Effective Neutrosophic Soft Expert Set and Its Application

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Abstract

The neutrosophic soft set emerges as a highly valuable and efficient adaptation of soft sets, specifically addressing parameterized values of alternatives. However, numerous decision-making algorithms rooted in neutrosophic soft sets often neglect the external factors impacting their effectiveness. This paper introduces the innovative concept of an effective neutrosophic soft expert set, meticulously crafted to encapsulate external influences on both neutrosophic soft sets and expert opinions within a unified model. This eliminates the necessity for additional operations. Notably, our groundbreaking approach seamlessly amalgamates the strengths of the neutrosophic soft expert set and the effective set, resulting in heightened efficiency and realism in this domain. The article comprehensively explores the fundamental operations of an effective neutrosophic soft expert set, elucidating these processes through apt examples. Finally, the paper showcases the practical application of this concept in decision-making problems, providing algorithms and illustrative examples to underscore its efficacy.
Keywords: Neutrosophic set; neutrosophic soft set; soft expert set; neutrosophic soft expert set; effective neutrosophic soft set; effective neutrosophic soft expert set.

1 introduction

Zadeh[1] introduced the concept of a fuzzy set to handle uncertain data, quantifying the membership degree for each element. Atanassov[2] extended this with intuitionistic fuzzy sets. Neutrosophic sets, established by Smarandache,[3,4] provide a framework for handling imprecise, indeterminate, and inconsistent data. Molodtsov[5] introduced soft sets as a novel approach to managing uncertainties. The concept of soft sets, along with various operations and applications, was explored by Maji et al.[6,7] Roy and Maji[8] utilized fuzzy soft sets in decision-making challenges, while Majumdar and Samanta[9] generalized the theory of fuzzy soft sets.

The concept of a soft expert set, considered as one of the valuable extensions of soft sets, is discussed in.[10] This concept was further extended to a fuzzy soft expert set as discussed in.[11] Maji[12] introduced the concept of a neutrosophic soft set, combining elements from both soft sets and neutrosophic sets. In a related development, the idea of an intuitionistic neutrosophic soft set was presented by Said and Smarandache,[13] along with some operations and properties associated with this concept. Furthermore, Sahin et al.[14] introduced the concept of neutrosophic soft expert sets.

Various generalizations of the NSES model were also defined including the generalized neutrosophic soft expert set (GNSES),[15] Q-neutrosophic soft expert set (Q-NSES) in,[16] Generalized Q-neutrosophic soft expert set (GQ-NSES) in,[17] and Time Q-neutrosophic soft expert set (TQ-NSES) in.[18]

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Many researchers have conducted investigations to examine its properties and uses in its practical application such as[19–21] . As development of neutrosophic soft set, the concept of mapping of complex interval neutrosophic soft sets (MI-CNSSs)[22] was introduced with its properties and theorems supported by concrete examples. Also, the concept of Q-complex neutrosophic set presented in[23] by extending the ranges of the membership functions in Q-neutrosophic set (Q-NS) from [0, 1] to the unit circle in the complex plane.

Saqlain et al.[24] introduced the concept of a neutrosophic hypersoft set, which includes aggregate operators, serving as a generalization of hypersoft sets. Furthermore, they introduced generalized aggregate operators for the neutrosophic hypersoft set in,[25] along with its associated operations and properties. They also presented the necessity and possibility operations on the neutrosophic hypersoft set, supported by illustrative examples.

In addition to this, Ihsan et al.[26] introduced the idea of neutrosophic hypersoft expert sets to tackle challenges associated with scenarios involving a single expert and the management of parameterized sets corresponding to distinct attributes. They thoroughly examined the operations and outcomes of this concept, providing clarification through practical examples.

In 2022, Alkhazaleh established the effective fuzzy soft set (EFSS)[27] to measure the influence of external effectiveness on soft sets. Also, he studied the operations, properties, and applications of EFSS. Moreover, the introduction of Effective Fuzzy Soft Expert Sets (ENSES)[28] serves the purpose of knowing expert opinions within a single model. The idea of an effective neutrosophic soft set (ENSS), as outlined in,[29] was introduced to enhance the concept of EFSS. This improvement entails taking into account three independent membership functions that represent the degree of truth (T), indeterminacy (I), and falsity (F), rendering it a more robust and pragmatic approach.

Within the scope of our study, we establish the concept of an expert soft set within the context of the effective neutrosophic soft set. This incorporation allows us to account for the influence of external effectiveness on neutrosophic soft sets and to capture the viewpoints of experts within a unified model. Significantly, our novel approach smoothly combines the advantages of both the soft expert set and the effective neutrosophic soft set, providing a flexible and all-encompassing way to represent uncertainty. To facilitate comprehension, we clarify basic operations related to the effective neutrosophic soft expert set by using practical examples and propositions. This research contributes to the advancement of mathematical models for managing uncertainty and bridges the divide between theory and practical application, providing valuable insights and practical solutions. By introducing a comprehensive framework that merges components of neutrosophic sets and soft expert sets, this study presents a novel approach to handling uncertainty and ambiguity in decision-making processes.

2 Preliminary

Within this section, we provide essential definitions required for this paper. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1. [5] A soft set over U is a pair (F, A) where F is a mapping $F : A \to P\,(U)$

Definition 2.2. [5] The power set of all fuzzy subsets of U represented by I^U A fuzzy soft set over U is a pair (F, E), where F is a mapping given by $F : A \to I^U$.

Definition 2.3. [3] Neutrosophic set
A neutrosophic set A be the universe of discourse X is defined as $A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$. where $T, I, F : X \to [0, 1]$ and $\leq T_A\,(x) + I_A(x) +\ F_A(x) \leq 3$ and $T_A(x)$ is the truth-membership function, $I_A(x)$ is an indeterminacy-membership and $F_A(x)$ f is a falsity-membership function.

Definition 2.4. [12] Consider $P\,(U)$ indicate the set of all neutrosophic sets of U. The soft neutrosophic set over U is the collection (F, A), where F is a mapping provided by $F : A \to P\,(U)$.

Definition 2.5. [14] Let X a set of experts (agents), and $O = \{agree = 1,\ disagree = 0\}$ a set of opinions. Let $Z = E \times X \times O$ and A $\subseteq$ Z. A pair (F, A) is called a neutrosophic soft expert set over U, where F is mapping given by $F : A \to N(U)$ where $N(U)$ denotes the power neutrosophic set of U.

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Definition 2.6. [14] An agree-NSES $(F,Z)^1_\Lambda$ over U is an NSE subset of $(F,Z)_\Lambda$ defined as follows:

$$(F,Z)^1_\Lambda = \{F_1(\alpha): \ \alpha \in E \times X \times \{1\}\}.$$

Definition 2.7. [14] An agree-NSES $(F,Z)^0_\Lambda$ over U is an NSE subset of $(F,Z)_\Lambda$ defined as follows:

$$(F,Z)^0_\Lambda = \{F_1(\alpha): \ \alpha \in E \times X \times \{0\}\}.$$

Definition 2.8. [27] A functional effective set Λ in a universe of discourse A is a fuzzy set defined as $\Lambda : A \to [0,1]$ and the set of effective parameters is represented by A,such that the membership values perhaps modified after implementation effective set and having a positive (or no) effect on membership values and given by

$$\Lambda = \{\langle a, \delta_\Lambda(a)\rangle : a \in A\}.$$

Definition 2.9. [27] Complement of Effective set
The effective set Λ^c, obtained from effective set Λ over the set of effective parameters A , where c represents any fuzzy complement.

Definition 2.10. [29] Let U represent the initial universal set and E represent a set of parameters. Let A represent a set of effective parameters, and let Λ represent the effective set over A. Let $N(U)$ indicate the set of all neutrosophic subsets of U, and a collection of $(F,E)_\Lambda$ known as an effective neutrosophic soft set (ENSS) over U, and F is a mapping provided by $F : E \to N(U)$ and given as follows:

$$F(s)_\Lambda = \left\{ \frac{x_j}{< T_U(x_j)_\Lambda, I_U(x_j)_\Lambda, \ F_U(x_j)_\Lambda >} : x_j \in U, \ e \in E \right\}. \tag{1}$$

Where

$$T_U(x_j)_\Lambda = \begin{cases} T_U(x_j) + \left[\frac{(1-T_U(x_j))\sum_k \delta_{\Lambda_{x_j}}(a_k)}{|A|}\right], & \text{if } T_U(x_j) \in (0,1) \\ T_U(x_j), & \text{O.W} \end{cases}$$

$$I_U(x_j)_\Lambda = I(x_j)$$

$$F_U(x_j)_\Lambda = \begin{cases} F_U(x_j) - \left[\frac{F_U(x_j)\sum_k \delta_{\Lambda_{x_j}}(a_k)}{|A|}\right], & \text{if } F_U(x_j) \in (0,1) \\ F_U(x_j), & \text{O.W} \end{cases}$$

Definition 2.11. [29] Let $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then $(F,E_1)_{\Lambda_1}$ is said to be ENS subset of $(G,E_2)_{\Lambda_2}$ if:

1. $E_1 \subset E_2$
2. $\Lambda_1(x) \le \Lambda_2(x)$
3. $T_{F_{\Lambda_1}(e)}(x) \le T_{G_{\Lambda_2}(e)}(x), I_{F_{\Lambda_1}(e)}(x) \le I_{G_{\Lambda_2}(e)}(x), F_{F_{\Lambda_1}(e)}(x) \ge F_{G_{\Lambda_2}(e)}(x)$

$\forall e \in E_1, x \in U$, and denoted it by $(F,E_1)_{\Lambda_1} \subseteq (G,E_1)_{\Lambda_2}$.

Definition 2.12. [29] The $\Lambda_{complement}$ of ENSS $(F,E)_\Lambda$ is the ENSS denoted by $(F,E)_{\Lambda^c}$, where Λ^c represents any fuzzy complement of Λ.
To compute $\Lambda_{complement}$ of ENSS, we preserve the neutrosophic soft set (F,E) in its original form and compute Λ^c.Then use Equation 1 to calculate a new ENSS.

Definition 2.13. [29] The $Soft_{complement}$ of the ENSS $(F,E)_\Lambda$is the ENSS denoted by $(F^c,E)_\Lambda$, where F^c represents neutrosophic soft complement of F.
To compute $Soft_{complement}$ of ENSS, we preserve the effective set Λ unchanged and compute F^c .Then we use Equation 1 to calculate a new ENSS.

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Definition 2.14. [29] The $Total_{complement}$ of ENSS $(F,E)_\Lambda$ is the ENSS denoted by $(F^c,E)_{\Lambda^c}$, where F^c represents neutrosophic soft complement of F and Λ^c represents any fuzzy complement of Λ.
To compute $Total_{complement}$ of ENSS, we compute Λ^c and F^c .Then we use Equation 1 to calculate a new ENSS.

Definition 2.15. [29] Let $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then the union of $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ is the ENSS and denoted by $(H,E)_{\Lambda_s}$ where $E=E_1\cup E_2$ and $\forall v\in E$, is given as follows:

$$H_{\Lambda_s}(v)=\begin{cases} F_{\Lambda_s}(v), & \text{if } v\in E_1-E_2\\ G_{\Lambda_s}(v), & \text{if } v\in E_2-E_1\\ (F\cup G)_{\Lambda_s}(v), & \text{if } v\in E_1\cap E_2\end{cases}$$

Such that s is any s-norm and $\cup$ represents the union of neutrosophic soft expert sets F and G.

Definition 2.16. [29] Let $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then the intersection of $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ is the ENSS and denoted by $(K,E)_{\Lambda_t}$ where $E=E_1\cap E_2$ and $\forall v\in E$, is given as follows:

$$K_{\Lambda_t}(v)=\begin{cases} F_{\Lambda_t}(v), & \text{if } v\in E_1-E_2\\ G_{\Lambda_t}(v), & \text{if } v\in E_2-E_1\\ (F\cap G)_{\Lambda_t}(v), & \text{if } v\in E_1\cap E_2\end{cases}$$

Such that t is any t-norm and $\cap$ represents the intersection of neutrosophic soft expert sets F and G.

Definition 2.17. [29] Consider $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ be ENSSs defined over the universe U. Then "$(F,E_1)_{\Lambda_1}$ AND $(G,E_2)_{\Lambda_2}$" denoted by $(F,E_1)_{\Lambda_1}\wedge(G,E_2)_{\Lambda_2}$ and given by $(F,E_1)_{\Lambda_1}\wedge(G,E_2)_{\Lambda_2}=(K,E_1\times E_2)_{\Lambda_t}$ where $K_{\Lambda_t}(\alpha,\beta)=(F(\alpha)\cap G(\beta))_{\Lambda_t}:\forall(\alpha,\beta)\in E_1\times E_2$, where t is any t-norm and H_{Λ_t} is the effective neutrosophic soft expert union between F_{Λ_1} and G_{Λ_2}.

