δ-separation Axioms on Fuzzy Hypersoft Topological Spaces

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Abstract

In this article, the concept of fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)-separation axioms in fuzzy hypersoft topological spaces are introduced by developing fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)-neighbourhood with respect to fuzzy hypersoft points. Also, the properties and relations between fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- T_i- spaces $(i = 0, 1, 2, 3, 4)$ are discussed.

Keywords: FH_yS δ (resp. semi, pre; δ semi & δ pre)-neighbourhood; FH_yS δ (resp. semi, pre, δ semi & δ pre)-separation axioms; FH_yS δ (resp. semi, pre, δ semi & δ pre)- T_i- space $(i = 0, 1, 2, 3, 4)$.
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1 Introduction

The real world decision making problems in medical diagnosis, management, computer science, engineering, artificial intelligence, economics,social sciences, environmental science and sociology contains more uncertain and inadequate data. The traditional mathematical methods cannot deal with these kind of problems due to the imprecise data. To deal the problems with uncertainty, Zadeh[20] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang[7] as fuzzy topological space. The soft set theory was introduced by Molodstov[10] in 1999 to deal uncertainties in which a soft set is a collection of approximate descriptions of an object. Soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory have several applications in different fields such as decision making, optimization, forecasting, data analysis etc. Consequently, the soft topological spaces were developed by Shabir and Naz.[15]

Smarandache[18] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Abbas et. al.[3] defined the basic operations on hypersoft sets and hypersoft point in all the universe of discourses. Ajay and Charisma[4] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology. Ajay et.al.[5] defined fuzzy hypersoft semi-open sets and developed an application in multiattribute group decision making.

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Saha[13] defined δ-open sets in fuzzy topological spaces. Vadivel et al.[19] introduced δ-open sets in neutrosophic topological spaces. The neutrosophic soft δ-topology was developed by Acikgoz and Esenbel.[1] In 2019, the separation axioms on neutrosophic soft topological spaces were studied by Aras et al.[6] The soft b-separation axioms were introduced by Khattak et al.[9] and pre-separation axioms were developed by Acikgoz et al.[2] in neutrosophic soft topological spaces. Revathi et al.[12] developed neutrosophic soft e-separation axioms in neutrosophic soft topological spaces. Ozturk[11] introduced separation axioms in fuzzy hypersoft topological spaces.

The goal of this paper is to introduce fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)-neighbourhood and fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)-separation axioms in fuzzy hypersoft topological spaces using fuzzy hypersoft points. In addition, the characteristics of fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- T_i- spaces $(i = 0, 1, 2, 3, 4)$ and relations between them are studied.

2 Preliminaries

The definitions of fuzzy set,[20] soft set,[10] hypersoft set[18] and fuzzy hypersoft set[3] are considered in this paper.

Definition 2.1. [3] Let $(\tilde{\Phi}, \wedge_1)$ and $(\tilde{\Psi}, \wedge_2)$ be two FH_ySs's over $\mathfrak{M}$. Then $(\tilde{\Phi}, \wedge_1)$ is the fuzzy hypersoft subset of $(\tilde{\Psi}, \wedge_2)$ if $\mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}) \leq \mu_{\tilde{\Psi}(\mathfrak{q})}(\mathfrak{m})$.

It is denoted by $(\tilde{\Phi}, \wedge_1) \subseteq (\tilde{\Psi}, \wedge_2)$.

Definition 2.2. [3] Let $(\tilde{\Phi}, \wedge_1)$ and $(\tilde{\Psi}, \wedge_2)$ be FH_ySs's over $\mathfrak{M}$. $(\tilde{\Phi}, \wedge_1)$ is equal to $(\tilde{\Psi}, \wedge_2)$ if $\mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}) = \mu_{\tilde{\Psi}(\mathfrak{q})}(\mathfrak{m})$.

Definition 2.3. [3] A FH_ySs $(\tilde{\Phi}, \wedge)$ over $\mathfrak{M}$ is called null fuzzy hypersoft set if $\mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}) = 0$, $\forall \mathfrak{q} \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{0}_{(\mathfrak{M},Q)}$.

A FH_ySs $(\tilde{\Psi}, \wedge)$ over $\mathfrak{M}$ is called absolute fuzzy hypersoft set if $\mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}) = 1$ $\forall \mathfrak{q} \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{1}_{(\mathfrak{M},Q)}$.

Clearly, $\tilde{0}^c_{(\mathfrak{M},Q)} = \tilde{1}_{(\mathfrak{M},Q)}$ and $\tilde{1}^c_{(\mathfrak{M},Q)} = \tilde{0}_{(\mathfrak{M},Q)}$.

Definition 2.4. [3] Let $(\tilde{\Phi}, \wedge)$ be FH_ySs over $\mathfrak{M}$. $(\tilde{\Phi}, \wedge)^c$ is the complement of $(\tilde{\Phi}, \wedge)$ if $\mu^C_{\tilde{H}(\mathfrak{q})}(\mathfrak{m}) = \tilde{1}^c_{(\mathfrak{M},Q)} - \mu_{\tilde{H}(\mathfrak{q})}(\mathfrak{m})$ where $\forall \mathfrak{q} \in \wedge$ and $\forall \mathfrak{m} \in \mathfrak{M}$. It is clear that $((\tilde{\Phi}, \wedge)^c)^c = (\tilde{\Phi}, \wedge)$.

