On Weak Fuzzy Complex Pythagoras Quadruples

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Abstract

In this work, we study the generating of Pythagoras quadruples in the sets of weak fuzzy complex integers and anti-weak fuzzy complex integers, where we present sufficient and necessary conditions for generating Pythagoras quadruples in the mentioned sets. Also, we present many examples to clarify our work's validity and novelty.

Keywords: weak fuzzy complex integer; anti-weak fuzzy complex integer; Pythagoras quadruple; Diophantine equation

1. Introduction

An important way in mathematics to pose new problems is to find generalizations of traditional structures, where new generalizations open many research doors.

Perhaps the most famous set that has been given attention to generalizing is the set of real numbers, as it has been generalized through complex numbers, split-complex numbers, and weak fuzzy complex numbers [1-4,7].

Weak fuzzy complex numbers were defined recently in [1], by the following formula:

$\{a + b\varepsilon;\ \varepsilon^2 = t \in]0,1[, a, b \in R\}$. Anti-weak fuzzy complex numbers are defined as follows:

$\{a + b\varepsilon;\ \varepsilon^2 = t > 1, a, b \in R\}$. These two sets are commutative rings with unity (1).

In [5-6], weak fuzzy complex numbers were used to define a generalization of real vector spaces and inner products.

The concept of weak fuzzy complex Diophantine equations began in [8], where weak fuzzy complex integer Pythagoras triples and anti-weak fuzzy complex integer Pythagoras triples were generated and presented.

This has motivated us to study the generating of weak fuzzy complex and anti-weak fuzzy complex integer Pythagoras quadruples. Also, we illustrate some examples to explain the novelty of our work [9,10,11].

2. Main Discussion

Definition:
Let (X, Y, Z, T) be a 4-triple in the set of weak fuzzy complex integers, i.e.
$X = x_1 + x_2\varepsilon, Y = y_1 + y_2\varepsilon, Z = z_1 + z_2\varepsilon, T = t_1 + t_2\varepsilon$, where $x_i, y_i, z_i, t_i \in Z, \varepsilon^2 = c \in]0,1[$
It is called a Pythagoras quadruple if it is a solution of the non-linear Diophantine $X^2 + Y^2 + Z^2 = T^2$.

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Theorem.

The non-linear Diophantine equation $X^2 + Y^2 + Z^2 = T^2$ is equivalent to:

$$\begin{cases} {x_1}^2 + {x_2}^2c + {y_1}^2 + {y_2}^2c + {z_1}^2 + {z_2}^2c = {t_1}^2 + {t_2}^2c & (1) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (2) \end{cases}$$

Proof.

The proof holds directly from the definition of weak fuzzy complex numbers.

Definition.

The equation $X^2 + Y^2 + Z^2 = T^2$.

1). It is called of type (1) if $\sqrt{c} = \frac{a}{b} \in Q, gcd(a,b) = 1$.

2). It is called type (2) if $\sqrt{c} \notin Q; c = \frac{a}{b} \in Q, gcd(a,b) = 1$.

It is called type (3) if $c \notin Q$.

Solving equation of type $(\mathbf{1})$.

Consider $X^2 + Y^2 + Z^2 = T^2$ of type (1), with $c = \frac{a}{b} \in Q, gcd(a,b) = 1$

Theorem.

(X, Y, Z, T) is a weak fuzzy complex Pythagoras quadruple of type (1) if and only if:

$L_1 = \left(x_1 + x_2\sqrt{c}, y_1 + y_2\sqrt{c}, z_1 + z_2\sqrt{c}, t_1 + t_2\sqrt{c}\right)$

$L_2 = \left(x_1 - x_2\sqrt{c}, y_1 - y_2\sqrt{c}, z_1 - z_2\sqrt{c}, t_1 - t_2\sqrt{c}\right)$

Are two Pythagoras quadruples in Q.

Proof.

