Some Properties of Fuzzy Semi-Open Sets in Fuzzy Bi-Spaces

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Abstract

The idea of fuzzy "semi-open sets" within the framework of fuzzy fields was proposed in the theory of fuzzy topology. This investigation delves deeper into the concept, specifically examining fuzzy "semi-open sets" concerning ω̃1 (ω̃2) with respect to ω̃2 (ω̃1). Additionally, we explore pairs of fuzzy "semi-open sets" in the context of a fuzzy bi-space and analyse their implications on results applicable in bi-topological spaces.

Keywords: Fuzzy poin; fuzzy set; fuzzy semi-open set; fuzzy bi-space

1. Introduction

The concept of fuzzy sets plays a big role in many studies related to many scientific fields, such as pure mathematics, engineering, and even computer science. This pattern of new sets was generalized by several related logical concepts, specifically the neutrosophic sets introduced by Smarandache. These combinations have been used similarly to their fuzzy counterparts in artificial intelligence, cryptography, and even in Intelligent Systems.

In 1940, A.D. Alexandroff introduced the idea of a fuzzy σ-field, an extension of the fuzzy field concept, where combinations of 'fuzzy open sets' are considered as fuzzy open, rather than limiting it to only countable combinations. In 2001, Lahiri and Das further advanced this concept by generalizing fuzzy bitopological spaces into fuzzy bispaces. The idea of "fuzzy semi-open sets" was first proposed by N. Levine in 1963 within the context of a fuzzy topological space. Later, S. Bose extends this notion to a fuzzy bitopological space (X̃, (P̃, Q̃)), utilizing P̃-fuzzy 'semi-open sets' in relation to Q̃. Subsequently, Lahiri and Das explore this idea within spaces, critically evaluating its generalization aspects. In this investigation, the exploration revolves around the notion of $\omega_1(\omega_2)$ fuzzy semi-open sets concerning $\omega_2(\omega_1)$ and various other properties within the context of a fuzzy field. Illustrative cases are presented to showcase how multiple findings, as documented in [3], are influenced in a fuzzy field. Additionally, a criterion has been established for a fuzzy field to qualify as a fuzzy space, involving $\omega_1(\omega_2)$ fuzzy semi-open sets with respect to $\omega_2(\omega_1)$.

2. Basic concepts of fuzzy set

Statement 1.1 :[1] In a nonempty set R, $\tilde{G}$ is described by a membership mapping $\mu_{\tilde{G}}: R \rightarrow [0,1]$. The representation of this fuzzy set is outlined below:

$$\tilde{G} = \{((R,, \mu\tilde{G}(r)) :: r \in R, \mu\tilde{G}(r) \leq 1\}\}$$

The depiction of the set containing all fuzzy sets within the set R is articulated as I^r.

Statement 1.2 : [1] The support of $\tilde{G}$ includes all r in R where the membership value $\mu(\tilde{G})(r)$ is greater than 0, denoted as $S(\tilde{G})$.

Statement 1.3 :[2] The set $\tilde{P}rx$ in 'X' is defined by its unique membership function.

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$\tilde{P}^r_x = \begin{cases} r, if\ x = y \\ 0, if\ x \neq y \end{cases}$, where $0 < r \leq 1$, y is the support of $\tilde{P}^r_x$ (x) .

Statement 1.4 :[2] If the support S((G)̃) of a fuzzy set (G)̃ comprises an infinite set of elements, then the fuzzy set is deemed infinite.

Remark 1.5 : 1) R is termed when all its elements have a membership value of 1, denoted as μ r (R) = 1, for every r in R. In this case, R is referred to as a crisp set.

2) μ∅̃ (r) equals 0 for every empty set ∅ in R. This function represents an empty set and is symbolized as ∅̃ .