Definition 2.5. [3] Let $(\tilde{\Phi}, \wedge_1)$ and $(\tilde{\Psi}, \wedge_2)$ be FH_ySs's over $\mathfrak{M}$. Extended union $(\tilde{\Phi}, \wedge_1) \cup (\tilde{\Psi}, \wedge_2)$ is defined as

$$\mu((\tilde{\Phi}, \wedge_1) \cup (\tilde{\Psi}, \wedge_2)) = \begin{cases} \mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}) & \text{if } \mathfrak{q} \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{\Psi}(\mathfrak{q})}(\mathfrak{m}) & \text{if } \mathfrak{q} \in \wedge_2 - \wedge_1 \\ max\{\mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}), \mu_{\tilde{\Psi}(\mathfrak{q})}(\mathfrak{m})\} & \text{if } \mathfrak{q} \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.6. [3,4] Let $(\tilde{\Phi}, \wedge_1)$ and $(\tilde{\Psi}, \wedge_2)$ be FH_ySs's over $\mathfrak{M}$. Extended intersection $(\tilde{\Phi}, \wedge_1) \cap (\tilde{\Psi}, \wedge_2)$ is defined as

$$\mu((\tilde{\Phi}, \wedge_1) \cap (\tilde{\Psi}, \wedge_2)) = \begin{cases} \mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}) & \text{if } \mathfrak{q} \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{\Psi}(\mathfrak{q})}(\mathfrak{m}) & \text{if } \mathfrak{q} \in \wedge_2 - \wedge_1 \\ min\{\mu_{\tilde{\Phi}(\mathfrak{q})}(\mathfrak{m}), \mu_{\tilde{\Psi}(\mathfrak{q})}(\mathfrak{m})\} & \text{if } \mathfrak{q} \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.7. [4] Let $(\mathfrak{M}, Q)$ be the family of all FH_ySs's over $\mathfrak{M}$ and $\tilde{\tau} \subseteq FH_ySs(\mathfrak{M}, Q)$. Then $\tilde{\tau}$ is said to be a fuzzy hypersoft topology (in short, FH_ySt) on $\mathfrak{M}$ if

(i) $\tilde{0}_{(\mathfrak{M},Q)}$ and $\tilde{1}_{(\mathfrak{M},Q)}$ belongs to $\tilde{\tau}$

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(ii) the union of any number of FH_ySs's in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(iii) the intersection of finite number of FH_ySs's in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

Then $(\mathfrak{M}, Q, \tilde{\tau})$ is known as a fuzzy hypersoft toplogical space (in short, FH_ySts) over $\mathfrak{M}$. Each member of $\tilde{\tau}$ is said to be fuzzy hypersoft open set (in short, FH_ySos). A FH_ySs $(\tilde{\Phi}, \wedge)$ is known as a fuzzy hypersoft closed set (in short, FH_yScs) if its complement $(\tilde{\Phi}, \wedge)^c$ is FH_ySos.

Definition 2.8. [4] Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be a FH_ySs in $\mathfrak{M}$. Then,

(i) the fuzzy hypersoft interior (in short, FH_ySint) of $(\tilde{\Phi}, \wedge)$ is defined as $FH_ySint(\tilde{\Phi}, \wedge) = \cup\{(\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$ where $(\tilde{\Psi}, \wedge)$ is $FH_ySos\}$.

(ii) the fuzzy hypersoft closure (in short, FH_yScl) of $(\tilde{\Phi}, \wedge)$ is defined as $FH_yScl(\tilde{\Phi}, \wedge) = \cap\{(\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \supseteq (\tilde{\Phi}, \wedge)$ where $(\tilde{\Psi}, \wedge)$ is $FH_yScs\}$.

Definition 2.9. [5] Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be a FH_ySs in $\mathfrak{M}$. Then, $(\tilde{\Phi}, \wedge)$ is called the fuzzy hypersoft semiopen set (in short, FH_ySSos) if $(\tilde{\Phi}, \wedge) \subseteq FH_yScl(int(\tilde{\Phi}, \wedge))$.

A FH_ySs $(\tilde{\Phi}, \wedge)$ is called a fuzzy hypersoft semiclosed set (in short, FH_ySScs) if its complement $(\tilde{\Phi}, \wedge)^c$ is a FH_ySSos.

Definition 2.10. [3] Let FH_yS's $(\tilde{\Phi}, \wedge)$ be the family of all FH_yS's over $\mathfrak{M}$ and let $\mathfrak{m} \in \mathfrak{M}$, $0 \leq \varphi \leq 1$, $\mathfrak{q} \in Q$. Then the FH_ySs $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is known as fuzzy hypersoft point (in short, FH_ySp) and is defined as follows: For each $\mathfrak{n} \in \mathfrak{M}$,

$$\mathfrak{m}^{\mathfrak{q}}_{\varphi}(\mathfrak{q}')(\mathfrak{n}) = \begin{cases} \varphi \text{ if } \mathfrak{q}' = \mathfrak{q} \text{ and } \mathfrak{n} = \mathfrak{m} \\ \\ 0 \text{ if } \mathfrak{q}' \neq \mathfrak{q} \text{ or } \mathfrak{n} \neq \mathfrak{m}. \end{cases}$$

Definition 2.11. [11] Let $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}$ be two FH_ySp's. For the FH_ySp's $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}$ over a common universe $\mathfrak{M}$, we say that FH_ySp's are distinct points, if $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap \mathfrak{n}^{\mathfrak{q}'}_{\varphi'} = 0_{(\mathfrak{M},Q)}$. It is clear that $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}$ are distinct FH_ySp's iff $\mathfrak{m} \neq \mathfrak{n}$ and $\mathfrak{q}' \neq \mathfrak{q}$.

Definition 2.12. [11] Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. A FH_yS's $(\tilde{\Phi}, \wedge)$ in $(\mathfrak{M}, Q, \tilde{\tau})$ is called a fuzzy hypersoft neighbourhood (in short, FH_ySnbd) of the FH_ySp $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$, if there exists a FH_ySos $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$.

Definition 2.13. [11] Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. Let $(\tilde{\Phi}, \wedge)$ be a FH_ySs over $\mathfrak{M}$ and $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ be a FH_ySp over $\mathfrak{M}$.

(i) $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is a fuzzy hypersoft interior point (in short, FH_ySintp) of $(\tilde{\Phi}, \wedge)$, if $(\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$ for some $(\tilde{\Psi}, \wedge) \in FH_ySnbd$ of the FH_ySp $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$.

(ii) $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is a fuzzy hypersoft adherent point (in short, FH_ySadhp) of $(\tilde{\Phi}, \wedge)$, if $(\tilde{\Psi}, \wedge) \bigcap (\tilde{\Phi}, \wedge) \notin 0_{(\mathfrak{M},Q)}$ for any $(\tilde{\Psi}, \wedge) \in FH_ySnbd$ of the FH_ySp $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$.

