}$. Thus, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_3$ space.

Definition 3.19: A $\mathcal{N}_{\mathcal{S}}\mathcal{STS}(\mathcal{X},\mathcal{T},\mathcal{Q})$ over $\mathcal{X}$ is called neutrosophic softδ-ß-normal (Concisely, $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß $-$ normal) space, if for each pair of disjoint $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$'s $(\tilde{\mathcal{F}}_1,\mathcal{Q}),(\tilde{\mathcal{F}}_2,\mathcal{Q})$, there exists disjoint $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{O}s$'s $(\tilde{\mathcal{K}}_1,\mathcal{Q}),(\tilde{\mathcal{K}}_2,\mathcal{Q})$ (s.t) $(\tilde{\mathcal{F}}_1,\mathcal{Q}) \subseteq (\tilde{\mathcal{K}}_1,\mathcal{Q})$ and $(\tilde{\mathcal{F}}_2,\mathcal{Q}) \subseteq (\tilde{\mathcal{K}}_2,\mathcal{Q})$, also $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is called neutrosophic soft$\delta-$ß – $\mathcal{T}_4-$ space(Concisely,$\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_4$)space if it's both a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß-normal and $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_1$-space.

Theorem3.20: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. Then, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_4$- space *iff* for every $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$ $(\tilde{\mathcal{F}},\mathcal{Q})$ and $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{O}s$ $(\tilde{\mathcal{K}},\mathcal{Q})$ with $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\tilde{\mathcal{K}},\mathcal{Q})$, there exists a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{O}s$ $(\widetilde{\mathcal{W}},\mathcal{Q})$ such that $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq \mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq (\tilde{\mathcal{K}},\mathcal{Q})$.

Proof: Assume, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_4$- space, $(\tilde{\mathcal{F}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$, and $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\tilde{\mathcal{K}},\mathcal{Q}) \in \mathcal{T}$. In this case $(\tilde{\mathcal{K}},\mathcal{Q})^c$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$ & $(\tilde{\mathcal{F}},\mathcal{Q})\cap(\tilde{\mathcal{K}},\mathcal{Q})^c = 0_{(\mathcal{X},\mathcal{Q})}$. Since $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_4$, so $\exists$ $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{O}S$'s $(\widetilde{\mathcal{W}}_1,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_2,\mathcal{Q})$ (s. t) $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_1,\mathcal{Q})$&$(\tilde{\mathcal{K}},\mathcal{Q})^c \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_1,\mathcal{Q}) \cap (\widetilde{\mathcal{W}}_2,\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$. Hence, $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})^c \subseteq (\tilde{\mathcal{K}},\mathcal{Q})$, since $(\widetilde{\mathcal{W}}_2,\mathcal{Q})^c$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{C}s$ and $(\widetilde{\mathcal{W}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})^c$. Consequently, $(\tilde{\mathcal{F}},\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_1,\mathcal{Q}) \subseteq \mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}}_1,\mathcal{Q}) \subseteq (\tilde{\mathcal{K}},\mathcal{Q})$.

Conversely: Presume that, $(\tilde{\mathcal{F}}_1,\mathcal{Q})$&$(\tilde{\mathcal{F}}_2,\mathcal{Q})$ are two disjoint $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{CS}$'s. So, $(\tilde{\mathcal{F}}_1,\mathcal{Q}) \subseteq (\tilde{\mathcal{F}}_2,\mathcal{Q})^c$. Via the condition of theorem, $\exists$ $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{O}s$ $(\widetilde{\mathcal{W}},\mathcal{Q})$ (s. t) $(\tilde{\mathcal{F}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq \mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q}) \subseteq (\tilde{\mathcal{F}}_2,\mathcal{Q})^c$. Consequently, $(\widetilde{\mathcal{W}},\mathcal{Q})$, & $\left(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q})\right)^c$ are $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{OS}$'s & $(\tilde{\mathcal{F}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}},\mathcal{Q}), (\tilde{\mathcal{F}}_2,\mathcal{Q}) \subseteq \left(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q})\right)^c$ and $(\widetilde{\mathcal{W}},\mathcal{Q}) \cap \left(\mathcal{N}_{\mathcal{S}}\mathcal{SCl}(\widetilde{\mathcal{W}},\mathcal{Q})\right)^c = 0_{(\mathcal{X},\mathcal{Q})}$.

Therefore, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß – $\mathcal{T}_4$-space.

Theorem 3.21: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. If $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta$-ß-$\mathcal{T}_i$-spaces, so the $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ $((\tilde{\mathcal{F}},\mathcal{Q}),\mathcal{T}_{(\tilde{\mathcal{F}},\mathcal{Q})},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta$-ß-$\mathcal{T}_i$- spaces-($i$ =0,1,2,3).

Proof: Suppose, $x^u_{(\vartheta,\xi,\mu)}$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in ((\tilde{\mathcal{F}},\mathcal{Q}),\tau_{(\tilde{\mathcal{F}},\mathcal{Q})},\mathcal{Q})$ (s.t) $x^u_{(\vartheta,\xi,\mu)} \cap z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} = 0_{(\mathcal{X},\mathcal{Q})}$. In that case $\exists$ $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{OS}$'s $(\tilde{\mathcal{F}}_1,\mathcal{Q})$&$(\tilde{\mathcal{F}}_2,\mathcal{Q})$ satisfying the cases of $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta$-ß-$\mathcal{T}_i$- spaces, (s.t) $x^u_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}}_1,\mathcal{Q})$ & $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{F}}_2,\mathcal{Q})$. As aresult, $x^u_{(\vartheta,\xi,\mu)} \in (\tilde{\mathcal{F}}_1,\mathcal{Q}) \cap (\tilde{\mathcal{F}},\mathcal{Q})$ and , $z^{u^*}_{(\vartheta^*,\xi^*,\mu^*)} \in (\tilde{\mathcal{F}}_2,\mathcal{Q}) \cap (\tilde{\mathcal{F}},\mathcal{Q})$.

