



Multi-criteria group decision making method in Pythagorean interval-valued neutrosophic fuzzy soft soft using VIKOR approach

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Abstract

In contrast to the Pythagorean interval valued fuzzy soft set and the neutrosophic interval valued fuzzy soft set, the Pythagorean neutrosophic interval valued fuzzy soft set is a generalization of these sets. We discuss aggregating PyNIVFS decision matrixes by using aggregated operations. The VIKOR method, which is an extension of neutrosophic fuzzy soft sets, is a powerful method for evaluating multi-criteria group decision making. The score function in this approach is based on the aggregation of the VIKOR method to a PyNIVFS-positive and negative solution. Optimal alternatives are introduced under closeness. An investment company plans to purchase some shares of the top five companies in the stock exchange to invest some money on. In order to minimize this factor, they decided to invest some of their cash in percentages of 30 dollars, 25 dollars, 20 dollars, 15 dollars and 10 dollars in accordance with the top five ranked companies in order to minimize the effect of this factor.

Keywords: PyNIVFS set; MCGDM; VIKOR; aggregation operator.

1 Introduction

Decision-makers are having a hard time identifying optimal solutions to real-world systems because they are becoming increasingly complex. There is still the possibility of choosing the most suitable option despite the difficulty of choosing between the alternatives. A number of firms have difficulty creating opportunities, objectives, and constraints based on their viewpoints. The decision-making (DM) process should consider both individuals and groups of objectives simultaneously. There are many different issues that are related to MADM that are dealt with on a daily basis. The result of this is that we need to improve our DM abilities as a result. Decision making involves deciding among options what is the most appropriate alternative. Hwang et al.¹ that the use of multiple criteria decision making (MCDM) can be beneficial. MCDM problem expressed

as a matrix

$$\mathcal{D}_{m \times n} = \begin{matrix} & \mathcal{D}_1 & \mathcal{D}_2 & \dots & \mathcal{D}_n \\ \Omega_1 & \left(\begin{matrix} \tau_{11} & \tau_{12} & \dots & \tau_{1n} \\ \tau_{21} & \tau_{22} & \dots & \tau_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{m1} & \tau_{m2} & \dots & \tau_{mn} \end{matrix} \right) \\ \Omega_2 & & & & \\ \vdots & & & & \\ \Omega_m & & & & \end{matrix}$$

where $\Omega_1, \Omega_2, \dots, \Omega_m$ are alternatives and $\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_n$ are criteria and τ_{ij} is the rating of i to \mathcal{D}_j . In 2013, A fuzzy soft set area with interval values was investigated by Xiao et al.² for MAGDM problems under uncertain environments. This field of study has been studied by a variety of researchers using a variety of methods. There are several uncertain theories proposed by them to deal with the uncertainties, including fuzzy set (FS),³ intuitionistic FS (IFS),⁴ interval valued FS (IVFS),⁵ vague set,⁶ Pythagorean FS (PyFS),⁷ IVPFS,⁸ spherical FS (SFS).⁹ A membership grade (MG) indicates how well a FS fits into the specified set with ranging from 0 to one. An IFS is defined by Atanassov⁴ as having a total of membership grade (MG) and non-membership grade (NMG) less than one. The sum of the MG and NMG is sometimes greater than one when a DM method is applied. Yager of⁷ developed PFS, which is characterized by a square sum of its MG and NMG is more than one. In order to generalize IFS, Yager used PFS to build a model.

This is insufficient for demonstrating neutrality (neither favor nor disfavor). It was developed by Cuong et al.¹⁰ with a total grade no higher than one for three pointers such as positive, neutral, and negative. As a result, it would be appropriate for the DM method to use this set over IFS or PFS for selective applications.¹¹⁻¹⁷ Liu et al. first presented the concept of an aggregation operator (AO) in generalized PFS.¹⁸ A PIVFS algorithm for the problem of identifying truth membership grades (TMGs), indeterminacy membership grades (IMGs) and false membership grades (FMGs) with AOs^{8,19-21} have the feature that the sum of the three grades (TMG, IMG, and FMG) is greater than one. It has been suggested by Ashraf et al.⁹ that the SFS should contain the following graph: this diagram shows that the sum of the squares of the TMG, IMG and FMG should be not exceeds one. An application of Pythagorean fuzzy soft set theory to real life was discussed by Peng et al.²² in 2015. Recent studies on complex Pythagorean FSs using pattern recognition by Ullah et al.²³ Jana et al.²⁴ examined bipolar intuitionistic fuzzy soft sets with applications. The area of robust single valued neutrosophic soft AOs based on MCDM with bipolar fuzzy soft structure was introduced by Jana et al.²⁶ in 2019 in regards to robust single valued neutrosophic soft operators with extension on MCDM.²⁵ The Pythagorean fuzzy dombi AOs were introduced by Jana et al. in 2019.²⁷ Using trapezoidal neutrosophic AOs for MADM, Jana et al.²⁸ extrapolated their results and demonstrated their use in MADM. The MCDM process developed by Jana et al.²⁹ are developing is based on single value neutrosophic dombi power aggregation operators that are applied to the MCDM process of 2021. The concepts of MCDM based on SVTrN aggregation functions have been introduced by Jana et al.³⁰ as an approach for MCDMs.

