



Neutrosophic Divisor Point of A Straight Line Segment With A Given Ratio

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Abstract

This paper is dedicated to study the neutrosophic divisor point with a known ratio, where we use the principals of neutrosophic Euclidean geometry to get the desired results, and we illustrate many examples that explain the novelty of our work.

Keywords: Neutrosophic Cartesian coordinates; Neutrosophic polar coordinates; Neutrosophic vectors.

1. Introduction

Neutrosophic logic is a generalization of intuitionistic fuzzy logic by adding an indeterminacy I with property $I=I^2$. Neutrosophic set concept has wide applications in different areas of science, such as decision making [7,20], health care [8,21], machine learning [9], artificial intelligence [10], soft computing [22], industry [23], and statistics [11].

On the other hand, neutrosophic sets played an interesting role in pure mathematics such as topology and analysis [12,13], spaces [1,2], and algebraic structures [3,4,5,6].

Neutrosophic spaces theory began with Agboola et.al [14], where they studied neutrosophic vector spaces and their properties. Recently, many studies have been carried out on these spaces, where AH-subspaces and homomorphisms were presented [15]. In [16,17,18,19], Hatip et. al studied neutrosophic modules (a generalized form of neutrosophic spaces) with their substructures such as homomorphisms and AH-submodules.

In[38] the concept of the neutrosophic plane with n neutrosophic dimensions is obtained. In addition, Euclidean geometric concepts are extended neutrosophically such as neutrosophic distance, neutrosophic midpoint, and circles.

This work is considered the first study in neutrosophic geometry by defining the neutrosophic vectors, and neutrosophic lines and their concepts based on neutrosophic numbers and spaces.

2. Preliminaries.

Definition 2.1 [28]: Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such that $0 \cdot I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers n .

Definition 2.2 [29]: Let $w_1 = a_1 + b_1I, w_2 = a_2 + b_2I$ Then we have:

$$\frac{w_1}{w_2} = \frac{a_1}{a_2} + \frac{a_1b_2 - a_2b_1}{a_2(a_2 + b_2)}$$

Definition 2.3[10]: Let K be a field, the neutrosophic file generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 2.4[38]: Let $M = R(I)^2 = R(I) \times R(I), V = R^2 \times R^2$ be the neutrosophic plane with N -dimensions and the Cartesian product of the classical Euclidean space R^2 with itself, then AH isometry map defined as follows:

$$f: M \rightarrow V; f(a + bI, c + dI) = ((a, a + b), (c, c + d))$$

3. Main discussion

Definition 3.1: The divisor point of a straight line segment with a given ratio.

Let point $C(c_1 + c_2I, c_3 + c_4I)$ be a divisor of the line segment \overline{AB} with a known ratio of $\lambda = \lambda_1 + \lambda_2I$, where $A(a_1 + a_2I, a_3 + a_4I), B(b_1 + b_2I, b_3 + b_4I)$, then we have written.

$$\frac{\overline{AC}}{\overline{CB}} = \lambda \implies \overline{AC} = \lambda \overline{CB}$$

Now, if C lie between A and B , then we have.

$$([(c_1 - a_1) + I(c_2 - a_2), (c_3 - a_3) + I(c_4 - a_4)]) = (\lambda_1 + \lambda_2I)([(b_1 - c_1) + I(b_2 - c_2), (b_3 - c_3) + I(b_4 - c_4)])$$

Then use a AH-isometry, we have.

$$T([(c_1 - a_1) + I(c_2 - a_2), (c_3 - a_3) + I(c_4 - a_4)]) \\ = T(\lambda_1 + \lambda_2I) T([(b_1 - c_1) + I(b_2 - c_2), (b_3 - c_3) + I(b_4 - c_4)])$$

Then.

$$([(c_1 - a_1), (c_1 + c_2) - (a_1 + a_2)], [(c_3 - a_3), (c_3 + c_4) - (a_3 + a_4)]) \\ = [\lambda_1, (\lambda_1 + \lambda_2)]([(b_1 - c_1), (b_1 + b_2) - (c_1 + c_2)], [(b_3 - c_3), (b_3 + b_4) - (c_3 + c_4)])$$

