



Synergizing Neutrosophy and Randomized Blocks Design: Development and Analytical Insights

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Abstract

The design of the experiment is a strategy for effectively examining the relationship between input design parameters and process output and developing a greater understanding. A randomized block design is an experimental design that has two primary factors and is widely used in agriculture, environment, biological, animal, and food sciences, where experimental material is heterogeneous and precise. In a randomised block design, one or more observations may lose their true significance due to an accident, poor handling, pest infestations in agricultural trials, or other factors. It is prudent to treat this value as missing and estimate it. In today's practical situations, uncertainty and inaccuracies are inevitable in most research areas. It is important to handle such data, which can lead to inaccurate and unreliable results. Neutrosophy is the branch of philosophy that provides an efficient method to study impreciseness among the data. Some of the common sources of Neutrosophy in randomised block design are incorrect blocking factor selection, measurement error, subjective factors, and natural variability. It is paramount to handle the Neutrosophy in a randomised block design; otherwise, it may lead to various problems, like a high risk of false positives. In this paper, the Neutrosophic Randomised Block Design (NRBD) is introduced to tackle data impreciseness. The study also, outlines a methodology for estimating missing observations in NRBD and presents its analysis. Additionally, the study compares the efficiency of NRBD to that of the Neutrosophic Completely Randomised Design (NCRD).

Keywords: Design of Experiment, Randomised Block Design; Neutrosophy; Neutrosophic Randomised Block Design; Neutrosophic Completely Randomised design

1. Introduction

In field experimentation [1, 2], if the entire experimental area is not homogenous and fertility gradient runs only in one direction, then a straight forward method of reducing the variability of the experimental material entails stratifying or dividing the entire area into relatively homogeneous strata or blocks. N M Ranka and L Sharma [3] focused on the detailed process of choosing an experimental design depending on the experimental goal and the number of factors to be investigated. Now if the treatments are randomly applied to relatively homogenous units within each block are replicated over all the blocks, the design is randomised block design. Despite its agricultural origin, the randomised block design is widely used in many types of studies. For example, to determine the differences in productivity of makes of machines etc.

Due to accident, mishandling, attack of pest (for agricultural experiments) or some other reason one or more observation may be lost. It is important to estimate the missing observation as it is difficult to perform the least square analysis and resulting normal equations with the missing observations.

F Smarchande [4], explains the neutrosophic data as a type of information that incorporates the concept of uncertainty, indeterminacy, and incompleteness. In design of experiments (DOE), this type of data can be used to model and analyze

complex systems that exhibit a high degree of uncertainty and variability. The main advantage of using neutrosophic data in DOE is that it allows for a more comprehensive representation of the system being studied, considering not only the known and definite information, but also the uncertain and indeterminate aspects of the system.

S Kumari, et.al [5] introduced the neutrosophic completely randomized design, which provides a flexible way of handling imprecise elements in a completely randomised design. K. Gnanapriya and et.al [6], describes the approach for statistical analysis of RBD through fuzzy ranking method using the location of median value. A. AIAita and et.al [7], introduced neutrosophic split plot design and its analysis.

A single missing observation in a neutrosophic randomized block design can have a significant impact on the results of the experiment. The presence of missing data can lead to bias in the estimated treatment effects and reduce the precision of the results. To minimize the impact of missing observations, it is important to carefully consider the underlying reasons for the missing data and choose an appropriate approach to handle the missing observations.

One common approach to handling missing data in neutrosophic randomized block designs is multiple imputation. This involves generating multiple imputed datasets by using statistical models to estimate the missing observations, and then combining the results from each imputed dataset to obtain a final estimate of the treatment effects.

In general, the choice of approach to handle missing data in NRBD will depend on the specific characteristics of the missing data and the goals of the experiment. It is important to carefully consider the underlying reasons for the missing data and choose an appropriate approach to minimize the impact of missing observations on the results of the experiment.

This paper introduces a neutrosophic perspective to randomized block design, designated as Neutrosophic Randomized Block Design (NRBD). In Section 2, the mathematical model for NRBD is formulated, accompanied by a comprehensive test procedure. Section 3 details the process for carrying out a post hoc test using the least significant difference method. Additionally, Section 4 outlines a procedure for estimating missing observations in NRBD. Section 5 compares the efficiency of NRBD to NCRD. A numerical illustration validating the proposed design is provided in Section 6.