Definition 2.18. [29] Consider $(F,E_1)_{\Lambda_1}$ and $(G,E_2)_{\Lambda_2}$ be ENSSs defined over the universe U. Then "$(F,E_1)_{\Lambda_1}$ OR $(G,E_2)_{\Lambda_2}$" denoted by $(F,E_1)_{\Lambda_1}\vee(G,E_2)_{\Lambda_2}$ and given by $(F,E_1)_{\Lambda_1}\vee(G,E_2)_{\Lambda_2}=(H,E_1\times E_2)_{\Lambda_s}$ where $H_{\Lambda_s}(\alpha,\beta)=(\psi(\alpha)\cup\phi(\beta))_{\Lambda_s}:\forall(\alpha,\beta)\in E_1\times E_2$, where s is any s-norm and H_{Λ_t} is the effective neutrosophic soft expert union between F_{Λ_1} and G_{Λ_2}.

3 Effective Neutrosophic Soft Expert Set (ENSES).

This section provides a general definition of ENSES as well as several instances and properties. Let U be a universe, E a set of parameters, X a set of experts, and $O=\{agree=1,\ disagree=0\}$ a set of opinions. Let $Z=E\times X\times O$ and $S\subseteq Z$.

Definition 3.1. A functional effective set Λ in a universe of discourse A is a fuzzy set defined as
$\Lambda: U\times X\longrightarrow I^A$. and the set of effective parameters is represented by A, such that the membership values perhaps modified after implementation effective set and having a positive (or no) effect on membership values and given by

$$\Lambda=\{(u,x),\delta_\Lambda(a):a\in A\}$$

Definition 3.2. A pair $(F,S)_\Lambda$ is called an effective neutrosophic soft expert set (ENSES) over U, provided that $N(U)$ denotes the set of all neutrosophic subsets of U and the set of effective parameters is represented by A such that Λ be the effective set over A. Then F is a mapping given by $F:Z\to N(U)$ and defined as follows:

$$F(s)_\Lambda=\left\{\frac{u_j}{<T_U(u_j)_\Lambda, I_U(u_j)_\Lambda,\ F_U(u_j)_\Lambda>}:u_j\in U,\ s\in S\right\}.$$

For all $s\in S$ and for all $a_k\in A$, we have:

$$T_U(u_j)_\Lambda=\begin{cases} T_U(u_j)+\left[\frac{(1-T_U(u_j))\sum_k\delta_{\Lambda_{x_j}}(a_k)}{|A|}\right], & \text{if } T_U(u_j)\in(0,1)\\ T_U(u_j), & \text{O.W}\end{cases}$$

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$I_U\left(u_j\right)_\Lambda = I\left(u_j\right)$

$$F_U\left(u_j\right)_\Lambda = \begin{cases} F_U\left(u_j\right) - \left[\frac{F_U(u_j)\sum_k \delta_{\Lambda_{u_j}}(a_k)}{|A|}\right], & \text{if } F_U\left(u_j\right) \in (0,1) \\ F_U\left(u_j\right), & \text{O.W} \end{cases}$$

Example 3.3. The set of universe is represented by $U = \{u_1, u_2, u_3\}$, the set of parameters is represented by $E = \{e_1, e_2, e_3\}$, the set of experts is represented by $X = \{p, q, r\}$, the set of opinion is represented by $O = \{agree = 1, disagree = 0\}$ and the set of effective parameters is represented by $A = \{a_1, a_2, a_3, a_4\}$. consider the experts provide the effective set over the universe A for all $\{u_1, u_2, u_3\}$ as follows:

$$\begin{aligned}
&\Lambda\left(u_1, p\right) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.6}\right\}, \Lambda\left(u_2, p\right) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0.8}\right\}, \\
&\Lambda\left(u_3, p\right) = \left\{\frac{a_1}{0.5}, \frac{a_2}{1}, \frac{a_3}{0.8}, \frac{a_4}{1}\right\}, \Lambda\left(u_1, q\right) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0.1}, \frac{a_3}{0.7}, \frac{a_4}{0.6}\right\}, \\
&\Lambda\left(u_2, q\right) = \left\{\frac{a_1}{0.5}, \frac{a_2}{0}, \frac{a_3}{0.4}, \frac{a_4}{0.1}\right\}, \Lambda\left(u_3, q\right) = \left\{\frac{a_1}{0.5}, \frac{a_2}{0.3}, \frac{a_3}{0.8}, \frac{a_4}{0.7}\right\}, \\
&\Lambda\left(u_1, r\right) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.2}, \frac{a_3}{0.2}, \frac{a_4}{0}\right\}, \Lambda\left(u_2, r\right) = \left\{\frac{a_1}{0}, \frac{a_2}{0.1}, \frac{a_3}{0.3}, \frac{a_4}{0.5}\right\}, \\
&\Lambda\left(u_3, r\right) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.1}, \frac{a_3}{0.7}, \frac{a_4}{0.2}\right\}.
\end{aligned}$$

Let F be the neutrosophic expert set (NSES) defined as follows:

$$F\left(e_1, p, 1\right) = \left\{\frac{u_1}{< 0.2, 0, 0.7 >}, \frac{u_2}{< 0.7, 0.3, 0.1 >}, \frac{u_3}{< 0.7, 0.2, 0.3 >}\right\}$$

$$F\left(e_1, q, 1\right) = \left\{\frac{u_1}{< 0.4, 0.3, 0.7 >}, \frac{u_2}{< 0.1, 0.3, 0.4 >}, \frac{u_3}{< 0.1, 0.2, 0.6 >}\right\}$$

$$F\left(e_1, r, 1\right) = \left\{\frac{u_1}{< 0.3, 0.5, 0.6 >}, \frac{u_2}{< 0.7, 0.3, 0.4 >}, \frac{u_3}{< 0.4, 0.1, 0.2 >}\right\}$$

$$F\left(e_2, p, 1\right) = \left\{\frac{u_1}{< 0.6, 0.1, 0.2 >}, \frac{u_2}{< 0.7, 0.2, 0.3 >}, \frac{x_3}{< 0.6, 0.1, 0.3 >}\right\}$$

$$F\left(e_2, q, 1\right) = \left\{\frac{u_1}{< 0.7, 0.1, 0.2 >}, \frac{u_2}{< 0.6, 0.7, 0.2 >}, \frac{u_3}{< 0.7, 0.4, 0.1 >}\right\}$$

$$F\left(e_2, r, 1\right) = \left\{\frac{u_1}{< 0.3, 0.2, 0.8 >}, \frac{u_2}{< 0.4, 0.1, 0.7 >}, \frac{u_3}{< 0.5, 0.2, 0.1 >}\right\}$$

$$F\left(e_3, p, 1\right) = \left\{\frac{u_1}{< 0.4, 0.2, 0.1 >}, \frac{u_2}{< 0.5, 0.7, 0.1 >}, \frac{u_3}{< 0.1, 0, 0.4 >}\right\}$$

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$$F\left(e_3,q,1\right)=\left\{\frac{u_1}{<0.4,0.1,0.7>},\frac{u_2}{<0.5,0.3,0.8>},\frac{u_3}{<0.3,0.4,0.6>}\right\}$$

$$F\left(e_3,r,1\right)=\left\{\frac{u_1}{<0.5,0.8,0.2>},\frac{u_2}{<0.5,0,0.4>},\frac{u_3}{<0.6,0.3,0.1>}\right\}$$

$$F\left(e_1,p,0\right)=\left\{\frac{u_1}{<0.3,0,0.5>},\frac{u_2}{<0.2,0.4,0.6>},\frac{u_3}{<0.8,0.1,0.4>}\right\}$$

$$F\left(e_1,q,0\right)=\left\{\frac{u_1}{<0.6,0.2,0.3>},\frac{u_2}{<0.8,0.2,0.5>},\frac{u_3}{<0.6,0.5,0.9>}\right\}$$

$$F\left(e_1,r,0\right)=\left\{\frac{u_1}{<0.6,0.4,0.9>},\frac{u_2}{<0.3,0.7,0.9>},\frac{u_3}{<0.2,0,0.6>}\right\}$$

$$F\left(e_2,p,0\right)=\left\{\frac{u_1}{<0.4,0.8,0.1>},\frac{u_2}{<0.7,0.6,0.4>},\frac{x_3}{<0.4,0.1,0.6>}\right\}$$

$$F\left(e_2,q,0\right)=\left\{\frac{u_1}{<0.2,0.4,0.8>},\frac{u_2}{<0.5,0.3,0.7>},\frac{u_3}{<0.8,0.5,0.3>}\right\}$$

$$F\left(e_2,r,0\right)=\left\{\frac{u_1}{<0.9,0.4,0.2>},\frac{u_2}{<0.4,0.7,0.9>},\frac{u_3}{<0.1,0,0.9>}\right\}$$

$$F\left(e_3,p,0\right)=\left\{\frac{u_1}{<0.7,0.2,0.4>},\frac{u_2}{<0.5,0.3,0.8>},\frac{u_3}{<0.8,0.4,0.5>}\right\}$$

$$F\left(e_3,q,0\right)=\left\{\frac{u_1}{<0.5,0.1,0.9>},\frac{u_2}{<0.8,0.3,0.4>},\frac{u_3}{<0.6,0.1,0.9>}\right\}$$

$$F\left(e_3,r,0\right)=\left\{\frac{u_1}{<0.3,0.2,0.5>},\frac{u_2}{<0.9,0.1,0.4>},\frac{u_3}{<0.4,0.1,0.7>}\right\}$$

Then by applying Definition 3.2, we get:

$$\begin{aligned}F\left(e_1,p,1\right)\left(u_j\right)_\Lambda=&\left\{\frac{u_1}{<0.2+[(1-0.2)(0.4+0+1+0.6)/4],0,0.7-[(0.7)(0.4+0+1+0.6)/4]>},\right.\\&\frac{u_2}{<0.7+[(1-0.7)(0.7+0+0.2+0.8)/4],0.3,0.1-[(0.1)(0.7+0+0.2+0.8)/4]>},\\&\left.\frac{u_3}{<0.7+[(1-0.7)(0.5+1+0.8+1)/4],0.2,0.3-[(0.3)(0.5+1+0.8+1)/4]>}\right\}.\\&=\left\{\frac{u_1}{<0.6,0,0.35>},\frac{u_2}{<0.83,0.3,0.06>},\frac{u_3}{<0.95,0.2,0.05>}\right\}\end{aligned}$$

Similarly, we get the following ENSES:

$$\begin{aligned}(F,Z)_\Lambda=&\left\{\left((e_1,p,1),\frac{u_1}{<0.6,0,0.35>},\frac{u_2}{<0.83,0.3,0.06>},\frac{u_3}{<0.95,0.2,0.05>}\right),\right.\\&\left((e_1,q,1),\frac{u_1}{<0.64,0.3,0.42>},\frac{u_2}{<0.33,0.3,0.3>},\frac{u_3}{<0.62,0.2,0.26>}\right),\\&\left((e_1,r,1),\frac{u_1}{<0.42,0.5,0.5>},\frac{u_2}{<0.77,0.3,0.31>},\frac{u_3}{<0.61,0.1,0.13>}\right),\\&\left((e_2,p,1),\frac{u_1}{<0.8,0.1,0.1>},\frac{u_2}{<0.83,0.2,0.17>},\frac{u_3}{<0.93,0.1,0.05>}\right),\\&\left((e_2,q,1),\frac{u_1}{<0.82,0.1,0.12>},\frac{u_2}{<0.7,0.7,0.15>},\frac{u_3}{<0.87,0.4,0.04>}\right),\end{aligned}$$