$X^2 = (x_1 + x_2\varepsilon)^2 = {x_1}^2 + {x_2}^2c + 2x_1x_2\varepsilon$

$Y^2 = (y_1 + y_2\varepsilon)^2 = {y_1}^2 + {y_2}^2c + 2y_1y_2\varepsilon$

$Z^2 = (z_1 + z_2\varepsilon)^2 = {z_1}^2 + {z_2}^2c + 2z_1z_2\varepsilon$

$T^2 = (t_1 + t_2\varepsilon)^2 = {t_1}^2 + {t_2}^2c + 2t_1t_2\varepsilon$

Thus:

$$\begin{cases} {x_1}^2 + {x_2}^2c + {y_1}^2 + {y_2}^2c + {z_1}^2 + {z_2}^2c = {t_1}^2 + {t_2}^2c & (1) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (2) \end{cases}$$

We multiply (2) by $\sqrt{c}$ and compute $(1) + (2)\sqrt{c}, (1) - (2)\sqrt{c}$, to get:

$$\begin{cases} \left(x_1 + x_2\sqrt{c}\right)^2 + \left(y_1 + y_2\sqrt{c}\right)^2 + \left(z_1 + z_2\sqrt{c}\right)^2 = \left(t_1 + t_2\sqrt{c}\right)^2 & (I) \\ \left(x_1 - x_2\sqrt{c}\right)^2 + \left(y_1 - y_2\sqrt{c}\right)^2 + \left(z_1 - z_2\sqrt{c}\right)^2 = \left(t_1 - t_2\sqrt{c}\right)^2 & (II) \end{cases}$$

Which implies the proof.

Remark.

To solve equation of type (1), we multiply (I) and (II) by $b^2 \neq 0$, hence:

$$\begin{cases} (bx_1 + ax_2)^2 + (by_1 + ay_2)^2 + (bz_1 + az_2)^2 = (bt_1 + at_2)^2 & (\acute{I}) \\ (bx_1 - ax_2)^2 + (by_1 - ay_2)^2 + (bz_1 - az_2)^2 = (bt_1 - at_2)^2 & (\acute{I}I) \end{cases}$$

Now, suppose that:

$L_1 = (X_1, Y_1, Z_1, T_1), L_2 = (X_2, Y_2, Z_2, T_2)$

$$\begin{cases} 2b \backslash X_1 + X_2, 2a \backslash X_1 - X_2 \\ 2b \backslash Y_1 + Y_2, 2a \backslash Y_1 - Y_2 \\ 2b \backslash Z_1 + Z_2, 2a \backslash Z_1 - Z_2 \\ 2b \backslash T_1 + T_2, 2a \backslash T_1 - T_2 \end{cases}$$

on the other hand, we get:

$$X = \left(\frac{X_1 + X_2}{2b}\right) + \varepsilon\left(\frac{X_1 - X_2}{2a}\right), Y = \left(\frac{Y_1 + Y_2}{2b}\right) + \varepsilon\left(\frac{Y_1 - Y_2}{2a}\right), Z = \left(\frac{Z_1 + Z_2}{2b}\right) + \varepsilon\left(\frac{Z_1 - Z_2}{2a}\right), T = \left(\frac{T_1 + T_2}{2b}\right) + \varepsilon\left(\frac{T_1 - T_2}{2a}\right)$$

example.

Consider $\varepsilon^2 = c = \frac{1}{9}, \sqrt{t} = \frac{1}{3}, a = 1, b = 3$, we have:

$L_1 = (3,6,6,9), L_2 = (9,0,0,9)$ are two pythagoras quadruples t in Z.

$$\begin{cases} 2b = 6 \backslash 12, 2a = 2 \backslash -6 \\ 2b = 6 \backslash 6, 2a = 2 \backslash 6 \\ 2b = 6 \backslash 6, 2a = 2 \backslash 6 \\ 2b = 6 \backslash 18, 2a = 2 \backslash 0 \end{cases}$$

so that (L_1, Ls_2) gives a weak fuzzy complex pythagoras quadruple follows:

$X = 2 - 3\varepsilon$

$Y = 1 + 3\varepsilon$

$Z = 1 + 3\varepsilon$

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$T = 3$

Solving equation of type(3).

Consider $c \notin Q$, then $X^2 + Y^2 + Z^2 = T^2$ is equivalent to:

$$\begin{cases} (x_1{}^2 + y_1{}^2 + z_1{}^2) + c(x_2{}^2 + y_2{}^2 + z_2{}^2) = t_1{}^2 + ct_2{}^2 \quad (1) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \quad (2) \end{cases}$$

Which is equivalent to:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 = t_1{}^2 \quad (I) \\ x_2{}^2 + y_2{}^2 + z_2{}^2 = t_2{}^2 \quad (II) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \quad (III) \end{cases}$$

thus

$L_1 = (x_1, y_1, z_1, t_1), L_2 = (x_2, y_2, z_2, t_2)$ are two Pythagoras quadruples in Z.

