A Study on Interval Valued Temporal Neutrosophic Fuzzy Sets

K. Rajesh[1*], Sharmila Rathod[2], Jyoti Kundale[3], Nilesh Rathod[4], M. Clement Joe Anand[5], Utpal Saikia[6], Mohit Tiwari[7], Nivetha Martin[8]

[1*]Department of Mathematics, Government Arts and Science College for Women, Krishnagiri, Tamil Nadu – 635 108, India.
[2]Department of Computer Engineering, Mcts Rajiv Gandhi Institute of Technology, Andheri West, Mumbai – 400053, India.
[3]Department of Information Technology, Ramrao Adik Institute of Technology, Nerul, Navi Mumbai – 400706, India.
[4]Department of Artificial Intelligence and Machine Learning, DJ Sanghvi College of Engineering, Mumbai – 400056, India.
[5]Department of Mathematics, Mount Carmel College (Autonomous), Bengaluru-560052, Karnataka, India.
[6]Department of Mathematics, Silapathar College, Dhemaji, Assam – 787059, India.
[7]Department of Computer Science and Engineering, Bharati Vidyapeeth's College of Engineering, Delhi -110063, India.
[8]Department of Mathematics, Arul Anandar College (Autonomous), Karumathur-625514, Tamil Nadu, India.

Emails: rajeshagm@gmail.com; sharmila.gaikwad@mctrgit.ac.in; jyojadhav04@gmail.com; nileshpop@gmail.com; arjoemi@gmail.com; utpalsaikiajorhat@gmail.com; mohit.tiwari@bharatividyapeeth.edu; nivetha.martin710@gmail.com

Abstract

In this research, we introduce the Interval Valued Temporal Neutrosophic Fuzzy Sets (IVTNFS) and some of its basic operations. Also, examine some of their properties. The Neutrosophic Fuzzy Sets of membership and non-membership values are not always possible up to our satisfaction, but the IVTNFS part has a more important role here, because the time movement with an interval in NFS gave the best solution to making a decision, deciding their careers in our real-life situation.

Keywords: Intuitionistic Fuzzy Sets; Temporal Intuitionistic Fuzzy Sets; Neutrosophic Fuzzy Sets; Interval Valued Neutrosophic Fuzzy Sets and Interval Valued Intuitionistic Fuzzy Sets.

Subject Classification: 03E72, 03B20, 03D45, 03F55.

1. Introduction

Zadeh L has introduced fuzzy subset idea in the beginning instance, many authors have lately explained fuzzy subset directions, encompassing soft set, hazy set, rough set, etc. whereas the interval-valued fuzzy sets [6] were expanded the IVFS and IVFSs. The idea of IVFS was proposed by Pal and Shyamal with interval-valued fuzzy matrices and demonstrated several properties of it. Intuitionistic fuzzy sets (IFSs) [1] were first described by Atanassov K. T, which are fuzzy subsets as well as a superior simplification of fuzzy sets. Since the initiation of IFS, plentiful researchers have shown their interest in the theory and utilized it in several domains, including model detection, apparatus learning, likeness processing, decision building and others. Lots of authors illustrate numerous consequences utilizing this intuitionistic fuzzy sets concept. Atanassov K.T. introduced the latest operations that