2. Neutrosophic Randomised Block Design (NRBD)

A neutrosophic randomized block design is a type of experimental design that incorporates the concept of neutrosophy, which is the study of uncertainty, indeterminacy, and incompleteness. Neutrosophic randomized block designs are used to study complex systems that exhibit a high degree of uncertainty and variability and aim to provide a more comprehensive understanding of these systems than traditional statistical methods.

In a NRBD, the experiment is divided into a series of blocks, each of which represents a group of similar units or conditions. Within each block, the units are randomly assigned to different treatments, and the responses of the units are measured and analyzed to determine the effects of the treatments.

A. Mathematical Model

Consider a neutrosophic random variable (NRV) X_N from a neutrosophic normal distribution with neutrosophic mean, say μ_N and neutrosophic standard deviation, say σ_N . Here $X_N \in [X_L, X_U]$, $\mu_N \in [\mu_L, \mu_U]$ and $\sigma_N \in [\sigma_L, \sigma_U]$.

The mathematical model for NRBD is as follows.

$$Y_{N(ij)} = \mu_N + \alpha_{N(i)} + \beta_{N(j)} + e_{N(ij)} ; i = 1, 2, 3, 4, \dots, t_N \text{ and } j = 1, 2, 3, \dots, r_N \quad (1)$$

Y_N : neutrosophic response

$Y_{N(ij)}$: neutrosophic response of the j^{th} block receiving the i^{th} treatment

μ_N : neutrosophic general mean effect

$\alpha_{N(i)}$: the effect due to i^{th} neutrosophic treatment

$\beta_{N(j)}$: the effect due to j^{th} neutrosophic block

$e_{N(ij)}$: neutrosophic error effect

t_N : neutrosophic number of treatments

r_N : neutrosophic number of blocks

NSST: neutrosophic sum of squares between treatments

NSSB: neutrosophic sum of squares between blocks

NSSE: neutrosophic sum of squares due to error

NTSS: neutrosophic total sum of squares

NMSSE : neutrosophic mean sum of squares due to error

B. Test Procedure

i) Stating Hypotheses

Null hypothesis 1

$H_0^1 : \alpha_{N(i)} = 0$ There is no additional effect due to neutrosophic treatments.

Alternative hypothesis 1

$H_1^1 : \alpha_{N(i)} \neq 0$ There is additional effect due to neutrosophic treatments.

Null hypothesis 2

$H_0^2 : \beta_{N(j)} = 0$ There is no additional effect due to neutrosophic blocks.

Alternative hypothesis 1

$H_1^2 : \beta_{N(j)} \neq 0$ There is additional effect due to neutrosophic blocks.

ii) Least Square Estimates of the Neutrosophic Parameters

Using the principles of maxima and minima, least square estimates of the neutrosophic parameters has been estimated.

$$\widehat{\mu}_N = \overline{Y_{N(. .)}} \quad (2)$$

$$\widehat{\alpha}_{N(i)} = \overline{Y_{N(i .)}} - \overline{Y_{N(. .)}} \quad (3)$$

$$\widehat{\beta}_{N(j)} = \overline{Y_{N(. j)}} - \overline{Y_{N(. .)}} \quad (4)$$

$$\widehat{e}_{N(ij)} = Y_{N(ij)} - \overline{Y_{N(i .)}} - \overline{Y_{N(. j)}} + \overline{Y_{N(. .)}} \quad (5)$$

iii) Construction of Test Statistic

Under null hypotheses

Lemma 1 : Let X_1, X_2, \dots, X_n be i.i.d $N_N(0, \sigma_N^2)$ distributed random variables and suppose that

$$\sum_{i=1}^n X_i^2 = Q_1 + Q_2 + Q_3 + \dots + Q_k, \quad (6)$$

Where $Q_1, Q_2, Q_3, \dots, Q_k$ are positive semi – definite quadratic forms in X_1, X_2, \dots, X_n

$$Q_i = X' A_i X, \quad i = 1, 2, \dots, k \quad (7)$$

Set $m_i = \text{rank}(A_i)$

If $m_1 + m_2 + \dots + m_k = n$ then

Then $Q_1, Q_2, Q_3, \dots, Q_k$ are independent and $Q_i \sim \sigma_N^2 \chi_{N_i}^2(m_i)$

Lemma 2:

If $\chi_{N_1}^2$ and $\chi_{N_2}^2$ are two independent neutrosophic chi-square variate with n_{N_1} and n_{N_2} degrees of freedom respectively, then the ratio of two chi-square variate divided by its degrees of freedom follows neutrosophic F distribution.