Statement 1.6: [1] Given a fuzzy point $\tilde{P}^r_x$ and a nonempty set (X)̃, $\tilde{P}^r_x$ is within (C)̃, or (C)̃ contains $\tilde{P}^r_x$ if $\mu\tilde{P}^r_x$ ≤ μ(C)̃(x) for all x ∈ X, denoted by X ∈ S((C)̃).

Statement 1.7 :[1] Consider (P)̃ and (Q)̃ within a universal set S.

1) $\tilde{P} \leq \tilde{Q}$ iff $\mu\tilde{P}$ (s) $\leq \mu\tilde{Q}$ (S) , ∀ s ∈ S.
2) $\tilde{P} = \tilde{Q}$ iff $\mu\tilde{P}$ (s) = $\mu\tilde{Q}$ (S) , ∀ s ∈ S.
3) $\tilde{P}^C$ is the fuzzy set negation $\tilde{P}$ with membership assignment $\mu\tilde{P}^C$ = 1 - $\mu\tilde{P}$ (s) .
4) $\tilde{C}$ = =$\tilde{P}$ ∪ $\tilde{Q}$ iff $\mu\tilde{C}$ (s) = Max {{ $\mu\tilde{P}$ (s) , ,$\mu\tilde{Q}$ (s) }, ,∀ s ∈ S.
5) $\tilde{D}$ == $\tilde{P}$ ∩ $\tilde{Q}$ iff $\mu\tilde{D}$ (s)= =Min {{ $\mu\tilde{P}$ (s) , $\mu\tilde{Q}$ (s) }, ∀ s ∈ S.
6) More generally , for a collection of fuzzy sets { $\tilde{P}p$: $p \in \wedge$ *where ∧ is the any index set* } *the union* $\tilde{C}$ = ∪ $\tilde{P}p$.

The intersection (D)̃ is obtained as the junction of (P)̃g and is described separately.

$\mu\tilde{C}$ ((s) = sup {{ $\mu\tilde{P}y$((s) ,, s ∈ S }} , $\mu\tilde{D}$ ((s)= inf { $\mu\tilde{P}p$(s): s ∈ S}.

3. Preliminaries

According to Statement 2.1 [1], A ‘fuzzy set' is labeled as a fuzzy ‘Alexandroff ‘space (denoted by (R)̃), or more simply referred to as a space, when there occurs a system of fuzzy sets R)̃ within it that satisfies the specified conditions:

(1) The junction of a enumerable n of fuzzy array from F̃ results in an array within F.

(2) combination of a limited n of fuzzy array from F̃ results in an array within F.

(3) fuzzy null array ∅̃ is considered an array in F.

(4) entire fuzzy array X̃ is considered an array in F.

Arrays in F are denoted as fuzzy ‘closed sets’, and its complement sets are called FOS. Instead of closed arrays in the space description, ‘fuzzy open sets’ can be used, provided they adhere to conditions such as fuzzy enumerable summation, fuzzy limited intersection property, and the stipulation that both R̃ and the empty array essential for it to be open. The assembly of all such fuzzy open neighborhoods is represented as φ̃, and the fuzzy field is denoted as ((R̃, ,T̃). It's crucial to note that typically, φ̃ is not a fuzzy mathematical structure, as evident in cases where R̃ == S̃, the fuzzy array of real quantities, and φ̃ includes all fuzzy σ arrays in S̃.

Statement 2.2 [1] associates every fuzzy array Õ in (R̃, ,S̃) with its fuzzy closure Õ, expressed as the combination of all closed fuzzy closed sets that include Õ. Often, the ‘fuzzy closure’ of a set M̃ is represented by φ̃. k(Õ) or ‘simply’ k.

1) $\overline{O \cup N} = \bar{O} \cup \bar{N}$; 2) $O \leq \bar{O}$3) $\bar{O} = \bar{\bar{O}}$4) $\bar{\emptyset} = \emptyset$

Statement 2.3 [7] defines the core of a fuzzy array Ã in((B̃,C̃) as the combination of all fuzzy neighborhood that are included within Ã. This concept is represented by φ̃ intÃ or simply intÃ in clear contexts.