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

were defined on intuitionistic fuzzy sets. Supriya Kumar De, Ranjit Biswas along with Akhil Ranjan Roy [7] suggested certain operations over intuitionistic fuzzy sets, proposed medical diagnostics via intuitionistic fuzzy sets (IFS). The Temporal intuitionistic fuzzy sets orginated by Attanassov K. T [5].
Further, the notion of Interval-Valued Intuitionistic Fuzzy Sets [IVIFS] was first introduced by George Gargov and Krassimir T Atanasov, which is an oversimplification of the equality of IFS as well as IVFS. With the subsequent advance of IVIFS, many researchers expressed their attention to the theory and also used it in a diversity of domains. Later Krassimir T Atanassov introduced the operators over interval-valued intuitionistic fuzzy sets in 1994 [4]. Florentin Smarandache and Broumi, proposed a new operation of interval-valued intuitionistic hesitant fuzzy sets. Power harmonic weighted aggregation operator on single-valued trapezoidal neutrosophic numbers and interval-valued neutrosophic sets introduced by Janani [8]. Interval-Valued Fermatean Neutrosophic Shortest Path Problem via Score Function studied, all these above are associated with temparal neutrosophic sets.
Florentin Smarandache [12-13] introduced Neutrosophic Set (NS) and their extensions like, Neutrosophic probability, Neutrosophic set, Neutrosophic logic, the Multi-Moora method, Single valued neutrosophic sets and Bipolar neutrosophic sets. The NS uses hesitancy as an independent measure of the membership and non-membership information. Hence the concept of NS is considered as a generalization of FS, IFS, and interval-valued sets. Interval Valued Neutrosophic Fuzzy Set was introduced by Florentin Smarandache, where the fuzzy membership grade of each part is connected with neutrosophic components, i.e., truth, indeterminacy, and falsity membership grades. The assimilation of neutrosophic components to FS is required to manage the real life information which is both uncertain and unpredictable in the environment. Certain level operators over temporal intuitionistic fuzzy sets [11] and also established some of their properties. Solving shortest path problems using an ant colony algorithm with triangular neutrosophic arc weights [3], Complex fermatean neutrosophic graph and application to decision making. Decision Making [2] explained by Broumi.
The neutrosophic fuzzy sets where proposed the fuzzy membership grade of each element is associated with neutrosophic mechanism, i.e., truth, indeterminacy, and falsity membership grades. The incorporation of neutrosophic components to Fuzzy Sets is necessary to handle the real life in turn which are both uncertain and inconsistent in nature. In various real life troubles, the membership degree of a FS cannot be totally assured due to the inaccurate and conflicting characteristic of human accomplishment. Therefore, it is more rational to engage neutrosophic fuzzy components to delegate the membership degree. From this view point, the authors propose the interval valued temporal neutrosophic fuzzy set (IVTNFS), in additional; the membership position of the neutrosophic components can also be expressed with IVNFS. The Interval Valued Neutrosophic Fuzzy Sets of membership and non-membership values are not always possible up to our satisfaction, but the IVTNFS part has a more important role here, because the time moment of IVNFS gives the best solution to finding the shortest distance in making a decision, decide their careers and so on. Particularly in the case of medical diagnosis, there is a fair chance of the existence of a non-zero hesitation part at each moment of evaluation by using this concept.
The rest of the paper is designed as follows: Section 2 gives some basic definition's. In Section 3, we define the Interval Valued Temporal Neutrosophic Fuzzy Sets. Also, we establish some relationships among the existing sets. This paper is concluded in section 4.

2. Preliminaries

Neutrosophic Fuzzy Sets is important in real life situation and its most important in decision making like career determination, image processing, medical diagnosis, etc. In our reality, we find things that cannot be precisely defined and that contain an indeterminacy part. This is the reason for studying neutrosophic fuzzy sets and their extensions.

Definition 2.1[29] Let G be a non-empty set. A Fuzzy Set A in G is characterized by its membership function and $\mu_A(r)$ is interpreted as the degree of membership of the element G in fuzzy set A, for each $r \in G$. It is clear that A is completely determined by the set of tuples

$$A = \{< r, \mu_A(r) > \mid r \in G\}$$

Definition 2.2[1] Let G be a non-empty set. An intuitionistic fuzzy set (IFS) A in G is defined as an object of the following form.

$$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}$$

where the functions $\mu_A: G \to [0,1]$ and $\nu_A: G \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $r \in G$, respectively, and for every $r \in G$.

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

Definition 2.3[9] Let G be a non-empty set and A Temporal Intuitionistic Fuzzy Sets (TIFS) is defined as the following object of the form.

$$A = \{< r, \mu_A(r,t), \nu_A(r,t) > \mid < r,t > \in G \times T\}$$

where

i) $A \subset G$ is a fixed set.

ii) $\mu_A(r,t) + \nu_A(r,t) \leq 1$ For every $< r,t > \in G \times T$.

$\mu_A(r,t)$ and $\nu_A(r,t)$ are the degree of membership and the degree of non-membership of the element $r \in G$, respectively, and the time – moment $t \in T$.

