Generating Weak Fuzzy Complex and Anti Weak Fuzzy Complex Integer Solutions for Pythagoras Diophantine Equation $X^2 + Y^2 = Z^2$

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Abstract

In this paper, we find necessary and sufficient conditions for a weak fuzzy complex integer triple (X, Y, Z) to be a pythagoras triple, and for an anti-weak fuzzy complex integer triple (X, Y, Z) to be a Pythagoras triple, where we prove that the non-linear Fermat's Diophantine equation has three different types of solutions according to the value of ε^2. All types will be solved and discussed in terms of theorems and examples that explains how the algorithms work.

Keywords: weak fuzzy complex integer; anti-weak fuzzy complex integer; Pythagoras triple; Pythagoras Diophantine equation.

1. Introduction.

The concept of weak fuzzy numbers was defined in [1] as a new generalization of real numbers in a similar approach of split-complex numbers [2-4]. This generalization has many applications and relationships with other algebraic structures. Pythagoras triples are solutions of the non-linear Diophantine equation $X^2 + Y^2 = Z^2$, they were studied by many authors with many interesting results and generalizations with many applications in algebra and computer science, see [5, 8-12].

Weak fuzzy complex numbers were very useful in the study of other algebraic structures such as spaces and products [6-7].

In this paper, we close a research gap by solving the non-linear Diophantine equation $X^2 + Y^2 = Z^2$ in the set of integer weak fuzzy complex numbers, where we discuss the all three type of the equation $X^2 + Y^2 = Z^2$ according to the values of $\varepsilon^2 = t \in]0,1[$, and we proved solutions for every possible case. Also, we discuss the anti-weak fuzzy complex integer solutions for the same equation for $\varepsilon^2 = t > 1$.

For definitions and basic concepts of weak fuzzy complex numbers/integers, see [1, 6-7].

2. Main discussion

Definition:

A weak fuzzy complex Pythagoras triple (X, Y, Z) is defined as follows:

$X^2 + Y^2 = Z^2; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon, Z = z_1 + z_2\varepsilon$, where $x_i, y_i, z_i \in Z, \varepsilon^2 = t \in]0,1[$.

Theorem.

The weak fuzzy complex Diophantine equation $X^2 + Y^2 = Z^2$ is equivalent to:

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$$\begin{cases}\left(x_1+\sqrt{t}x_2\right)^2+\left(y_1+\sqrt{t}y_2\right)^2=\left(z_1+\sqrt{t}z_2\right)^2\\ \left(x_1-\sqrt{t}x_2\right)^2+\left(y_1-\sqrt{t}y_2\right)^2=\left(z_1-\sqrt{t}z_2\right)^2\end{cases}$$

Proof.

We have:

$X^2=(x_1+x_2\varepsilon)^2={x_1}^2+{x_2}^2t+2x_1x_2\varepsilon$

$Y^2=(y_1+y_2\varepsilon)^2={y_1}^2+{y_2}^2t+2y_1y_2\varepsilon$

$Z^2=(z_1+z_2\varepsilon)^2={z_1}^2+{z_2}^2t+2z_1z_2\varepsilon$

Thus,

$$\begin{cases}{x_1}^2+{x_2}^2t+{y_1}^2+{y_2}^2t={z_1}^2+{z_2}^2t \quad (1)\\ 2x_1x_2+2y_1y_2=2z_1z_2 \quad (2)\end{cases}$$

We multiply (2) by $\sqrt{t}$, and compute $(1)+\sqrt{t}\,(2)$, $(1)-\sqrt{t}\,(2)$, then:

$$\begin{cases}{x_1}^2+{x_2}^2t+2\sqrt{t}x_1x_2+{y_1}^2+{y_2}^2t+2\sqrt{t}y_1y_2={z_1}^2+{z_2}^2t+2\sqrt{t}\,z_1z_2\\ {x_1}^2+{x_2}^2t-2\sqrt{t}x_1x_2+{y_1}^2+{y_2}^2t-2\sqrt{t}y_1y_2={z_1}^2+{z_2}^2t-2\sqrt{t}\,z_1z_2\end{cases}$$

Which is equivalent to:

$$\begin{cases}\left(x_1+\sqrt{t}x_2\right)^2+\left(y_1+\sqrt{t}y_2\right)^2=\left(z_1+\sqrt{t}z_2\right)^2\\ \left(x_1-\sqrt{t}x_2\right)^2+\left(y_1-\sqrt{t}y_2\right)^2=\left(z_1-\sqrt{t}z_2\right)^2\end{cases}$$

Thus, the proof is complete.