It has been studied by many researchers that the VIKOR method is an effective way to solve DM problems. Boran et al.³¹ in 2009, multi-criterion intuitionistic fuzzy group decision-making (GDM) is applied to TOPSIS. The bipolar fuzzy TOPSIS for GDM was proposed by Akram et al.³² The search results of the proposed m -polar fuzzy linguistic TOPSIS approach for MCGDM were discussed by Adeel et al.³³ after 2019. GDM based on TOPSIS has been discussed by Eraslan et al.³⁴ PFSs based on TOPSIS were proposed to be extended to MCDM by Zhang et al.³⁵ With regard to the concept of single-valued neutrosophic MADM, Peng et al.³⁶ initiated the concept by combining MABAC and TOPSIS in 2018. Zulqarnain et al.³⁷ discussed the TOPSIS extension to interval valued intuitionistic fuzzy soft sets (IVIFSS), highlighting the possibility of extending TOPSIS. As well as discussing IVIFSS's, a new type of correlation coefficient, he discussed a new type of regression coefficient. Based on the TOPSIS method, distances to a positive ideal solution (PIS) and a negative ideal solution (NIS) are computed under relative closeness, and a preference order is calculated to substitute the two distance measures with each other to find the optimum solution. The VIKOR method focuses on ranking and selecting among a set of alternative solutions for a problem that has inconsistent criteria, and it calculates compromise solutions for a problem that has inconsistent criteria, which will help the decision-makers to reach a final decision.^{38,39} The VIKOR method, based on fuzzy logic, was discussed by Opricovic et al.⁴⁰ Using a public transportation problem as an example, Tzeng et al.⁴¹ compared VIKOR with TOPSIS methods in the context of a comparison. Recently, Palanikumar et al.⁴²⁻⁴⁹ discussed many AOs based on Neutrosophic logic, SFSs, soft sets are introduced.

It is argued that the purpose of the current article is to extend MCDM techniques which are used to describe Pythagorean fuzzy soft sets, under VIKOR, to PyNIVFS sets under VIKOR and to derive some of its properties based on that. Accordingly, the paper is divided into four sections, which can be found in the following sections. A brief introduction can be found in section 1. There is a brief description of PyNIVFS set in Section 2. The section 3 discusses the concept of MCGDM in the light of PyNIVFS-VIKOR using AO,

with several examples provided. There is a conclusion provided in the section 4.

2 Preliminaries

Definition 2.1. Let X be a PIVFS in \mathbb{U} is of the form $\tilde{X} = \{x, \langle \tilde{\rho}_X^t(x), \tilde{\rho}_X^f(x) \rangle | x \in \mathbb{U}\}$, $\tilde{\rho}_X^t(x) = [\rho_X^{tl}(x), \rho_X^{tu}(x)]$ and $\tilde{\rho}_X^f(x) = [\rho_X^{fl}(x), \rho_X^{fu}(x)]$ denotes the membership degree and non-membership degree of X respectively. Here $\tilde{\rho}_X^t$ and $\tilde{\rho}_X^f$ are function from \mathbb{U} into $\mathbb{D}[0, 1]$ and $0 \leq (\tilde{\rho}_X^t(x))^2 + (\tilde{\rho}_X^f(x))^2 \leq 1$ it is observed that $0 \leq (\rho_X^{tu}(x))^2 + (\rho_X^{fu}(x))^2 \leq 1$.

Definition 2.2. The neutrosophic set (NS) $X = \{x, \langle \rho_X^t(x), \rho_X^f(x) \rangle | x \in \mathbb{U}\}$, where $\rho_X^t(x)$ and $\rho_X^f(x)$ are called membership degree and non membership degree of X respectively. Here ρ_X^t and ρ_X^f are function from \mathbb{U} into $[0, 1]$ and $0 \leq (\rho_X^t(x))^3 + (\rho_X^f(x))^3 \leq 1$.

Definition 2.3. The neutrosophic interval valued set $\tilde{X} = \{x, \langle \tilde{\sigma}_X^t(x), \tilde{\sigma}_X^i(x), \tilde{\sigma}_X^f(x) \rangle | x \in \mathbb{U}\}$, where $\tilde{\sigma}_X^t(x) = [\sigma_X^{tl}(x), \sigma_X^{tu}(x)]$, $\tilde{\sigma}_X^i(x) = [\sigma_X^{Il}(x), \sigma_X^{Iu}(x)]$ and $\tilde{\sigma}_X^f(x) = [\sigma_X^{fl}(x), \sigma_X^{fu}(x)]$ represents the degree of truth, indeterminacy and falsity-membership of X respectively. Consider the mapping $\tilde{\sigma}_X^t : \mathbb{U} \rightarrow D[0, 1]$, $\tilde{\sigma}_X^i : \mathbb{U} \rightarrow D[0, 1]$, $\tilde{\sigma}_X^f : \mathbb{U} \rightarrow D[0, 1]$ and $0 \leq (\tilde{\sigma}_X^t(x))^2 + (\tilde{\sigma}_X^i(x))^2 + (\tilde{\sigma}_X^f(x))^2 \leq 2$. Here $\tilde{X} = \left([\sigma_X^{tl}, \sigma_X^{tu}], [\sigma_X^{Il}, \sigma_X^{Iu}], [\sigma_X^{fl}, \sigma_X^{fu}] \right)$ is called a Pythagorean neutrosophic interval valued number (PyNIVN).