Definition 2.4[4] An Interval Valued Intuitionistic Fuzzy Set (IVIFS) A in G is given by

$$A = \{< r, M_A(r,t), N_A(r,t) > | r \in G\}$$

where M_A: G → [0,1], N_A: G → [0,1]. The intervals $M_A(r,t)$ and $N_A(r,t)$ denote the degree of membership and the degree of non-membership of the element G to the set A, where $M_A(r,t) = [M_{AL}(r,t), M_{AU}(r,t)]$ and $N_A(r,t) = [N_{AL}(r,t), N_{AU}(r,t)]$ with the condition that $M_{AU}(r,t) + N_{AU}(r,t) \leq 1$ for all $r \in G$.

Definition 2.5[19] Let G be a universal set and $r \in G$. A Neutrosophic Sets (NS) A in G is characterized by a truth, indeterminacy, and falsity membership function which are, respectively denoted as T_A, I_A and F_A and it is denoted as the following form

$$A = \{< r, T_A(r), I_A(r), F_A(r) >| r \in G\}$$

The functions $T_A(r), I_A(r)$ and $F_A(r)$ are real standard or non-standard subsets of $]0^-, 1^+[$, .i.e., $T_A(r) : G \to]0^-, 1^+[, I_A(r) : G \to]0^-, 1^+[$ and $F_A(r) : G \to]0^-, 1^+[$ No restriction is applied on the sum of $T_A(r), I_A(r)$ and $F_A(r)$, so

$$0^- \leq \sup T_A(r) + \sup I_A(r) + \sup F_A(r) \leq 3^+$$

For a fixed $r \in G$. $T_A(r), I_A(r)$ and $F_A(r)$ is called neutrosophic number.

Definition 2.6[22] Let G be a set of objects and $A = \{r, \ \mu_A(r) \mid r \in G\}$, where $\mu_A(r) : G \to [0,1]$ be a fuzzy set. Then a Neutrosophic Fuzzy Sets (NFS) A in G defined by

$$A = \{< r, \mu_A(r), T_A(r,\mu), I_A(r,\mu), F_A(r) >| r \in G\}$$

where each membership value is expressed by a truth, indeterminacy, and falsity membership function which are respectively denoted as $T_A(r,\mu), I_A(r,\mu)$ and $F_A(r,\mu)$. Moreover T_A, I_A and F_A are real standard or non-standard subsets of $]0^-, 1^+[$, i.e., $T_A(r) : G \to]0^-, 1^+[, I_A(r) : G \to]0^-, 1^+[$ and $F_A(r) : G \to]0^-, 1^+[$ No restriction is applied on the sum of $T_A(r), I_A(r)$ and $F_A(r)$, so

$$0^- \leq \sup T_A(r) + \sup I_A(r) + \sup F_A(r) \leq 3^+$$

For a fixed $r \in G$. $\mu_A(r), T_A(r), I_A(r)$ and $F_A(r)$ is called neutrosophic fuzzy number(NFN).

Definition 2.7[26] Let G be a non empty set. Then an Interval Valued Neutrosophic Sets (IVNS) A is an object of the following form

$$A = \{< r, [\inf T_A(r), \sup T_A(r)], [\inf I_A(r), \sup I_A(r)], [\inf F_A(r), \sup F_A(r)] >| r \in G\}$$

where the functions $T_A(r), I_A(r)$ & $F_A(r) : G \to]0^-, 1^+[$ and $0 \leq \sup T_A(r) + \sup I_A(r) + \sup F_A(r) \leq 3$

We denote the class of all interval valued neutrosophic sets on G by IVNSG.

Definition 2.8[6] Let A, B be two interval valued neutrosophic sets on G. Then

(i) A is called a subset of B, denoted by $A \subseteq B$ if

$$\inf T_A(r) \leq \inf T_B(r), \sup T_A(r) \leq \sup T_B(r), \inf I_A(r) \leq \inf I_B(r), \sup I_A(r) \leq \sup I_B(r),$$
$$\inf F_A(r) \leq \inf F_B(r), \sup F_A(r) \leq \sup F_B(r) \ \forall\, r \in G$$

(ii) The intersection of A and B is denoted by $A \cap B$ and is defined by

$$A \cap B = \{r, < [\min[\inf T_A(r), \inf T_B(r)], \min[\sup T_A(r), \sup T_B(r)]],$$
$$[\max[\inf I_A(r), \inf I_B(r)], \max[\sup I_A(r), \sup I_B(r)]],$$
$$[\max[\inf F_A(r), \inf F_B(r)], \max[\sup F_A(r), \sup F_B(r)]] > | r \in G\}$$