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$$\left((e_2,r,1),\frac{u_1}{<0.42,0.2,0.66>},\frac{u_2}{<0.54,0.1,0.54>},\frac{u_3}{<0.68,0.2,0.07>}\right),$$
$$\left((e_3,p,1),\frac{u_1}{<0.7,0.2,0.05>},\frac{u_2}{<0.71,0.7,0.6>},\frac{u_3}{<0.84,0,0.07>}\right),$$
$$\left((e_3,q,1),\frac{u_1}{<0.64,0.1,0.42>},\frac{u_2}{<0.63,0.3,0.6>},\frac{u_3}{<0.7,0.4,0.26>}\right),$$
$$\left((e_3,r,1),\frac{u_1}{<0.59,0.8,0.17>},\frac{u_2}{<0.61,0,0.49>},\frac{u_3}{<0.74,0.3,0.07>}\right),$$
$$\left((e_1,p,0),\frac{u_1}{<0.65,0,0.25>},\frac{u_2}{<0.54,0.4,0.35>},\frac{u_3}{<0.97,0.1,0.07>}\right),$$
$$\left((e_1,q,0),\frac{u_1}{<0.76,0.2,0.18>},\frac{u_2}{<0.85,0.2,0.38>},\frac{u_3}{<0.83,0.5,0.38>}\right),$$
$$\left((e_1,r,0),\frac{u_1}{<0.67,0.4,0.74>},\frac{u_2}{<0.46,0.7,0.7>},\frac{u_3}{<0.48,0,0.39>}\right),$$
$$\left((e_2,p,0),\frac{u_1}{<0.7,0.8,0.05>},\frac{u_2}{<0.83,0.6,0.23>},\frac{u_3}{<0.9,0.1,0.12>}\right),$$
$$\left((e_2,q,0),\frac{u_1}{<0.52,0.4,0.48>},\frac{u_2}{<0.63,0.3,0.53>},\frac{u_3}{<0.92,0.5,0.13>}\right),$$
$$\left((e_2,r,0),\frac{u_1}{<0.92,0.4,0.17>},\frac{u_2}{<0.54,0.7,0.7>},\frac{u_3}{<0.42,0,0.59>}\right),$$
$$\left((e_3,p,0),\frac{u_1}{<0.85,0.2,0.2>},\frac{u_2}{<0.71,0.3,0.46>},\frac{u_3}{<0.97,0.4,0.13>}\right),$$
$$\left((e_3,q,0),\frac{u_1}{<0.7,0.1,0.54>},\frac{u_2}{<0.85,0.3,0.3>},\frac{u_3}{<0.83,0.1,0.38>}\right),$$
$$\left((e_3,r,0),\frac{u_1}{<0.42,0.2,0.41>},\frac{u_2}{<0.92,0.1,0.31>},\frac{u_3}{<0.61,0.1,0.5>}\right)\Bigg\}$$

Definition 3.4. Consider $(F,S_1)_{\Lambda_1}$ and $(G,S_2)_{\Lambda_2}$ be two ENSESs over the common universe U. Then $(F,S_1)_{\Lambda_1}$ is an ENSE subset of $(G,S_2)_{\Lambda_2}$ if:

1. $S_1 \subset S_2$
2. $\Lambda_1(x)$ fuzzy subset of $\Lambda_2(x)$
3. $T_{F_{\Lambda_1}(e)}(x) \leq T_{G_{\Lambda_2}(e)}(x), I_{F_{\Lambda_1}(e)}(x) \leq I_{G_{\Lambda_2}(e)}(x), F_{F_{\Lambda_1}(e)}(x) \geq F_{G_{\Lambda_2}(e)}(x)$

$\forall e \in S_1, x \in U$ and denote it by $(F,S_1)_{\Lambda_1} \subseteq (G,S_1)_{\Lambda_2}$.

Definition 3.5. Consider $(F,S_1)_{\Lambda_1}$ and $(G,S_2)_{\Lambda_2}$ be two ENSESs over the common universe U. Then $(F,S_1)_{\Lambda_1}$ is said to be equal to $(G,S_2)_{\Lambda_2}$ and denoted by $(F,S_1)_{\Lambda_1} = (G,S_2)_{\Lambda_2}$ if $(F,S_1)_{\Lambda_1}$ is an ENSE subset of $(G,S_2)_{\Lambda_2}$ and $(G,S_2)_{\Lambda_2}$ is an ENSE subset of $(F,S_1)_{\Lambda_1}$.

Definition 3.6. An agree-ENSES $(F,Z)^1_\Lambda$ over U is an ENSE subset of $(F,Z)_\Lambda$ defined as follows:

$$(F,Z)^1_\Lambda = \{F_1(\alpha) : \alpha \in E \times X \times \{1\}\}.$$

Example 3.7. Consider Example 3.3. The agree effective neutrosophic soft expert set $(F,Z)^1_\Lambda$ over U is

$$(F,Z)^1_\Lambda = \Bigg\{\left((e_1,p,1),\frac{u_1}{<0.6,0,0.35>},\frac{u_2}{<0.83,0.3,0.06>},\frac{u_3}{<0.95,0.2,0.05>}\right),$$
$$\left((e_1,q,1),\frac{u_1}{<0.64,0.3,0.42>},\frac{u_2}{<0.33,0.3,0.3>},\frac{u_3}{<0.62,0.2,0.26>}\right),$$

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$$\left((e_1, r, 1), \frac{u_1}{< 0.42, 0.5, 0.5 >}, \frac{u_2}{< 0.77, 0.3, 0.31 >}, \frac{u_3}{< 0.61, 0.1, 0.13 >}\right),$$
$$\left((e_2, p, 1), \frac{u_1}{< 0.8, 0.1, 0.1 >}, \frac{u_2}{< 0.83, 0.2, 0.17 >}, \frac{u_3}{< 0.93, 0.1, 0.05 >}\right),$$
$$\left((e_2, q, 1), \frac{u_1}{< 0.82, 0.1, 0.12 >}, \frac{u_2}{< 0.7, 0.7, 0.15 >}, \frac{u_3}{< 0.87, 0.4, 0.04 >}\right),$$
$$\left((e_2, r, 1), \frac{u_1}{< 0.42, 0.2, 0.66 >}, \frac{u_2}{< 0.54, 0.1, 0.54 >}, \frac{u_3}{< 0.68, 0.2, 0.07 >}\right),$$
$$\left((e_3, p, 1), \frac{u_1}{< 0.7, 0.2, 0.05 >}, \frac{u_2}{< 0.71, 0.7, 0.6 >}, \frac{u_3}{< 0.84, 0, 0.07 >}\right),$$
$$\left((e_3, q, 1), \frac{u_1}{< 0.64, 0.1, 0.42 >}, \frac{u_2}{< 0.63, 0.3, 0.6 >}, \frac{u_3}{< 0.7, 0.4, 0.26 >}\right),$$
$$\left((e_3, r, 1), \frac{u_1}{< 0.59, 0.8, 0.17 >}, \frac{u_2}{< 0.61, 0, 0.49 >}, \frac{u_3}{< 0.74, 0.3, 0.07 >}\right)\Bigg\}$$

Definition 3.8. Disagree-ENSES $(F, Z)^0_\Lambda$ over U is an ENSE subset of $(F, Z)_\Lambda$ defined as follows:

$$(F, Z)^0_\Lambda = \{F_1(\alpha) : \alpha \in E \times X \times \{0\}\}.$$

Example 3.9. Consider Example 3.3. The disagree effective neutrosophic soft expert set $(F, Z)^0_\Lambda$ over U is

$$(F, Z)^0_\Lambda = \Bigg\{ \left((e_1, p, 0), \frac{u_1}{< 0.65, 0, 0.25 >}, \frac{u_2}{< 0.54, 0.4, 0.35 >}, \frac{u_3}{< 0.97, 0.1, 0.07 >}\right),$$
$$\left((e_1, q, 0), \frac{u_1}{< 0.76, 0.2, 0.18 >}, \frac{u_2}{< 0.85, 0.2, 0.38 >}, \frac{u_3}{< 0.83, 0.5, 0.38 >}\right),$$
$$\left((e_1, r, 0), \frac{u_1}{< 0.67, 0.4, 0.74 >}, \frac{u_2}{< 0.46, 0.7, 0.7 >}, \frac{u_3}{< 0.48, 0, 0.39 >}\right),$$
$$\left((e_2, p, 0), \frac{u_1}{< 0.7, 0.8, 0.05 >}, \frac{u_2}{< 0.83, 0.6, 0.23 >}, \frac{u_3}{< 0.9, 0.1, 0.12 >}\right),$$
$$\left((e_2, q, 0), \frac{u_1}{< 0.52, 0.4, 0.48 >}, \frac{u_2}{< 0.63, 0.3, 0.53 >}, \frac{u_3}{< 0.92, 0.5, 0.13 >}\right),$$
$$\left((e_2, r, 0), \frac{u_1}{< 0.92, 0.4, 0.17 >}, \frac{u_2}{< 0.54, 0.7, 0.7 >}, \frac{u_3}{< 0.42, 0, 0.59 >}\right),$$
$$\left((e_3, p, 0), \frac{u_1}{< 0.85, 0.2, 0.2 >}, \frac{u_2}{< 0.71, 0.3, 0.46 >}, \frac{u_3}{< 0.97, 0.4, 0.13 >}\right),$$
$$\left((e_3, q, 0), \frac{u_1}{< 0.7, 0.1, 0.54 >}, \frac{u_2}{< 0.85, 0.3, 0.3 >}, \frac{u_3}{< 0.83, 0.1, 0.38 >}\right),$$
$$\left((e_3, r, 0), \frac{u_1}{< 0.42, 0.2, 0.41 >}, \frac{u_2}{< 0.92, 0.1, 0.31 >}, \frac{u_3}{< 0.61, 0.1, 0.5 >}\right)\Bigg\}$$

Definition 3.10. The $\Lambda_{complement}$ of ENSES $(F, S)_\Lambda$ is the ENSES denoted by $(F, S)_{\Lambda^c}$, where Λ^c represents any fuzzy complement of Λ.
To compute $\Lambda_{complement}$ of ENSS,we preserve the neutrosophic soft set (F, E) unchanged and compute Λ^c. Lastly, employ Definition 3.2 on the ENSES.

Definition 3.11. The $Soft - Expert_{complement}$ of the ENSS $(F, S)_\Lambda$is the ENSES denoted by $(F^c, E)_\Lambda$ where, F^c represents neutrosophic soft complement of F.
To compute $Soft - Expert_{complement}$ of ENSS,we preserve the effective set Λ^c unchanged and compute F^c. Lastly, employ Definition 3.2 on the ENSES.

Definition 3.12. The $Total_{complement}$ of the ENSES $(F, S)_\Lambda$ is the ENSES denoted by $(F^c, S)_{\Lambda^c}$ where F^c represents neutrosophic soft complement of F and Λ^c represents any fuzzy complement of Λ.
To compute $Total_{complement}$ of ENSS, we compute F^c and Λ^c Lastly, employ Definition 3.2 on the ENSES.

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Example 3.13. Consider Example 3.3 and let $S = \{(e_1, p, 1), (e_3, r, 1), (e_2, q, 0)\} \subset Z$.

$$(F,S) = \left\{ \left((e_1,p,1), \frac{u_1}{<0.2,0,0.7>}, \frac{u_2}{<0.7,0.3,0.1>}, \frac{u_3}{<0.7,0.2,0.3>} \right), \left((e_3,r,1), \frac{u_1}{<0.5,0.8,0.2>}, \frac{u_2}{<0.5,0,0.4>}, \frac{u_3}{<0.6,0.3,0.1>} \right), \left((e_2,q,0), \frac{u_1}{<0.2,0.4,0.8>}, \frac{u_2}{<0.5,0.3,0.7>}, \frac{u_3}{<0.8,0.5,0.3>} \right) \right\}$$

We get Λ^c by applying the basic fuzzy complement over Λ as follows:

$$\Lambda^c(u_1,p) = \left\{ \frac{a_1}{0.6}, \frac{a_2}{1}, \frac{a_3}{0}, \frac{a_4}{0.4} \right\}, \Lambda^c(u_2,p) = \left\{ \frac{a_1}{0.3}, \frac{a_2}{1}, \frac{a_3}{0.8}, \frac{a_4}{0.2} \right\},$$
$$\Lambda^c(u_3,p) = \left\{ \frac{a_1}{0.5}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0} \right\}, \Lambda^c(u_1,q) = \left\{ \frac{a_1}{0.8}, \frac{a_2}{0.9}, \frac{a_3}{0.3}, \frac{a_4}{0.4} \right\},$$
$$\Lambda^c(u_2,q) = \left\{ \frac{a_1}{0.5}, \frac{a_2}{1}, \frac{a_3}{0.6}, \frac{a_4}{0.9} \right\}, \Lambda^c(u_3,q) = \left\{ \frac{a_1}{0.5}, \frac{a_2}{0.7}, \frac{a_3}{0.2}, \frac{a_4}{0.3} \right\},$$
$$\Lambda^c(u_1,r) = \left\{ \frac{a_1}{0.7}, \frac{a_2}{0.8}, \frac{a_3}{0.8}, \frac{a_4}{1} \right\}, \Lambda^c(u_2,r) = \left\{ \frac{a_1}{1}, \frac{a_2}{0.9}, \frac{a_3}{0.7}, \frac{a_4}{0.5} \right\},$$
$$\Lambda^c(u_3,r) = \left\{ \frac{a_1}{0.6}, \frac{a_2}{0.9}, \frac{a_3}{0.3}, \frac{a_4}{0.8} \right\}$$

We get F^c by applying neutrosophic soft expert set complement over F as follows:

$$(F^c,S) = \left\{ \left((e_1,p,1), \frac{u_1}{<0.7,0,0.2>}, \frac{u_2}{<0.1,0.3,0.7>}, \frac{u_3}{<0.3,0.2,0.7>} \right), \left((e_3,r,1), \frac{u_1}{<0.2,0.8,0.5>}, \frac{u_2}{<0.4,0,0.5>}, \frac{u_3}{<0.1,0.3,0.6>} \right), \left((e_2,q,0), \frac{u_1}{<0.8,0.4,0.2>}, \frac{u_2}{<0.7,0.3,0.5>}, \frac{u_3}{<0.3,0.5,0.8>} \right) \right\}$$