By computing $(I) + (II) + (III)$ and $(I) + (II) - 2(III)$, we get:

$$\begin{cases} (x_1 + x_2)^2 + (y_1 + y_2)^2 + (z_1 + z_2)^2 = (t_1 + t_2)^2 \\ (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = (t_1 - t_2)^2 \end{cases}$$

So that, $L_1 + L_2, L_1 - L_2$ are two Pythagoras quadruples in Z.

Example.

For $\varepsilon^2 = \frac{1}{\sqrt{3}} \notin Q$, we have $L_1 = (3,6,6,9), L_2 = (6,12,12,18)$, and:

$L_1 + L_2 = (9,18,18,27), L_1 - L_2 = (-3,-6,-6,-9)$ are two Pythagoras quadruples in Z, hence L_1, L_2 give us a weak fuzzy complex Pythagoras quadruple as follows:

$X = 3 + 6\varepsilon, Y = 6 + 12\varepsilon, Z = 6 + 12\varepsilon, T = 9 + 18\varepsilon$.

Solving equation of type (2).

Consider that $\varepsilon^2 = c = \frac{a}{b} \in Q, gcd(a, b) = 1, \sqrt{c} \notin Q$, the equation $X^2 + Y^2 + Z^2 = T^2$ is equivalent to:

$$\begin{cases} (x_1{}^2 + y_1{}^2 + z_1{}^2) + c(x_2{}^2 + y_2{}^2 + z_2{}^2) = t_1{}^2 + ct_2{}^2 \quad (1) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \quad (2) \end{cases}$$

We multiply (1) by $b \neq 0$ to get: Which equivalent to:

$$\begin{cases} b(x_1{}^2 + y_1{}^2 + z_1{}^2) + a(x_2{}^2 + y_2{}^2 + z_2{}^2) = bt_1{}^2 + at_2{}^2 \quad (I) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \quad (II) \end{cases}$$

Thus:

$$\begin{cases} b(x_1{}^2 + y_1{}^2 + z_1{}^2 - t_1{}^2) + a(x_2{}^2 + y_2{}^2 + z_2{}^2 - t_2{}^2) = 0 \quad (\acute{I}) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \quad (\acute{I}I) \end{cases}$$

We put $S_1 = x_1{}^2 + y_1{}^2 + z_1{}^2 - t_1{}^2, S_2 = x_2{}^2 + y_2{}^2 + z_2{}^2 - t_2{}^2$, hence:

$$\begin{cases} bS_1 + aS_2 = 0 \quad (\acute{I}) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \quad (\acute{I}I) \end{cases}$$

$(\acute{I})$ is linear Diophantine equation in two variables, so that $S_1 = ka, S_2 = -kb$, where $k \in Z$,this implies that:

$$\begin{cases} \frac{S_1}{S_2} = -\frac{a}{b} = -c < 0; \ S_1 \neq 0, S_2 \neq 0 \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \end{cases}$$

Or:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 = t_1{}^2, x_2{}^2 + y_2{}^2 + z_2{}^2 = t_2{}^2 \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \end{cases}$$

We have two possible cases:

Case (1).

For $S_1 = S_2 = 0$, then $L_1 = (x_1, y_1, z_1, t_1), L_2 = (x_2, y_2, z_2, t_2)$ are two Pythagoras quadruples in$Z$.

Also, $L_1 + L_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2, t_1 + t_2), L_1 - L_2 = (x_1 - x_2, y_1 - y_2, z_1 - z_2, t_1 - t_2)$ are two Pythagoras quadruples in$Z$, (it can be proved a similar argument of type(3) equation).

Case (2).

If $S_1 \neq 0, S_2 \neq 0$, then we should chose $L_1 = (x_1, y_1, z_1, t_1), L_2 = (x_2, y_2, z_2, t_2)$ such that:

$$\begin{cases} \frac{x_1{}^2 + y_1{}^2 + z_1{}^2 - t_1{}^2}{x_2{}^2 + y_2{}^2 + z_2{}^2 - t_2{}^2} = -c < 0 \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \end{cases}$$

Example.