Definition 2.4. Let \mathbb{U} and E be the universe and set of parameter respectively. The pair $(\tilde{\Delta}, \tilde{X})$ or $\tilde{\Delta}_X$ is called a PyNIVFS set on \mathbb{U} if $X \subseteq E$ and $\tilde{\Delta} : X \rightarrow PyNIVF^{\mathbb{U}}$, $PyNIVF^{\mathbb{U}}$ is denote the set of all Pythagorean neutrosophic interval valued fuzzy subsets of \mathbb{U} . That is

$$\tilde{\Delta}_X = \left\{ \left(e, \left\{ \frac{x}{([\sigma_{\tilde{\Delta}_X}^{tl}(x), \sigma_{\tilde{\Delta}_X}^{tu}(x)]), [\sigma_{\tilde{\Delta}_X}^{Il}(x), \sigma_{\tilde{\Delta}_X}^{Iu}(x)]}, [\sigma_{\tilde{\Delta}_X}^{fl}(x), \sigma_{\tilde{\Delta}_X}^{fu}(x)]} \right\} \right) : e \in X, x \in \mathbb{U} \right\}.$$

Remark 2.5. If we write $\tilde{p}_{ij} = \tilde{\sigma}_{\tilde{\Delta}_X}^t(e_j)(x_i)$, $\tilde{q}_{ij} = \tilde{\sigma}_{\tilde{\Delta}_X}^i(e_j)(x_i)$ and $\tilde{r}_{ij} = \tilde{\sigma}_{\tilde{\Delta}_X}^f(e_j)(x_i)$, where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ then the PyNIVFS set $\tilde{\Delta}_X$ defined in matrix form:

$$\tilde{\Delta}_X = [(\tilde{p}_{ij}, \tilde{q}_{ij}, \tilde{r}_{ij})]_{m \times n} = \begin{bmatrix} (\tilde{p}_{11}, \tilde{q}_{11}, \tilde{r}_{11}) & (\tilde{p}_{12}, \tilde{q}_{12}, \tilde{r}_{12}) & \dots & (\tilde{p}_{1n}, \tilde{q}_{1n}, \tilde{r}_{1n}) \\ (\tilde{p}_{21}, \tilde{q}_{21}, \tilde{r}_{21}) & (\tilde{p}_{22}, \tilde{q}_{22}, \tilde{r}_{22}) & \dots & (\tilde{p}_{2n}, \tilde{q}_{2n}, \tilde{r}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{p}_{m1}, \tilde{q}_{m1}, \tilde{r}_{m1}) & (\tilde{p}_{m2}, \tilde{q}_{m2}, \tilde{r}_{m2}) & \dots & (\tilde{p}_{mn}, \tilde{q}_{mn}, \tilde{r}_{mn}) \end{bmatrix}$$

The matrix is known as Pythagorean neutrosophic interval valued soft matrix (PyNIVISM).

3 PyNIVFS-VIKOR using AO

Algorithm (PyNIVFS-VIKOR)

Step-1: Suppose that $\mathcal{D} = \{\mathcal{D}_i : i \in \mathbb{N}\}$ be the decision makers, $\mathcal{C} = \{\tau_i : i \in \mathbb{N}\}$ be the alternatives and $D = \{e_i : i \in \mathbb{N}\}$ is a parameters.

Step-2: Determine the weighted parameter matrix and linguistic variables

$$\mathcal{P} = [\omega_{ij}^l, \omega_{ij}^u]_{n \times m} = \begin{bmatrix} [\omega_{11}^l, \omega_{11}^u] & [\omega_{12}^l, \omega_{12}^u] & \dots & [\omega_{1m}^l, \omega_{1m}^u] \\ [\omega_{21}^l, \omega_{21}^u] & [\omega_{22}^l, \omega_{22}^u] & \dots & [\omega_{2m}^l, \omega_{2m}^u] \\ \vdots & \vdots & \ddots & \vdots \\ [\omega_{i1}^l, \omega_{i1}^u] & [\omega_{i2}^l, \omega_{i2}^u] & \dots & [\omega_{im}^l, \omega_{im}^u] \\ \vdots & \vdots & \ddots & \vdots \\ [\omega_{n1}^l, \omega_{n1}^u] & [\omega_{n2}^l, \omega_{n2}^u] & \dots & [\omega_{nm}^l, \omega_{nm}^u] \end{bmatrix}$$

where $[\omega_{ij}^l, \omega_{ij}^u]$ is the weight and assigned to the expert \mathcal{D}_i to e_j .

Step-3: Form the weighted normalized decision matrix as

$$\widehat{\mathcal{N}} = [\widehat{\xi}_{ij}^l, \widehat{\xi}_{ij}^u]_{n \times m} = \begin{bmatrix} [\xi_{11}^l, \xi_{11}^u] & [\xi_{12}^l, \xi_{12}^u] & \dots & [\xi_{1m}^l, \xi_{1m}^u] \\ [\xi_{21}^l, \xi_{21}^u] & [\xi_{22}^l, \xi_{22}^u] & \dots & [\xi_{2m}^l, \xi_{2m}^u] \\ \vdots & \vdots & \ddots & \vdots \\ [\xi_{i1}^l, \xi_{i1}^u] & [\xi_{i2}^l, \xi_{i2}^u] & \dots & [\xi_{im}^l, \xi_{im}^u] \\ \vdots & \vdots & \ddots & \vdots \\ [\xi_{n1}^l, \xi_{n1}^u] & [\xi_{n2}^l, \xi_{n2}^u] & \dots & [\xi_{nm}^l, \xi_{nm}^u] \end{bmatrix}$$

Here $[\widehat{\xi}_{ij}^l, \widehat{\xi}_{ij}^u] = \left[\frac{\omega_{ij}^l}{\sqrt{\sum_{i=1}^n \omega_{ij}^{2u}}}, \frac{\omega_{ij}^u}{\sqrt{\sum_{i=1}^n \omega_{ij}^{2l}}} \right]$ is the normalized criteria and find the weighted vector $\mathcal{W} =$

$([m_1^l, m_1^u], [m_2^l, m_2^u], \dots, [m_m^l, m_m^u])$, where $[m_i^l, m_i^u] = \left[\frac{\omega_i^l}{\sqrt{\sum_{l=1}^n \omega_{li}^u}}, \frac{\omega_i^u}{\sqrt{\sum_{l=1}^n \omega_{li}^l}} \right]$ mean relative weight of the

j^{th} parameter and $[\omega_j^l, \omega_j^u] = \left[\frac{\sum_{i=1}^n \widehat{\xi}_{ij}^l}{n}, \frac{\sum_{i=1}^n \widehat{\xi}_{ij}^u}{n} \right]$.