(iii) The union of A and B is denoted by $A \cup B$ and is defined by

$$A \cup B = \{r, < [\max[\inf T_A(r), \inf T_B(r)], \max[\sup T_A(r), \sup T_B(r)]],$$

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

$$[\min[\inf I_A(r), \inf I_B(r)], \min[\sup I_A(r), \sup I_B(r)]],$$
$$[\min[\inf F_A(r), \inf F_B(r)], \min[\sup F_A(r), \sup F_B(r)]] > | r \in G \}$$

(iv) The complement of A is denoted by A^c and is defined by

$$A^c = \{ < [\inf F_A(r), \sup F_A(r)], [1 - \sup I_A(r), 1 - \inf I_A(r)], [\inf T_A(r), \sup T_A(r)] > | r \in G\}$$

3. Interval Valued Temporal Neutrosophic Fuzzy Sets

In this section, we define the Interval Valued Temporal Neutrosophic Fuzzy Sets and its fundamental operations in addition that, we establish some relation along with the existing sets.

Definition 3.1 Let G be a non- empty set, then the Interval Valued Temporal Neutrosophic Fuzzy Sets (IVNFS) A in G is define in the following form

$$A = \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)],$$
$$[\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

where the functions $\mu_A(r,t), T_A(r,t), I_A(r,t)$ & $F_A(r,t) : G \to]0^-, 1^+[$ and

$$0 \le \sup \mu_A(r,t) + \sup T_A(r,t) + \sup I_A(r,t) + \sup F_A(r,t) \le 3$$

Definition 3.2 Let A, B be two interval valued temporal neutrosophic fuzzy sets on G. Then A is called a subset of B, denoted by $A \subseteq B$ if

$$\inf T_A(r,t) \le \inf T_B(r,t), \sup T_A(r,t) \le \sup T_B(r,t), \inf I_A(r,t) \le \inf I_B(r,t), \sup I_A(r,t) \le \sup I_B(r,t),$$
$$\inf F_A(r,t) \le \inf F_B(r,t), \sup F_A(r,t) \le \sup F_B(r,t) \ \forall (r,t) \in G \times T$$

Definition 3.3 Let A, B be two interval valued temporal neutrosophic fuzzy sets on G. Then A is called the intersection of A and B is denoted by $A \cap B$ and the union of A and B is denoted by $A \cup B$ respectively and it is define by the following form

$$A \cap B = \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\sup T_A(r,t), \sup T_B(r,t)]],$$
$$[\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup I_A(r,t), \sup I_B(r,t)]],$$
$$[\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > | (r,t) \in G \times T\}$$
$$A \cup B = \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]],$$
$$[\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]],$$
$$[\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > | (r,t) \in G \times T\}$$

Definition 3.4 Let A, B be two interval valued temporal neutrosophic fuzzy sets on G. Then A is called the complement of A is denoted by A^c and it is define by the following form

$$A^c = \{(r,t), < [\inf F_A(r,t), \sup F_A(r,t)], [1 - \sup I_A(r,t), 1 - \inf I_A(r,t)],$$
$$[\inf T_A(r,t), \sup T_A(r,t)] > | (r,t) \in G \times T\}$$

Proposition 3.5 *Let A, B and C be two interval valued temporal neutrosophic fuzzy sets on G. Then for every IVTNFSs, we have the following*

i. $A \cup B = B \cup A$
ii. $A \cap B = B \cap A$
iii. $(A \cup B) \cup C = A \cup (B \cup C)$
iv. $(A \cap B) \cap C = A \cap (B \cap C)$
v. $\overline{(\bar{A} \cup \bar{B})} = A \cap B$
vi. $\overline{(\bar{A} \cap \bar{B})} = A \cup B$

Proof: Let

$$A = \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)],$$
$$[\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

and

$$B = \{ < (r,t), [\inf \mu_B(r,t), \sup \mu_B(r,t)], [\inf T_B(r,t), \sup T_B(r,t)],$$
$$[\inf I_B(r,t), \sup I_B(r,t)], [\inf F_B(r,t), \sup F_B(r,t)] > | (r,t) \in G \times T\}$$