Theorem 2.14. [11] Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be a FH_ySs over $\mathfrak{M}$. Then

(i) $FH_ySint(\tilde{\Phi}, \wedge) = \bigcup\{\mathfrak{m}^{\mathfrak{q}}_{\varphi} : \mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is a FH_ySintp of $(\tilde{\Phi}, \wedge)\}$.

(ii) $FH_yScl(\tilde{\Phi}, \wedge) = \bigcup\{\mathfrak{m}^{\mathfrak{q}}_{\varphi} : \mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is a FH_ySadhp of $(\tilde{\Phi}, \wedge)\}$.

Definition 2.15. [11] Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be an arbitrary FH_yS's . Then $\tilde{\tau}_{(\tilde{\Phi},\wedge)} = \{(\tilde{\Phi}, \wedge) \cap (\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \in \tilde{\tau}\}$ is called FH_ySt on $(\tilde{\Phi}, \wedge)$ and $\big((\tilde{\Phi}, \wedge), \tilde{\tau}_{(\tilde{\Phi},\wedge)}, Q\big)$ is known as a fuzzy hypersoft topological subspace (in short, FH_yStss) of $(\mathfrak{M}, Q, \tilde{\tau})$.

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3 Fuzzy hypersoft δ-separation axioms

Definition 3.1. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. A FH_ySs $(\tilde{\Phi}, \wedge)$ is said to be a fuzzy hypersoft regular open set (in short, FH_ySros) if $(\tilde{\Phi}, \wedge) = FH_ySint(FH_釐yScl(\tilde{\Phi}, \wedge))$. The complement of FH_ySros is known as a fuzzy hypersoft regular closed set (in short, FH_ySrcs) in $\mathfrak{M}$.

Definition 3.2. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be a FH_ySs on $\mathfrak{M}$. Then the fuzzy hypersoft

(i) δ-interior (in short, FH_ySint) of $(\tilde{\Phi}, \wedge)$ is defined by $FH_yS\delta int(\tilde{\Phi}, \wedge) = \bigcup\{(\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ is a FH_ySros in $\mathfrak{M}\}$

(ii) δ-closure (in short, FH_yScl) of $(\tilde{\Phi}, \wedge)$ is defined by $FH_yS\delta cl(\tilde{\Phi}, \wedge) = \bigcap\{(\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \supseteq (\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ is a FH_ySrcs in $\mathfrak{M}\}$

Definition 3.3. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. An FH_ySs $(\tilde{\Phi}, \wedge)$ is said to be a fuzzy hypersoft

(i) semi-regular if $(\tilde{\Phi}, \wedge)$ is both FH_ySSos and FH_ySScs.

(ii) pre open set (in short, $FH_yS\mathcal{P}os$) if $(\tilde{\Phi}, \wedge) \subseteq FH_ySint(FH_yScl(\tilde{\Phi}, \wedge)$

(iii) δ-open set (in short, $FH_yS\delta os$) if $(\tilde{\Phi}, \wedge) = FH_yS\delta int(\tilde{\Phi}, \wedge)$

(iv) δ-pre open set (in short, $FH_yS\delta\mathcal{P}os$) if $(\tilde{\Phi}, \wedge) \subseteq FH_ySint(FH_yS\delta cl(\tilde{\Phi}, \wedge))$

(v) δ-semi open set (in short, $FH_yS\delta\mathcal{S}os$) if $(\tilde{\Phi}, \wedge) \subseteq FH_yScl(FH_yS\delta int(\tilde{\Phi}, \wedge))$

The complement of $FH_yS\delta os$ (resp. $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{P}os$ & $FH_yS\delta\mathcal{S}os$) is called a $FH_yS\delta$ (resp. FH_yS pre, $FH_yS\delta$ pre & $FH_yS\delta$ semi) closed set (in short, $FH_yS\delta cs$ (resp. $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{P}cs$ & $FH_yS\delta\mathcal{S}cs$)) in $\mathfrak{M}$.

The family of all $FH_yS\delta os$ (resp. $FH_yS\delta cs$, FH_ySros, FH_ySrcs, $FH_yS\mathcal{P}os$, $FH_yS\mathcal{P}cs$ $FH_yS\delta\mathcal{P}os$, $FH_yS\delta\mathcal{P}cs$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{S}cs$) of $\mathfrak{M}$ is denoted by $FH_yS\delta OS(\mathfrak{M})$ (resp. $FH_yS\delta CS(\mathfrak{M})$, FH_yS $rOS(\mathfrak{M})$, $FH_ySrOS(\mathfrak{M})$, $FH_yS\mathcal{P}OS(\mathfrak{M})$, $FH_yS\mathcal{P}CS(\mathfrak{M})$, $FH_yS\delta\mathcal{P}OS(\mathfrak{M})$, $FH_yS\delta\mathcal{P}CS(\mathfrak{M})$, FH_y $S\delta\mathcal{S}OS(\mathfrak{M})$ & $FH_yS\delta\mathcal{S}CS(\mathfrak{M})$).

Definition 3.4. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be a FH_ySs on $\mathfrak{M}$. Then the fuzzy hypersoft

(i) δ-pre (resp. δ-semi) interior (in short, $FH_yS\delta\mathcal{P}int$ (resp. $FH_yS\delta\mathcal{S}int$)) of $(\tilde{\Phi}, \wedge)$ is defined by $FH_yS\delta\mathcal{P}int(\tilde{\Phi}, \wedge) = \bigcup\{(\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ is a $FH_yS\delta\mathcal{P}os$ (resp. $FH_yS\delta\mathcal{S}os$) in $\mathfrak{M}\}$

(ii) δ-pre (resp. δ-semi) closure (in short, $FH_yS\delta\mathcal{P}cl$ (resp. $FH_yS\delta\mathcal{S}cl$)) of $(\tilde{\Phi}, \wedge)$ is defined by $FH_yS\delta\mathcal{P}$ $cl(\tilde{\Phi}, \wedge) = \bigcap\{(\tilde{\Psi}, \wedge) : (\tilde{\Psi}, \wedge) \supseteq (\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ is a $FH_yS\delta\mathcal{P}cs$ (resp. $FH_yS\delta\mathcal{S}cs$) in $\mathfrak{M}\}$

Definition 3.5. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. A FH_yS's $(\tilde{\Phi}, \wedge)$ in $(\mathfrak{M}, Q, \tilde{\tau})$ is known as a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- neighbourhood (in short, $FH_yS\delta$(resp. semi, pre, δ semi & δ pre)-nbd) of the FH_ySp $\mathfrak{m}_{\varphi}^{\mathfrak{q}} \in (\tilde{\Phi}, \wedge)$, if there exists a $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}_{\varphi}^{\mathfrak{q}} \in (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$.