Step-4: The PyNIVFS decision matrix is given by

$$\mathcal{D}_i = [c_{jk}^{li}, c_{jk}^{ui}]_{l \times m} = \begin{bmatrix} [c_{11}^{li}, c_{11}^{ui}] & [c_{12}^{li}, c_{12}^{ui}] & \dots & [c_{1m}^{li}, c_{1m}^{ui}] \\ [c_{21}^{li}, c_{21}^{ui}] & [c_{22}^{li}, c_{22}^{ui}] & \dots & [c_{2m}^{li}, c_{2m}^{ui}] \\ \vdots & \vdots & \ddots & \vdots \\ [c_{j1}^{li}, c_{j1}^{ui}] & [c_{j2}^{li}, c_{j2}^{ui}] & \dots & [c_{jm}^{li}, c_{jm}^{ui}] \\ \vdots & \vdots & \ddots & \vdots \\ [c_{l1}^{li}, c_{l1}^{ui}] & [c_{l2}^{li}, c_{l2}^{ui}] & \dots & [c_{lm}^{li}, c_{lm}^{ui}] \end{bmatrix}$$

where $[c_{jk}^{li}, c_{jk}^{ui}]$ represents i^{th} decision maker $[\mathcal{D}_i^l, \mathcal{D}_i^u]$ for each i . The aggregating matrix $[\mathcal{X}^l, \mathcal{X}^u] = \frac{[\mathcal{D}_1^l, \mathcal{D}_1^u] + [\mathcal{D}_2^l, \mathcal{D}_2^u] + \dots + [\mathcal{D}_n^l, \mathcal{D}_n^u]}{n} = [x_{jk}^l, x_{jk}^u]_{l \times m}$.

Step-5: Find the weighted PyNIVFS decision matrix

$$[\mathcal{Y}^l, \mathcal{Y}^u] = [\tau_{jk}^l, \tau_{jk}^u]_{l \times m} = \begin{bmatrix} [\tau_{11}^l, \tau_{11}^u] & [\tau_{12}^l, \tau_{12}^u] & \dots & [\tau_{1m}^l, \tau_{1m}^u] \\ [\tau_{21}^l, \tau_{21}^u] & [\tau_{22}^l, \tau_{22}^u] & \dots & [\tau_{2m}^l, \tau_{2m}^u] \\ \vdots & \vdots & \ddots & \vdots \\ [\tau_{j1}^l, \tau_{j1}^u] & [\tau_{j2}^l, \tau_{j2}^u] & \dots & [\tau_{jm}^l, \tau_{jm}^u] \\ \vdots & \vdots & \ddots & \vdots \\ [\tau_{l1}^l, \tau_{l1}^u] & [\tau_{l2}^l, \tau_{l2}^u] & \dots & [\tau_{lm}^l, \tau_{lm}^u] \end{bmatrix}$$

where $[\tau_{jk}^l, \tau_{jk}^u] = [m_k^l \times x_{jk}^l, m_k^u \times x_{jk}^u]$.

Step-6: Calculate the values for PyNIVFSV-PIS and PyNIVFSV-NIS. Now,

PyNIVFSV-PIS = $([\tau_1^{l+}, \tau_1^{u+}], [\tau_2^{l+}, \tau_2^{u+}], \dots, [\tau_l^{l+}, \tau_l^{u+}])$
 $= \left\{ (\vee_k [\tau_{jk}^l, \tau_{jk}^u], \wedge_k [\tau_{jk}^l, \tau_{jk}^u], \wedge_k [\tau_{jk}^l, \tau_{jk}^u]) : j = 1, 2, \dots, l \right\}$ and

PyNIVFSV-NIS = $([\tau_1^{l-}, \tau_1^{u-}], [\tau_2^{l-}, \tau_2^{u-}], \dots, [\tau_l^{l-}, \tau_l^{u-}])$
 $= \left\{ (\wedge_k [\tau_{jk}^l, \tau_{jk}^u], \vee_k [\tau_{jk}^l, \tau_{jk}^u], \vee_k [\tau_{jk}^l, \tau_{jk}^u]) : j = 1, 2, \dots, l \right\}$.

Here PyNIVFS union \vee and PyNIVFS intersection \wedge .

Step-7: Determine the values of utility $[S_i^l, S_i^u]$, individual regret $[\mathcal{R}_i^l, \mathcal{R}_i^u]$ and compromise $[Q_i^l, Q_i^u]$, where

$$[S_i^l, S_i^u] = \left[\sum_{j=1}^m m_j^l \cdot \left(\sqrt{\frac{\tau_{ij}^{2l} - \tau_j^{2u+}}{\tau_j^{2u+} - \tau_j^{2l-}}} \right), \sum_{j=1}^m m_j^u \cdot \left(\sqrt{\frac{\tau_{ij}^{2u} - \tau_j^{2l+}}{\tau_j^{2l+} - \tau_j^{2u-}}} \right) \right]$$

$$\text{and } [\mathcal{R}_i^l, \mathcal{R}_i^u] = \left[\max_{j=1}^m m_j^l \cdot \left(\sqrt{\frac{\tau_{ij}^{2l} - \tau_j^{2u+}}{\tau_j^{2u+} - \tau_j^{2l-}}} \right), \sum_{j=1}^m m_j^u \cdot \left(\sqrt{\frac{\tau_{ij}^{2u} - \tau_j^{2l+}}{\tau_j^{2l+} - \tau_j^{2u-}}} \right) \right]$$

$$\text{and } [Q_i^l, Q_i^u] = \left[\kappa \left(\frac{S_i^l - S_i^{u-}}{S_i^{u+} - S_i^{l-}} \right) + (1 - \kappa) \left(\frac{\mathcal{R}_i^l - \mathcal{R}_i^{u-}}{\mathcal{R}_i^{u+} - \mathcal{R}_i^{l-}} \right), \kappa \left(\frac{S_i^u - S_i^{l-}}{S_i^{l+} - S_i^{u-}} \right) + (1 - \kappa) \left(\frac{\mathcal{R}_i^u - \mathcal{R}_i^{l-}}{\mathcal{R}_i^{l+} - \mathcal{R}_i^{u-}} \right) \right].$$