We get $\Lambda_{complement}$, $Soft-Expert_{complement}$ and $Total_{complement}$ respectively by applying Definitions 3.10, 3.11 and 3.12 as follows: $\Lambda_{complement}\left((F,S)_\Lambda\right) =$

$$(F,S)_{\Lambda^c} = \left\{ \left((e_1,p,1), \frac{u_1}{<0.6,0,0.35>}, \frac{u_2}{<0.87,0.3,0.04>}, \frac{u_3}{<0.75,0.2,0.25>} \right), \left((e_3,r,1), \frac{u_1}{<0.91,0.8,0.04>}, \frac{u_2}{<0.89,0,0.09>}, \frac{u_3}{<0.86,0.3,0.04>} \right), \left((e_2,q,0), \frac{u_1}{<0.86,0.4,0.32>}, \frac{u_2}{<0.88,0.3,0.18>}, \frac{u_3}{<0.89,0.5,0.17>} \right) \right\}$$

$Soft-Expert_{complement}\left((F,S)_\Lambda\right) =$

$$(F^c,S)_{\Lambda} = \left\{ \left((e_1,p,1), \frac{u_1}{<0.85,0,0.1>}, \frac{u_2}{<0.48,0.3,0.4>}, \frac{u_3}{<0.88,0.2,0.12>} \right), \left((e_3,r,1), \frac{u_1}{<0.34,0.8,0.41>}, \frac{u_2}{<0.54,0,39>}, \frac{u_3}{<0.42,0.3,0.39>} \right), \left((e_2,q,0), \frac{u_1}{<0.88,0.4,0.12>}, \frac{u_2}{<0.78,0.3,0.38>}, \frac{u_3}{<0.7,0.5,0.34>} \right) \right\}$$

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$Total_{complement}\left((F,S)_{\Lambda}\right)=$

$$\begin{aligned}(F^c,S)_{\Lambda^c}=&\left\{\left((e_1,p,1),\frac{u_1}{<0.85,0,0.1>},\frac{u_2}{<0.6,0.3,0.3>},\frac{u_3}{<0.42,0.2,0.58>}\right),\right.\\&\left((e_3,r,1),\frac{u_1}{<0.86,0.8,0.09>},\frac{u_2}{<0.87,0,0.11>},\frac{u_3}{<0.69,0.3,0.21>}\right),\\&\left.\left((e_2,q,0),\frac{u_1}{<0.92,0.4,0.08>},\frac{u_2}{<0.93,0.3,0.13>},\frac{u_3}{<0.6,0.5,0.46>}\right)\right\}\end{aligned}$$

Proposition 3.14. *Let $(F,S)_{\Lambda}$ be ENSES over the U.Then*

1. $Total_{complement}\left(Total_{complement}\ (F,S)_{\Lambda}\right)=(F,S)_{\Lambda}$
2. $\Lambda_{complement}\ \left(\Lambda_{complement}\ (F,S)_{\Lambda}\right)=(F,S)_{\Lambda}$
3. $Soft-Expert_{complement}\ \left(Soft-Expert_{complement}\ (F,S)_{\Lambda}\right)=(F,S)_{\Lambda}$

Proof. 1) Let $(F,E)_{\Lambda}$ be ENSES over the U.

$$\text{Let } (F,S)_{\Lambda}=\left\{\left(s,\frac{u_j}{<T_U(u_j),I_U(u_j),\ F_U(u_j)>}\right)_{\Lambda}\right\}.$$

For all $s\in S, u_j\in U$ and for all $a_k\in A$, we have:

$$\begin{aligned}Total_{complement}\left((F,S)_{\Lambda}\right)\ =\ (F,S)^c_{\Lambda^c}=&\left\{\left(s,\frac{u_j}{<F_U(u_j),I_U(u_j)_{\Lambda^c},\ T_U(u_j)>}\right)_{\Lambda^c}\right\}\\Total_{complement}\left(Total_{complement}\left((F,S)_{\Lambda}\right)\right)=&\left\{\left(s,\frac{u_j}{<F_U(u_j),I_U(u_j)_{\Lambda^c},\ T_U(u_j)>}\right)_{(\Lambda^c)^c}\right\}\\=&\left\{\left(s,\frac{u_j}{<F_U(u_j),I_U(u_j)_{\Lambda^c},\ T_U(u_j)>}\right)_{\Lambda}\right\}\\=&(F,S)_{\Lambda}\end{aligned}$$

Then $\left((F,,S)^c_{\Lambda^c}\right)^c=(F,S)_{\Lambda}$
The proof of 2 and 3 can be easily obtained from relative definitions.

Definition 3.15. The union of two ENSSs $(F,S_1)_{\Lambda_1}$ and $(G,S_2)_{\Lambda_2}$ over the common universe U is the ENSES $(H,S)_{\Lambda_s}$ where $S=S_1\cup S_2$ and $\forall\nu\in S$, is given as follows:

$$H_{\Lambda_s}(\nu)=\begin{cases}F_{\Lambda_s}(\nu), & \text{if } \nu\in S_1-S_2\\ G_{\Lambda_s}(\nu), & \text{if } \nu\in S_2-S_1\\ (F\cup G)_{\Lambda_s}, & \text{if } \nu\in S_1\cap S_2\end{cases}$$

Such that s is s-norm and $\cup$ represents the union of neutrosophic soft experts F and G.

Example 3.16. Let

$$\begin{aligned}&\Lambda_1(u_1,p)=\left\{\frac{a_1}{0.4},\frac{a_2}{0},\frac{a_3}{1},\frac{a_4}{0.6}\right\},\Lambda_1(u_2,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{0},\frac{a_3}{0.2},\frac{a_4}{0.8}\right\},\\&\Lambda_1(u_3,p)=\left\{\frac{a_1}{0.5},\frac{a_2}{1},\frac{a_3}{0.8},\frac{a_4}{1}\right\},\Lambda_1(u_1,q)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.7},\frac{a_4}{0.6}\right\},\\&\Lambda_1(u_2,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0},\frac{a_3}{0.4},\frac{a_4}{0.1}\right\},\Lambda_1(u_3,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.3},\frac{a_3}{0.8},\frac{a_4}{0.7}\right\},\\&\Lambda_1(u_1,r)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.2},\frac{a_3}{0.2},\frac{a_4}{0}\right\},\Lambda_1(u_2,r)=\left\{\frac{a_1}{0},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.5}\right\},\\&\Lambda_1(u_3,r)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.7},\frac{a_4}{0.2}\right\}.\end{aligned}$$

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$$\begin{aligned}
&\Lambda_2(u_1,p)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.7}\right\},\Lambda_2(u_2,p)=\left\{\frac{a_1}{0.8},\frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.6}\right\},\\
&\Lambda_2(u_3,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{0.9},\frac{a_3}{0.6},\frac{a_4}{0.2}\right\}\Lambda_2(u_1,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.3},\frac{a_3}{0.6},\frac{a_4}{0.5}\right\},\\
&\Lambda_2(u_2,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.2},\frac{a_3}{0.6},\frac{a_4}{0.3}\right\},\Lambda_2(u_3,q)=\left\{\frac{a_1}{0.6},\frac{a_2}{0.2},\frac{a_3}{0.5},\frac{a_4}{0.8}\right\},\\
&\Lambda_2(u_1,r)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.1},\frac{a_4}{0.5}\right\},\Lambda_2(u_2,r)=\left\{\frac{a_1}{0},\frac{a_2}{0.2},\frac{a_3}{0.1},\frac{a_4}{0.6}\right\},\\
&\Lambda_2(u_3,r)=\left\{\frac{a_1}{0.7},\frac{a_2}{0.2},\frac{a_3}{0.6},\frac{a_4}{0.1}\right\}.
\end{aligned}$$

Define (F,S_1) and (G,S_2) as follows:

$$\begin{aligned}
(F,S_1)=&\left\{\left((e_1,p,1),\frac{u_1}{<0.2,0,0.7>},\frac{u_2}{<0.7,0.3,0.1>},\frac{u_3}{<0.7,0.2,0.3>}\right),\right.\\
&\left.\left((e_3,r,1),\frac{u_1}{<0.5,0.8,0.2>},\frac{u_2}{<0.5,0,0.4>},\frac{u_3}{<0.6,0.3,0.1>}\right)\right\}\\
(G,S_2)=&\left\{\left((e_1,p,1),\frac{u_1}{<0.3,0.1,0.8>},\frac{u_2}{<0.8,0.2,0.3>},\frac{u_3}{<0.3,0.1,0.6>}\right),\right.\\
&\left.\left((e_1,q,0),\frac{u_1}{<0.6,0.2,0.3>},\frac{u_2}{<0.8,0.2,0.5>},\frac{u_3}{<0.6,0.5,0.9>}\right)\right\}
\end{aligned}$$

Clearly, (F,S_1) and (G,S_2) be two NSESs.
We get Λ_s by applying the basic fuzzy union as follows:

$$\begin{aligned}
&\Lambda_s(u_1,p)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{1},\frac{a_4}{0.7}\right\},\Lambda_s(u_2,p)=\left\{\frac{a_1}{0.8},\frac{a_2}{0},\frac{a_3}{0.2},\frac{a_4}{0.8}\right\},\\
&\Lambda_s(u_3,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{1},\frac{a_3}{0.8},\frac{a_4}{1}\right\},\Lambda_s(u_1,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.3},\frac{a_3}{0.7},\frac{a_4}{0.6}\right\},\\
&\Lambda_s(u_2,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.2},\frac{a_3}{0.6},\frac{a_4}{0.3}\right\},\Lambda_s(u_3,q)=\left\{\frac{a_1}{0.6},\frac{a_2}{0.3},\frac{a_3}{0.8},\frac{a_4}{0.8}\right\},\\
&\Lambda_s(u_1,r)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.2},\frac{a_3}{0.2},\frac{a_4}{0.5}\right\},\Lambda_s(u_2,r)=\left\{\frac{a_1}{0},\frac{a_2}{0.2},\frac{a_3}{0.3},\frac{a_4}{0.6}\right\},\\
&\Lambda_s(u_3,r)=\left\{\frac{a_1}{0.6},\frac{a_2}{0.3},\frac{a_3}{0.8},\frac{a_4}{0.8}\right\}.
\end{aligned}$$

Also, by using the neutrosophic soft expert union, we have the following NSES (H,S), where $S=S_1\cup S_2$ as follows:

$$\begin{aligned}
(H,S)=&\left\{\left((e_1,p,1),\frac{u_1}{<0.3,0,0.7>},\frac{u_2}{<0.8,0.2,0.1>},\frac{u_3}{<0.7,0.1,0.3>}\right),\right.\\
&\left((e_3,r,1),\frac{u_1}{<0.5,0.8,0.2>},\frac{u_2}{<0.5,0,0.4>},\frac{u_3}{<0.6,0.3,0.1>}\right),\\
&\left.\left((e_1,q,0),\frac{u_1}{<0.6,0.2,0.3>},\frac{u_2}{<0.8,0.2,0.5>},\frac{u_3}{<0.6,0.5,0.9>}\right)\right\}
\end{aligned}$$

Then, by using Definition 3.15 and Definition 3.2 , we get the ENSES $(H,S)_{\Lambda_s}$ as follows:

$$\begin{aligned}
(H,S)_{\Lambda_s}=&\left\{\left((e_1,p,1),\frac{u_1}{<0.69,0,0.32>},\frac{u_2}{<0.89,0.2,0.06>},\frac{u_3}{<0.96,0.1,0.04>}\right),\right.\\
&\left((e_3,r,1),\frac{u_1}{<0.66,0.8,0.14>},\frac{u_2}{<0.64,0,0.29>},\frac{u_3}{<0.78,0.3,0.06>}\right),\\
&\left.\left((e_1,q,0),\frac{u_1}{<0.8,0.2,0.15>},\frac{u_2}{<0.88,0.2,0.3>},\frac{u_3}{<0.85,0.5,0.34>}\right)\right\}
\end{aligned}$$

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Definition 3.17. The intersection of two ENSSs $(F,S_1)_{\Lambda_1}$ and $(G,S_2)_{\Lambda_2}$ over the common universe U is the ENSES $(K,E)_{\Lambda_t}$ where $S=S_1\cup S_2$ and $\forall\nu\in S$, $(K,S)_{\Lambda_t}$ is given as follows:

$$K_{\Lambda_t}(\nu)=\begin{cases}F_{\Lambda_t}(\nu), & \text{if } \nu\in S_1-S_2\\ G_{\Lambda_t}(\nu), & \text{if } \nu\in S_2-S_1\\ (F\cap G)_{\Lambda_t}, & \text{if } \nu\in S_1\cap S_2\end{cases}$$

Such that t is t-norm and $\cap$ represents the intersection of neutrosophic soft experts F and G.