Statement 2.4 [6] states that a non-empty fuzzy set R̃, equipped with 2 randomal fuzzy topological A and B , is referred to as a fuzzy bitopological structure and is represented by (R̃, Ã, B̃).

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Statement 2.5 [8] defines a fuzzy bispace as follows: If $\tilde{R}$ is a nonempty fuzzy set and there are two collections of fuzzy sets, namely ω_1 and ω_2, like (($\tilde{R}$, ω_1) and (($\tilde{R}$, ω_2) form 2 fields, $\tilde{R}$ is termed a fuzzy bitopological field and is represented by (($\tilde{R}$, ω_1, ω_2).

Statement 3.1 defined as follows: ((R, ω_1, ω_2), ($\tilde{G}$) is considered ω_1 fuzzy semi-open regarding ω_2 if there is a ω_1 'open set' $\tilde{O}$ in a manner that $\tilde{O}$ is contained in $\tilde{G}$ and $\tilde{G}$ is contained in the closure of $\tilde{O}$ with respect to ω_2. Likewise, ($\tilde{G} \leq \tilde{R}$) is ω_2 fuzzy semi-open regarding ω_1 if there is a ω_2 'open set' $\tilde{O}$ such that $\tilde{O}$ is contained in $\tilde{G}$ and $\tilde{G}$ is contained in the closure of $\tilde{O}$ regarding ω_1.

We use the term 'pairwise fuzzy semi-open' to characterize a fuzzy set $\tilde{G}$ that exhibits both ω_1 semi-openness regarding ω_2 and ω_2 semi-openness regarding ω_1. It's essential to note that an ω_1 (ω_2) 'fuzzy open set' is ω_1 (ω_2) semi-open in the context of ω_2 (ω_1). In our discussions, (($\tilde{X}$, ω_1, ω_2), $\tilde{R}$ signifies the fuzzy set of decimal-numbers, $\tilde{Q}$ represents fuzzy array of ratios, and $\tilde{N}$ denotes the fuzzy array of n numbers. Unless explicitly stated otherwise.

In a (($\tilde{R}$, ω_1, ω_2), if $\tilde{G}$ is a fuzzy set of $\tilde{R}$ and is ω_1 semi-open with respect to ω_2, then the closure of $\tilde{G}$ concerning ω_2 equals the closure of ω_1 core regarding ω_2.

Proof: Suppose $\tilde{G}$ is ω_1 semi-open with respect to ω_2 in an ω_1 fuzzy S-open $\tilde{O}$ such that $\tilde{O} \leq \tilde{G} \leq \omega_2 (\bar{O})$. $\tilde{O} \leq \omega_1$ int $\tilde{G}$. Therefore, $\tilde{G} \leq \omega_2$ k$\tilde{O} \leq \omega_2$ k(ω_1 int $\tilde{G}$), and hence ω_2 k$\tilde{G} \leq \omega_2$ k(ω_2 k(ω_1 int $\tilde{G}$)) = ω_2 k((ω_1 int $\tilde{G}$)). Also, ω_2 k((ω_1 int $\tilde{G}$) $\leq \omega_2$ k$\tilde{G}$. Hence, ω_2 kG = ω_2 k((ω_1 int $\tilde{G}$)).

Corollary 3.3: While $\tilde{G}$ is ω_1 semi-open with respect to ω_2 and $\tilde{G}$ is not equal to $\tilde{\emptyset}$, then ω_1 int $\tilde{G}$ is not equal to $\tilde{\emptyset}$.

Corollary 3.4: If $\tilde{G}$ is ω_1 semi-open with respect to ω_2 and $\tilde{G} \leq \tilde{H}$, then $\tilde{G} \leq \omega_2$ k((ω_1 int $\tilde{H}$)).