Definition.

A weak fuzzy complex Pythagoras triple (X,Y,Z) is called of type (1) if $\sqrt{t}=\frac{a}{b}\in Q, gcd(a,b)=1$.

It is called of type (2) if $\sqrt{t}\notin Q; t=\frac{a}{b}\in Q, gcd(a,b)=1$.

It is called of type (3) if $\sqrt{t}\notin Q; t\notin Q$.

3. Algorithm for finding solutions for type (1) equation.

Let $\sqrt{t}=\frac{a}{b}\in Q, gcd(a,b)=1$, then $X^2+Y^2=Z^2$ is equivalent to:

$$\begin{cases}\left(x_1+\frac{a}{b}x_2\right)^2+\left(y_1+\frac{a}{b}y_2\right)^2=\left(z_1+\frac{a}{b}z_2\right)^2 \quad (1)\\ \left(x_1-\frac{a}{b}x_2\right)^2+\left(y_1-\frac{a}{b}y_2\right)^2=\left(z_1-\frac{a}{b}z_2\right)^2 \quad (2)\end{cases}$$

We multiply $(1),(2)$ by $b^2\neq 0$, to get:

$$\begin{cases}(bx_1+ax_2)^2+(by_1+ay_2)^2=(bz_1+az_2)^2 \quad (1)\\ (bx_1-ax_2)^2+(by_1-ay_2)^2=(bz_1-az_2)^2 \quad (2)\end{cases}$$

This mean that:

$T_1=(bx_1+ax_2, by_1+ay_2, bz_1+az_2)$

$T_2=(bx_1-ax_2, by_1-ay_2, bz_1-az_2)$

Are two classical Pythagoras triples.

Now, let $T_1=(X_1,Y_1,Z_1), T_2=(X_2,Y_2,Z_2)$ are two Pythagoras triples in Z, then they give us a weak fuzzy complex Pythagoras triple if and only if:

$$\begin{cases}2b\setminus X_1+X_2, 2a\setminus X_1-X_2\\ 2b\setminus Y_1+Y_2, 2a\setminus Y_1-Y_2\\ 2b\setminus Z_1+Z_2, 2a\setminus Z_1-Z_2\end{cases}$$

on the other hand, we get:

$$X=\left(\frac{X_1+X_2}{2b}\right)+\varepsilon\left(\frac{X_1-X_2}{2a}\right), Y=\left(\frac{Y_1+Y_2}{2b}\right)+\varepsilon\left(\frac{Y_1-Y_2}{2a}\right), Z=\left(\frac{Z_1+Z_2}{2b}\right)+\varepsilon\left(\frac{Z_1-Z_2}{2a}\right)$$

example.

Consider $\varepsilon^2=t=\frac{1}{9}, \sqrt{t}=\frac{1}{3}\in Q$, then $X^2+Y^2=Z^2$ is Pythagoras Diophantine equation of type (1).

Consider the following two classical Pythagoras triples.

$T_1=(6,8,10), T_2=(24,10,26)$ are two pythagoras triples in Z, where $X_1=6, Y_1=8, Z_1=10, X_2=24, Y_2=10, Z_2=26$, and:

$$\begin{cases}X_1+X_2=30, X_1-X_2=-18\\ Y_1+Y_2=18, Y_1-Y_2=-2\\ Z_1+Z_2=36, Z_1-Z_2=-16\end{cases}$$

also,

$$\begin{cases}2b=6\setminus 30, 2a=2\setminus -18\\ 2b=6\setminus 18, 2a=2\setminus -2\\ 2b=6\setminus 36, 2a=2\setminus -16\end{cases}$$