Theorem 3.6. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$ and $(\tilde{\Phi}, \wedge)$ be a FH_yS's on $\mathfrak{M}$. Then $(\tilde{\Phi}, \wedge)$ is a $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) iff $(\tilde{\Phi}, \wedge)$ is a $FH_yS\delta$- (resp. semi, pre, δ semi & δ pre)nbd of its FH_ySp's.

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Proof. Let $(\tilde{\Phi}, \wedge)$ be a $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$) and $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$. Then, $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$. Thus $(\tilde{\Phi}, \wedge)$ is a $FH_ySδ$ (resp. semi, pre, δ semi & δ pre)-nbd of $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$.

Conversely, let $(\tilde{\Phi}, \wedge)$ be a $FH_ySδ$ (resp. semi, pre, δ semi & δ pre)-nbd of its FH_ySp's. Let $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$. Since $(\tilde{\Phi}, \wedge)$ is a $FH_ySδ$ (resp. semi, pre, δ semi & δ pre)-nbd of the FH_ySp $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$, there exists $(\tilde{\Psi}, \wedge) \in \tilde{\tau}$ such that $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$. Since $(\tilde{\Phi}, \wedge) = \bigcup\{\mathfrak{m}^{\mathfrak{q}}_{\varphi} : \mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)\}$, it follows that $(\tilde{\Phi}, \wedge)$ is a union of $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$)'s. Then $(\tilde{\Phi}, \wedge)$ is a $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$). □

The $FH_ySδ$ (resp. semi, pre, δ semi & δ pre)-nbd system of a FH_ySp $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ denoted by $\bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$, is the family of all its $FH_ySδ$ (resp. semi, pre, δ semi & δ pre)-nbd's.

Theorem 3.7. The $FH_ySδ$ (resp. semi, pre, δ semi & δ pre)-nbd system $\bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$ at $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ in a FH_ySts $(\mathfrak{M}, Q, \tilde{\tau})$ has the following properties:

(i) If $(\tilde{\Phi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$, then $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$.

(ii) If $(\tilde{\Phi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$ and $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Omega}, \wedge)$, then $(\tilde{\Omega}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$.

(iii) $(\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$, then $(\tilde{\Phi}, \wedge) \cap (\tilde{\Psi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$.

(iv) If $(\tilde{\Phi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$, then there exists a $(\tilde{\Psi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$ such that $(\tilde{\Psi}, \wedge) \in \bigcup(\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}, Q)$ for each $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\tilde{\Psi}, \wedge)$.

Proof. The proofs of (i), (ii) and (iii) directly follow from the Definition 3.5.

(iv) Let $(\tilde{\Phi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$. Then there exists a $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$) $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$. Then by Theorem 3.6, $(\tilde{\Psi}, \wedge) \in \bigcup(\mathfrak{m}^{\mathfrak{q}}_{\varphi}, Q)$. So for each $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\tilde{\Psi}, \wedge)$, $(\tilde{\Psi}, \wedge) \in (\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}, Q)$.

□

Definition 3.8. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. Let $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $z^{\mathfrak{q}'}_{\varphi'}$ be distinct FH_ySp's. If there exist $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$ and $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap (\tilde{\Psi}, \wedge) = 0_{(\mathfrak{M},Q)}$ or $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\tilde{\Psi}, \wedge)$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \cap (\tilde{\Phi}, \wedge) = 0_{(\mathfrak{M},Q)}$, then $(\mathfrak{M}, Q, \tilde{\tau})$ is known as a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- T_0- space (in short, $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_0- space).

Definition 3.9. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. Let $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}$ be distinct FH_ySp's. If there exist $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$, $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap (\tilde{\Psi}, \wedge) = 0_{(\mathfrak{M},Q)}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\tilde{\Psi}, \wedge)$, $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \cap (\tilde{\Phi}, \wedge) = 0_{(\mathfrak{M},Q)}$, then $(\mathfrak{M}, Q, \tilde{\tau})$ is known as a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- T_1- space (in short, $FH_ySδ$- (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$) T_1- space).

Definition 3.10. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. Let $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}$ be distinct FH_ySp's. If there exist $FH_ySδos$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_ySδSos$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \in (\tilde{\Phi}, \wedge)$, $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\tilde{\Psi}, \wedge)$ and $(\tilde{\Phi}, \wedge) \cap (\tilde{\Psi}, \wedge) = 0_{(\mathfrak{M},Q)}$, then $(\mathfrak{M}, Q, \tilde{\tau})$ is known as a fuzzy hypersoft δ- (resp. semi, pre, δ semi & δ pre) T_2- space (in short, $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_2- space).