The reverse aspect of Axiom 3.2 has been proven to be accurate in reference [3]. Nevertheless, in the context of a fuzzy bispace, this assertion might not remain valid, as shown below:

Example 3.5: Consider $\tilde{R}$ = [0,2] and the group {G} of all enumerable fuzzy sets of non-repeating n in [0,1]. Let ω_1 include ($\tilde{G_i}$) ∪ {√2}, $\tilde{R}$, and $\tilde{\emptyset}$, and let ω_2 consist of fuzzy sets ($\tilde{G_i}$) along with $\tilde{R}$ and $\tilde{\emptyset}$. This forms a fuzzy bispace ($\tilde{R}$, ω_1, ω_2). Now, consider a fuzzy set $\tilde{G}$ = [0, 1] ∪ {√}. Here, ω_2 k $\tilde{G}$ = $\tilde{R}$, and ω_1 int $\tilde{A}$ comprises non-repeating n. in [0,1] along with √. Thus, ω_2 k(ω_1 int $\tilde{A}$) = $\tilde{R}$. Hence, ω_2 k $\tilde{G}$ equals ω_2 k(ω_1 int $\tilde{G}$)). However, this holds for any ω_1

The $\tilde{G}$, defined as ($\tilde{R}$, $\tilde{\emptyset}$), does not satisfy ω_2 k $\tilde{G}$ = ($\tilde{G}$) ∪ ($\tilde{Q_1}$) ∪ [1, 2], while ($\tilde{Q_1}$) represents the group of repeating n in [0, 1]. It is evident that ω_2 k $\tilde{G}$ does not encompass ($\tilde{A}$). Consequently, there is no ω_1 $\tilde{G}$ like $\tilde{G} \leq \tilde{A} \leq \omega_1$ k $\tilde{G}$. Hence, $\tilde{A}$ is not ω_1 semi-open with respect to ω_2.

In the fuzzy bitopology ($\tilde{R}$, ω_1, ω_2), if ω_2 k($\tilde{A}$) = ω_2 k(ω_1 int($\tilde{A}$)), then $\tilde{A}$ is considered ω_1 with respect to ω_2 for every fuzzy set $\tilde{A}$ in $\tilde{X}$, given that condition ($\tilde{C_1}$) is satisfied: The random combination of ω_1 fuzzy S-open are ω_1 with respect to ω_2.

Validation: Consider $\tilde{O}$ = ω_1 int $\tilde{A}$. Due to condition ($\tilde{C_1}$), $\tilde{O}$ is ω_1 semi-open with respect to ω_2. Thus, there occurs a ω_2 fuzzy S-open $\tilde{G}$ such that $\tilde{G} \leq \tilde{O} \leq \omega_2$ k $\tilde{G}$. Since ω_2 k $\tilde{O}$ = ω_2 k(ω_1 int $\tilde{A}$) = ω_2 k $\tilde{A}$ and $\tilde{O} \leq \omega_2$ k $\tilde{G}$, we have ω_2 k $\tilde{O} \leq \omega_2$ k(ω_2 k $\tilde{G}$) = ω_2 k $\tilde{G}$, and hence ω_2 k $\tilde{A} \leq \omega_2$ k $\tilde{G}$. Hence, $\tilde{G} \leq \tilde{O} \leq \tilde{A} \leq \omega_2$ k $\tilde{A} \leq \omega_2$ k $\tilde{G}$, and so $\tilde{A}$ is ω_1 semi-open with respect to ω_2.

Remark 3.7: The example provided in Ex 3.8 illustrates that there occurs a fts that does not qualify as a fts even though state ($\tilde{C1}$) is met.