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so that (T_1, T_2) gives a weak fuzzy complex pythagoras triple as follows:

$$X = \left(\frac{30}{6}\right) + \varepsilon\left(\frac{-18}{2}\right) = 5 - 9\varepsilon$$
$$Y = \left(\frac{18}{6}\right) + \varepsilon\left(\frac{-2}{2}\right) = 3 - \varepsilon$$
$$Z = \left(\frac{36}{6}\right) + \varepsilon\left(\frac{-16}{2}\right) = 6 - 8\varepsilon$$

4. Algorithm for solving type (3) equation.

Consider $t \notin Q$, then $X^2 + Y^2 = Z^2$ is written as follows:

$$\begin{cases} {x_1}^2 + {y_1}^2 + t({x_2}^2 + {y_2}^2) = {z_1}^2 + t{z_2}^2 & (1) \\ x_1x_2 + y_1y_2 = z_1z_2 & (2) \end{cases}$$

Since $x_i, y_i, z_i \in Z$ and $t \in R - Q$, then:

$$\begin{cases} {x_1}^2 + {y_1}^2 = {z_1}^2 & (1) \\ {x_2}^2 + {y_2}^2 = {z_2}^2 & (2) \\ {x_3}^2 + {y_3}^2 = {z_3}^2 & (3) \end{cases}$$

this means that $(x_1, y_1, z_1), (x_2, y_2, z_2)$ are two classical Pythagoras triples in Z.

From equation (3), we get:

$(x_1x_2 + y_1y_2)^2 = {z_1}^2{z_2}^2$, hence

${x_1}^2{x_2}^2 + {y_1}^2{y_2}^2 + 2x_1x_2y_1y_2 = {z_1}^2{z_2}^2$, thus:

${x_1}^2{x_2}^2 + ({z_1}^2 - {x_1}^2)({z_2}^2 - {x_2}^2) + 2x_1x_2y_1y_2 = {z_1}^2{z_2}^2$,

So that:

$2{x_1}^2{x_2}^2 - {z_1}^2{x_2}^2 - {x_1}^2{z_2}^2 + 2x_1x_2y_1y_2 = 0$,

$2x_1x_2(x_1x_2 + y_1y_2) - {x_1}^2{z_2}^2 - {z_1}^2{x_2}^2 = 0$,

$2x_1x_2(z_1z_2) - {x_1}^2{z_2}^2 - {z_1}^2{x_2}^2 = 0$

$(x_1z_2 - x_2z_1)^2 = 0 \Longrightarrow \frac{x_1}{x_2} = \frac{z_1}{z_2}$

On the other hand, we have:

${x_1}^2{z_2}^2 = {z_1}^2{x_2}^2 \Longrightarrow {x_1}^2({x_2}^2 + {y_2}^2) = {x_2}^2({x_1}^2 + {y_1}^2)$, thus:

${x_1}^2{y_2}^2 = {y_1}^2{x_2}^2$, hence $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$ under the assumption of $x_1, x_2, y_1, y_2, z_1, z_2 > 0$.

From this point of view, if $T_1 = (X_1, Y_1, Z_1), T_2 = (X_2, Y_2, Z_2)$ are two classical Pythagoras triples in Z, then they give us a weak fuzzy complex Pythagoras triple of type (3) if and only if:

$\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = c \in Q$.

Example.

For $t = \frac{1}{\sqrt{3}} \notin Q$, then $X^2 + Y^2 = Z^2$ is an equation of type (3).

Consider the following Pythagoras triples in Z, $T_1 = (3,4,5), T_2 = (6,8,10)$, then $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = \frac{1}{2} \in Q$.

This implies that (T_1, T_2) give us a weak fuzzy complex Pythagoras triple with respect to $t = \frac{1}{3}$.

$X = 3 + 6\varepsilon, Y = 4 + 8\varepsilon, Z = 5 + 10\varepsilon$.