Example 3.11. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ be the FH_yS initial universe and the attribute be $Q = Q_1 \times Q_2$. The attribute is given as:

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$$Q_1 = \{a_1, a_2\} \;\&\; Q_2 = \{b_1\} \text{ and}$$
$$\wedge = \{\mathfrak{q}_1 = (a_1, b_1) \;\&\; \mathfrak{q}_2 = (a_2, b_1)\}.$$

Let $\mathfrak{m}^{\mathfrak{q}_1}_{1(0.8)}$, $\mathfrak{m}^{\mathfrak{q}_2}_{1(0.2)}$, $\mathfrak{m}^{\mathfrak{q}_1}_{2(0.3)}$ and $\mathfrak{m}^{\mathfrak{q}_2}_{2(0.5)}$ be FH_ySp's. Let $(\mathfrak{M}, Q)$ be the class of FH_yS sets. Let the FH_ySs's $(\tilde{\Phi}_1, \wedge)$, $(\tilde{\Phi}_2, \wedge)$, $(\tilde{\Phi}_3, \wedge)$, $(\tilde{\Phi}_4, \wedge)$, $(\tilde{\Phi}_5, \wedge)$, $(\tilde{\Phi}_6, \wedge)$, $(\tilde{\Phi}_7, \wedge)$ and $(\tilde{\Phi}_8, \wedge)$ over the universe $\mathfrak{M}$ be

$$(\tilde{\Phi}_1, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_2, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0}, \frac{\mathfrak{m}_2}{0}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_3, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_4, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_5, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_6, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_7, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0}\} \rangle \end{Bmatrix}$$

$$(\tilde{\Phi}_8, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\tilde{\tau} = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{\Phi}_1, \wedge), (\tilde{\Phi}_2, \wedge), (\tilde{\Phi}_3, \wedge), (\tilde{\Phi}_4, \wedge), (\tilde{\Phi}_5, \wedge), (\tilde{\Phi}_6, \wedge), (\tilde{\Phi}_7, \wedge), (\tilde{\Phi}_8, \wedge)\}$ is FH_ySts.

Hence, $(\mathfrak{M}, Q, \tilde{\tau})$ is a FH_ySts over $\mathfrak{M}$. Here, $(\tilde{\Phi}_6, \wedge)$ and $(\tilde{\Phi}_8, \wedge)$ are $FH_yS\ \delta os$'s. Also, $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$- T_0- space but not a $FH_yS\delta$- T_1- space because for FH_ySp's $\mathfrak{m}^{\mathfrak{q}_1}_{1(0.8)}$ and $\mathfrak{m}^{\mathfrak{q}_2}_{2(0.5)}$, $(\mathfrak{M}, Q, \tilde{\tau})$ is not a $FH_yS\delta$- T_1- space.

Example 3.12. Consider a set of natural numbers $\mathfrak{M} = N$ and a parameter set $Q = \{\wedge\}$. Let the FH_ySp's be $n^{\mathfrak{q}}_{\varphi_n}$. Now we can take φ_n appropriate values and the FH_ySp's $n^{\mathfrak{q}}_{\varphi_n}$, $m^{\mathfrak{q}}_{\varphi_m}$ are distinct FH_ySp's iff $n \neq m$. It is obvious that there is one-to-one compatibility between the set of natural numbers and the set of FH_ySp's $N^{\mathfrak{q}} = \{n^{\mathfrak{q}}_{\varphi_n}\}$. Here we define cofinite topology on this set. Then FH_ySs's $(\tilde{\Phi}, \wedge)$ is a $FH_yS\delta os$ iff the finite FH_ySp's are discarded from $N^{\mathfrak{q}}$. Hence, $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$- T_1- space but not a $FH_yS\delta$- T_2- space.

Theorem 3.13. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. Then $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_1- space iff each FH_ySp is a $FH_yS\delta cs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$ & $FH_yS\delta\mathcal{P}cs$).

Proof. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_1- space and $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ be an arbitrary FH_ySp. Let $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\mathfrak{m}^{\mathfrak{q}}_{\varphi})^c$. Then $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ and $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'}$ are distinct FH_ySp's. Thus $\mathfrak{m} \neq \mathfrak{n}$ or $\mathfrak{q}' \neq \mathfrak{q}$. Since $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_1- space, there exists a $FH_yS\delta os$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{n}^{\mathfrak{q}'}_{(\alpha',\beta',\gamma')} \in (\tilde{\Psi}, \wedge)$ and $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap (\tilde{\Psi}, \wedge) = 0_{(\mathfrak{M},Q)}$. Since $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap (\tilde{\Psi}, \wedge) = 0_{(\mathfrak{M},Q)}$, we have $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\tilde{\Psi}, \wedge) \subseteq (\mathfrak{m}^{\mathfrak{q}}_{\varphi})^c$. Thus $(\mathfrak{m}^{\mathfrak{q}}_{\varphi})^c$ is a $FH_yS\delta os$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$), ie, $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is a $FH_yS\delta cs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$ & $FH_yS\delta\mathcal{P}cs$).

Conversely, suppose that each FH_ySp $\mathfrak{m}^{\mathfrak{q}}_{\varphi}$ is a $FH_yS\delta cs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$ & $FH_yS\delta\mathcal{P}cs$). Then $(\mathfrak{m}^{\mathfrak{q}}_{\varphi})^c$ is a $FH_yS\delta os$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$). Let $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap \mathfrak{n}^{\mathfrak{q}'}_{\varphi'} = 0_{(\mathfrak{M},Q)}$. Thus, $\mathfrak{n}^{\mathfrak{q}'}_{\varphi'} \in (\mathfrak{m}^{\mathfrak{q}}_{\varphi})^c$ and $\mathfrak{m}^{\mathfrak{q}}_{\varphi} \cap (\mathfrak{m}^{\mathfrak{q}}_{\varphi})^c = 0_{(\mathfrak{M},Q)}$. So $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_1- space on $\mathfrak{M}$. □

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Theorem 3.14. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. Then $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_2- space iff for distinct FH_ySp's $\mathfrak{m}_\varphi^\mathfrak{q}$ and $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$, there exists a $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) $(\tilde{\Phi}, \wedge)$ containing $\mathfrak{m}_\varphi^\mathfrak{q}$ but not $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ such that $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ does not belong to $FH_yScl(\tilde{\Phi}, \wedge)$.

Proof. Let $\mathfrak{m}_\varphi^\mathfrak{q}$ and $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ be two FH_ySp's in $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_2- space $(\mathfrak{M}, Q, \tilde{\tau})$. Then there exist disjoint $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$)'s $(\tilde{\Phi}, \wedge)$ and $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Phi}, \wedge)$, $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'} \in (\tilde{\Psi}, \wedge)$. Since $\mathfrak{m}_\varphi^\mathfrak{q} \cap \mathfrak{n}_{\varphi'}^{\mathfrak{q}'} = 0_{(\mathfrak{M},Q)}$ and $(\tilde{\Phi}, \wedge) \cap (\tilde{\Psi}, \wedge) = 0_{(\mathfrak{M},Q)}$, $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ does not belong to $(\tilde{\Phi}, \wedge)$. It implies that $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ does not belong to $FH_yScl(\tilde{\Phi}, \wedge)$.