Example 3.18. Consider Example 3.16. We get Λ_t by applying the basic fuzzy intersection as follows:

$$\Lambda_t(u_1,p)=\left\{\frac{a_1}{0.2},\frac{a_2}{0},\frac{a_3}{0.3},\frac{a_4}{0.6}\right\},\Lambda_t(u_2,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.6}\right\},$$
$$\Lambda_t(u_3,p)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.9},\frac{a_3}{0.6},\frac{a_4}{0.2}\right\},\Lambda_t(u_1,q)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.6},\frac{a_4}{0.5}\right\},$$
$$\Lambda_t(u_2,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0},\frac{a_3}{0.4},\frac{a_4}{0.1}\right\},\Lambda_t(u_3,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.2},\frac{a_3}{0.5},\frac{a_4}{0.7}\right\},$$
$$\Lambda_t(u_1,r)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.1},\frac{a_3}{0.1},\frac{a_4}{0}\right\},\Lambda_t(u_2,r)=\left\{\frac{a_1}{0},\frac{a_2}{0.1},\frac{a_3}{0.1},\frac{a_4}{0.5}\right\},$$
$$\Lambda_t(u_3,r)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.6},\frac{a_4}{0.1}\right\}$$

Also, by using the neutrosophic soft expert intersection, we have the following NSES (K,S), where $S=S_1\cup S_2$ as follows:

$$\begin{aligned}(K,S)=&\left\{\left((e_1,p,1),\frac{u_1}{<0.2,0.1,0.8>},\frac{u_2}{<0.7,0.3,0.3>},\frac{u_3}{<0.3,0.2,0.6>}\right),\right.\\&\left((e_3,r,1),\frac{u_1}{<0.5,0.8,0.2>},\frac{u_2}{<0.5,0,0.4>},\frac{u_3}{<0.6,0.3,0.1>}\right),\\&\left.\left((e_1,q,0),\frac{u_1}{<0.6,0.2,0.3>},\frac{u_2}{<0.8,0.2,0.5>},\frac{u_3}{<0.6,0.5,0.9>}\right)\right\}\end{aligned}$$

Then, by using Definition 3.17 and Definition 3.2 , we get the ENSES $(K,S)_{\Lambda_t}$ as follows:

$$\begin{aligned}(K,S)_{\Lambda_t}=&\left\{\left((e_1,p,1),\frac{u_1}{<0.42,0.1,0.58>},\frac{u_2}{<0.81,0.3,0.2>},\frac{u_3}{<0.69,0.2,0.27>}\right),\right.\\&\left((e_3,r,1),\frac{u_1}{<0.56,0.8,0.18>},\frac{u_2}{<0.59,0,0.33>},\frac{u_3}{<0.72,0.3,0.07>}\right),\\&\left.\left((e_1,q,0),\frac{u_1}{<0.74,0.2,0.2>},\frac{u_2}{<0.85,0.2,0.39>},\frac{u_3}{<0.79,0.5,0.47>}\right)\right\}\end{aligned}$$

Proposition 3.19. *Consider $(F,S_1)_{\Lambda_1}$ and $(G,S_2)_{\Lambda_2}$ be ENSESs over the universe U.Then*

1. $(F,S_1)_{\Lambda_1}\cup(G,S_2)_{\Lambda_2}=(G,S_2)_{\Lambda_2}\cup(F,S_1)_{\Lambda_1}$
2. $(F,S_1)_{\Lambda_1}\cap(G,S_2)_{\Lambda_2}=(G,S_2)_{\Lambda_2}\cap(F,S_1)_{\Lambda_1}$

Proof. The proof is straightforward.

Definition 3.20. Let $(F,S_1)_{\Lambda_1}$ and $(G,S_2)_{\Lambda_2}$ be two ENSESs over the common universe U. Then "$(F,S_1)_{\Lambda_1}$ AND $(G,S_2)_{\Lambda_2}$" denoted by $(F,S_1)_{\Lambda_1}\wedge(G,S_2)_{\Lambda_2}$ and given by

$$(F,S_1)_{\Lambda_1}\wedge(G,S_2)_{\Lambda_2}=(K,S_1\times S_2)_{(\Lambda_1\wedge\Lambda_2)}$$

Where $K_{(\Lambda_1\wedge\Lambda_2)}(\beta,\gamma)=(F(\beta)\cap G(\gamma))_{(\Lambda_1\wedge\Lambda_2)}:\forall(\beta,\gamma)\in S_1\times S_2$, $\Lambda_1\wedge\Lambda_2$ is a fazzy AND and $\cap$ is an effective neutrosophic soft expert intersection between F and G.

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Example 3.21. Let

$$\begin{aligned}
&\Lambda_1(u_1,p)=\left\{\frac{a_1}{0.4},\frac{a_2}{0},\frac{a_3}{1},\frac{a_4}{0.6}\right\},\Lambda_1(u_2,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{0},\frac{a_3}{0.2},\frac{a_4}{0.8}\right\},\\
&\Lambda_1(u_3,p)=\left\{\frac{a_1}{0.5},\frac{a_2}{1},\frac{a_3}{0.8},\frac{a_4}{1}\right\},\Lambda_1(u_1,q)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.7},\frac{a_4}{0.6}\right\},\\
&\Lambda_1(u_2,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0},\frac{a_3}{0.4},\frac{a_4}{0.1}\right\},\Lambda_1(u_3,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.3},\frac{a_3}{0.8},\frac{a_4}{0.7}\right\},\\
&\Lambda_1(u_1,r)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.2},\frac{a_3}{0.2},\frac{a_4}{0}\right\},\Lambda_1(u_2,r)=\left\{\frac{a_1}{0},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.5}\right\},\\
&\Lambda_1(u_3,r)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.7},\frac{a_4}{0.2}\right\}\\
&\Lambda_2(u_1,p)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.7}\right\},\Lambda_2(u_2,p)=\left\{\frac{a_1}{0.8},\frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.6}\right\},\\
&\Lambda_2(u_3,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{0.9},\frac{a_3}{0.6},\frac{a_4}{0.2}\right\},\Lambda_2(u_1,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.3},\frac{a_3}{0.6},\frac{a_4}{0.5}\right\},\\
&\Lambda_2(u_2,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.2},\frac{a_3}{0.6},\frac{a_4}{0.3}\right\},\Lambda_2(u_3,q)=\left\{\frac{a_1}{0.6},\frac{a_2}{0.2},\frac{a_3}{0.5},\frac{a_4}{0.8}\right\},\\
&\Lambda_2(u_1,r)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.1},\frac{a_4}{0.5}\right\},\Lambda_2(u_2,r)=\left\{\frac{a_1}{0},\frac{a_2}{0.2},\frac{a_3}{0.1},\frac{a_4}{0.6}\right\},\\
&\Lambda_2(u_3,r)=\left\{\frac{a_1}{0.7},\frac{a_2}{0.2},\frac{a_3}{0.6},\frac{a_4}{0.1}\right\}.
\end{aligned}$$

Define (F,S_1) and (G,S_2) as follows:

$$\begin{aligned}
(F,S_1)=&\left\{\left((e_1,p,1),\frac{u_1}{<0.2,0,0.7>},\frac{u_2}{<0.7,0.3,0.1>},\frac{u_3}{<0.7,0.2,0.3>}\right),\right.\\
&\left.\left((e_3,r,1),\frac{u_1}{<0.5,0.8,0.2>},\frac{u_2}{<0.5,0,0.4>},\frac{u_3}{<0.6,0.3,0.1>}\right)\right\}\\
(G,S_2)=&\left\{\left((e_1,p,1),\frac{u_1}{<0.3,0.1,0.8>},\frac{u_2}{<0.8,0.2,0.3>},\frac{u_3}{<0.3,0.1,0.6>}\right),\right.\\
&\left.\left((e_1,q,0),\frac{u_1}{<0.6,0.2,0.3>},\frac{u_2}{<0.8,0.2,0.5>},\frac{u_3}{<0.6,0.5,0.9>}\right)\right\}
\end{aligned}$$

By using fuzzy AND (minimum), we have the following effective sets:

$$\begin{aligned}
&(\Lambda_1\wedge\Lambda_2)(u_1,p,p)=\left\{\frac{a_1}{0.2},\frac{a_2}{0},\frac{a_3}{0.3},\frac{a_4}{0.6}\right\},(\Lambda_1\wedge\Lambda_2)(u_2,p,p)=\left\{\frac{a_1}{0.7},\frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.6}\right\},\\
&(\Lambda_1\wedge\Lambda_2)(u_3,p,p)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.9},\frac{a_3}{0.6},\frac{a_4}{0.2}\right\},(\Lambda_1\wedge\Lambda_2)(u_1,p,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0},\frac{a_3}{0.6},\frac{a_4}{0.5}\right\},\\
&(\Lambda_1\wedge\Lambda_2)(u_2,p,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0},\frac{a_3}{0.2},\frac{a_4}{0.3}\right\},(\Lambda_1\wedge\Lambda_2)(u_3,p,q)=\left\{\frac{a_1}{0.5},\frac{a_2}{0.2},\frac{a_3}{0.5},\frac{a_4}{0.8}\right\},\\
&(\Lambda_1\wedge\Lambda_2)(u_1,r,p)=\left\{\frac{a_1}{0.2},\frac{a_2}{0.1},\frac{a_3}{0.2},\frac{a_4}{0}\right\},(\Lambda_1\wedge\Lambda_2)(u_2,r,p)=\left\{\frac{a_1}{0},\frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.5}\right\},\\
&(\Lambda_1\wedge\Lambda_2)(u_3,r,p)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.6},\frac{a_4}{0.2}\right\},(\Lambda_1\wedge\Lambda_2)(u_1,r,q)=\left\{\frac{a_1}{0.3},\frac{a_2}{0.2},\frac{a_3}{0.2},\frac{a_4}{0}\right\},\\
&(\Lambda_1\wedge\Lambda_2)(u_2,r,q)=\left\{\frac{a_1}{0},\frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.3}\right\},(\Lambda_1\wedge\Lambda_2)(u_3,r,q)=\left\{\frac{a_1}{0.4},\frac{a_2}{0.1},\frac{a_3}{0.5},\frac{a_4}{0.2}\right\}.
\end{aligned}$$

Now, by using the neutrosophic soft expert intersection, we have $(F,S_1)\wedge(G,S_2)=(K,S_1\times S_2)$ Where,

$$\begin{aligned}
(K,S_1\times S_2)=&\left\{\left((e_1,p,1),(e_1,p,1),\frac{u_1}{<0.3,0.1,0.8>},\frac{u_2}{<0.7,0.3,0.3>},\frac{u_3}{<0.3,0.2,0.6>}\right),\right.\\
&\left((e_1,p,1),(e_1,q,0),\frac{u_1}{<0.5,0.2,0.7>},\frac{u_2}{<0.7,0.3,0.5>},\frac{u_3}{<0.6,0.5,0.8>}\right),
\end{aligned}$$

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$$\left((e_3, r, 1), (e_1, p, 1), \frac{u_1}{<0.3, 0.8, 0.8>}, \frac{u_2}{<0.5, 0.2, 0.4>}, \frac{u_3}{<0.3, 0.3, 0.6>} \right),$$
$$\left((e_3, r, 1), (e_1, q, 0), \frac{u_1}{<0.5, 0.8, 0.3>}, \frac{u_2}{<0.5, 0.2, 0.5>}, \frac{u_3}{<0.6, 0.5, 0.8>} \right) \Bigg\}$$

Then, by using Definition 3.20 and Definition 3.2 we get the ENSES $(K, S_1 \times S_2)_{\Lambda_t}$ as follows:

$$\begin{aligned}(K, S_1 \times S_2)_{(\Lambda_1 \wedge \Lambda_2)} = \Bigg\{ &\left((e_1, p, 1), (e_1, p, 1), \frac{u_1}{<0.49, 0.1, 0.58>}, \frac{u_2}{<0.81, 0.3, 0.2>}, \frac{u_3}{<0.69, 0.2, 0.27>} \right),\\ &\left((e_1, p, 1), (e_1, q, 0), \frac{u_1}{<0.69, 0.2, 0.44>}, \frac{u_2}{<0.77, 0.3, 0.39>}, \frac{u_3}{<0.8, 0.5, 0.4>} \right),\\ &\left((e_3, r, 1), (e_1, p, 1), \frac{u_1}{<0.39, 0.8, 0.7>}, \frac{u_2}{<0.58, 0.2, 0.34>}, \frac{u_3}{<0.53, 0.3, 0.41>} \right),\\ &\left((e_3, r, 1), (e_1, q, 0), \frac{u_1}{<0.59, 0.8, 0.25>}, \frac{u_2}{<0.59, 0.2, 0.41>}, \frac{u_3}{<0.72, 0.5, 0.56>} \right) \Bigg\}\end{aligned}$$

Definition 3.22. Let $(F, S_1)_{\Lambda_1}$ and $(G, S_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then "$(F, S_1)_{\Lambda_1}$ OR $(G, S_2)_{\Lambda_2}$" denoted by $(F, S_1)_{\Lambda_1} \vee (G, S_2)_{\Lambda_2}$ and defined by

$$(F, S_1)_{\Lambda_1} \vee (G, S_2)_{\Lambda_2} = (H, S_1 \times S_2)_{(\Lambda_1 \vee \Lambda_2)}$$

Where $H_{(\Lambda_1 \vee \Lambda_2)}(\beta, \gamma) = (F(\beta) \cup G(\gamma))_{(\Lambda_1 \vee \Lambda_2)} : \forall (\beta, \gamma) \in S_1 \times S_2$, $\Lambda_1 \vee \Lambda_2$ is a fuzzy OR and $\cup$ is an effective neutrosophic soft expert union between F and G.