5. Algorithm for solving equation of type (2).

Consider that $\frac{a}{b} = t \in Q, \sqrt{t} \notin Q$, then $X^2 + Y^2 = Z^2$ is of type (2), and it is written as follows:

$$\begin{cases} {x_1}^2 + {y_1}^2 + t({x_2}^2 + {y_2}^2) = {z_1}^2 + t{z_2}^2 \\ x_1x_2 + y_1y_2 = z_1z_2 \end{cases}$$

Which equivalent to:

$$\begin{cases} b({x_1}^2 + {y_1}^2 - {z_1}^2) + a({x_2}^2 + {y_2}^2 - {z_2}^2) = 0 & (1) \\ x_1x_2 + y_1y_2 = z_1z_2 & (2) \end{cases}$$

Put $L_1 = {x_1}^2 + {y_1}^2 - {z_1}^2, L_2 = {x_2}^2 + {y_2}^2 - {z_2}^2$, then (1) takes the following formula:

$bL_1 + aL_2 = 0; gcd\,(a, b) = 1$.

It is a linear Diophantine equation in two variables L_1, L_2.

It is solvable since $gcd\,(a, b) = 1$, and $L_1 = ka, L_2 = -kb$, where $k \in Z$, hence:

$\frac{L_1}{L_2} = -\frac{a}{b} = -t$ if $L_1, L_2 \neq 0$, or $L_1 = L_2 = 0$.

We have possible cases:

Case (1).

If $L_1 = L_2 = 0$, then $(x_1, y_1, z_1), (x_2, y_2, z_2)$ are two Pythagoras triple in Z with $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = c \in Q$ (the proof is exactly similar of type (3) equation).

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Case (2).

If $L_1 \neq 0, L_2 \neq 0$, then:

$$\begin{cases} \frac{L_1}{L_2} = -\frac{a}{b} = -t < 0 \quad (1) \\ x_1x_2 + y_1y_2 = z_1z_2 \quad (2) \end{cases}$$

Then, $(x_1, y_1, z_1), (x_2, y_2, z_2)$ are not Pythagoras triples in Z.

Example.

For $t = \frac{1}{10}, \sqrt{t} \notin Q$, consider the following triples:

$T_1 = (X_1, Y_1, Z_1) = (1,3,2), T_2 = (X_2, Y_2, Z_2) = (2,6,10)$

We have:

$$\frac{L_1}{L_2} = \frac{{x_1}^2 + {y_1}^2 - {z_1}^2}{{x_2}^2 + {y_2}^2 - {z_2}^2} = \frac{6}{-60} = \frac{-1}{10} = -t$$

Also, $x_1x_2 + y_1y_2 = 20 = z_1z_2$, this implies that:

$X = 1 + 2\varepsilon, Y = 3 + 6\varepsilon, Z = 2 + 10\varepsilon$ is a Pythagoras weak fuzzy complex triple of type (2).

By easy computing, we find:

$$X^2 = 1 + \frac{4}{10} + 4\varepsilon = \frac{14}{10} + 4\varepsilon$$

$$Y^2 = 9 + \frac{36}{10} + 36\varepsilon = \frac{126}{10} + 36\varepsilon$$

$$Z^2 = 4 + \frac{100}{10} + 40\varepsilon = 14 + 40\varepsilon$$

It is easy to see that $X^2 + Y^2 = Z^2$.

Definition:

An anti-weak fuzzy complex Pythagoras triple (X, Y, Z) is defined as follows:

$X^2 + Y^2 = Z^2; X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon, Z = z_1 + z_2\varepsilon$, where $x_i, y_i, z_i \in Z, \varepsilon^2 = t > 1$.

Theorem.

The anti-weak fuzzy complex Diophantine equation $X^2 + Y^2 = Z^2$ is equivalent to:

$$\begin{cases} \left(x_1 + \sqrt{t}x_2\right)^2 + \left(y_1 + \sqrt{t}y_2\right)^2 = \left(z_1 + \sqrt{t}z_2\right)^2 \\ \left(x_1 - \sqrt{t}x_2\right)^2 + \left(y_1 - \sqrt{t}y_2\right)^2 = \left(z_1 - \sqrt{t}z_2\right)^2 \end{cases}$$

Proof.