Then,

$$\begin{cases} c_1 - a_1 = \lambda_1(b_1 - c_1) \\ (c_3 - a_3) = \lambda_1(b_3 - c_3) \\ (c_1 + c_2) - (a_1 + a_2) = (\lambda_1 + \lambda_2)[(b_1 + b_2) - (c_1 + c_2)] \\ (c_3 + c_4) - (a_3 + a_4) = (\lambda_1 + \lambda_2)[(b_3 + b_4) - (c_3 + c_4)] \end{cases}$$

$$\implies \begin{cases} \lambda_1 = \frac{b_1 - c_1}{c_1 - a_1} \\ \lambda_1 = \frac{b_3 - c_3}{c_3 - a_3} \\ \lambda_1 + \lambda_2 = \frac{(c_1 + c_2) - (a_1 + a_2)}{(b_1 + b_2) - (c_1 + c_2)} \\ \lambda_1 + \lambda_2 = \frac{(c_3 + c_4) - (a_3 + a_4)}{(b_3 + b_4) - (c_3 + c_4)} \end{cases}$$

$$\implies \begin{cases} c_1 = \frac{a_1 + \lambda_1 b_1}{1 + \lambda_1} \\ c_3 = \frac{a_3 + \lambda_1 b_3}{1 + \lambda_1} \\ c_2 = \frac{(\lambda_1 + \lambda_2)(b_1 + b_2 - c_1) + (a_1 + a_2) - c_1}{1 + \lambda_1 + \lambda_2} \\ c_4 = \frac{(\lambda_1 + \lambda_2)(b_3 + b_4 - c_3) + (a_3 + a_4) - c_3}{1 + \lambda_1 + \lambda_2} \end{cases}$$

Example 3.2: Let $A(-2 + I, 6 + I), B(2 + I, -4 + b_4I), \lambda = \frac{1}{2} + I$, find $C(c_1 + c_2I, c_3 + c_4I)$.

Solution. We have. $a_1 = -2, a_2 = 1, b_1 = 2, b_2 = 1, \lambda_1 = \frac{1}{2}, \lambda_2 = 1$, Hence.

$$\left\{ \begin{array}{l} c_1 = \frac{a_1 + \lambda_1 b_1}{1 + \lambda_1} \\ c_3 = \frac{a_3 + \lambda_1 b_3}{1 + \lambda_1} \\ c_2 = \frac{(\lambda_1 + \lambda_2)(b_1 + b_2 - c_1) + (a_1 + a_2) - c_1}{1 + \lambda_1 + \lambda_2} \\ c_4 = \frac{(\lambda_1 + \lambda_2)(b_3 + b_4 - c_3) + (a_3 + a_4) - c_3}{1 + \lambda_1 + \lambda_2} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} c_1 = \frac{-2}{3} \\ c_3 = \frac{8}{3} \\ c_2 = \frac{31}{15} \\ c_4 = \frac{-5}{3} \end{array} \right. \Rightarrow C \left(\frac{-2}{3} + \frac{31}{15}I, \frac{8}{3} - \frac{5}{3}I \right)$$

Now, If we use AH-isometry we have. $T(C) = T \left(\frac{-2}{3} + \frac{31}{15}I, \frac{8}{3} - \frac{5}{3}I \right) = \left(\left(\frac{-2}{3}, \frac{21}{15} \right), \left(\frac{8}{3}, 1 \right) \right)$

Definition 3.3: The Area of A Neutrosophic Circle.

Let a neutrosophic circle by following:

$$([x_0 + x_1I] - [a + bI])^2 + ([y_0 + y_1I] - [c + dI])^2 = (r_1 + r_2I)^2$$

Definition 3.4: A neutrosophic polar coordinates.

Let O be a fixed point of the plane $(R(I))^2$, then each point M of this plane is given two coordinates, then first is it's distance from O it given as follows $OM = \rho_1 + \rho_2I$, and the second is the angle made by the vector \overrightarrow{OM} with the axis $O[x_1 + x_2I]$ it given as follows $\theta_1 + \theta_2I$.