Consider the fuzzy bispace ($\tilde{R}$, ω_1, ω_2) with $\tilde{R}$ = [0,2], where ω_1 consists of ($\tilde{Gi}$), $\tilde{X}$, and $\tilde{\emptyset}$, and ω_2 consists of ($\tilde{Fi}$) $\tilde{X}$, where {($\tilde{Gi}$)} and {($\tilde{Fi}$)} represent enumerable fuzzy sets of non-repeating n. in [0,1] and [1,2]. This forms a fuzzy double field but not a fts space. Contemplate all ω_1 {($\tilde{Gi}$)}. ($\tilde{Gi}$) represents the non-repeating n in [0,1]. However, for any ω_1 ($\tilde{Gi}$), ω_2 k ($\tilde{Gi}$) = [0, 1] ∪ ($\tilde{Q_2}$), where ($\tilde{Q_2}$) represents the fuzzy set of repeating n in [1,2]. Consequently, ($\tilde{Gi}$) ≤ U($\tilde{Gi}$) ≤ ω_2 k ($\tilde{Gi}$). This implies that U$\tilde{G}$ is ω_1 semi-open with respect to ω_2 even though it is not ω_1 fuzzy open.

Axiom 3.9 asserts that the enumerable combination of ω_1 'semi-open' sets in relation to ω_2 is ω_1 'semi-open' in relation to ω_2. The Validation involves considering an enumerable set of ω_1 'semi-open' sets {$\tilde{A}_n$: n ≤ N} in relation to ω_2. For each n < N, there occurs a corresponding ω_1 fuzzy 'S-open set' $\tilde{O}_n$ such that $\tilde{O}_n$ is contained within $\tilde{A}_n$, and $\tilde{A}_n$ is within ω_2 k($\tilde{O}_n$). This implies that the collection {$\tilde{O}_n$: n ≤ N} forms a ω_1 'fuzzy open set', denoted as $\tilde{O}$. Therefore, the combination of {$\tilde{A}_n$: n ≤ N} is ω_1 in relation to ω_2 because $\tilde{O}$ is not greater than or equal to the combination of {$\tilde{A}_n$: n ≤ N}, and the combination of {$\tilde{A}_n$: n ≤ N} is not greater than or equal to ω_2 k($\tilde{O}$).

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Remark 3.10: According to [3], it was demonstrated that in a fuzzy double topological field, the arbitrary combination of ω_1 semi-open sets with respect to ω_2 results in ω_1 with respect to ω_2. However, this assertion may not hold in a fuzzy double field, as demonstrated in 'Example 3.11' below.

Example 3.11 illustrates the case of R̃ = X̃. In this scenario, $\tilde{\omega}_1$ 'fuzzy open sets' consist of R̃, along with all (Gĩ) sets, where {G̃} represents the group of enumerable fuzzy sets of non-repeating n in X̃. The $\tilde{\omega}_2$ fuzzy S-open sets include X̃, ∅̃, and all (F̃)σ sets in X̃. Notably, $\tilde{\omega}_2$ fuzzy closed sets correspond to the (Gs̃) sets. For any fuzzy set G̃ in X̃, $\tilde{\omega}_2$ k G̃ equals G̃, demonstrating that any (Ã) that is $\tilde{\omega}_1$ semi-open with respect to $\tilde{\omega}_2$ in X̃ must be a $\tilde{\omega}_1$ S-open. Consequently, $\tilde{\omega}_1$ 'fuzzy open sets' are the exclusive $\tilde{\omega}_1$ semi-open sets with respect to $\tilde{\omega}_2$. The combination of all $\tilde{\omega}_1$ 'fuzzy open sets' (Gĩ) ((Gĩ) ≠ X̃) represents the fuzzy set.

The additional condition (C1̃) becomes crucial to ensure the outcome stated in Thm 3.9 for random combinations. Theorem 3.12 establishes that the random combination of $\tilde{\omega}_1$ sets with respect to $\tilde{\omega}_2$ results in $\tilde{\omega}_1$ with respect to $\tilde{\omega}_2$ if and only if condition (C1̃) is met.