For $\varepsilon^2 = c = \frac{1}{3}, \sqrt{c} \notin Q$, we chose:

$L_1 = (1,1,1,0), T_2 = (0,0,0,3)$.

$S_1 = 1 + 1 + 1 - 0 = 3, S_2 = 0 + 0 + 0 - 9 = -9, \frac{S_1}{S_2} = -\frac{1}{3} = -c$, $x_1x_2 + y_1y_2 + z_1z_2 = 0 = t_1t_2$, thus:

$X = 1, Y = 1, Z = 1, T = 3\varepsilon$ is a weak fuzzy complex Pythagoras quadruple (2).

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Definition:

Let (X,Y,Z,T) be a 4-triple in the set of weak fuzzy complex integers ,i.e.

$X=x_1+x_2\varepsilon, Y=y_1+y_2\varepsilon, Z=z_1+z_2\varepsilon, T=t_1+t_2\varepsilon$, where $x_i,y_i,z_i,t_i\in Z, \varepsilon^2=c>1$

It is called a Pythagoras quadruple if it a solution of the non-linear Diophantine $X^2+Y^2+Z^2=T^2$.

Theorem.

The non-linear Diophantine equation $X^2+Y^2+Z^2=T^2$ is equivalent to:

$$\begin{cases} {x_1}^2+{x_2}^2c+{y_1}^2+{y_2}^2c+{z_1}^2+{z_2}^2c={t_1}^2+{t_2}^2c & (1)\\ x_1x_2+y_1y_2+z_1z_2=t_1t_2 & (2)\end{cases}$$

Proof.

The proof holds directly from the definition of weak fuzzy complex numbers.

Definition.

The equation $X^2+Y^2+Z^2=T^2$.

1). It is called of type (1) if $\sqrt{c}=\frac{a}{b}\in Q, gcd(a,b)=1$.

2). It is called of type (2) if $\sqrt{c}\notin Q; c=\frac{a}{b}\in Q, gcd(a,b)=1$.

It is called of type (3) if $c\notin Q$.

Solving equation of type (1).

Consider $X^2+Y^2+Z^2=T^2$ of type (1), with $c=\frac{a}{b}\in Q, gcd(a,b)=1$

Theorem.

(X,Y,Z,T) is a weak fuzzy complex Pythagoras quadruple of type (1) if and only if:

$L_1=\left(x_1+x_2\sqrt{c}, y_1+y_2\sqrt{c}, z_1+z_2\sqrt{c}, t_1+t_2\sqrt{c}\right)$

$L_2=\left(x_1-x_2\sqrt{c}, y_1-y_2\sqrt{c}, z_1-z_2\sqrt{c}, t_1-t_2\sqrt{c}\right)$

Are two Pythagoras quadruples in Q.

Proof.

$X^2=(x_1+x_2\varepsilon)^2={x_1}^2+{x_2}^2c+2x_1x_2\varepsilon$

$Y^2=(y_1+y_2\varepsilon)^2={y_1}^2+{y_2}^2c+2y_1y_2\varepsilon$

$Z^2=(z_1+z_2\varepsilon)^2={z_1}^2+{z_2}^2c+2z_1z_2\varepsilon$

$T^2=(t_1+t_2\varepsilon)^2={t_1}^2+{t_2}^2c+2t_1t_2\varepsilon$

Thus:

$$\begin{cases} {x_1}^2+{x_2}^2c+{y_1}^2+{y_2}^2c+{z_1}^2+{z_2}^2c={t_1}^2+{t_2}^2c & (1)\\ x_1x_2+y_1y_2+z_1z_2=t_1t_2 & (2)\end{cases}$$

We multiply (2) by $\sqrt{c}$ and compute $(1)+(2)\sqrt{c}, (1)-(2)\sqrt{c}$, to get:

$$\begin{cases} \left(x_1+x_2\sqrt{c}\right)^2+\left(y_1+y_2\sqrt{c}\right)^2+\left(z_1+z_2\sqrt{c}\right)^2=\left(t_1+t_2\sqrt{c}\right)^2 & (I)\\ \left(x_1-x_2\sqrt{c}\right)^2+\left(y_1-y_2\sqrt{c}\right)^2+\left(z_1-z_2\sqrt{c}\right)^2=\left(t_1-t_2\sqrt{c}\right)^2 & (II)\end{cases}$$

Which implies the proof.