Example 3.23. Consider Example 3.21, By using fuzzy OR (maximum), we have the following effective sets:

$$\begin{aligned}
&(\Lambda_1 \vee \Lambda_2)(u_1, p, p) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.1}, \frac{a_3}{1}, \frac{a_4}{0.7} \right\}, (\Lambda_1 \vee \Lambda_2)(u_2, p, p) = \left\{ \frac{a_1}{0.8}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0.8} \right\},\\
&(\Lambda_1 \vee \Lambda_2)(u_3, p, p) = \left\{ \frac{a_1}{0.7}, \frac{a_2}{1}, \frac{a_3}{0.8}, \frac{a_4}{1} \right\}, (\Lambda_1 \vee \Lambda_2)(u_1, p, q) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.3}, \frac{a_3}{1}, \frac{a_4}{0.6} \right\},\\
&(\Lambda_1 \vee \Lambda_2)(u_2, p, q) = \left\{ \frac{a_1}{0.7}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.8} \right\}, (\Lambda_1 \vee \Lambda_2)(u_3, p, q) = \left\{ \frac{a_1}{0.6}, \frac{a_2}{1}, \frac{a_3}{0.8}, \frac{a_4}{0.8} \right\},\\
&(\Lambda_1 \vee \Lambda_2)(u_1, r, p) = \left\{ \frac{a_1}{0.3}, \frac{a_2}{0.2}, \frac{a_3}{0.3}, \frac{a_4}{0.7} \right\}, (\Lambda_1 \vee \Lambda_2)(u_2, r, p) = \left\{ \frac{a_1}{0.8}, \frac{a_2}{0.1}, \frac{a_3}{0.3}, \frac{a_4}{0.6} \right\},\\
&(\Lambda_1 \vee \Lambda_2)(u_3, r, p) = \left\{ \frac{a_1}{0.7}, \frac{a_2}{0.9}, \frac{a_3}{0.7}, \frac{a_4}{0.2} \right\}, (\Lambda_1 \vee \Lambda_2)(u_1, r, q) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.3}, \frac{a_3}{0.6}, \frac{a_4}{0.5} \right\},\\
&(\Lambda_1 \vee \Lambda_2)(u_2, r, q) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.5} \right\}, (\Lambda_1 \vee \Lambda_2)(u_3, r, q) = \left\{ \frac{a_1}{0.6}, \frac{a_2}{0.2}, \frac{a_3}{0.7}, \frac{a_4}{0.8} \right\}.
\end{aligned}$$

Now, by using the neutrosophic soft expert union, we have $(F, S_1) \vee (G, S_2) = (H, S_1 \times S_2)$
Where,

$$\begin{aligned}(H, S_1 \times S_2) = \Bigg\{ &\left((e_1, p, 1), (e_1, p, 1), \frac{u_1}{<0.5, 0, 0.7>}, \frac{u_2}{<0.8, 0.2, 0.1>}, \frac{u_3}{<0.7, 0.1, 0.5>} \right),\\ &\left((e_1, p, 1), (e_1, q, 0), \frac{u_1}{<0.6, 0, 0.3>}, \frac{u_2}{<0.8, 0.2, 0.1>}, \frac{u_3}{<0.7, 0.2, 0.5>} \right),\\ &\left((e_3, r, 1), (e_1, p, 1), \frac{u_1}{<0.5, 0.1, 0.2>}, \frac{u_2}{<0.8, 0, 0.3>}, \frac{u_3}{<0.6, 0.1, 0.5>} \right),\\ &\left((e_3, r, 1), (e_1, q, 0), \frac{u_1}{<0.6, 0.2, 0.2>}, \frac{u_2}{<0.8, 0, 0.4>}, \frac{u_3}{<0.6, 0.3, 0.5>} \right) \Bigg\}\end{aligned}$$

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Then, by using Definition 3.22 and Definition 3.2 we get the ENSES $(H, S_1 \times S_2)_{(\Lambda_1 \vee \Lambda_2)}$ as follows:

$$\begin{aligned}(K, S_1 \times S_2)_{(\Lambda_1 \vee \Lambda_2)} = & \left\{ \left((e_1, p, 1), (e_1, p, 1), \frac{u_1}{<0.77, 0, 0.31>}, \frac{u_2}{<0.89, 0.2, 0.06>}, \frac{u_3}{<0.96, 0.1, 0.05>} \right), \right. \\ & \left((e_1, p, 1), (e_1, q, 0), \frac{u_1}{<0.83, 0, 0.13>}, \frac{u_2}{<0.92, 0.2, 0.04>}, \frac{u_3}{<0.94, 0.2, 0.1>} \right), \\ & \left((e_3, r, 1), (e_1, p, 1), \frac{u_1}{<0.69, 0.1, 0.13>}, \frac{u_2}{<0.89, 0, 0.17>}, \frac{u_3}{<0.85, 0.1, 0.19>} \right), \\ & \left. \left((e_3, r, 1), (e_1, q, 0), \frac{u_1}{<0.78, 0.2, 0.11>}, \frac{u_2}{<0.85, 0, 0.23>}, \frac{u_3}{<0.83, 0.3, 0.21>} \right) \right\}\end{aligned}$$

4 An application of ENSS in decision making problem.

In this section, we will discuss the use of an effective neutrosophic soft expert set in decision-making situations. We obtain the following algorithm for ENSES by combining the algorithms of Sahin et al.[14] and Al-Hijjawi et al.[29]

4.1 New Algorithn

1. Take the neutrosophic soft expert set (H, C) from (F, A) and (G, B) as requested.
2. Take two effective sets Λ_1 and Λ_2 over A for the NSESs (F, A), (G, B) in that order.
3. Find Λ_s from Λ_1 and Λ_2.
4. Find the required table H_{Λ_s}.
5. Find agree ENSES and disagree ENSS.
6. Find $u_{ij} = T + I - F$
7. Find $c_{j=\sum_i u_{ij}}$ for agree ENSES.
8. Find $k_{j=\sum_i u_{ij}}$ for disagree ENSES
9. Find $s_j = c_j - k_j$.
10. Find $s_m = max\ s_j$.

4.2 Application in a decision-making problem.

The set of universe is represented by $U = \{u_1, u_2, u_3\}$, the set of parameters is represented by $E = \{e_1, e_2, e_3\}$, the set of experts is represented by $X = \{p, q, r\}$, the set of opinion is represented by $O = \{agree = 1, disagree = 0\}$ and the set of effective parameters is represented by $A = \{a_1, a_2, a_3, a_4\}$. consider the experts provide the effective set over A for all $\{u_1, u_2, u_3\}$ as follows: a_1 : Each part was created at the original factory,a_2 : It was reassembled at the original factory, a_3 : It was not owned by multiple people and a_4 : The latest version of the software is running.

$$\begin{aligned}&\Lambda_1(u_1, p) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.6}\right\}, \Lambda_1(u_2, p) = \left\{\frac{a_1}{0.6}, \frac{a_2}{1}, \frac{a_3}{0.2}, \frac{a_4}{0.9}\right\}, \\ &\Lambda_1(u_3, p) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.1}, \frac{a_3}{0}, \frac{a_4}{0}\right\}, \Lambda_1(u_1, q) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0.1}, \frac{a_3}{0.7}, \frac{a_4}{0.6}\right\}, \\ &\Lambda_1(u_2, q) = \left\{\frac{a_1}{0.8}, \frac{a_2}{1}, \frac{a_3}{0.4}, \frac{a_4}{0.2}\right\}, \Lambda_1(u_3, q) = \left\{\frac{a_1}{0.1}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0.1}\right\}, \\ &\Lambda_1(u_1, r) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0.2}, \frac{a_3}{0.2}, \frac{a_4}{0}\right\}, \Lambda_1(u_2, r) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.4}, \frac{a_3}{1}, \frac{a_4}{0.3}\right\}, \\ &\Lambda_1(u_3, r) = \left\{\frac{a_1}{0}, \frac{a_2}{0.1}, \frac{a_3}{0}, \frac{a_4}{0.1}\right\}\end{aligned}$$

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$$\Lambda_2(u_1,p)=\left\{\frac{a_1}{0.2},\ \frac{a_2}{0.1},\frac{a_3}{0.3},\frac{a_4}{0.7}\right\},\Lambda_2(u_2,p)=\left\{\frac{a_1}{0.8},\ \frac{a_2}{1},\frac{a_3}{0.8},\frac{a_4}{0.3}\right\},$$
$$\Lambda_2(u_3,p)=\left\{\frac{a_1}{0.2},\ \frac{a_2}{0.1},\frac{a_3}{0.1},\frac{a_4}{0}\right\},\Lambda_2(u_1,q)=\left\{\frac{a_1}{0.4},\ \frac{a_2}{0.3},\frac{a_3}{0.6},\frac{a_4}{0.5}\right\},$$
$$\Lambda_2(u_2,q)=\left\{\frac{a_1}{0.3},\ \frac{a_2}{1},\frac{a_3}{0.7},\frac{a_4}{1}\right\},\Lambda_2(u_3,q)=\left\{\frac{a_1}{0},\ \frac{a_2}{0},\frac{a_3}{0.1},\frac{a_4}{0.2}\right\}$$
$$\Lambda_2(u_1,r)=\left\{\frac{a_1}{0.4},\ \frac{a_2}{0.1},\frac{a_3}{0.1},\frac{a_4}{0.5}\right\},\Lambda_2(u_2,r)=\left\{\frac{a_1}{0.5},\ \frac{a_2}{0.7},\frac{a_3}{0.9},\frac{a_4}{0.7}\right\},$$
$$\Lambda_2(u_3,r)=\left\{\frac{a_1}{0},\ \frac{a_2}{0.1},\frac{a_3}{0},\frac{a_4}{0.2}\right\}$$

Let (F,S_1) and (G,S_2) be the neutrosophic soft expert set (NSES) defined as follows:

$$\begin{aligned}(F,S_1)=\Bigg\{&\left((e_1,p,1),\frac{u_1}{<0.3,0.2,0.4>},\frac{u_2}{<0.5,0.1,0.3>},\frac{u_3}{<0.3,0.1,0.5>}\right),\\
&\left((e_1,q,1),\frac{u_1}{<0.6,0.2,0.2>},\frac{u_2}{<0.6,0.2,0.4>},\frac{u_3}{<0.7,0.1,0.2>}\right),\\
&\left((e_1,r,1),\frac{u_1}{<0.3,0.3,0.5>},\frac{u_2}{<0.1,0.4,0.6>},\frac{u_3}{<0.1,0.4,0.8>}\right),\\
&\left((e_2,p,1),\frac{u_1}{<0.3,0.2,0.4>},\frac{u_2}{<0.4,0.2,0.3>},\frac{u_3}{<0.7,0.2,0.3>}\right),\\
&\left((e_2,q,1),\frac{u_1}{<0.3,0.2,0.4>},\frac{u_2}{<0.5,0.2,0.7>},\frac{u_3}{<0.4,0.2,0.3>}\right),\\
&\left((e_2,r,1),\frac{u_1}{<0.4,0.2,0.3>},\frac{u_2}{<0.5,0.1,0.3>},\frac{u_3}{<0.7,0.3,0.4>}\right),\\
&\left((e_1,p,0),\frac{u_1}{<0.5,0.2,0.4>},\frac{u_2}{<0.5,0.1,0.3>},\frac{u_3}{<0.3,0.1,0.2>}\right),\\
&\left((e_1,q,0),\frac{u_1}{<0.7,0.1,0.2>},\frac{u_2}{<0.6,0.2,0.2>},\frac{u_3}{<0.2,0.4,0.6>}\right),\\
&\left((e_1,r,0),\frac{u_1}{<0.2,0.1,0.7>},\frac{u_2}{<0.2,0.4,0.6>},\frac{u_3}{<0.2,0.2,0.6>}\right),\\
&\left((e_2,p,0),\frac{u_1}{<0.1,0.2,0.8>},\frac{u_2}{<0.1,0.4,0.6>},\frac{u_3}{<0.5,0.3,0.3>}\right),\\
&\left((e_2,q,0),\frac{u_1}{<0.4,0.2,0.3>},\frac{u_2}{<0.3,0.4,0.6>},\frac{u_3}{<0.5,0.3,0.6>}\right),\\
&\left((e_2,r,0),\frac{u_1}{<0.1,0.3,0.2>},\frac{u_2}{<0.5,0.2,0.1>},\frac{u_3}{<0.7,0.1,0.1>}\right)\Bigg\}\end{aligned}$$

$$\begin{aligned}(G,S_2)=\Bigg\{&\left((e_1,p,1),\frac{u_1}{<0.4,0.1,0.2>},\frac{u_2}{<0.5,0.4,0.1>},\frac{u_3}{<0.1,0.3,0.7>}\right),\\
&\left((e_1,q,1),\frac{u_1}{<0.9,0.4,0.1>},\frac{u_2}{<0.4,0.4,0.2>},\frac{u_3}{<0.7,0.1,0.3>}\right),\\
&\left((e_1,r,1),\frac{u_1}{<0.2,0.1,0.5>},\frac{u_2}{<0.7,0.5,0.1>},\frac{u_3}{<0.7,0.3,0.8>}\right),\\
&\left((e_3,p,1),\frac{u_1}{<0.2,0.1,0.5>},\frac{u_2}{<0.6,0.1,0.1>},\frac{u_3}{<0.8,0.3,0.4>}\right),\end{aligned}$$