Proof: Firstly, suppose the combination of $\tilde{\omega}_1$ sets concerning $\tilde{\omega}_2$ forms a $\tilde{\omega}_1$ set in relation to $\tilde{\omega}_2$. Since every $\tilde{\omega}_1$ S-open is $\tilde{\omega}_1$ with respect to $\tilde{\omega}_2$, the combined set of $\tilde{\omega}_1$ 'fuzzy open sets' also satisfies being $\tilde{\omega}_1$ in relation to $\tilde{\omega}_2$. This fulfills condition (C1̃).

Conversely, if condition (C1̃) holds, let {Ã} represent any arbitrary collection of $\tilde{\omega}_1$ semi-open sets in relation to $\tilde{\omega}_2$, and consider Ã = ⋃ Ã_i. For each i, there occurs an $\tilde{\omega}_1$ 'fuzzy open set' (G̃_i) such that (G̃_i) is not greater than or equal to (Ã_i), and (Ã_i) is not greater than or equal to $\tilde{\omega}_2$ k(G̃_i). Therefore, U(G̃) ≤ U(Ã_i) = Ã ≤ U($\tilde{\omega}_2$ k(G̃_i)) ≤ $\tilde{\omega}_2$ k(U(G̃_i)). Since U(G̃_i), by assumption, is $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$, there is an $\tilde{\omega}_1$ 'fuzzy open set' (G̃) such that (G̃) ≤ U(G̃) ≤ $\tilde{\omega}_2$ k(G̃). Therefore, (G̃) ≤ U(G̃_i) ≤ Ã ≤ U($\tilde{\omega}_2$ k(G̃)) ≤ $\tilde{\omega}_2$ k(U(G̃)) ≤ $\tilde{\omega}_2$ k($\tilde{\omega}_2$ k(G̃)) = $\tilde{\omega}_2$ k(G̃). This proves that Ã is $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$.

Axiom 3.13: Let G̃ be $\tilde{\omega}_1$ s.o. with respect to $\tilde{\omega}_2$ in a fuzzy bispace (R̃, $\tilde{\omega}_1$, $\tilde{\omega}_2$), and let (G̃) is not greater than or equal to (H̃) is not greater than or equal to $\tilde{\omega}_2$ k(G̃). Then (H̃) is $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$.

Validation: Since G̃ is $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$, there occurs an $\tilde{\omega}_1$ 'fuzzy open set' (Q̃) such that (Q̃) is not greater than or equal to (G̃) is not greater than or equal to $\tilde{\omega}_2$ k(Q̃). Therefore, (Q̃) is not greater than or equal to (G̃) is not greater than or equal to (H̃) is not greater than or equal to $\tilde{\omega}_2$ k(G̃) is not greater than or equal to $\tilde{\omega}_2$ k($\tilde{\omega}_2$ k(Q̃)) = $\tilde{\omega}_2$ k(Q̃), and hence (H̃) is $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$.

Axiom 3.14: Let (H̃, $\tilde{\omega}_1$, $\tilde{\omega}_2$) be a fuzzy bispace, and let G̃ ≤ Q̃ ≤ H̃. If G̃ is pairwise semi-open in R̃, it is pairwise semi-open in Q̃.

Proof: Let G̃ be $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$. Then there occurs an $\tilde{\omega}_1$ 'fuzzy open set' Ũ such that Ũ ≤ G̃ ≤ $\tilde{\omega}_2$ k Ũ. Let (UQ̃) = (Q̃) ∩ Ũ, which is $\tilde{\omega}_1$ open in Q̃. So (UQ̃) = Q̃ ∩ Ũ ≤ Q̃ ∩ G̃ ≤ Q̃ ∩ $\tilde{\omega}_2$ k Ũ = $\tilde{\omega}_2$ k UQ in Q̃. Interchanging the role of $\tilde{\omega}_1$ and $\tilde{\omega}_2$, we get the result. We denote the class of all $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$ by $\tilde{\omega}_1$ F.S.O.(H̃) $\tilde{\omega}_2$.