Remark.

To solve equation of type (1), we multiply (I) and (II) by $b^2\neq 0$, hence:

$$\begin{cases} (bx_1+ax_2)^2+(by_1+ay_2)^2+(bz_1+az_2)^2=(bt_1+at_2)^2 & (\acute{I})\\ (bx_1-ax_2)^2+(by_1-ay_2)^2+(bz_1-az_2)^2=(bt_1-at_2)^2 & (\acute{I}I)\end{cases}$$

Now, suppose that:

$L_1=(X_1,Y_1,Z_1,T_1), L_2=(X_2,Y_2,Z_2,T_2)$

$$\begin{cases} 2b\setminus X_1+X_2, 2a\setminus X_1-X_2\\ 2b\setminus Y_1+Y_2, 2a\setminus Y_1-Y_2\\ 2b\setminus Z_1+Z_2, 2a\setminus Z_1-Z_2\\ 2b\setminus T_1+T_2, 2a\setminus T_1-T_2\end{cases}$$

on the other hand, we get:

$$X=\left(\frac{X_1+X_2}{2b}\right)+\varepsilon\left(\frac{X_1-X_2}{2a}\right), Y=\left(\frac{Y_1+Y_2}{2b}\right)+\varepsilon\left(\frac{Y_1-Y_2}{2a}\right), Z=\left(\frac{Z_1+Z_2}{2b}\right)+\varepsilon\left(\frac{Z_1-Z_2}{2a}\right), T=\left(\frac{T_1+T_2}{2b}\right)+\varepsilon\left(\frac{T_1-T_2}{2a}\right)$$

Example.

Consider $\varepsilon^2=c=9, \sqrt{t}=3, a=3, b=1$, we have:

$L_1=(3,6,6,9), L_2=(9,0,0,9)$ are two Pythagoras quadruples in Z.

$$\begin{cases} 2b=2\setminus 12, 2a=6\setminus -6\\ 2b=2\setminus 6, 2a=6\setminus 6\\ 2b=2\setminus 6, 2a=6\setminus 6\\ 2b=2\setminus 18, 2a=6\setminus 0\end{cases}$$

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so that (L_1, L_2) gives a weak fuzzy complex pythagoras quadruple follows:
$X = 6 - \varepsilon$
$Y = 3 + \varepsilon$
$Z = 3 + \varepsilon$
$T = 9$

Solving equation of type (3).

Consider $c \notin Q$, then $X^2 + Y^2 + Z^2 = T^2$ is equivalent to:

$$\begin{cases} (x_1{}^2 + y_1{}^2 + z_1{}^2) + c(x_2{}^2 + y_2{}^2 + z_2{}^2) = t_1{}^2 + ct_2{}^2 & (1) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (2) \end{cases}$$

Which is equivalent to:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 = t_1{}^2 & (I) \\ x_2{}^2 + y_2{}^2 + z_2{}^2 = t_2{}^2 & (II) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (III) \end{cases}$$

thus

$L_1 = (x_1, y_1, z_1, t_1), L_2 = (x_2, y_2, z_2, t_2)$ are two Pythagoras quadruples in Z.

By computing $(I) + (II) + (III)$ and $(I) + (II) - 2(III)$, we get:

$$\begin{cases} (x_1 + x_2)^2 + (y_1 + y_2)^2 + (z_1 + z_2)^2 = (t_1 + t_2)^2 \\ (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = (t_1 - t_2)^2 \end{cases}$$

So that, $L_1 + L_2, L_1 - L_2$ are two Pythagoras quadruples in Z.

Example.

For $\varepsilon^2 = \sqrt{3} \notin Q$, we have $L_1 = (3,6,6,9), L_2 = (6,12,12,18)$, and:
$L_1 + L_2 = (9,18,18,27), L_1 - L_2 = (-3,-6,-6,-9)$ are two Pythagoras quadruples in Z, hence L_1, L_2 give us a weak fuzzy complex Pythagoras quadruple as follows:
$X = 3 + 6\varepsilon, Y = 6 + 12\varepsilon, Z = 6 + 12\varepsilon, T = 9 + 18\varepsilon$.