As well, the $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta-$ ß$\mathcal{OS}$'s $(\tilde{\mathcal{F}}_1,\mathcal{Q}) \cap (\tilde{\mathcal{F}},\mathcal{Q})$& $(\tilde{\mathcal{F}}_2,\mathcal{Q}) \cap (\tilde{\mathcal{F}},\mathcal{Q})$ in $\mathcal{T}_{(\tilde{\mathcal{F}},\mathcal{Q})}$ satisfy the cases of a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta$-ß-$\mathcal{T}_i$- spaces-($i$ = 0,1,2,3).

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Theorem 3.22: Let $(\mathcal{X},\mathcal{T},\mathcal{Q})$ be $\mathcal{N}_{\mathcal{S}}\mathcal{STS}$ over $\mathcal{X}$. If $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß $-\mathcal{T}_4$ space and $(\widetilde{\mathcal{K}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß$\mathcal{C}s$ in$(\mathcal{X},\mathcal{T},\mathcal{Q})$,hen $((\widetilde{\mathcal{K}},\mathcal{Q}),\mathcal{T}_{(\widetilde{\mathcal{K}},\mathcal{Q})},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß $-\mathcal{T}_4$-space.

Proof: Presume that, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß $-\mathcal{T}_4$-space and $(\widetilde{\mathcal{K}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß$\mathcal{C}s$ in $(\mathcal{X},\mathcal{T},\mathcal{Q})$. Assume that, $(\widetilde{\mathcal{K}}_1,\mathcal{Q})$ & $(\widetilde{\mathcal{K}}_2,\mathcal{Q})$ are two $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß$\mathcal{CS}$'s in $((\widetilde{\mathcal{K}},\mathcal{Q}),\mathcal{T}_{(\widetilde{\mathcal{K}},\mathcal{Q})},\mathcal{Q})$ (s. t) $(\widetilde{\mathcal{K}}_1,\mathcal{Q}) \cap (\widetilde{\mathcal{K}}_2,\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$. When, $(\widetilde{\mathcal{K}},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß$\mathcal{C}s$ *in* $(\mathcal{X},\mathcal{T},\mathcal{Q})$, $(\widetilde{\mathcal{K}}_1,\mathcal{Q})$& $(\widetilde{\mathcal{K}}_2,\mathcal{Q})$ are $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß$\mathcal{CS}$'s *in* $(\mathcal{X},\mathcal{T},\mathcal{Q})$. Since, $(\mathcal{X},\mathcal{T},\mathcal{Q})$ is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß $-\mathcal{T}_4$-space, so $\exists\ \mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß$\mathcal{OS}$'s $(\widetilde{\mathcal{W}}_1,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_2,\mathcal{Q})$ (s. t) $(\widetilde{\mathcal{K}}_1,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_1,\mathcal{Q})$&$(\widetilde{\mathcal{K}}_2,\mathcal{Q}) \subseteq (\widetilde{\mathcal{W}}_2,\mathcal{Q})$ and $(\widetilde{\mathcal{W}}_1,\mathcal{Q})\cap(\widetilde{\mathcal{W}}_2,\mathcal{Q}) = 0_{(\mathcal{X},\mathcal{Q})}$.

In that case $(\widetilde{\mathcal{K}}_1,\mathcal{Q}) = (\widetilde{\mathcal{W}}_1,\mathcal{Q})\cap(\widetilde{\mathcal{K}},\mathcal{Q})$&$(\widetilde{\mathcal{K}}_2,\mathcal{Q}) = (\widetilde{\mathcal{W}}_2,\mathcal{Q})\cap(\widetilde{\mathcal{K}},\mathcal{Q})$ and

$((\widetilde{\mathcal{W}}_1,\mathcal{Q})\cap(\widetilde{\mathcal{K}},\mathcal{Q}))\cap((\widetilde{\mathcal{W}}_2,\mathcal{Q})\cap(\widetilde{\mathcal{K}},\mathcal{Q})) = 0_{(\mathcal{X},\mathcal{Q})}$. This is implies$((\widetilde{\mathcal{K}},\mathcal{Q}),\mathcal{T}_{(\widetilde{\mathcal{K}},\mathcal{Q})},\mathcal{Q})$is a $\mathcal{N}_{\mathcal{S}}\mathcal{S}\delta -$ ß $-\mathcal{T}_4$-space.

4. Conclusion

The study of generalized neutrosophic soft sets and generalized neutrosophic soft topological spaces is very significant in the study of possible applications in classical and non-classical logic". Therefore, the main target of offer this manuscript is to employing the idea of neutrosophic soft *δ-ß*-open sets to study another extension of neutrosophic soft separation structures called, neutrosophic soft *δ-ß*-separation axioms. Also, some essential properties and various characterizations concerning of neutrosophic soft *δ-ß*-$\mathcal{T}_i$-spaces (i =0, 1, 2, 3, 4) have been investigated. "We hope that the findings in this manuscript will aid scholars in support and developing the research on neutrosophic soft topology in order to promote a general framework for their practical applications in mathematics and other sciences .

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