Axiom 3.15: Let H̃ = {(Hr̃)} be a group of sets in R̃, (i) $\tilde{\omega}_1$ ≤ H̃, and (ii) H̃ ≤ H̃ and H̃ ≤ Q̃ ≤ $\tilde{\omega}_2$ k H̃ implies Q̃ ≤ (H̃), then $\tilde{\omega}_1$ F.S.O.((R̃) $\tilde{\omega}_2$ ≤ H̃.

Proof: If G̃ is within the scope of $\tilde{\omega}_1$ F.S.O.((R̃) $\tilde{\omega}_2$, there is a corresponding $\tilde{\omega}_1$ Ũ. This means Ũ is not greater than or equal to (G̃) and (G̃) is not greater than or equal to $\tilde{\omega}_2$ k(Ũ). Consequently, when (Ũ) is not greater than or equal to H̃ and Ũ is not greater than or equal to (G̃) and also not greater than or equal to $\tilde{\omega}_2$ k(Ũ), it implies that (G̃) is not greater than or equal to H̃. Hence, the conclusion follows.

We define the set {$\tilde{\omega}_1$ int((G̃)): G̃ ≤ $\tilde{\omega}_1$ F.S.O.((R̃) $\tilde{\omega}_2$)} as $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.(R)) $\tilde{\omega}_2$. Similar classes can be named by swapping the roles of $\tilde{\omega}_1$ and $\tilde{\omega}_2$. However, it's clear that $\tilde{\omega}_1$ is not greater than or equal to $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.(R)) $\tilde{\omega}_2$. Nonetheless, generally, $\tilde{\omega}_1$ may not match $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.((R̃) $\tilde{\omega}_2$), as illustrated in the subsequent instances.

Evaluate the fuzzy double space (R̃,, $\tilde{\omega}$, $\tilde{\varphi}_1$, $\tilde{\varphi}_2$) described in Ex 3.8. In this case, for any $\tilde{\omega}_1$ 'fuzzy open set' (Ũi), the closure $\tilde{\varphi}_2$ k(Ũi) is [0, 1] ∪ Q2, where Q2 represents the fuzzy set of rational numbers in the interval [1, 2]. Let H̃ be the fuzzy set [0, 1]. For any $\tilde{\omega}_1$ 'fuzzy open set' (Ũi), (Ũi) is not greater than or equal to H̃, and H̃ is not greater than or equal to [0, 1] ∪ Q2 = $\tilde{\varphi}_2$ k(Ũi). This implies that H̃ is $\tilde{\omega}_1$ "s.o.w.r.to" $\tilde{\omega}_2$, i.e., H̃ is within $\tilde{\omega}_1$ F.S.O.(R̃,) $\tilde{\omega}_2$. However, $\tilde{\omega}_1$ int(H̃) represents the fuzzy set of all non-repeating n in the interval [0, 1], which is not $\tilde{\omega}_1$ fo. The equality $\tilde{\omega}_1$ = $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.((R̃) $\tilde{\omega}_2$) applies only when an extra requirement is met.

Axiom 3.17: In the fuzzy space (R̃, $\tilde{\omega}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$), the equality $\tilde{\omega}_1$ = $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.(R̃,) $\tilde{\omega}_2$) is true 'if and only if condition' C2 is met. Condition C2 states that for any fuzzy set (G̃) less than (R̃), which is $\tilde{\omega}_1$ with respect to $\tilde{\omega}_2$, there is a largest $\tilde{\omega}_1$ 'fuzzy open set' Õ such that Õ is 'less than or equal' to (G̃) and (G̃) is 'less than or equal' to $\tilde{\omega}_2$ k(Õ).