$$F_N = \frac{\chi_{N_1}^2/n_{N_1}}{\chi_{N_2}^2/n_{N_2}} \sim F_N(n_{N_1}, n_{N_2}) \tag{8}$$

Lemma 3: Expectation of Neutrosophic Sum of Squares due to Error is $\sigma_{N(e)}^2$ and Expectation of Neutrosophic Sum of Squares due to Treatment or Blocks is greater $\sigma_{N(e)}^2$ i.e.

$$E(NMSSE) = \sigma_{N(e)}^2, E(NMSST) > \sigma_{N(e)}^2 \text{ and } E(NMSSB) > \sigma_{N(e)}^2$$

Where

$$NSSE(\text{ Neutrosophic Sum of Squares due Error }) = \sum_{i=1}^{t_N} \sum_{j=1}^{r_N} (Y_{N(ij)} - \overline{Y_{N(i.)}} - \overline{Y_{N(. j)}} - \overline{Y_{N(. .)}})^2$$

$$NSST(\text{ Neutrosophic Sum of Squares due Treatment }) = r_N \sum_{i=1}^{t_N} (\overline{Y_{N(i.)}} - \overline{Y_{N(. .)}})^2$$

$$NSST(\text{ Neutrosophic Sum of Squares due Blocks }) = t_N \sum_{j=1}^{r_N} (\overline{Y_{N(. j)}} - \overline{Y_{N(. .)}})^2$$

Neutrosophic Mean Sum of Squares = Neutrosophic Sum of Squares / Degrees of Freedom

We get,

$$F_{N1} = \frac{\frac{NSST}{\sigma_{N(e)}^2} / t_N - 1}{\frac{NSSE}{\sigma_{N(e)}^2} / (t_N - 1)(r_N - 1)} = \frac{\frac{NSST}{t_N - 1}}{\frac{NSSE}{(t_N - 1)(r_N - 1)}} = \frac{NMSST}{NMSSE} \sim F_N(t_N - 1, (t_N - 1)(r_N - 1)) \tag{9}$$

$$F_{N2} = \frac{\frac{NSSB}{\sigma_{N(e)}^2} / r_N - 1}{\frac{NSSE}{\sigma_{N(e)}^2} / (t_N - 1)(r_N - 1)} = \frac{\frac{NSSB}{r_N - 1}}{\frac{NSSE}{(t_N - 1)(r_N - 1)}} = \frac{NMSSB}{NMSSE} \sim F_N(r_N - 1, (t_N - 1)(r_N - 1)) \tag{10}$$

iv) Decision Rule

Using p_N value approach [8] the following decision is taken as given below.

1. If maximum P_N value $\leq \alpha$, then reject H_0 at α level of significance.
2. If minimum P_N value $> \alpha$, then do not reject H_0 at α level of significance.
3. If minimum P_N value $< \alpha <$ maximum P_N value then there is an indeterminacy. Thus

(i) $\frac{(\alpha - \text{minimum } P_N \text{ value})}{\text{maximum } P_N \text{ value} - \text{minimum } P_N \text{ value}}$ is the chance of rejecting H_0 at α level of significance,

and

(ii) $\frac{(\text{maximum } P_N \text{ value} - \alpha)}{\text{maximum } P_N \text{ value} - \text{minimum } P_N \text{ value}}$ is the chance of not rejecting H_0 at α level of significance.

v) Table 1: NANOVA table for NRBD

Sources of Variation	NSS	df _N	NMS	F _N ratio
Neutrosophic Treatments	NSST	t _N -1	$\frac{NSST}{t_N - 1}$	$F_{N1} = \frac{NMSST}{NMSSE}$
Neutrosophic Blocks	NSSB	r _N -1	$\frac{NSSB}{r_N - 1}$	$F_{N2} = \frac{NMSSB}{NMSSE}$
Error	NSSE	(t _N -1)(r _N -1)	$\frac{NSSTE}{(t_N - 1)(r_N - 1)}$	

Total	NTSS	$(t_N-1)(r_N-1)-1$		
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3. Neutrosophic Least Significant Difference (NLSD) in NRBD

If $H_{N0^1} : \alpha_{N(i)} = 0$ i. e there is no additional effect on output response due to neutrosophic treatments, is rejected at α level of significance, we are required to find which pair of neutrosophic treatment means differ significantly. The p^{th} and q^{th} neutrosophic treatment differ significantly if

$$|\bar{Y}_{N(p)} - \bar{Y}_{N(q)}| > \text{Neutrosophic } t_{\frac{\alpha}{2}, (t_N-1)(r_N-1)} \times \sqrt{NMSSE \left(\frac{2}{r_N}\right)} \tag{11}$$

Also, If $H_{N0^2} : \beta_{N(j)} = 0$ i. e There is no additional effect on output response due to neutrosophic blocks, is rejected at α level of significance, we are required to find which pair of neutrosophic blocks means differ significantly.