Hence $Q = \frac{Q_i^l + Q_i^u}{2}$, where $[S_i^{l+}, S_i^{u+}] = \max_i [S_i^l, S_i^u]$, $[S_i^{l-}, S_i^{u-}] = \min_i [S_i^l, S_i^u]$, $[\mathcal{R}_i^{l+}, \mathcal{R}_i^{u+}] = \max_i [\mathcal{R}_i^l, \mathcal{R}_i^u]$ and $[\mathcal{R}_i^{l-}, \mathcal{R}_i^{u-}] = \min_i [\mathcal{R}_i^l, \mathcal{R}_i^u]$. A coefficient of decision mechanism is the real number κ . It has been found that the role of κ in compromise solution (CS) is to determine whether CSs can be majority. If $\kappa > 0.5$, majority solutions can be consensus. If $\kappa = 0.5$, and veto solutions can be CSs, where $[m_j^l, m_j^u]$ denotes weight

of the j^{th} parameter.

Step-8: Derive a CS by ranking the choices. The ranking list should be arranged in increasing order of Q_i . When Q_i ranks lowest and meets both $C1$ and $C2$, the alternative τ_α will be declared a CS:

$C1$ admissible: If τ_α and τ_β represent top alternatives in Q , then $Q(\tau_\beta) - Q(\tau_\alpha) \geq \frac{1}{n-1}$. Here n is the number of parameters.

$C2$ admissible: The alternative τ_α is ranked by $[S_i^l, S_i^u] = \frac{S_i^l + S_i^u}{2}$ and (or) $[R_i^l, R_i^u] = \frac{R_i^l + R_i^u}{2}$.

If $C1$ and $C2$ are not simultaneously satisfied, then there exist multiple CSs:

(i) If $C1$ is true, then both alternatives τ_α and τ_β are called the CSs:

(ii) If $C1$ is false, then the alternatives $\tau_\alpha, \tau_\beta, \dots, \tau_\xi$ are called the CSs, where τ_ξ is founded by $Q(\tau_\xi) - Q(\tau_\alpha) \geq \frac{1}{n-1}$.

Example 3.1. An investment company plans to purchase some shares of the top five companies in the stock exchange to invest some money on. In order to minimize this factor, they decided to invest some of their cash in percentages of 30 dollars, 25 dollars, 20 dollars, 15 dollars and 10 dollars in accordance with the top five ranked companies in order to minimize the effect of this factor.

Step-1: Suppose $[\mathcal{D}^l, \mathcal{D}^u] = \{[\mathcal{D}_i^l, \mathcal{D}_i^u] : (i \text{ goes to } 1 \text{ to } 5)\}$ is a decision makers, $\mathcal{C} = \{\tau_i : i = 1, 2, \dots, 10\}$ is the collection of companies/alternatives and $D = \{e_i : i = 1, 2, \dots, 5\}$ is a family of parameters, where $e_1 =$ Momentum, $e_2 =$ Value, $e_3 =$ Growth, $e_4 =$ Volatility, $e_5 =$ Quality.

Step-2: Linguistic variables in tabular form

Linguistic variables	IVFweights
Very Good Value(VGV)	[0.9, 0.95]
Good Value(GV)	[0.8, 0.9]
Average Value(AV)	[0.65, 0.8]
Poor Value(PV)	[0.5, 0.65]
Very Poor Value(VPV)	[0.35, 0.5]

Form the weighted parameter matrix is given as

$$\begin{aligned}
 \mathcal{P} &= [\omega_{ij}^l, \omega_{ij}^u]_{5 \times 5} \\
 &= \begin{bmatrix} PV & VPV & VGV & VPV & GV \\ AV & VPV & PV & VGV & AV \\ VGV & AV & VGV & PV & VPV \\ VPV & VGV & AV & GV & PV \\ AV & PV & GV & AV & VPV \end{bmatrix} \\
 &= \begin{bmatrix} [0.5, 0.65] & [0.35, 0.5] & [0.9, 0.95] & [0.35, 0.5] & [0.8, 0.9] \\ [0.65, 0.8] & [0.35, 0.5] & [0.5, 0.65] & [0.9, 0.95] & [0.65, 0.8] \\ [0.9, 0.95] & [0.65, 0.8] & [0.9, 0.95] & [0.5, 0.65] & [0.35, 0.5] \\ [0.35, 0.5] & [0.9, 0.95] & [0.65, 0.8] & [0.8, 0.9] & [0.5, 0.65] \\ [0.65, 0.8] & [0.5, 0.65] & [0.8, 0.9] & [0.65, 0.8] & [0.35, 0.5] \end{bmatrix}
 \end{aligned}$$

where $[\omega_{ij}^l, \omega_{ij}^u]$ denotes the weight to $[\mathcal{D}_i^l, \mathcal{D}_i^u]$.