Now, we recall $(\rho_1 + \rho_2I, \theta_1 + \theta_2I)$ the neutrosophic Pole coordinates of point M .

Definition 3.5: The relationship between a neutrosophic Cartesian coordinates and a neutrosophic pole coordinates.

We have.

$$x_1 + x_2I = (\rho_1 + \rho_2I)\cos(\theta_1 + \theta_2I), y_1 + y_2I = (\rho_1 + \rho_2I)\sin(\theta_1 + \theta_2I)$$

Now, we have.

$$\begin{aligned} (x_1 + x_2I)^2 + (y_1 + y_2I)^2 &= (\rho_1 + \rho_2I)^2 \\ \Rightarrow \rho_1 + \rho_2I &= \sqrt{(x_1 + x_2I)^2 + (y_1 + y_2I)^2} \\ \Rightarrow \rho_1 + \rho_2I &= \sqrt{(x_1y_1)^2 + I[(x_1 + x_2)^2 + (y_1 + y_2)^2 - (y_1 + y_2I)^2 - (x_1^2 + y_1^2)]} \\ \theta_1 + \theta_2I &= \tan^{-1} \left(\frac{y_1 + y_2I}{x_1 + x_2I} \right) = \tan^{-1} \left(\frac{y_1}{x_1} + \frac{y_1x_2 - x_1y_2I}{x_1(x_1 + x_2)} \right) \text{ or } \tan(\theta_1 + \theta_2I) = \frac{y_1}{x_1} + \frac{y_1x_2 - x_1y_2I}{x_1(x_1 + x_2)}. \end{aligned}$$

Theorem 3.6: Let $(R(I))^2$ be the neutrosophic plane with two dimensions, Let $M(x_1 + x_2I, y_1 + y_2I)$ be a neutrosophic point, then the neutrosophic point $M(x_1 + x_2I, y_1 + y_2I)$ is equivalent to the direct product of the following two classical points.

$$M_1(x_1, y_1), M_2(x_1 + x_2, y_1 + y_2).$$

Proof. Consider the point $M(x_1 + x_2I, y_1 + y_2I)$ by computing its direct image with the 2-dimensional AH-isometry, we get: $T(M) = T(x_1 + x_2I, y_1 + y_2I)$, thus

$$\left((x_1, x_1 + x_2), (y_1, y_1 + y_2) \right), \text{ so that } M_1(x_1, y_1), M_2(x_1 + x_2, y_1 + y_2).$$

Remark 3.7: we can defined a neutrosophic pole coordinate of neutrosophic point $M(x_1 + x_2I, y_1 + y_2I)$ as follows:

$$\left\{ \begin{array}{l} \rho_1 = \sqrt{x_1^2 + y_1^2}, \theta_1 = \tan^{-1} \left(\frac{y_1}{x_1} \right) \\ \rho_2 = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}, \theta_2 = \tan^{-1} \left(\frac{y_1 + y_2}{x_1 + x_2} \right) \end{array} \right.$$

Example 3.8: Find a neutrosophic pole coordinate to point $M\left(\frac{1}{2} + \frac{\sqrt{3}}{2}I, 1 + I\right)$.

Solution.

By Remark 3.7, we have.

$$\rho_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1, \theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\rho_2 = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2} = \sqrt{4 + \sqrt{3}}, \theta_2 = \tan^{-1}\left(\frac{y_1 + y_2}{x_1 + x_2}\right) = \tan^{-1}\left(\frac{1 + \sqrt{3}}{3}\right)$$

Then,

$$M\left(1 + \sqrt{4 + \sqrt{3}}I, \frac{\pi}{3} + \tan^{-1}\left(\frac{1 + \sqrt{3}}{3}\right)I\right)$$

Definition 3.9: Let $\vec{u}(a_1 + a_2I, b_1 + b_2I), \vec{v}(a_3 + a_4I, b_3 + b_4I)$ be two aneutrosophic vectors, then we have \vec{u}, \vec{v} is Linear dependence if:

$$\frac{a_1 + a_2I}{a_3 + a_4I} = \frac{b_1 + b_2I}{b_3 + b_4I} \Leftrightarrow \frac{a_1}{a_3} = \frac{b_1}{b_3} \wedge \frac{a_1a_4 - a_3a_2}{a_3(a_3 + a_4)} = \frac{b_1b_4 - b_3b_2}{b_3(b_3 + b_4)}$$