Solving equation of type (2).

Consider that $\varepsilon^2 = c = \frac{a}{b} \in Q, gcd(a,b) = 1, \sqrt{c} \notin Q$, the equation $X^2 + Y^2 + Z^2 = T^2$ is equivalent to:

$$\begin{cases} (x_1{}^2 + y_1{}^2 + z_1{}^2) + c(x_2{}^2 + y_2{}^2 + z_2{}^2) = t_1{}^2 + ct_2{}^2 & (1) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (2) \end{cases}$$

We multiply (1) by $b \neq 0$ to get:

$$\begin{cases} b(x_1{}^2 + y_1{}^2 + z_1{}^2) + a(x_2{}^2 + y_2{}^2 + z_2{}^2) = bt_1{}^2 + at_2{}^2 & (I) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (II) \end{cases}$$

Thus:

$$\begin{cases} b(x_1{}^2 + y_1{}^2 + z_1{}^2 - t_1{}^2) + a(x_2{}^2 + y_2{}^2 + z_2{}^2 - t_2{}^2) = 0 & (\acute{I}) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (\acute{I}I) \end{cases}$$

We put $S_1 = x_1{}^2 + y_1{}^2 + z_1{}^2 - t_1{}^2, S_2 = x_2{}^2 + y_2{}^2 + z_2{}^2 - t_2{}^2$, hence:

$$\begin{cases} bS_1 + aS_2 = 0 & (\acute{I}) \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 & (\acute{I}I) \end{cases}$$

$(\acute{I})$ is a linear Diophantine equation in two variables, so that $S_1 = ka, S_2 = -kb$, where $k \in Z$,this implies that:

$$\begin{cases} \frac{S_1}{S_2} = -\frac{a}{b} = -c < 0; \; S_1 \neq 0, S_2 \neq 0 \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \end{cases}$$

Or:

$$\begin{cases} x_1{}^2 + y_1{}^2 + z_1{}^2 = t_1{}^2, x_2{}^2 + y_2{}^2 + z_2{}^2 = t_2{}^2 \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \end{cases}$$

We have two possible cases:

Case (1).

For $S_1 = S_2 = 0$, then $L_1 = (x_1, y_1, z_1, t_1), L_2 = (x_2, y_2, z_2, t_2)$ are two Pythagoras quadruples in$Z$.

Also, $L_1 + L_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2, t_1 + t_2), L_1 - L_2 = (x_1 - x_2, y_1 - y_2, z_1 - z_2, t_1 - t_2)$ are two Pythagoras quadruples in$Z$, (it can be proved a similar argument of type(3) equation).

Case (2).

If $S_1 \neq 0, S_2 \neq 0$, then we should chose $L_1 = (x_1, y_1, z_1, t_1), L_2 = (x_2, y_2, z_2, t_2)$ such that:

$$\begin{cases} \frac{x_1{}^2 + y_1{}^2 + z_1{}^2 - t_1{}^2}{x_2{}^2 + y_2{}^2 + z_2{}^2 - t_2{}^2} = -c < 0 \\ x_1x_2 + y_1y_2 + z_1z_2 = t_1t_2 \end{cases}$$

Example.

For $\varepsilon^2 = c = 3, \sqrt{c} \notin Q$, we chose:
$L_2 = (1,1,1,0), L_1 = (0,0,0,3)$.

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$S_2 = 1+1+1-0 = 3, S_1 = 0+0+0-9 = -9, \frac{S_1}{S_2} = -3 = -c, x_1x_2 + y_1y_2 + z_1z_2 = 0 = t_1t_2$, thus: $X = \varepsilon, Y = \varepsilon, Z = \varepsilon, T = 3$ is a weak fuzzy complex Pythagoras quadruple (2).

3. Conclusion

In this paper, we have studied the problem of generating weak fuzzy complex and anti-weak fuzzy complex integer Pythagoras quadruples, where we have presented necessary and sufficient conditions for such quadruples.

In the future, we aim to study other types of weak fuzzy complex and anti-weak fuzzy complex Diophantine equations and their solutions.

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