Step-3: As a result, the normalized weighted decision matrix can be formed as follows:

$$\begin{aligned}
 \hat{\mathcal{N}} &= [\hat{\xi}_{ij}^l, \hat{\xi}_{ij}^u]_{5 \times 5} \\
 &= \begin{bmatrix} [0.2959, 0.3847] & [0.2229, 0.3185] & [0.4693, 0.4954] & [0.2012, 0.2875] & [0.5194, 0.5843] \\ [0.3847, 0.4735] & [0.2229, 0.3185] & [0.2607, 0.339] & [0.5175, 0.5462] & [0.422, 0.5194] \\ [0.5326, 0.5622] & [0.414, 0.5095] & [0.4693, 0.4954] & [0.2875, 0.3737] & [0.2272, 0.3246] \\ [0.2071, 0.2959] & [0.5732, 0.6051] & [0.339, 0.4172] & [0.46, 0.5175] & [0.3246, 0.422] \\ [0.3847, 0.4735] & [0.3185, 0.414] & [0.4172, 0.4693] & [0.3737, 0.46] & [0.2272, 0.3246] \end{bmatrix}
 \end{aligned}$$

and weighted vector can be written as

$$W = ([0.0976, 0.1436], [0.103, 0.1575], [0.092, 0.1182], [0.0968, 0.1366], [0.1027, 0.1641]).$$

Step-4: The matrix $[\mathcal{X}^l, \mathcal{X}^u] = \frac{[\mathcal{D}_1^l, \mathcal{D}_1^u] + [\mathcal{D}_2^l, \mathcal{D}_2^u] + \dots + [\mathcal{D}_5^l, \mathcal{D}_5^u]}{5}$

$$= \begin{bmatrix} ([0.75, 0.8], [0.6, 0.65], [0.5, 0.55]) & ([0.27, 0.6], [0.75, 0.8], [0.66, 0.85]) & ([0.58, 0.6], [0.6, 0.65], [0.71, 0.75]) \\ ([0.65, 0.8], [0.7, 0.75], [0.5, 0.75]) & ([0.8, 0.85], [0.85, 0.9], [0.3, 0.4]) & ([0.85, 0.9], [0.45, 0.5], [0.5, 0.55]) \\ ([0.75, 0.8], [0.8, 0.82], [0.8, 0.81]) & ([0.7, 0.75], [0.75, 0.8], [0.35, 0.4]) & ([0.74, 0.75], [0.43, 0.45], [0.54, 0.55]) \\ ([0.7, 0.78], [0.6, 0.65], [0.6, 0.65]) & ([0.64, 0.7], [0.55, 0.65], [0.67, 0.75]) & ([0.8, 0.85], [0.7, 0.75], [0.51, 0.55]) \\ ([0.75, 0.8], [0.65, 0.68], [0.35, 0.45]) & ([0.45, 0.5], [0.66, 0.7], [0.85, 0.9]) & ([0.7, 0.75], [0.6, 0.65], [0.5, 0.7]) \\ ([0.82, 0.85], [0.85, 0.88], [0.45, 0.5]) & ([0.64, 0.66], [0.45, 0.5], [0.75, 0.8]) & ([0.5, 0.65], [0.65, 0.7], [0.8, 0.85]) \\ ([0.6, 0.7], [0.75, 0.77], [0.35, 0.4]) & ([0.5, 0.57], [0.49, 0.55], [0.8, 0.85]) & ([0.85, 0.9], [0.4, 0.45], [0.75, 0.9]) \\ ([0.55, 0.6], [0.8, 0.89], [0.7, 0.75]) & ([0.5, 0.55], [0.45, 0.8], [0.85, 0.9]) & ([0.48, 0.5], [0.6, 0.65], [0.7, 0.75]) \\ ([0.65, 0.8], [0.7, 0.88], [0.6, 0.75]) & ([0.61, 0.65], [0.75, 0.8], [0.7, 0.75]) & ([0.65, 0.7], [0.45, 0.5], [0.85, 0.9]) \\ ([0.55, 0.7], [0.85, 0.95], [0.35, 0.45]) & ([0.75, 0.8], [0.64, 0.65], [0.75, 0.8]) & ([0.55, 0.65], [0.54, 0.55], [0.75, 0.8]) \\ \dots & \dots & \dots \\ ([0.7, 0.75], [0.75, 0.8], [0.59, 0.65]) & ([0.7, 0.75], [0.8, 0.85], [0.7, 0.75]) \\ ([0.45, 0.48], [0.8, 0.85], [0.5, 0.75]) & ([0.7, 0.75], [0.4, 0.45], [0.8, 0.85]) \\ ([0.43, 0.45], [0.82, 0.85], [0.4, 0.45]) & ([0.72, 0.75], [0.42, 0.7], [0.7, 0.88]) \\ ([0.8, 0.85], [0.65, 0.7], [0.65, 0.7]) & ([0.55, 0.65], [0.5, 0.7], [0.75, 0.8]) \\ ([0.6, 0.65], [0.75, 0.8], [0.55, 0.65]) & ([0.75, 0.85], [0.6, 0.7], [0.5, 0.85]) \\ ([0.7, 0.75], [0.8, 0.85], [0.6, 0.7]) & ([0.6, 0.64], [0.58, 0.6], [0.6, 0.65]) \\ ([0.69, 0.7], [0.8, 0.9], [0.45, 0.5]) & ([0.7, 0.75], [0.8, 0.85], [0.4, 0.65]) \\ ([0.7, 0.75], [0.9, 0.95], [0.15, 0.35]) & ([0.56, 0.6], [0.9, 0.93], [0.3, 0.4]) \\ ([0.64, 0.75], [0.55, 0.65], [0.6, 0.65]) & ([0.45, 0.6], [0.85, 0.9], [0.6, 0.7]) \\ ([0.49, 0.6], [0.85, 0.9], [0.7, 0.75]) & ([0.6, 0.65], [0.75, 0.78], [0.8, 0.85]) \end{bmatrix}$$