Theorem 3.10: Let $(R(I))^2$ be a neutrosophic plane with two dimensions, Let $\vec{u}(a_1 + a_2I, b_1 + b_2I), \vec{v}(a_3 + a_4I, b_3 + b_4I)$ be two a neutrosophic vectors, then the neutrosophic vectors $\vec{u}(a_1 + a_2I, b_1 + b_2I), \vec{v}(a_3 + a_4I, b_3 + b_4I)$ is equivalent to the direct product of the following two classical Linear dependence vectors.

$$\vec{u}_1(a_1, b_1), \vec{v}_1(a_3, b_3) \wedge \vec{u}_2(a_1 + a_2, b_1 + b_2), \vec{v}_2(a_3 + a_4, b_3 + b_4).$$

Proof. Consider the neutrosophic vectors $\vec{u}(a_1 + a_2I, b_1 + b_2I), \vec{v}(a_3 + a_4I, b_3 + b_4I)$ by computing its direct image with the 2-dimensional AH-isometry, we get:

$$T(\vec{u}) = T(a_1 + a_2I, b_1 + b_2I), T(\vec{v}) = T(a_3 + a_4I, b_3 + b_4I)$$

Thus:

$$((a_1, a_1 + a_2), (b_1, b_1 + b_2)), ((a_3, a_3 + a_4), (b_3, b_3 + b_4))$$

so that, $\vec{u}_1(a_1, b_1), \vec{u}_2(a_1 + a_2, b_1 + b_2), \vec{v}_1(a_3, b_3), \vec{v}_2(a_3 + a_4, b_3 + b_4)$

then.

\vec{u}, \vec{v} is Linear dependence, so:

$\vec{u}_1(a_1, b_1), \vec{v}_1(a_3, b_3)$ is Linear dependence and $\vec{u}_2(a_1 + a_2, b_1 + b_2), \vec{v}_2(a_3 + a_4, b_3 + b_4)$ is Linear dependence.

Example 3.11: Prove \vec{u}, \vec{v} is Linear dependence, where $\vec{u}(2 + 2I, 6 + 4I), \vec{v}(1 + I, 3 + 2I)$.

Solution.

By definition 3.9, we have.

$$\left. \begin{array}{l} \frac{a_1}{a_3} = \frac{2}{1} = 2 \\ \frac{b_1}{b_3} = \frac{6}{3} = 2 \end{array} \right\} \Rightarrow \frac{a_1}{a_3} = \frac{b_1}{b_3}$$

$$\left. \begin{array}{l} \frac{a_1a_4 - a_3a_2}{a_3(a_3 + a_4)} = \frac{0}{2} = 0 \\ \frac{b_1b_4 - b_3b_2}{b_3(b_3 + b_4)} = \frac{0}{15} = 0 \end{array} \right\} \Rightarrow \frac{a_1a_4 - a_3a_2}{a_3(a_3 + a_4)} = \frac{b_1b_4 - b_3b_2}{b_3(b_3 + b_4)}$$

Then, \vec{u}, \vec{v} is Linear dependence.

Definition 3.12: Let $\vec{u}(a_1 + a_2I, b_1 + b_2I)$, $\vec{v}(a_3 + a_4I, b_3 + b_4I)$ be two a neutrosophic vectors, then.

$$\vec{u} \cdot \vec{v} = ([a_1a_3, (a_1a_4 + a_2a_4 + a_2a_3)I], [b_1b_3, (b_1b_4 + b_2b_4 + b_2b_3)I])$$

$$\vec{u} \perp \vec{v} \Leftrightarrow \vec{u} \cdot \vec{v} = 0.$$

4. Conclusion

In this article, we have studied coordinate pole of a neutrosophic point, and Cartesian point. Also, we presented the neutrosophic divisor point of a straight line segment with a given ratio.

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