We have:

$X^2 = (x_1 + x_2\varepsilon)^2 = {x_1}^2 + {x_2}^2t + 2x_1x_2\varepsilon$

$Y^2 = (y_1 + y_2\varepsilon)^2 = {y_1}^2 + {y_2}^2t + 2y_1y_2\varepsilon$

$Z^2 = (z_1 + z_2\varepsilon)^2 = {z_1}^2 + {z_2}^2t + 2z_1z_2\varepsilon$

Thus,

$$\begin{cases} {x_1}^2 + {x_2}^2t + {y_1}^2 + {y_2}^2t = {z_1}^2 + {z_2}^2t \quad (1) \\ 2x_1x_2 + 2y_1y_2 = 2z_1z_2 \quad (2) \end{cases}$$

We multiply (2) by $\sqrt{t}$, and compute $(1) + \sqrt{t}\,(2), (1) - \sqrt{t}\,(2)$, then:

$$\begin{cases} {x_1}^2 + {x_2}^2t + 2\sqrt{t}x_1x_2 + {y_1}^2 + {y_2}^2t + 2\sqrt{t}y_1y_2 = {z_1}^2 + {z_2}^2t + 2\sqrt{t}\,z_1z_2 \\ {x_1}^2 + {x_2}^2t - 2\sqrt{t}x_1x_2 + {y_1}^2 + {y_2}^2t - 2\sqrt{t}y_1y_2 = {z_1}^2 + {z_2}^2t - 2\sqrt{t}\,z_1z_2 \end{cases}$$

Which is equivalent to:

$$\begin{cases} \left(x_1 + \sqrt{t}x_2\right)^2 + \left(y_1 + \sqrt{t}y_2\right)^2 = \left(z_1 + \sqrt{t}z_2\right)^2 \\ \left(x_1 - \sqrt{t}x_2\right)^2 + \left(y_1 - \sqrt{t}y_2\right)^2 = \left(z_1 - \sqrt{t}z_2\right)^2 \end{cases}$$

Thus, the proof is complete.

Definition.

An anti- weak fuzzy complex Pythagoras triple (X, Y, Z) is called of type (1) if $\sqrt{t} = \frac{a}{b} \in Q, gcd(a, b) = 1$.

It is called of type (2) if $\sqrt{t} \notin Q; t = \frac{a}{b} \in Q, gcd(a, b) = 1$.

It is called of type (3) if $\sqrt{t} \notin Q; t \notin Q$.

6. Algorithm for finding solutions for type (1) equation.

Let $\sqrt{t} = \frac{a}{b} \in Q, gcd(a, b) = 1$, then $X^2 + Y^2 = Z^2$ is equivalent to:

$$\begin{cases} \left(x_1 + \frac{a}{b}x_2\right)^2 + \left(y_1 + \frac{a}{b}y_2\right)^2 = \left(z_1 + \frac{a}{b}z_2\right)^2 \quad (1) \\ \left(x_1 - \frac{a}{b}x_2\right)^2 + \left(y_1 - \frac{a}{b}y_2\right)^2 = \left(z_1 - \frac{a}{b}z_2\right)^2 \quad (2) \end{cases}$$

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We multiply $(1),(2)$ by $b^2 \neq 0$, to get:

$$\begin{cases}(bx_1+ax_2)^2+(by_1+ay_2)^2=(bz_1+az_2)^2 & (1)\\ (bx_1-ax_2)^2+(by_1-ay_2)^2=(bz_1-az_2)^2 & (2)\end{cases}$$

This means that:

$T_1=(bx_1+ax_2, by_1+ay_2, bz_1+az_2)$

$T_2=(bx_1-ax_2, by_1-ay_2, bz_1-az_2)$

Are two classical Pythagoras triples.

Now, let $T_1=(X_1,Y_1,Z_1), T_2=(X_2,Y_2,Z_2)$ are two Pythagoras triples in Z, then they give us an anti-weak fuzzy complex Pythagoras triple if and only if:

$$\begin{cases}2b \setminus X_1+X_2, 2a \setminus X_1-X_2\\ 2b \setminus Y_1+Y_2, 2a \setminus Y_1-Y_2\\ 2b \setminus Z_1+Z_2, 2a \setminus Z_1-Z_2\end{cases}$$

on the other hand, we get:

$$X=\left(\frac{X_1+X_2}{2b}\right)+\varepsilon\left(\frac{X_1-X_2}{2a}\right), Y=\left(\frac{Y_1+Y_2}{2b}\right)+\varepsilon\left(\frac{Y_1-Y_2}{2a}\right), Z=\left(\frac{Z_1+Z_2}{2b}\right)+\varepsilon\left(\frac{Z_1-Z_2}{2a}\right)$$

Example.