Then by the definition of union,

$$A \cup B = \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]],$$
$$[\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]],$$

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

$$[\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\}$$
$$= \{(r,t), < [\max[\inf T_B(r,t), \inf T_A(r,t)], \max[\sup T_B(r,t), \sup T_A(r,t)]],$$
$$[\min[\inf I_B(r,t), \inf I_A(r,t)], \min[\sup I_B(r,t), \sup I_A(r,t)]],$$
$$[\min[\inf F_B(r,t), \inf F_A(r,t)], \min[\sup F_B(r,t), \sup F_A(r,t)]] > |(r,t) \in G \times T\}$$
$$= B \cup A$$

Which is proved (i). by the definition of intersection,

$$A \cap B = \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\text{supe}\, T_A(r,t), \sup T_B(r,t)]],$$
$$[\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup Ih_A(r,t), \sup I_B(r,t)]],$$
$$[\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\}$$
$$= \{(r,t), < [\min[\inf T_B(r,t), \inf T_A(r,t)], \min[\sup T_B(r,t), \sup T_A(r,t)]],$$
$$[\max[\inf I_B(r,t), \inf I_A(r,t)], \max[\sup I_B(r,t), \sup I_A(r,t)]],$$
$$[\max[\inf F_B(r,t), \inf F_A(r,t)], \max[\sup F_B(r,t), \sup F_A(r,t)]] > |(r,t) \in G \times T\}$$
$$= B \cap A$$

hence its proved the part (ii) and the third part of the proof as follows

$$(A \cup B) \cup C = \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]],$$
$$[\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]],$$
$$[\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\} \cup C$$
$$= \{(r,t),$$
$$< [\max[\max[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \max[\max[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$$
$$[\min[\min[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \min[\min[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$$
$$[\min[\min[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \min[\min[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$$
$$>\}$$
$$= \{(r,t), < [\max[\inf T_A(r,t), \max[\inf T_B(r,t), \inf T_C(r,t)]], \max[\sup T_A(r,t), \max[\sup T_B(r,t), \sup T_C(r,t)]],$$
$$[\min[\inf I_A(r,t), \min[\inf I_B(r,t), \inf I_C(r,t)]], \min[\sup I_A(r,t), \min[\sup I_B(r,t), \sup I_C(r,t)]],$$
$$[\min[\inf F_A(r,t), \min[\inf F_B(r,t), \inf F_C(r,t)]], \min[\sup F_A(r,t), \min[\sup F_B(r,t), \sup F_C(r,t)]] >\}$$
$$= A \cup \{(r,t), < [\max[\inf T_B(r,t), \inf T_C(r,t)], \max[\sup T_B(r,t), \sup T_C(r,t)]],$$
$$[\min[\inf I_B(r,t), \inf I_C(r,t)], \min[\sup I_B(r,t), \sup I_C(r,t)]],$$
$$[\min[\inf F_B(r,t), \inf F_C(r,t)], \min[\sup F_B(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$$
$$= A \cup (B \cup C)$$

hence its proved the part (iii) and the fourth part of the proof as follows

$$(A \cap B) \cap C = \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\sup T_A(r,t), \sup T_B(r,t)]],$$
$$[\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup I_A(r,t), \sup I_B(r,t)]],$$
$$[\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\} \cup C$$
$$= \{(r,t), < [\min[\min[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \min[\min[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$$
$$[\max[\max[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \max[\max[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$$
$$[\max[\max[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \max[\max[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$$
$$>\}$$
$$= \{(r,t), < [\min[\inf T_A(r,t), \min[\inf T_B(r,t), \inf T_C(r,t)]], \min[\sup T_A(r,t), \min[\sup T_B(r,t), \sup T_C(r,t)]],$$
$$[\max[\inf I_A(r,t), \max[\inf I_B(r,t), \inf I_C(r,t)]], \max[\sup I_A(r,t), \max[\sup I_B(r,t), \sup I_C(r,t)]],$$
$$[\max[\inf F_A(r,t), \max[\inf F_B(r,t), \inf F_C(r,t)]], \max[\sup F_A(r,t), \max[\sup F_B(r,t), \sup F_C(r,t)]] >\}$$
$$= A \cap \{(r,t), < [\min[\inf T_B(r,t), \inf T_C(r,t)], \min[\sup T_B(r,t), \sup T_C(r,t)]],$$
$$[\max[\inf I_B(r,t), \inf I_C(r,t)], \max[\sup I_B(r,t), \sup I_C(r,t)]],$$
$$[\max[\inf F_B(r,t), \inf F_C(r,t)], \max[\sup F_B(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$$
$$= A \cap (B \cap C)$$