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Proof: Suppose $\tilde{\omega}_1 = \tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.($\tilde{R}$,) $\tilde{\omega}_2$), and let $\tilde{G}$ be a fuzzy set on $\tilde{R}$, concerning $\tilde{\omega}_2$. This implies that $\tilde{\omega}_1$ int($\tilde{G}$) is either not greater than or equal to $\tilde{\omega}_1$. As stated in Axiom 3.2, if $\tilde{O}$ is a $\tilde{\omega}_1$ 'fuzzy open set such that' $\tilde{O}$ is within $\tilde{G}$ and $\tilde{G}$ is within $\tilde{\omega}_2$ k($\tilde{O}$), then $\tilde{O}$ is also not greater than or equal to $\tilde{\omega}_1$ int($\tilde{G}$). Thus, $\tilde{\omega}_1$ int($\tilde{G}$) signifies the most extensive $\tilde{\omega}_1$ 'fuzzy open set' contained in $\tilde{G}$, ensuring $\tilde{\omega}_1$ int($\tilde{G}$) is not greater than or equal to $\tilde{G}$ and $\tilde{\omega}_2$ k($\tilde{\omega}_1$ int($\tilde{G}$)). By defining $\tilde{O}$ as $\tilde{\omega}_1$ int($\tilde{G}$), we establish $\tilde{O}$ is not greater than or equal to ($\tilde{G}$) and $\tilde{\omega}_2$ k($\tilde{O}$).

If $\tilde{G}$ is within the scope of $\tilde{\omega}_1$ F.S.O.($\tilde{R}$,) $\tilde{\omega}_2$, there occurs a largest $\tilde{\omega}_1$ 'fuzzy open set' $\tilde{O}$ that fulfills $\tilde{O} \leq (\tilde{G}) \leq \tilde{\omega}_2$ k($\tilde{O}$) (condition 1). Suppose ($\tilde{O}$) is not equal to $\tilde{\omega}_1$ int($\tilde{G}$); in that case, there is a 'fuzzy open set' $\tilde{H}$ such that $\tilde{H}$ is within $\tilde{G}$ and not within $\tilde{O}$. However, the union of $\tilde{O}$ and $\tilde{H}$ forms a $\tilde{\omega}_1$ fuzzy open set, contradicting $\tilde{O}$'s maximality in satisfying condition (1). Therefore, $\tilde{\omega}_1$ int($\tilde{G}$) = 0, establishing it as a $\tilde{\omega}_1$ fuzzy open set. Consequently, $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.($\tilde{R}$,) $\tilde{\omega}_2$) is not greater than or equal to $\tilde{\omega}_1$, leading to $\tilde{\omega}_1$ being equal to $\tilde{\omega}_1$ int($\tilde{\omega}_1$ F.S.O.($\tilde{R}$,) $\tilde{\omega}_2$).

Statement 3.18: In the context of a fuzzy bispace ($\tilde{R}$, $\tilde{\omega}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$), where $\tilde{M}$ and $\tilde{N}$ are 'non-empty fuzzy sets', $\tilde{N}$ and $\tilde{M}$ are considered (i) fuzzily mutually weakly isolated if there occur $\tilde{\omega}_1$ fuzzy open set $\tilde{U}$ and $\tilde{\omega}_2$ 'fuzzy open set' $\tilde{V}$ such that $\tilde{M}$ is a subset of ($\tilde{U}$), $\tilde{N}$ is a subset of ($\tilde{V}$), $\tilde{M}$ does not intersect with ($\tilde{V}$), and ($\tilde{N}$) does not intersect with ($\tilde{U}$), and (ii) fuzzily pairwise strongly apart if there occur $\tilde{\omega}_1$ 'fuzzy open set' $\tilde{U}$ and $\tilde{\omega}_2$ 'fuzzy open set' $\tilde{V}$ such that ($\tilde{M}$) is a subset of ($\tilde{U}$), $\tilde{N}$ is a subset of ($\tilde{V}$), and the intersection of ($\tilde{U}$) and ($\tilde{V}$) is empty.

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