Consider $\varepsilon^2=t=9, \sqrt{t}=2 \in Q$, then $X^2+Y^2=Z^2$ is Pythagoras Diophantine equation of type (1).

Consider the following two classical Pythagoras triples.

$T_1=(6,8,10), T_2=(10,24,26)$ are two pythagoras triples in Z, where $X_1=6, Y_1=8, Z_1=10, X_2=24, Y_2=10, Z_2=26$, and:

$$\begin{cases}X_1+X_2=16, X_1-X_2=-4\\ Y_1+Y_2=32, Y_1-Y_2=-16\\ Z_1+Z_2=36, Z_1-Z_2=-16\end{cases}$$

also,

$$\begin{cases}2b=2 \setminus 16, 2a=4 \setminus -4\\ 2b=2 \setminus 32, 2a=4 \setminus -16\\ 2b=2 \setminus 36, 2a=4 \setminus -16\end{cases}$$

so that (T_1,T_2) gives an anti-weak fuzzy complex pythagoras triple as follows:

$$X=\left(\frac{16}{2}\right)+\varepsilon\left(\frac{-4}{4}\right)=8-\varepsilon$$

$$Y=\left(\frac{32}{2}\right)+\varepsilon\left(\frac{-16}{4}\right)=16-4\varepsilon$$

$$Z=\left(\frac{36}{2}\right)+\varepsilon\left(\frac{-16}{4}\right)=18-4\varepsilon$$

7. Algorithm for solving type (3) equation.

Consider $t \notin Q$, then $X^2+Y^2=Z^2$ is written as follows:

$$\begin{cases}x_1{}^2+y_1{}^2+t(x_2{}^2+y_2{}^2)=z_1{}^2+tz_2{}^2 & (1)\\ x_1x_2+y_1y_2=z_1z_2 & (2)\end{cases}$$

Since $x_i, y_i, z_i \in Z$ and $t \in R-Q$, then:

$$\begin{cases}x_1{}^2+y_1{}^2=z_1{}^2 & (1)\\ x_2{}^2+y_2{}^2=z_2{}^2 & (2)\\ x_3{}^2+y_3{}^2=z_3{}^2 & (3)\end{cases}$$

This means that $(x_1,y_1,z_1),(x_2,y_2,z_2)$ are two classical Pythagoras triples in Z.

From equation (3), we get:

$(x_1x_2+y_1y_2)^2=z_1{}^2z_2{}^2$, hence

$x_1{}^2x_2{}^2+y_1{}^2y_2{}^2+2x_1x_2y_1y_2=z_1{}^2z_2{}^2$, thus:

$x_1{}^2x_2{}^2+(z_1{}^2-x_1{}^2)(z_2{}^2-x_2{}^2)+2x_1x_2y_1y_2=z_1{}^2z_2{}^2$,

So that:

$2x_1{}^2x_2{}^2-z_1{}^2x_2{}^2-x_1{}^2z_2{}^2+2x_1x_2y_1y_2=0$,

$2x_1x_2(x_1x_2+y_1y_2)-x_1{}^2z_2{}^2-z_1{}^2x_2{}^2=0$,

$2x_1x_2(z_1z_2)-x_1{}^2z_2{}^2-z_1{}^2x_2{}^2=0$

$(x_1z_2-x_2z_1)^2=0 \Rightarrow \frac{x_1}{x_2}=\frac{z_1}{z_2}$

On the other hand, we have:

$x_1{}^2z_2{}^2=z_1{}^2x_2{}^2 \Rightarrow x_1{}^2(x_2{}^2+y_2{}^2)=x_2{}^2(x_1{}^2+y_1{}^2)$, thus:

$x_1{}^2y_2{}^2=y_1{}^2x_2{}^2$, hence $\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2}$ under the assumption of $x_1,x_2,y_1,y_2,z_1,z_2>0$.