Which is proved (iv). by the definition of complement,

$$\bar{A} = \{(r,t), < [\inf F_A(r,t), \sup F_A(r,t)], [1 - \sup I_A(r,t), 1 - \inf I_A(r,t)],$$
$$[\inf T_A(r,t), \sup T_A(r,t)] > |(r,t) \in G \times T\}$$

And

$$\bar{B} = \{(r,t), < [\inf F_B(r,t), \sup F_B(r,t)], [1 - \sup I_B(r,t), 1 - \inf I_B(r,t)],$$
$$[\inf T_B(r,t), \sup T_B(r,t)] > |(r,t) \in G \times T\}$$

Take the union operation, we have

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

$$\bar{A} \cup \bar{B} = \{(r,t), < [\inf F_A(r,t), \sup F_A(r,t)], [1 - \sup I_A(r,t), 1 - \inf I_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)] > | (r,t) \in G \times T\} \cup \{(r,t), < [\inf F_B(r,t), \sup F_B(r,t)], [1 - \sup I_B(r,t), 1 - \inf I_B(r,t)], [\inf T_B(r,t), \sup T_B(r,t)] > | (r,t) \in G \times T\}$$

$$= \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]], [\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]], [\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > | (r,t) \in G \times T\}$$

$$\overline{(\bar{A} \cup \bar{B})} = \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\sup T_A(r,t), \sup T_B(r,t)]], [\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup I_A(r,t), \sup I_B(r,t)]], [\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > | (r,t) \in G \times T\}$$

$$= A \cap B$$

Hence proved (v). Appl the intersection operation, we have

$$\bar{A} \cap \bar{B} = \{(r,t), < [\inf F_A(r,t), \sup F_A(r,t)], [1 - \sup I_A(r,t), 1 - \inf I_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)] > | (r,t) \in G \times T\} \cap \{(r,t), < [\inf F_B(r,t), \sup F_B(r,t)], [1 - \sup I_B(r,t), 1 - \inf I_B(r,t)], [\inf T_B(r,t), \sup T_B(r,t)] > | (r,t) \in G \times T\}$$

$$= \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\sup T_A(r,t), \sup T_B(r,t)]], [\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup I_A(r,t), \sup I_B(r,t)]], [\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > | (r,t) \in G \times T\}$$

$$\overline{(\bar{A} \cap \bar{B})} = \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]], [\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]], [\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > | (r,t) \in G \times T\}$$

$$= A \cup B$$

Hence, this completes the proofs of the proposition.

Proposition 3.6 *The following law holds good for every IVTNFS A:*

i. A ∪ A = A

ii. A ∩ A = A

Proof: Let

$$A = \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

Take the union operation for the same set A, we have

$$= \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\} \cup \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

$$= \{(r,t), < [\max[\inf T_A(r,t), \inf T_A(r,t)], \max[\sup T_A(r,t), \sup T_A(r,t)]], [\min[\inf I_A(r,t), \inf I_A(r,t)], \min[\sup I_A(r,t), \sup I_A(r,t)]], [\min[\inf F_A(r,t), \inf F_A(r,t)], \min[\sup F_A(r,t), \sup F_A(r,t)]] > | (r,t) \in G \times T\}$$

$$= \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

$$= A$$

Take the intersection operation for the same set A, we have

$$= \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\} \cap \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

$$= \{(r,t), < [\min[\inf T_A(r,t), \inf T_A(r,t)], \min[\sup T_A(r,t), \sup T_A(r,t)]], [\max[\inf I_A(r,t), \inf I_A(r,t)], \max[\sup I_A(r,t), \sup I_A(r,t)]], [\max[\inf F_A(r,t), \inf F_A(r,t)], \max[\sup F_A(r,t), \sup F_A(r,t)]] > | (r,t) \in G \times T\}$$

$$= \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)], [\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > | (r,t) \in G \times T\}$$

$$= A.$$

This completes the proposition.