$$= [x_{jk}^l, x_{jk}^u]_{10 \times 5}$$

Step-5: The weighted PyNIVFS decision matrix $[\mathcal{Y}^l, \mathcal{Y}^u] = [m_k^l \times x_{jk}^l, m_k^u \times x_{jk}^u]$

$$= \begin{bmatrix} ([0.0732, 0.1149], [0.0585, 0.0933], [0.0488, 0.079]) & ([0.0278, 0.0945], [0.0773, 0.126], [0.068, 0.1339]) \\ ([0.0634, 0.1149], [0.0683, 0.1077], [0.0488, 0.1077]) & ([0.0824, 0.1339], [0.0876, 0.1417], [0.0309, 0.063]) \\ ([0.0732, 0.1149], [0.0781, 0.1177], [0.0781, 0.1163]) & ([0.0721, 0.1181], [0.0773, 0.126], [0.0361, 0.063]) \\ ([0.0683, 0.112], [0.0585, 0.0933], [0.0585, 0.0933]) & ([0.0659, 0.1102], [0.0567, 0.1024], [0.069, 0.1181]) \\ ([0.0732, 0.1149], [0.0634, 0.0976], [0.0342, 0.0646]) & ([0.0464, 0.0787], [0.068, 0.1102], [0.0876, 0.1417]) \\ ([0.08, 0.1221], [0.0829, 0.1264], [0.0439, 0.0718]) & ([0.0659, 0.1039], [0.0464, 0.0787], [0.0773, 0.126]) \\ ([0.0585, 0.1005], [0.0732, 0.1106], [0.0342, 0.0574]) & ([0.0515, 0.0898], [0.0505, 0.0866], [0.0824, 0.1339]) \\ ([0.0537, 0.0862], [0.0781, 0.1278], [0.0683, 0.1077]) & ([0.0515, 0.0866], [0.0464, 0.126], [0.0876, 0.1417]) \\ ([0.0634, 0.1149], [0.0683, 0.1264], [0.0585, 0.1077]) & ([0.0628, 0.1024], [0.0773, 0.126], [0.0721, 0.1181]) \\ ([0.0537, 0.1005], [0.0829, 0.1364], [0.0342, 0.0646]) & ([0.0773, 0.126], [0.0659, 0.1024], [0.0773, 0.126]) \\ \dots & \dots \\ ([0.0534, 0.0709], [0.0552, 0.0768], [0.0653, 0.0886]) & ([0.0629, 0.0956], [0.0726, 0.1092], [0.0571, 0.0888]) \\ ([0.0782, 0.1064], [0.0414, 0.0591], [0.046, 0.065]) & ([0.0436, 0.0655], [0.0775, 0.1161], [0.0484, 0.1024]) \\ ([0.0681, 0.0886], [0.0396, 0.0532], [0.0497, 0.065]) & ([0.0416, 0.0614], [0.0794, 0.1161], [0.0387, 0.0614]) \\ ([0.0736, 0.1005], [0.0644, 0.0886], [0.0469, 0.065]) & ([0.0775, 0.1161], [0.0629, 0.0956], [0.0629, 0.0956]) \\ ([0.0644, 0.0886], [0.0552, 0.0768], [0.046, 0.0827]) & ([0.0581, 0.0888], [0.0726, 0.1092], [0.0533, 0.0888]) \\ ([0.046, 0.0768], [0.0598, 0.0827], [0.0736, 0.1005]) & ([0.0678, 0.1024], [0.0775, 0.1161], [0.0581, 0.0956]) \\ ([0.0782, 0.1064], [0.0368, 0.0532], [0.069, 0.1064]) & ([0.0668, 0.0956], [0.0775, 0.1229], [0.0436, 0.0683]) \\ ([0.0442, 0.0591], [0.0552, 0.0768], [0.0644, 0.0886]) & ([0.0678, 0.1024], [0.0872, 0.1297], [0.0145, 0.0478]) \\ ([0.0598, 0.0827], [0.0414, 0.0591], [0.0782, 0.1064]) & ([0.062, 0.1024], [0.0533, 0.0888], [0.0581, 0.0888]) \\ ([0.0506, 0.0768], [0.0497, 0.065], [0.069, 0.0946]) & ([0.0474, 0.0819], [0.0823, 0.1229], [0.0678, 0.1024]) \\ \dots & \dots \\ ([0.0719, 0.1231], [0.0822, 0.1395], [0.0719, 0.1231]) & ([0.0719, 0.1231], [0.0822, 0.1395]) \\ ([0.0719, 0.1231], [0.0411, 0.0739], [0.0822, 0.1395]) & ([0.0719, 0.1231], [0.0822, 0.1395]) \\ ([0.074, 0.1231], [0.0431, 0.1149], [0.0719, 0.1444]) & ([0.0719, 0.1444], [0.0719, 0.1444]) \\ ([0.0565, 0.1067], [0.0514, 0.1149], [0.077, 0.1313]) & ([0.077, 0.1313], [0.077, 0.1313]) \\ ([0.077, 0.1395], [0.0616, 0.1149], [0.0514, 0.1395]) & ([0.0514, 0.1395], [0.0514, 0.1395]) \\ ([0.0616, 0.1051], [0.0596, 0.0985], [0.0616, 0.1067]) & ([0.0616, 0.1067], [0.0616, 0.1067]) \\ ([0.0719, 0.1231], [0.0822, 0.1395], [0.0411, 0.1067]) & ([0.0411, 0.1067], [0.0411, 0.1067]) \\ ([0.0575, 0.0985], [0.0924, 0.1527], [0.0308, 0.0657]) & ([0.0308, 0.0657], [0.0308, 0.0657]) \\ ([0.0462, 0.0985], [0.0873, 0.1477], [0.0616, 0.1149]) & ([0.0616, 0.1149], [0.0616, 0.1149]) \\ ([0.0616, 0.1067], [0.077, 0.128], [0.0822, 0.1395]) & ([0.0822, 0.1395], [0.0822, 0.1395]) \end{bmatrix}$$

$$= [\tau_{jk}^l, \tau_{jk}^u]_{10 \times 5}$$

Step-6: In the following table, you will be able to find PyNIVFSV-PIS and PyNIVFSV-NIS.