Conversely suppose that, for distinct FH_ySp's $\mathfrak{m}_\varphi^\mathfrak{q}$, $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$, there exists a $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) $(\tilde{\Phi}, \wedge)$ containing $\mathfrak{m}_\varphi^\mathfrak{q}$ but not $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ such that $\mathfrak{n}_{(\alpha',\beta',\gamma)'}^{\mathfrak{q}'}$ does not belong to $FH_yScl(\tilde{\Phi}, \wedge)$. Then $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'} \in (FH_yScl(\tilde{\Phi}, \wedge))^c$, *i.e.*, $(\tilde{\Phi}, \wedge)$ and $(FH_yScl(\tilde{\Phi}, \wedge))^c$ are disjoint $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$)'s containing $\mathfrak{m}_\varphi^\mathfrak{q}$, $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ respectively. □

Theorem 3.15. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_1- space for every FH_ySp $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Phi}, \wedge) \in \tilde{\tau}$. If there exists a $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) $(\tilde{\Psi}, \wedge)$ such that $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Psi}, \wedge) \subseteq FH_yScl(\tilde{\Psi}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$, then $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_2- space.

Proof. Suppose that $\mathfrak{m}_\varphi^\mathfrak{q} \cap \mathfrak{n}_{\varphi'}^{\mathfrak{q}'} = 0_{(\mathfrak{M},Q)}$. Since $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)-T_1- space, $\mathfrak{m}_\varphi^\mathfrak{q}$ and $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ are $FH_yS\delta cs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$ & $FH_yS\delta\mathcal{P}cs$)'s in $\tilde{\tau}$. Then $\mathfrak{m}_\varphi^\mathfrak{q} \in (\mathfrak{n}_{\varphi'}^{\mathfrak{q}'})^c \in \tilde{\tau}$. Thus there exists a $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$) $(\tilde{\Psi}, \wedge)$ in $\tilde{\tau}$ such that $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Psi}, \wedge) \subseteq FH_yScl(\tilde{\Psi}, \wedge) \subseteq (\mathfrak{n}_{\varphi'}^{\mathfrak{q}'})^c$. So, we have $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'} \in (FH_yScl(\tilde{\Psi}, \wedge))^c$, $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Psi}, \wedge)$ and $(\tilde{\Psi}, \wedge) \cap (FH_yScl(\tilde{\Psi}, \wedge))^c = 0_{(\mathfrak{M},Q)}$, *i.e.*, $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_2- space. □

Remark 3.16. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_i- space for $i = 0, 1, 2$. For each $\mathfrak{m} \neq \mathfrak{n}$, FH_ySp's $\mathfrak{m}_\varphi$ and $\mathfrak{n}_{\varphi'}$ have neighbourhoods satisfying conditions of δ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_i- space in FH_ySts $(\mathfrak{M}, \tilde{\tau}^\mathfrak{q})$ for each $\mathfrak{q} \in Q$ because $\mathfrak{m}_\varphi^\mathfrak{q}$ and $\mathfrak{n}_{\varphi'}^{\mathfrak{q}'}$ are distinct FH_ySp 's.

Definition 3.17. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. Let $(\tilde{\Phi}, \wedge)$ be a $FH_yS\delta cs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$ & $FH_yS\delta\mathcal{P}cs$) and $\mathfrak{m}_\varphi^\mathfrak{q} \cap (\tilde{\Phi}, \wedge) = 0_{(\mathfrak{M},Q)}$. If there exist $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta\mathcal{S}os$ & $FH_yS\delta\mathcal{P}os$)'s $(\tilde{\Upsilon}_1, \wedge)$ and $(\tilde{\Upsilon}_2, \wedge)$ such that $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Upsilon}_1, \wedge)$, $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Upsilon}_2, \wedge)$ and $(\tilde{\Upsilon}_1, \wedge) \cap (\tilde{\Upsilon}_2, \wedge) = 0_{(\mathfrak{M},Q)}$, then $(\mathfrak{M}, Q, \tilde{\tau})$ is known as a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- regular (in short, $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)-regular) space. $(\mathfrak{M}, Q, \tilde{\tau})$ is said to be a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- T_3- space (in short, $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_3- space) if it is both a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)-regular and $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_1- space.

Theorem 3.18. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be FH_ySts over $\mathfrak{M}$. $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_3- space iff for every $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Phi}, \wedge) \in \tilde{\tau}$, there exists $(\tilde{\Upsilon}, \wedge) \in \tilde{\tau}$ such that $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Upsilon}, \wedge) \subseteq FH_yScl(\tilde{\Upsilon}, \wedge) \subseteq (\tilde{\Phi}, \wedge)$.

Proof. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a $FH_yS\delta$- T_3- space and $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Phi}, \wedge) \in \tilde{\tau}$. Since $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_3- space for the FH_ySp $\mathfrak{m}_\varphi^\mathfrak{q}$ and $FH_yS\delta cs$ $(\tilde{\Phi}, \wedge)^c$, there exist $(\tilde{\Upsilon}_1, \wedge)$, $(\tilde{\Upsilon}_2, \wedge) \in \tilde{\tau}$ such that $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Upsilon}_1, \wedge)$, $(\tilde{\Phi}, \wedge)^c \subseteq (\tilde{\Upsilon}_2, \wedge)$ and $(\tilde{\Upsilon}_1, \wedge) \cap (\tilde{\Upsilon}_2, \wedge) = 0_{(\mathfrak{M},Q)}$. Then we have $\mathfrak{m}_\varphi^\mathfrak{q} \in (\tilde{\Upsilon}_1, \wedge) \subseteq (\tilde{\Upsilon}_2, \wedge)^c \subseteq (\tilde{\Phi}, \wedge)$. Since $(\tilde{\Upsilon}_2, \wedge)^c$ is a $FH_yS\delta cs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_yS\delta\mathcal{S}cs$ & $FH_yS\delta\mathcal{P}cs$), $FH_yScl(\tilde{\Upsilon}_1, \wedge) \subseteq (\tilde{\Upsilon}_2, \wedge)^c$.