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

Proposition 3.7 *Let A, B and C be the interval valued temporal neutrosophic fuzzy sets on G. Then for every IVTNFS, we have the following*

i. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

ii. $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Proof: Let

$A = \{ < (r,t), [\inf \mu_A(r,t), \sup \mu_A(r,t)], [\inf T_A(r,t), \sup T_A(r,t)],$
$[\inf I_A(r,t), \sup I_A(r,t)], [\inf F_A(r,t), \sup F_A(r,t)] > |(r,t) \in G \times T\}$

and

$B = \{ < (r,t), [\inf \mu_B(r,t), \sup \mu_B(r,t)], [\inf T_B(r,t), \sup T_B(r,t)],$
$[\inf I_B(r,t), \sup I_B(r,t)], [\inf F_B(r,t), \sup F_B(r,t)] > |(r,t) \in G \times T\}$

By the definition of union,

$A \cup B = \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]],$
$[\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]],$
$[\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\}$

By the definition of intersection,

$A \cap B = \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\text{supe}\, T_A(r,t), \sup T_B(r,t)]],$
$[\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup Ih_A(r,t), \sup I_B(r,t)]],$
$[\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\}$

i. LHS: $(A \cup B) \cap C$

$= \{(r,t), < [\max[\inf T_A(r,t), \inf T_B(r,t)], \max[\sup T_A(r,t), \sup T_B(r,t)]],$
$[\min[\inf I_A(r,t), \inf I_B(r,t)], \min[\sup I_A(r,t), \sup I_B(r,t)]],$
$[\min[\inf F_A(r,t), \inf F_B(r,t)], \min[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\} \cap C$

$= \{(r,t),$
$< [\min[\max[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \min[\max[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$
$[\max[\min[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \max[\min[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$
$[\max[\min[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \max[\min[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$
$>\}$

$= \{(r,t),$
$< [\max[\min[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \max[\min[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$
$[\min[\max[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \min[\max[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$
$[\min[\max[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \min[\max[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$
$>\}$

Now,

$A \cap C = \{(r,t), < [\min[\inf T_A(r,t), \inf T_C(r,t)], \min[\sup T_A(r,t), \sup T_C(r,t)]],$
$[\max[\inf I_A(r,t), \inf I_C(r,t)], \max[\sup I_A(r,t), \sup I_C(r,t)]],$
$[\max[\inf F_A(r,t), \inf F_C(r,t)], \max[\sup F_A(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$

$B \cap C = \{(r,t), < [\min[\inf T_B(r,t), \inf T_C(r,t)], \min[\sup T_B(r,t), \sup T_C(r,t)]],$
$[\max[\inf I_B(r,t), \inf I_C(r,t)], \max[\sup I_B(r,t), \sup I_C(r,t)]],$
$[\max[\inf F_B(r,t), \inf F_C(r,t)], \max[\sup F_B(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$

RHS: $(A \cap C) \cup (B \cap C)$

$= \{(r,t), < [\min[\inf T_A(r,t), \inf T_C(r,t)], \min[\sup T_A(r,t), \sup T_C(r,t)]],$
$[\max[\inf I_A(r,t), \inf I_C(r,t)], \max[\sup I_A(r,t), \sup I_C(r,t)]],$
$[\max[\inf F_A(r,t), \inf F_C(r,t)], \max[\sup F_A(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\} \cup$
$\{(r,t), < [\min[\inf T_B(r,t), \inf T_C(r,t)], \min[\sup T_B(r,t), \sup T_C(r,t)]],$
$[\max[\inf I_B(r,t), \inf I_C(r,t)], \max[\sup I_B(r,t), \sup I_C(r,t)]],$
$[\max[\inf F_B(r,t), \inf F_C(r,t)], \max[\sup F_B(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$

$= \{(r,t),$
$< [\max[\min[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \max[\min[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$
$[\min[\max[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \min[\max[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

$[\min[\max[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \min[\max[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$
$>\}$

Hence, $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

ii. LHS: $(A \cap B) \cup C$
$= \{(r,t), < [\min[\inf T_A(r,t), \inf T_B(r,t)], \min[\sup T_A(r,t), \sup T_B(r,t)]],$
$[\max[\inf I_A(r,t), \inf I_B(r,t)], \max[\sup I_A(r,t), \sup I_B(r,t)]],$
$[\max[\inf F_A(r,t), \inf F_B(r,t)], \max[\sup F_A(r,t), \sup F_B(r,t)]] > |(r,t) \in G \times T\} \cup C$