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$$\left((e_3, q, 1), \frac{u_1}{<0.5, 0.2, 0.3>}, \frac{u_2}{<0.4, 0.5, 0.1>}, \frac{u_3}{<0.3, 0.1, 0.6>} \right),$$
$$\left((e_3, r, 1), \frac{u_1}{<0.7, 0.1, 0.5>}, \frac{u_2}{<0.8, 0.2, 0.3>}, \frac{u_3}{<0.5, 0.2, 0.7>} \right),$$
$$\left((e_1, p, 0), \frac{u_1}{<0.7, 0.1, 0.1>}, \frac{u_2}{<0.3, 0.4, 0.6>}, \frac{u_3}{<0.7, 0.1, 0.3>} \right),$$
$$\left((e_1, q, 0), \frac{u_1}{<0.5, 0.5, 0.1>}, \frac{u_2}{<0.8, 0.2, 0.1>}, \frac{u_3}{<0.7, 0.1, 0.1>} \right),$$
$$\left((e_1, r, 0), \frac{u_1}{<0.2, 0.1, 0.5>}, \frac{u_2}{<0.4, 0.2, 0.7>}, \frac{u_3}{<0.3, 0.1, 0.5>} \right),$$
$$\left((e_3, p, 0), \frac{u_1}{<0.9, 0.4, 0.1>}, \frac{u_2}{<0.1, 0.2, 0.7>}, \frac{u_3}{<0.3, 0.3, 0.5>} \right),$$
$$\left((e_3, q, 0), \frac{u_1}{<0.5, 0.4, 0.7>}, \frac{u_2}{<0.2, 0.1, 0.5>}, \frac{u_3}{<0.5, 0.2, 0.1>} \right),$$
$$\left((e_3, r, 0), \frac{u_1}{<0.9, 0.3, 0.4>}, \frac{u_2}{<0.6, 0.1, 0.4>}, \frac{u_3}{<0.2, 0.1, 0.5>} \right) \Big\}$$

Now, we get Λ_s by applying the basic fuzzy union as follows:

$$\begin{aligned}
&\Lambda_s(u_1, p) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.1}, \frac{a_3}{1}, \frac{a_4}{0.7} \right\}, \Lambda_s(u_2, p) = \left\{ \frac{a_1}{0.8}, \frac{a_2}{1}, \frac{a_3}{0.8}, \frac{a_4}{0.9} \right\}, \\
&\Lambda_s(u_3, p) = \left\{ \frac{a_1}{0.3}, \frac{a_2}{0.1}, \frac{a_3}{0.1}, \frac{a_4}{0} \right\}, \Lambda_s(u_1, q) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.3}, \frac{a_3}{0.7}, \frac{a_4}{0.6} \right\}, \\
&\Lambda_s(u_2, q) = \left\{ \frac{a_1}{0.8}, \frac{a_2}{1}, \frac{a_3}{0.7}, \frac{a_4}{1} \right\}, \Lambda_s(u_3, q) = \left\{ \frac{a_1}{0.1}, \frac{a_2}{0}, \frac{a_3}{0.2}, \frac{a_4}{0.2} \right\}, \\
&\Lambda_s(u_1, r) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.2}, \frac{a_3}{0.2}, \frac{a_4}{0.5} \right\}, \Lambda_s(u_2, r) = \left\{ \frac{a_1}{0.6}, \frac{a_2}{0.7}, \frac{a_3}{1}, \frac{a_4}{0.7} \right\}, \\
&\Lambda_s(u_3, r) = \left\{ \frac{a_1}{0}, \frac{a_2}{0.1}, \frac{a_3}{0}, \frac{a_4}{0.2} \right\}.
\end{aligned}$$

By using the neutrosophic soft expert union, we have the following :

$$\begin{aligned}
(H, S) = \Big\{ &\left((e_1, p, 1), \frac{u_1}{<0.4, 0.1, 0.2>}, \frac{u_2}{<0.5, 0.1, 0.1>}, \frac{u_3}{<0.3, 0.1, 0.5>} \right), \\
&\left((e_1, q, 1), \frac{u_1}{<0.9, 0.2, 0.1>}, \frac{u_2}{<0.6, 0.2, 0.2>}, \frac{u_3}{<0.7, 0.1, 0.2>} \right), \\
&\left((e_1, r, 1), \frac{u_1}{<0.3, 0.1, 0.5>}, \frac{u_2}{<0.7, 0.4, 0.1>}, \frac{u_3}{<0.7, 0.3, 0.4>} \right), \\
&\left((e_2, p, 1), \frac{u_1}{<0.3, 0.2, 0.4>}, \frac{u_2}{<0.4, 0.2, 0.3>}, \frac{u_3}{<0.7, 0.2, 0.3>} \right), \\
&\left((e_2, q, 1), \frac{u_1}{<0.3, 0.2, 0.4>}, \frac{u_2}{<0.5, 0.2, 0.7>}, \frac{u_3}{<0.4, 0.2, 0.3>} \right), \\
&\left((e_2, r, 1), \frac{u_1}{<0.4, 0.2, 0.3>}, \frac{u_2}{<0.5, 0.1, 0.3>}, \frac{u_3}{<0.7, 0.3, 0.4>} \right), \\
&\left((e_3, p, 1), \frac{u_1}{<0.2, 0.1, 0.5>}, \frac{u_2}{<0.6, 0.1, 0.1>}, \frac{u_3}{<0.8, 0.3, 0.4>} \right),
\end{aligned}$$

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$$\left((e_3,q,1)\,,\frac{u_1}{<0.5,0.2,0.3>},\frac{u_2}{<0.4,0.5,0.1>},\frac{u_3}{<0.3,0.1,0.6>}\right),$$
$$\left((e_3,r,1)\,,\frac{u_1}{<0.7,0.1,0.5>},\frac{u_2}{<0.8,0.2,0.3>},\frac{u_3}{<0.5,0.2,0.7>}\right),$$
$$\left((e_1,p,0)\,,\frac{u_1}{<0.7,0.1,0.1>},\frac{u_2}{<0.5,0.1,0.3>},\frac{u_3}{<0.7,0.1,0.2>}\right),$$
$$\left((e_1,q,0)\,,\frac{u_1}{<0.7,0.1,0.1>},\frac{u_2}{<0.8,0.2,0.1>},\frac{u_3}{<0.7,0.1,0.1>}\right),$$
$$\left((e_1,r,0)\,,\frac{u_1}{<0.2,0.1,0.5>},\frac{u_2}{<0.4,0.2,0.6>},\frac{u_3}{<0.3,0.1,0.5>}\right),$$
$$\left((e_2,p,0)\,,\frac{u_1}{<0.1,0.2,0.8>},\frac{u_2}{<0.1,0.4,0.6>},\frac{u_3}{<0.5,0.3,0.3>}\right),$$
$$\left((e_2,q,0)\,,\frac{u_1}{<0.4,0.2,0.3>},\frac{u_2}{<0.3,0.4,0.6>},\frac{u_3}{<0.5,0.3,0.6>}\right),$$
$$\left((e_2,r,0)\,,\frac{u_1}{<0.1,0.3,0.2>},\frac{u_2}{<0.5,0.2,0.1>},\frac{u_3}{<0.7,0.1,0.1>}\right),$$
$$\left((e_3,p,0)\,,\frac{u_1}{<0.9,0.4,0.1>},\frac{u_2}{<0.1,0.2,0.7>},\frac{u_3}{<0.3,0.3,0.5>}\right),$$
$$\left((e_3,q,0)\,,\frac{u_1}{<0.5,0.4,0.7>},\frac{u_2}{<0.2,0.1,0.5>},\frac{u_3}{<0.5,0.2,0.1>}\right),$$
$$\left((e_3,r,0)\,,\frac{u_1}{<0.9,0.3,0.4>},\frac{u_2}{<0.6,0.1,0.4>},\frac{u_3}{<0.2,0.1,0.5>}\right)\Bigg\}$$

Now, by using Definition 3.15 and Definition 3.2 we get the ENSES as follows:

$$(H,S)_{\Lambda_s}=\Bigg\{\left((e_1,p,1)\,,\frac{u_1}{<0.73,0.1,0.09>},\frac{u_2}{<0.93,0.1,0.01>},\frac{u_3}{<0.38,0.1,0.43>}\right),$$
$$\left((e_1,q,1)\,,\frac{u_1}{<0.95,0.2,0.05>},\frac{u_2}{<0.95,0.2,0.02>},\frac{u_3}{<0.73,0.1,0.17>}\right),$$
$$\left((e_1,r,1)\,,\frac{u_1}{<0.53,0.1,0.34>},\frac{u_2}{<0.92,0.4,0.02>},\frac{u_3}{<0.72,0.3,0.37>}\right),$$
$$\left((e_2,p,1)\,,\frac{u_1}{<0.69,0.2,0.18>},\frac{u_2}{<0.92,0.2,0.03>},\frac{u_3}{<0.73,0.2,0.26>}\right),$$
$$\left((e_2,q,1)\,,\frac{u_1}{<0.65,0.2,0.2>},\frac{u_2}{<0.93,0.2,0.08>},\frac{u_3}{<0.47,0.2,0.26>}\right),$$
$$\left((e_2,r,1)\,,\frac{u_1}{<0.6,0.2,0.21>},\frac{u_2}{<0.87,0.1,0.07>},\frac{u_3}{<0.72,0.3,0.37>}\right),$$
$$\left((e_3,p,1)\,,\frac{u_1}{<0.64,0.1,0.23>},\frac{u_2}{<0.95,0.1,0.01>},\frac{u_3}{<0.82,0.3,0.35>}\right),$$
$$\left((e_3,q,1)\,,\frac{u_1}{<0.75,0.2,0.15>},\frac{u_2}{<0.92,0.5,0.01>},\frac{u_3}{<0.38,0.1,0.52>}\right),$$

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$$\left((e_3,r,1), \frac{u_1}{<0.8,0.1,0.34>}, \frac{u_2}{<0.95,0.2,0.07>}, \frac{u_3}{<0.53,0.2,0.64>}\right),$$
$$\left((e_1,p,0), \frac{u_1}{<0.87,0.1,0.05>}, \frac{u_2}{<0.93,0.1,0.03>}, \frac{u_3}{<0.73,0.1,0.17>}\right),$$
$$\left((e_1,q,0), \frac{u_1}{<0.85,0.1,0.05>}, \frac{u_2}{<0.97,0.2,0.01>}, \frac{u_3}{<0.73,0.1,0.08>}\right),$$
$$\left((e_1,r,0), \frac{u_1}{<0.46,0.1,0.34>}, \frac{u_2}{<0.85,0.2,0.15>}, \frac{u_3}{<0.35,0.1,0.46>}\right),$$
$$\left((e_2,p,0), \frac{u_1}{<0.6,0.2,0.36>}, \frac{u_2}{<0.88,0.4,0.07>}, \frac{u_3}{<0.5,0.3,0.26>}\right),$$
$$\left((e_2,q,0), \frac{u_1}{<0.7,0.2,0.15>}, \frac{u_2}{<0.91,0.4,0.07>}, \frac{u_3}{<0.56,0.3,0.52>}\right),$$
$$\left((e_2,r,0), \frac{u_1}{<0.39,0.3,0.14>}, \frac{u_2}{<0.87,0.2,0.02>}, \frac{u_3}{<0.72,0.1,0.09>}\right),$$
$$\left((e_3,p,0), \frac{u_1}{<0.96,0.4,0.05>}, \frac{u_2}{<0.88,0.2,0.08>}, \frac{u_3}{<0.38,0.3,0.43>}\right),$$
$$\left((e_3,q,0), \frac{u_1}{<0.75,0.4,0.35>}, \frac{u_2}{<0.9,0.1,0.06>}, \frac{u_3}{<0.56,0.2,0.08>}\right),$$
$$\left.\left((e_3,r,0), \frac{u_1}{<0.93,0.3,0.27>}, \frac{u_2}{<0.9,0.1,0.1>}, \frac{u_3}{<0.26,0.1,0.46>}\right)\right\}$$