The u^{th} and v^{th} neutrosophic blocks differ significantly if

$$|\bar{Y}_{N(.u)} - \bar{Y}_{N(.v)}| > \text{Neutrosophic } t_{\frac{\alpha}{2}, (t_N-1)(r_N-1)} \times \sqrt{NMSSE \left(\frac{2}{t_N}\right)} \tag{12}$$

The following decision is taken

1. If $\max\{\text{neutrosophic mean difference(MD)}\} > \max\{\text{neutrosophic least significant difference(neutrosophic LSD)}\}$, then pair of neutrosophic treatment/ blocks means differs.

2. If $\min\{\text{neutrosophic mean difference(MD)}\} < \min\{\text{neutrosophic LSD}\}$, then pair of neutrosophic treatment/ blocks means does not differs.

3. If $\min\{\text{neutrosophic LSD}\} < [\min \text{neutrosophic MD}, \max \text{neutrosophic MD}] < \max\{\text{neutrosophic LSD}\}$ then there is an indeterminacy. Thus

(i) $\frac{\min\{\text{NMD}\} - \min\{\text{NLSD}\}}{[\min\{\text{NMD}\} - \min\{\text{NLSD}\}] + [\max\{\text{NLSD}\} - \max\{\text{NMD}\}]}$ is the chance of pair of neutrosophic treatment/ blocks means does not differs and

(ii) $\frac{\max\{\text{NLSD}\} - \max\{\text{NMD}\}}{[\min\{\text{NMD}\} - \min\{\text{NLSD}\}] + [\max\{\text{NLSD}\} - \max\{\text{NMD}\}]}$ is the chance of pair of neutrosophic treatment/ blocks means differs.

4. If $\min\{\text{neutrosophic MD}\} < [\min \text{neutrosophic LSD}, \max \text{neutrosophic LSD}] < \max\{\text{neutrosophic MD}\}$ then there is an indeterminacy. Thus

(i) $\frac{\min\{\text{NLSD}\} - \min\{\text{NMD}\}}{[\min\{\text{NLSD}\} - \min\{\text{NMD}\}] + [\max\{\text{NMD}\} - \max\{\text{NLSD}\}]}$ is the chance of pair of neutrosophic treatment/ blocks means does not differs. and

(ii) $\frac{\max\{\text{NMD}\} - \max\{\text{NLSD}\}}{[\min\{\text{NLSD}\} - \min\{\text{NMD}\}] + [\max\{\text{NMD}\} - \max\{\text{NLSD}\}]}$ is the chance of pair of neutrosophic treatment/ blocks means differs.

4. Estimation of a missing observation in NRBD

Let [a, b] be the neutrosophic missing observation of j^{th} block from i^{th} treatment in a NRBD. Then neutrosophic missing observation is estimated using the formula given below

$$a = \frac{r_N y'_{jl} + t_N y'_{i.l} - y'_{..l}}{(r_N - 1)(t_N - 1)} \quad \text{and } b = \frac{r_N y'_{ju} + t_N y'_{i.u} - y'_{..u}}{(r_N - 1)(t_N - 1)} \tag{13}$$

Where $y'_{j,l}$ is lower total of known observations in j^{th} neutrosophic block.

$y'_{j,u}$ is upper total of known observations in j^{th} neutrosophic block.

$y'_{i,l}$ is lower total of known observation in i^{th} neutrosophic treatment.

$y'_{i,u}$ is upper total of known observations in i^{th} neutrosophic treatment.

$y'_{..u}$ is lower grand total of all known neutrosophic observations.

$y'_{..l}$ is upper grand total of all known neutrosophic observations.