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Conversely, let $\mathfrak{m}_{\varphi}^{\mathfrak{q}} \cap (\tilde{\Omega}, \wedge) = 0_{(\mathfrak{M},Q)}$ and $(\tilde{\Omega}, \wedge)$ be a $FH_ySδcs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}\ cs$ & $FH_ySδ\mathcal{P}cs$). Then $\mathfrak{m}_{\varphi}^{\mathfrak{q}} \in (\tilde{\Omega}, \wedge)^c$ and by hypotheis, we have $\mathfrak{m}_{\varphi}^{\mathfrak{q}} \in (\tilde{\Upsilon}, \wedge) \subseteq FH_yScl(\tilde{\Upsilon}, \wedge) \subseteq (\tilde{\Omega}, \wedge)^c$. Thus $\mathfrak{m}_{\varphi}^{\mathfrak{q}} \in (\tilde{\Upsilon}, \wedge)$, $(\tilde{\Omega}, \wedge) \subseteq (FH_yScl(\tilde{\Upsilon}, \wedge))^c$ and $(\tilde{\Upsilon}, \wedge) \cap (FH_yScl(\tilde{\Upsilon}, \wedge))^c = 0_{(\mathfrak{M},Q)}$. Hence $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_3- space. □

Definition 3.19. A FH_ySts $(\mathfrak{M}, Q, \tilde{\tau})$ over $\mathfrak{M}$ is known as a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)-normal (in short, $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)-normal) space, if for every pair of disjoint $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$)'s $(\tilde{\Phi_1}, \wedge)$, $(\tilde{\Phi_2}, \wedge)$, there exist disjoint $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Omega_1}, \wedge)$, $(\tilde{\Omega_2}, \wedge)$ such that $(\tilde{\Phi_1}, \wedge) \subseteq (\tilde{\Omega_1}, \wedge)$ and $(\tilde{\Phi_2}, \wedge) \subseteq (\tilde{\Omega_2}, \wedge)$. $(\mathfrak{M}, Q, \tilde{\tau})$ is said to be a fuzzy hypersoft δ (resp. semi, pre, δ semi & δ pre)- T_4- space (in short, $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space) if it is both a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)-normal and $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_1- space.

Theorem 3.20. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. Then $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space iff for each $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$) $(\tilde{\Phi}, \wedge)$ and $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$) $(\tilde{\Omega}, \wedge)$ with $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Omega}, \wedge)$, there exists a $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$) $(\tilde{\Upsilon}, \wedge)$ such that $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Upsilon}, \wedge) \subseteq FH_yScl(\tilde{\Upsilon}, \wedge) \subseteq (\tilde{\Omega}, \wedge)$.

Proof. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space. Let $(\tilde{\Phi}, \wedge)$ be a $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$) and let $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Omega}, \wedge) \in \tilde{\tau}$. Then $(\tilde{\Omega}, \wedge)^c$ is a $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$) and $(\tilde{\Phi}, \wedge) \cap (\tilde{\Omega}, \wedge)^c = 0_{(\mathfrak{M},Q)}$. Since $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space, there exist $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Upsilon_1}, \wedge)$ and $(\tilde{\Upsilon_2}, \wedge)$ such that $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Upsilon_1}, \wedge)$, $(\tilde{\Omega}, \wedge)^c \subseteq (\tilde{\Upsilon_2}, \wedge)$ and $(\tilde{\Upsilon_1}, \wedge) \cap (\tilde{\Upsilon_2}, \wedge) = 0_{(\mathfrak{M},Q)}$. Thus $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Upsilon_1}, \wedge) \subseteq (\tilde{\Upsilon_2}, \wedge)^c \subseteq (\tilde{\Omega}, \wedge)$, $(\tilde{\Upsilon_2}, \wedge)^c$ is a $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$) and $(\tilde{\Upsilon_1}, \wedge) \subseteq (\tilde{\Upsilon_2}, \wedge)^c$. So, $(\tilde{\Phi}, \wedge) \subseteq (\tilde{\Upsilon_1}, \wedge) \subseteq FH_yScl(\tilde{\Upsilon_1}, \wedge) \subseteq (\tilde{\Omega}, \wedge)$.

Conversely, let $(\tilde{\Phi_1}, \wedge)$, $(\tilde{\Phi_2}, \wedge)$ be two disjoint $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$)'s. Then $(\tilde{\Phi_1}, \wedge) \subseteq (\tilde{\Phi_2}, \wedge)^c$. By hypothesis, there exists a $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$) $(\tilde{\Upsilon}, \wedge)$ such that $(\tilde{\Phi_1}, \wedge) \subseteq (\tilde{\Upsilon}, \wedge) \subseteq FH_yScl(\tilde{\Upsilon_1}, \wedge) \subseteq (\tilde{\Phi_2}, \wedge)^c$. Thus $(\tilde{\Upsilon}, \wedge)$, $(FH_yScl(\tilde{\Upsilon}, \wedge))^c$ are $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$)'s and $(\tilde{\Phi_1}, \wedge) \subseteq (\tilde{\Upsilon}, \wedge)$, $(\tilde{\Phi_2}, \wedge) \subseteq (FH_yScl(\tilde{\Upsilon}, \wedge))^c$ and $(\tilde{\Upsilon}, \wedge) \cap (FH_yScl(\tilde{\Upsilon}, \wedge))^c = 0_{(\mathfrak{M},Q)}$. So $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space. □

Theorem 3.21. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. If $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_i- space, then the FH_ySts $\big((\tilde{\Phi}, \wedge), \tilde{\tau}_{(\tilde{\Phi},\wedge)}, Q\big)$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_i- space for $i = 0, 1, 2, 3$.