From this point of view, if $T_1=(X_1,Y_1,Z_1), T_2=(X_2,Y_2,Z_2)$ are two classical Pythagoras triples in Z, then they give us an anti-weak fuzzy complex Pythagoras triple of type (3) if and only if:

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$\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = c \in Q$.

Example.

For $t = \sqrt{3} \notin Q$, then $X^2 + Y^2 = Z^2$ is an equation of type (3).

Consider the following Pythagoras triples in Z, $T_1 = (3,4,5), T_2 = (6,8,10)$, then $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = \frac{1}{2} \in Q$.

This implies that (T_1, T_2) give us a weak fuzzy complex Pythagoras triple with respect to $t = 3$.

$X = 3 + 6\varepsilon, Y = 4 + 8\varepsilon, Z = 5 + 10\varepsilon$.

8. Algorithm for solving equation of type (2).

Consider that $\frac{a}{b} = t \in Q, \sqrt{t} \notin Q$, then $X^2 + Y^2 = Z^2$ is of type (2), and it is written as follows:

$$\begin{cases} x_1{}^2 + y_1{}^2 + t(x_2{}^2 + y_2{}^2) = z_1{}^2 + tz_2{}^2 \\ x_1x_2 + y_1y_2 = z_1z_2 \end{cases}$$

Which equivalent to:

$$\begin{cases} b(x_1{}^2 + y_1{}^2 - z_1{}^2) + a(x_2{}^2 + y_2{}^2 - z_2{}^2) = 0 \quad (1) \\ x_1x_2 + y_1y_2 = z_1z_2 \quad (2) \end{cases}$$

Put $L_1 = x_1{}^2 + y_1{}^2 - z_1{}^2, L_2 = x_2{}^2 + y_2{}^2 - z_2{}^2$, then (1) takes the following formula:

$bL_1 + aL_2 = 0; gcd\,(a, b) = 1$.

It is a linear Diophantine equation in two variables L_1, L_2.

It is solvable since $gcd\,(a, b) = 1$, and $L_1 = ka, L_2 = -kb$, where $k \in Z$, hence:

$\frac{L_1}{L_2} = -\frac{a}{b} = -t$ if $L_1, L_2 \neq 0$, or $L_1 = L_2 = 0$.

We have possible cases:

Case (1).

If $L_1 = L_2 = 0$, then $(x_1, y_1, z_1), (x_2, y_2, z_2)$ are two Pythagoras triple in Z with $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = c \in Q$ (the proof is exactly similar of type (3) equation).

Case (2).

If $L_1 \neq 0, L_2 \neq 0$, then:

$$\begin{cases} \frac{L_1}{L_2} = -\frac{a}{b} = -t < 0 \quad (1) \\ x_1x_2 + y_1y_2 = z_1z_2 \quad (2) \end{cases}$$

Then, $(x_1, y_1, z_1), (x_2, y_2, z_2)$ are not Pythagoras triples in Z.

Example.

For $t = 10, \sqrt{t} \notin Q$, consider the following triples:

$T_2 = (x_2, y_2, z_2) = (1,3,2), T_1 = (x_1, y_1, z_1) = (2,6,10)$

We have:

$$\frac{L_1}{L_2} = \frac{x_1{}^2 + y_1{}^2 - z_1{}^2}{x_2{}^2 + y_2{}^2 - z_2{}^2} = \frac{-60}{6} = -10 = -t$$

Also, $x_1x_2 + y_1y_2 = 20 = z_1z_2$, this implies that:

$X = 2 + \varepsilon, Y = 6 + 3\varepsilon, Z = 10 + 2\varepsilon$ is a Pythagoras anti-weak fuzzy complex triple of type (2).

9. Conclusion

In this paper, we studied for the first time the weak fuzzy complex and anti-weak fuzzy complex Pythagoras triples, where algorithms for generating weak fuzzy and anti-weak fuzzy complex integer solutions for the non-linear Diophantine equation $X^2 + Y^2 = Z^2$ are presented and handled by many related theorems and examples that clarify the novelty of this work.

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