5. Assessing Model Efficiency of NRBD over NCRD

In a design of experiment (DOE), the efficiency of a model refers to its ability to predict the relationship accurately and reliably between the independent variables and the dependent variables. A more efficient model is one that requires fewer trials to accurately capture the relationship between the variables and can make accurate predictions with fewer inputs. In this section a procedure to study the efficiency of NRBD over NCRD is formulated.

Efficiency of NRBD over NCRD can be obtained using the formula given below:

$$E_N = \frac{(r_N - 1) NMSSB + r_N (t_N - 1) NMSSE}{(r_N t_N - 1) NMSSE} \tag{14}$$

The following decision is taken

1. If {neutrosophic E-value} > 1, then gain in efficiency
2. If {neutrosophic E-value} < 1 then no gain in efficiency.
3. If {neutrosophic E-value} = 1 then both design are equally efficiency.
4. If $\min\{\text{neutrosophic E-value}\} < 1 < \max\{\text{neutrosophic E-value}\}$ then there is an indeterminacy. Thus

- (i) $\frac{(1 - \min\{\text{neutrosophic E-value}\})}{\max\{\text{neutrosophic E-value}\} - \min\{\text{neutrosophic E-value}\}}$ is the chance of no gain in efficiency and
- (ii) $\frac{(\max\{\text{neutrosophic E-value}\} - 1)}{\max\{\text{neutrosophic E-value}\} - \min\{\text{neutrosophic E-value}\}}$ is the chance of gain in efficiency.

6. Numerical Illustrations

Case Study 1: To test whether there is a significant difference in the makes of cars (A, B, C and D) and speed (25, 35, 50, 60 and 70 mph) in the petrol consumption rates. For this, the petrol consumptions rates of the different makes of car for suitable average speed were collected from Salem district are given in the table 2 below:

Table 2: Petrol consumptions rates of different makes of car and average speed

Makes of Car	Speeds of the cars in miles per hour (mph)				
	25	35	50	60	70
A	[17,25]	[18,22]	[15,21]	[15,21]	[12,21]
B	[18,22]	[15,22]	[12,21]	[14,20]	[12,17]
C	[17,25]	[15,22]	[a,b]	[13,17]	[12,17]
D	[12,21]	[14,20]	[12,21]	[13,17]	[10,17]

Solution: The estimated neutrosophic missing observation of 3rd block from 3rd treatment is [13,21] obtained using the formula given in 2.3.

Now, consider $\widetilde{\mu}_{N(i)}$ as the neutrosophic mean petrol consumption of the i^{th} makes of car.

The null hypothesis, $H_{NO}^1 : \widetilde{\mu}_{N(1)} = \widetilde{\mu}_{N(2)} = \widetilde{\mu}_{N(3)} = \widetilde{\mu}_{N(4)}$ against the alternative hypothesis H_{N1}^1 : Atleast one pair of $\widetilde{\mu}_{N(i)}$ differs

Also Consider $\widetilde{\mu}_{N(j)}$ as the neutrosophic mean petrol consumption of the j^{th} speed of car.

The null hypothesis, $H_{NO}^2 : \widetilde{\mu}_{N(1)} = \widetilde{\mu}_{N(2)} = \widetilde{\mu}_{N(3)} = \widetilde{\mu}_{N(4)} = \widetilde{\mu}_{N(5)}$ against the alternative hypothesis H_{N1}^1 : Atleast one pair of $\widetilde{\mu}_{N(i)}$ differs

On carrying out the neutrosophic two-way ANOVA [9], the results is obtained in table 3.

According to the decision rules given by F. Smarandache [8] , for any α level of significance, the null hypothesis is rejected if maximum $p_N < \alpha$, where p_N is the p value for the neutrosophic statistics.

From table 2, it is observed that maximum p_N value = 0.039 < 0.05, which indicates the rejection of the null hypothesis.

Hence, we conclude that there is an additional effect on petrol consumption due to different makes of car.

Also, for this data, maximum p_N -value = 0.0009 < 0.05, which indicates the rejection of the null hypothesis. Hence, we conclude that there is an additional effect on petrol consumption due to different speeds of car.

Table 3: NANOVA Table of Petrol Consumption

Sources of Variation	N S S	df _N	NMS	F _N Ratio	P _N Value
Treatments (Makes of Car)	(19.8,26.15)	3	(6.6,8.7167)	(3.8261,6.3012)	(0.0082,0.0391)
Blocks (Speed of Car)	(54.12,72.5)	4	(13.55,18.125)	(9.7925,10.507)	(0.0007,0.0009)
Error	(16.6,20.7)	12	(1.3833,1.725)		
Total	(96.95,113)	19			

Next, we use neutrosophic least significant difference to identify the pairs of Neutrosophic treatment and blocks that differs. We observe that makes of car A and D differs significantly. Also, the following pairs of speed of car differ significantly (25,50), (25,60), (25,70), (35,70) and (50,70).