Proof. Let $\mathfrak{m}_{\varphi}^{q}$, $\mathfrak{n}_{\varphi'}^{q'} \in \big((\tilde{\Phi}, \wedge), \tilde{\tau}_{(\tilde{\Phi},\wedge)}, Q\big)$ such that $\mathfrak{m}_{\varphi}^{q} \cap \mathfrak{n}_{\varphi'}^{q'} = 0_{(\mathfrak{M},Q)}$. Then there exist $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Phi_1}, \wedge)$ and $(\tilde{\Phi_2}, \wedge)$ satisfying the conditions of $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_i-space such that $\mathfrak{m}_{\varphi}^{q} \in (\tilde{\Phi_1}, \wedge)$, $\mathfrak{n}_{\varphi'}^{q'} \in (\tilde{\Phi_2}, \wedge)$. Thus, $\mathfrak{m}_{\varphi}^{q} \in (\tilde{\Phi_1}, \wedge) \cap (\tilde{\Phi}, \wedge)$ and $\mathfrak{n}_{\varphi'}^{q'} \in (\tilde{\Phi_2}, \wedge) \cap (\tilde{\Phi}, \wedge)$. Also, the $FH_ySδos$ (resp. $FH_yS\mathcal{S}os$, $FH_yS\mathcal{P}os$, $FH_ySδ\mathcal{S}os$ & $FH_ySδ\mathcal{P}os$)'s $(\tilde{\Phi_1}, \wedge) \cap (\tilde{\Phi}, \wedge)$, $(\tilde{\Phi_2}, \wedge) \cap (\tilde{\Phi}, \wedge)$ in $\tilde{\tau}_{(\tilde{\Phi},\wedge)}$ satisfy the conditions of $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_i- space for $i = 0, 1, 2, 3$. □

Theorem 3.22. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a FH_ySts over $\mathfrak{M}$. If $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space and $(\tilde{\Omega}, \wedge)$ is a $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$) in $(\mathfrak{M}, Q, \tilde{\tau})$, then $\big((\tilde{\Omega}, \wedge), \tilde{\tau}_{(\tilde{\Omega},\wedge)}, Q\big)$ is a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space.

Proof. Let $(\mathfrak{M}, Q, \tilde{\tau})$ be a $FH_ySδ$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_ySδ\mathcal{S}$, & $FH_ySδ\mathcal{P}$)- T_4- space and $(\tilde{\Omega}, \wedge)$ be a $FH_ySδcs$ (resp. $FH_yS\mathcal{S}cs$, $FH_yS\mathcal{P}cs$, $FH_ySδ\mathcal{S}cs$ & $FH_ySδ\mathcal{P}cs$) in $(\mathfrak{M}, Q, \tilde{\tau})$. Let $(\tilde{\Omega_1}, \wedge)$ and

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$(\tilde{\Omega_2}, \wedge)$ be two $FH_yS\delta cs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_yS\delta Scs$ & $FH_yS\delta\mathcal{P}cs$)'s in $\big((\tilde{\Omega}, \wedge), \tilde{\tau}_{(\tilde{\Omega},\wedge)}, Q\big)$ such that $(\tilde{\Omega_1}, \wedge) \cap (\tilde{\Omega_2}, \wedge) = 0_{(\mathfrak{M},Q)}$. When $(\tilde{\Omega}, \wedge)$ is a $FH_yS\delta cs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_yS\delta Scs$ & $FH_yS\delta\mathcal{P}cs$) in $(\mathfrak{M}, Q, \tilde{\tau})$, $(\tilde{\Omega_1}, \wedge)$ and $(\tilde{\Omega_2}, \wedge)$ are $FH_yS\delta cs$ (resp. FH_ySScs, $FH_yS\mathcal{P}cs$, $FH_yS\delta Scs$ & $FH_yS\delta\mathcal{P}cs$)'s in $(\mathfrak{M}, Q, \tilde{\tau})$. Since $(\mathfrak{M}, Q, \tilde{\tau})$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_4- space, there exist $FH_yS\delta os$ (resp. FH_ySSos, $FH_yS\mathcal{P}os$, $FH_yS\delta Sos$ & $FH_yS\delta\mathcal{P}os$)'s $(\tilde{\Upsilon}_1, \wedge)$ and $(\tilde{\Upsilon}_2, \wedge)$ such that $(\tilde{\Omega_1}, \wedge) \subseteq (\tilde{\Upsilon}_1, \wedge)$, $(\tilde{\Omega_2}, \wedge) \subseteq (\tilde{\Upsilon}_2, \wedge)$ and $(\tilde{\Upsilon}_1, \wedge) \cap (\tilde{\Upsilon}_2, \wedge) = 0_{(\mathfrak{M},Q)}$. Then $(\tilde{\Omega_1}, \wedge) = (\tilde{\Upsilon}_1, \wedge) \cap (\tilde{\Omega}, \wedge)$, $(\tilde{\Omega_2}, \wedge) = (\tilde{\Upsilon}_2, \wedge) \cap (\tilde{\Omega}, \wedge)$ and $\big((\tilde{\Upsilon}_1, \wedge) \cap (\tilde{\Omega}, \wedge)\big) \cap \big((\tilde{\Upsilon}_2, \wedge) \cap (\tilde{\Omega}, \wedge)\big) = 0_{(\mathfrak{M},Q)}$. Hence $\big((\tilde{\Omega}, \wedge), \tilde{\tau}_{(\tilde{\Omega},\wedge)}, Q\big)$ is a $FH_yS\delta$ (resp. $FH_yS\mathcal{S}$, $FH_yS\mathcal{P}$, $FH_yS\delta\mathcal{S}$, & $FH_yS\delta\mathcal{P}$)- T_4- space. □

4 Conclusions

In this paper, $FH_yS\delta$ (resp. semi, pre, δ semi & δ pre)-separation axioms in FH_ySts are introduced and studied using FH_ySp's. The relation and properties between $FH_yS\delta$ (resp. semi, pre, δ semi & δ pre)- T_i- spaces $(i = 0, 1, 2, 3, 4)$ are also discussed. The future work can involves the investigation of $FH_yS\delta$ (resp. semi, pre, δ semi & δ pre)- compactness, $FH_yS\delta$ (resp. semi, pre, δ semi & δ pre)- connectedness and FH_yS contra δ (resp. semi, pre, δ semi & δ pre)- continuous functions.

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