τ^{l+}, τ^{u+}	<i>PyNIVFSV – PIS</i>
τ_1^{l+}, τ_1^{u+}	([0.08, 0.1221], [0.0585, 0.0933], [0.0342, 0.0574])
τ_2^{l+}, τ_2^{u+}	([0.0824, 0.1339], [0.0464, 0.0787], [0.0309, 0.063])
τ_3^{l+}, τ_3^{u+}	([0.0782, 0.1064], [0.0368, 0.0532], [0.046, 0.065])
τ_4^{l+}, τ_4^{u+}	([0.0775, 0.1161], [0.0533, 0.0888], [0.0145, 0.0478])
τ_5^{l+}, τ_5^{u+}	([0.077, 0.1395], [0.0411, 0.0739], [0.0308, 0.0657])

τ^{l-}, τ^{u-}	<i>PyNIVFSV – NIS</i>
τ_1^{l-}, τ_1^{u-}	([0.0537, 0.0862], [0.0829, 0.1364], [0.0781, 0.1163])
τ_2^{l-}, τ_2^{u-}	([0.0278, 0.0787], [0.0876, 0.1417], [0.0876, 0.1417])
τ_3^{l-}, τ_3^{u-}	([0.0442, 0.0591], [0.0644, 0.0886], [0.0782, 0.1064])
τ_4^{l-}, τ_4^{u-}	([0.0416, 0.0614], [0.0872, 0.1297], [0.0678, 0.1024])
τ_5^{l-}, τ_5^{u-}	([0.0462, 0.0985], [0.0924, 0.1527], [0.0822, 0.1444])

Step-7: Taking $\kappa = 0.5$, we found that the values of $[S_i^l, S_i^u]$, $[R_i^l, R_i^u]$ and Q_i .

Alternative (y)	S_i^l, S_i^u	R_i^l, R_i^u	Q_i^l, Q_i^u	Q_i
$\tau_1 = [\tau_1^l, \tau_1^u]$	[0.4012, 0.5659]	[0.1002, 0.1457]	[0.7337, 0.7565]	0.7451
$\tau_2 = [\tau_2^l, \tau_2^u]$	[0.37, 0.5522]	[0.0922, 0.1262]	[0.2493, 0.3006]	0.2749
$\tau_3 = [\tau_3^l, \tau_3^u]$	[0.3723, 0.5141]	[0.0949, 0.1462]	[0.3619, 0.4534]	0.4077
$\tau_4 = [\tau_4^l, \tau_4^u]$	[0.3635, 0.5437]	[0.0923, 0.1309]	[0.2128, 0.3396]	0.2762
$\tau_5 = [\tau_5^l, \tau_5^u]$	[0.381, 0.5518]	[0.0863, 0.1486]	[0.1116, 0.7286]	0.4201
$\tau_6 = [\tau_6^l, \tau_6^u]$	[0.369, 0.5492]	[0.0874, 0.1226]	[0.0728, 0.2123]	0.1425
$\tau_7 = [\tau_7^l, \tau_7^u]$	[0.3671, 0.5348]	[0.0954, 0.1354]	[0.346, 0.3716]	0.3588
$\tau_8 = [\tau_8^l, \tau_8^u]$	[0.4417, 0.5865]	[0.0968, 0.147]	[0.8735, 0.9079]	0.8907
$\tau_9 = [\tau_9^l, \tau_9^u]$	[0.4074, 0.5966]	[0.1004, 0.1404]	[0.7807, 0.8431]	0.8119
$\tau_{10} = [\tau_{10}^l, \tau_{10}^u]$	[0.4098, 0.5936]	[0.0987, 0.1454]	[0.735, 0.9196]	0.8273

Step-8: The ranking of alternatives Q_i are $\tau_6 \leq \tau_2 \leq \tau_4 \leq \tau_7 \leq \tau_3 \leq \tau_5 \leq \tau_1 \leq \tau_9 \leq \tau_{10} \leq \tau_8$.
 Now, $Q(\tau_2) - Q(\tau_6) = 0.1324 \not\geq \frac{1}{4}$. Thus, the condition C1 is false, further $Q(\tau_3) - Q(\tau_6) = 0.2652 \geq \frac{1}{4}$.
 Therefore, we decide $\tau_6, \tau_2, \tau_4, \tau_7$ and τ_3 are multiple CSs. It is therefore imperative that the firm invests in the future 30% on τ_6 , 25% on τ_2 , 20% on τ_4 , 15% on τ_7 and 10% on τ_3 .

4 Conclusion:

There is a newly developed concept towards decision making under uncertainty known as PyNIVFS set presented in this communication. There are a few ideas pertaining to the use of VIKOR methods for MCDM in groups. It could be described as an extension of interval valued fuzzy soft sets and neutrosophic fuzzy soft sets. In this algorithm, linguistic VIKOR approaches are used using AOs based on PyNIVFS linguistic VIKOR algorithms. By using some techniques, we were able to determine the scores function values based on interactions with the PyNIVFS aggregation operator. It was discussed that the utility S values of each alternative, as well as individual regret R and compromise Q values based on the VIKOR approach were different for each alternative. With the help of VIKOR, we ranked the choices and derived the compromise solution based on the rank of choices. Furthermore, we have included various graphs to illustrate the rankings of alternative options that we are considering, as well as different types of statistical charts.

Conflicts of Interest: The authors declare no conflict of interest.

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