For the pair (35,50), there is 75% chance that it does not differs and 25% that it differs. For the pairs (50,60) and (60,70), there is 82% chance that it does not differs and 18 % that it differs.

Further, we observe that there is gain in efficiency of NRBD over NCRD by [185%, 200%]

Case Study 2: To test whether there is a significant difference in the typing machines and three typists while typing the number of words per minute (typing speed). For this, typing speed of four typing machines and three typists were recorded from an institution. While collecting the data, some of the output (typing speed) collected were not clear if it is obtained using Typing Machine 3 or 4.

Table 4 : Typing speed of different typist on different typing machines

Typing Machines	Typist		
	T1	T2	T3

M1	105	95	102
M2	99	95	92
M3 or M4	[83,88]	[a, b]	[80,87]

Solution: The estimated neutrosophic missing observation of 2nd block from 3rd treatment is [83,87] obtained using the formula given in 2.3.

Now, consider $\widetilde{\mu}_{N(i)}$ as the neutrosophic mean number of words per minute of the neutrosophic ith typing machine.

The null hypothesis, $H_{NO}^1 : \widetilde{\mu}_{N(1)} = \widetilde{\mu}_{N(2)} = \widetilde{\mu}_{N(3)} = \widetilde{\mu}_{N(4)}$ against the alternative hypothesis H_{N1}^1 : Atleast one pair of $\widetilde{\mu}_{N(i)}$ differs

Also Consider $\widetilde{\mu}_{N(j)}$ as the neutrosophic mean number of words per minute the jth typist.

The null hypothesis, $H_{NO}^2 : \widetilde{\mu}_{N(1)} = \widetilde{\mu}_{N(2)} = \widetilde{\mu}_{N(3)} = \widetilde{\mu}_{N(4)} = \widetilde{\mu}_{N(5)}$ against the alternative hypothesis.

H_{N1}^1 : Atleast one pair of $\widetilde{\mu}_{N(i)}$ differs

On carrying out the neutrosophic two way ANOVA(8), the results is obtained in table 5.

Table 5: NANOVA Table of Typing Speed (No. of words per minute)

Sources of Variation	NSS	df _N	NMS	F _N	P _N
Treatments (Typing Machines)	(270.22,554.67)	2	(135.11,277.33)	(14.30,26)	(0.0051,0.015)
Blocks (Typists)	(40.22,40.67)	2	(20.11,20.33)	(1.91,2.13)	(0.2345, 0.2621)
Error	(37.78,42.67)	4	(9.44,10.67)		
Total	(348.22,638)	8			

According to rules, the null hypothesis that there is no significant difference between the different neutrosophic typing machines will be rejected if maximum p_N-value ≤ α, where α is a level of significance. For this data, maximum p_N-value = 0.015 < 0.05, we reject the null hypothesis and conclude that there is an additional effect on typing speed due to different neutrosophic typing machines.

According to rules, the null hypothesis that there is no significant difference between the different typists will be accepted if minimum p_N-value > α, where α is a level of significance. For this data, minimum p_N-value = 0.2345 > 0.05, we accept the null hypothesis and conclude that there is no additional effect on typing speed due to different typists.

Next, we use neutrosophic least significant difference to identify the pairs of Neutrosophic treatment that differs. We observe that Typing machine M1 and (M3 or M4) differs significantly. Also, the following pairs of typing machine differ significantly (M2, M3 or M4).

Further, we observe that there is gain in efficiency of NRBD over NCRD by [16%, 20%]

7. Conclusion

The use of neutrosophic data in a randomized block design allows for a more comprehensive representation of the system being studied, considering not only the known and definite information, but also the uncertain and indeterminate aspects of the system. This can provide valuable insights into the relationships between variables and the effects of different treatments on the system and can help to improve the accuracy and reliability of experimental results. Overall, neutrosophic randomized block designs provide a flexible and powerful approach to study between variables and the effects of treatments. The use of neutrosophic data in design of experiments can provide valuable insights into complex systems and can help to improve the accuracy and reliability of experimental results.

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