Now we will make a comparison between the Sahin et al. algorithm[14] and the new algorithm 4.1.
Tables 1 and 2 show agree and disagree neutrosophic soft expert set $(H,S)^1$ and $(H,S)^0$ respectively. Tables 3 and 4 show $c_{j=\sum_i u_{ij}}$ for agree neutrosophic soft expert set $(H,\ S)^1$ and $k_{j=\sum_i u_{ij}}$ for disagree neutrosophic soft expert set $(H,S)^0$. From Tables 3 and 4 we find the values of $s_j = c_j - k_j$ as in Table 5. The maximum score is 3.3 for u_2. By Sahin et al. algorithm[14] for NSES the optimal decision is u_2.
Now we use the new algorithm 4.1, tables 6 and 7 show agree and disagree effective neutrosophic soft expert set $(H,S)^1_{\Lambda_s}$ and $(H,S)^0_{\Lambda_s}$ respectively. Tables 8 and 9 show $c_{j=\sum_i u_{ij}}$ for agree effective neutrosophic soft expert set $(H,S)^1_{\Lambda_s}$ and $k_{j=\sum_i u_{ij}}$ for disagree effective neutrosophic soft expert set $(H,S)^0_{\Lambda_s}$. From Tables 8 and 9 we find the values of $s_j = c_j - k_j$ as in Table 10. The maximum score is 0.85 for u_3.
The optimal decision for ENSES is u_3. Hence the ENSES changes our decision from u_2 to u_3.

5 Conclusions.

In this paper, we have introduced the concept of an effective neutrosophic soft expert set with some of its properties and examples. The advantage of this new concept lies in its capacity to account for the influence of external effectiveness on neutrosophic soft sets and to know expert viewpoints into one model. Additionally, we present an elucidation of elementary operations concerning the effective neutrosophic soft expert set, encompassing aspects like subsets, equal, complements, unions, intersections, as well as AND and OR operations. These operations are complemented by clarifications of their characteristics and practical examples.

Moreover, it's worth mentioning that, in the future, researchers and authors might employ this innovative concept in the domain of medical diagnosis to observe how external factors impact it.
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Table 1: Agree neutrosophic soft expert set $(H,\ S)^1$.

U	u_1	u_2	u_3
(e_1, p)	$\langle 0.4, 0.1, 0.2\rangle$	$\langle 0.5, 0.1, 0.1\rangle$	$\langle 0.3, 0.1, 0.5\rangle$
(e_1, q)	$\langle 0.9, 0.2, 0.1\rangle$	$\langle 0.6, 0.2, 0.2\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
(e_1, r)	$\langle 0.3, 0.1, 0.5\rangle$	$\langle 0.7, 0.4, 0.1\rangle$	$\langle 0.7, 0.3, 0.4\rangle$
(e_2, p)	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.7, 0.2, 0.3\rangle$
(e_2, q)	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.5, 0.2, 0.7\rangle$	$\langle 0.4, 0.2, 0.3\rangle$
(e_2, r)	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.7, 0.3, 0.4\rangle$
(e_3, p)	$\langle 0.2, 0.1, 0.5\rangle$	$\langle 0.6, 0.1, 0.1\rangle$	$\langle 0.8, 0.3, 0.4\rangle$
(e_3, q)	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.4, 0.5, 0.1\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
(e_3, r)	$\langle 0.7, 0.1, 0.5\rangle$	$\langle 0.8, 0.2, 0.3\rangle$	$\langle 0.5, 0.2, 0.7\rangle$

Table 2: Disgree neutrosophic soft expert set $(H,\ S)^0$.

U	u_1	u_2	u_3
(e_1, p)	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
(e_1, q)	$\langle 0.7, 0.1, 0.1\rangle$	$\langle 0.8, 0.2, 0.1\rangle$	$\langle 0.7, 0.1, 0.1\rangle$
(e_1, r)	$\langle 0.2, 0.1, 0.5\rangle$	$\langle 0.4, 0.2, 0.6\rangle$	$\langle 0.3, 0.1, 0.5\rangle$
(e_2, p)	$\langle 0.1, 0.2, 0.8\rangle$	$\langle 0.1, 0.4, 0.6\rangle$	$\langle 0.5, 0.3, 0.3\rangle$
(e_2, q)	$\langle 0.4, 0.2, 0.3\rangle$	$\langle 0.3, 0.4, 0.6\rangle$	$\langle 0.5, 0.3, 0.6\rangle$
(e_2, r)	$\langle 0.1, 0.3, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$	$\langle 0.7, 0.1, 0.1\rangle$
(e_3, p)	$\langle 0.9, 0.4, 0.1\rangle$	$\langle 0.1, 0.2, 0.7\rangle$	$\langle 0.3, 0.3, 0.5\rangle$
(e_3, q)	$\langle 0.5, 0.4, 0.7\rangle$	$\langle 0.2, 0.1, 0.5\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
(e_3, r)	$\langle 0.9, 0.3, 0.4\rangle$	$\langle 0.6, 0.1, 0.4\rangle$	$\langle 0.2, 0.1, 0.5\rangle$

Table 3: $c_{j=\sum_i u_{ij}}$ for agree neutrosophic soft expert set $(H,\ S)^1$

U	u_1	u_2	u_3
(e_1, p)	0.3	0.5	-0.1
(e_1, q)	1	0.6	0.6
(e_1, r)	-0.1	1	-0.4
(e_2, p)	0.1	0.3	0.6
(e_2, q)	0.1	0	0.3
(e_2, r)	0.3	0.3	0.6
(e_3, p)	-0.2	0.6	0.7
(e_3, q)	0.4	0.8	-0.2
(e_3, r)	0.3	0.7	0
$c_{j=\sum_i u_{ij}}$	$c_1 = 2.2$	$c_2 = 4.8$	$c_3 = 2.1$

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Table 4: $k_{j=\sum_i u_{ij}}$ for disagree neutrosophic soft expert set $(H,\ S)^0$

U	u_1	u_2	u_3
(e_1, p)	0.7	0.9	0.7
(e_1, q)	-0.2	0	-0.1
(e_1, r)	-0.5	-0.1	0.5
(e_2, p)	-0.5	-0.1	0.5
(e_2, q)	0.3	0.1	0.2
(e_2, r)	0.2	0.6	0.7
(e_3, p)	1.2	-0.4	0.1
(e_3, q)	0.2	-0.2	0.6
(e_3, r)	0.8	0.3	-0.2
$k_{j=\sum_i u_{ij}}$	$k_1 = 3.4$	$k_2 = 1.5$	$k_3 = 3.1$

Table 5: Score $s_j = c_j - k_j$ for neutrosophic soft expert set $(H,\ S)$.

j	c_j	k_j	s_j
1	2.2	3.4	-1.2
2	4.8	1.5	3.3
3	2.1	3.1	-1

Table 6: Agree effective neutrosophic soft expert set $(H, S)^1_{\Lambda_s}$

U	u_1	u_2	u_3
(e_1, p)	$\langle 0.73, 0.1, 0.09\rangle$	$\langle 0.93, 0.1, 0.01\rangle$	$\langle 0.38, 0.1, 0.43\rangle$
(e_1, q)	$\langle 0.95, 0.2, 0.05\rangle$	$\langle 0.95, 0.2, 0.02\rangle$	$\langle 0.73, 0.1, 0.17\rangle$
(e_1, r)	$\langle 0.53, 0.1, 0.34\rangle$	$\langle 0.92, 0.4, 0.02\rangle$	$\langle 0.72, 0.3, 0.37\rangle$
(e_2, p)	$\langle 0.69, 0.2, 0.18\rangle$	$\langle 0.92, 0.2, 0.03\rangle$	$\langle 0.73, 0.2, 0.26\rangle$
(e_2, q)	$\langle 0.65, 0.2, 0.2\rangle$	$\langle 0.93, 0.2, 0.08\rangle$	$\langle 0.47, 0.2, 0.26\rangle$
(e_2, r)	$\langle 0.6, 0.2, 0.21\rangle$	$\langle 0.87, 0.1, 0.07\rangle$	$\langle 0.72, 0.3, 0.37\rangle$
(e_3, p)	$\langle 0.64, 0.1, 0.23\rangle$	$\langle 0.95, 0.1, 0.01\rangle$	$\langle 0.82, 0.3, 0.35\rangle$
(e_3, q)	$\langle 0.75, 0.2, 0.15\rangle$	$\langle 0.92, 0.5, 0.01\rangle$	$\langle 0.38, 0.1, 0.52\rangle$
(e_3, r)	$\langle 0.8, 0.1, 0.34\rangle$	$\langle 0.95, 0.2, 0.07\rangle$	$\langle 0.53, 0.2, 0.64\rangle$

Table 7: Disagree effective neutrosophic soft expert set $(H, S)^0_{\Lambda_s}$

U	u_1	u_2	u_3
(e_1, p)	$\langle 0.87, 0.1, 0.05\rangle$	$\langle 0.93, 0.1, 0.03\rangle$	$\langle 0.73, 0.1, 0.17\rangle$
(e_1, q)	$\langle 0.85, 0.1, 0.05\rangle$	$\langle 0.97, 0.2, 0.01\rangle$	$\langle 0.73, 0.1, 0.08\rangle$
(e_1, r)	$\langle 0.46, 0.1, 0.34\rangle$	$\langle 0.85, 0.2, 0.15\rangle$	$\langle 0.35, 0.1, 0.46\rangle$
(e_2, p)	$\langle 0.6, 0.2, 0.36\rangle$	$\langle 0.88, 0.4, 0.07\rangle$	$\langle 0.56, 0.3, 0.26\rangle$
(e_2, q)	$\langle 0.7, 0.2, 0.15\rangle$	$\langle 0.91, 0.4, 0.07\rangle$	$\langle 0.56, 0.3, 0.52\rangle$
(e_2, r)	$\langle 0.39, 0.3, 0.14\rangle$	$\langle 0.87, 0.2, 0.02\rangle$	$\langle 0.72, 0.1, 0.09\rangle$
(e_3, p)	$\langle 0.96, 0.4, 0.05\rangle$	$\langle 0.88, 0.2, 0.08\rangle$	$\langle 0.38, 0.3, 0.43\rangle$
(e_3, q)	$\langle 0.75, 0.4, 0.35\rangle$	$\langle 0.9, 0.1, 0.06\rangle$	$\langle 0.56, 0.2, 0.08\rangle$
(e_3, r)	$\langle 0.93, 0.3, 0.27\rangle$	$\langle 0.9, 0.1, 0.1\rangle$	$\langle 0.26, 0.1, 0.46\rangle$

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Table 8: $c_{j=\sum_i u_{ij}}$ for agree effective neutrosophic soft expert set $(H, S)^1$

U	u_1	u_2	u_3
(e_1, p)	0.74	1.02	0.05
(e_1, q)	1.1	1.13	0.66
(e_1, r)	0.29	1.03	0.65
(e_2, p)	0.71	1.09	0.67
(e_2, q)	0.65	1.05	0.41
(e_2, r)	0.59	0.9	1.39
(e_3, p)	0.51	1.04	0.77
(e_3, q)	0.8	1.41	-0.04
(e_3, r)	0.56	1.08	0.09
$c_{j=\sum_i u_{ij}}$	$c_1 = 5.95$	$c_2 = 9.75$	$c_3 = 4.65$

Table 9: $k_{j=\sum_i u_{ij}}$ for disagree effective neutrosophic soft expert set $(H, S)^0$

U	u_1	u_2	u_3
(e_1, p)	0.92	1	0.66
(e_1, q)	0.9	1.16	0.65
(e_1, r)	0.22	0.9	-0.01
(e_2, p)	0.44	1.21	0.6
(e_2, q)	0.75	1.24	0.34
(e_2, r)	0.55	0.05	0.73
(e_3, p)	1.31	1	0.25
(e_3, q)	0.8	0.94	0.68
(e_3, r)	0.96	0.9	-0.1
$k_{j=\sum_i u_{ij}}$	$k_1 = 6.85$	$k_2 = 9.4$	$k_3 = 3.8$

Table 10: Score $s_j = c_j - k_j$ for effective neutrosophic soft expert set (H, S).

j	c_j	k_j	s_j
1	5.95	6.85	-0.9
2	9.75	9.4	0.35
3	4.65	3.8	0.85

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