$= \{(r,t),$
$< [\max[\min[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \max[\min[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$
$[\min[\max[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \min[\max[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$
$[\min[\max[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \min[\max[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$
$>\}$

Now,
$A \cup C = \{(r,t), < [\max[\inf T_A(r,t), \inf T_C(r,t)], \max[\sup T_A(r,t), \sup T_C(r,t)]],$
$[\min[\inf I_A(r,t), \inf I_C(r,t)], \min[\sup I_A(r,t), \sup I_C(r,t)]],$
$[\min[\inf F_A(r,t), \inf F_C(r,t)], \min[\sup F_A(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$

$B \cup C = \{(r,t), < [\max[\inf T_B(r,t), \inf T_C(r,t)], \max[\sup T_B(r,t), \sup T_C(r,t)]],$
$[\min[\inf I_B(r,t), \inf I_C(r,t)], \min[\sup I_B(r,t), \sup I_C(r,t)]],$
$[\min[\inf F_B(r,t), \inf F_C(r,t)], \min[\sup F_B(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$

RHS: (A ∪ C) ∩ (B ∪ C)
$= \{(r,t), < [\max[\inf T_A(r,t), \inf T_C(r,t)], \max[\sup T_A(r,t), \sup T_C(r,t)]],$
$[\min[\inf I_A(r,t), \inf I_C(r,t)], \min[\sup I_A(r,t), \sup I_C(r,t)]],$
$[\min[\inf F_A(r,t), \inf F_C(r,t)], \min[\sup F_A(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\} \cap$
$\{(r,t), < [\max[\inf T_B(r,t), \inf T_C(r,t)], \max[\sup T_B(r,t), \sup T_C(r,t)]],$
$[\min[\inf I_B(r,t), \inf I_C(r,t)], \min[\sup I_B(r,t), \sup I_C(r,t)]],$
$[\min[\inf F_B(r,t), \inf F_C(r,t)], \min[\sup F_B(r,t), \sup F_C(r,t)]] > |(r,t) \in G \times T\}$

$= \{(r,t),$
$< [\min[\max[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \min[\max[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$
$[\max[\min[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \max[\min[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$
$[\max[\min[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \max[\min[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$
$>\}$

$= \{(r,t),$
$< [\max[\min[\inf T_A(r,t), \inf T_B(r,t)], \inf T_C(r,t)], \max[\min[\sup T_A(r,t), \sup T_B(r,t)], \sup T_C(r,t)],$
$[\min[\max[\inf I_A(r,t), \inf I_B(r,t)], \inf I_C(r,t)], \min[\max[\sup I_A(r,t), \sup I_B(r,t)], \sup I_C(r,t)],$
$[\min[\max[\inf F_A(r,t), \inf F_B(r,t)], \inf F_C(r,t)], \min[\max[\sup F_A(r,t), \sup F_B(r,t)], \sup F_C(r,t)]$
$>\}$

Hence, (A ∩ B) ∪ C = (A ∪ C) ∩ (B ∪ C).

4. Conclusion

We have defined a new extension of Intuitionistic Fuzzy Sets, namely, Interval Valued Temporal Intuitionistic Fuzzy Sets and studied various basic operations like combination, connection, separation and complement. We have proved the commutatively and Associative of union and intersections and the distributive law of one over the other. Also, we have proved the idempotence law and demorgan's law. The defined IVTIFS is useful in many applications. It is open to check the newly defined IVTIFS in the real time applications such as medical diagnosis, electrol system, career determination and pattern recognition and so on.

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023

Acknowledgement

The authors are in truth grateful to the beloved Editor in Chief and well-regarded reviewers for all their valuable comments and guidelines. We do not receive any external funding.

Conflicts of Interest: The author has no conflicts of significance to argue regarding the article.

References

[1] Atanassov K. T., "Intuitionistic fuzzy sets", Fuzzy Sets and Systems, vol.20, pp. 87 – 96, 1986.

[2] Broumi, S., Mohanaselvi, S., Witczak, T., Talea, M., Bakali, A., & Smarandache, F. (2023). Complex fermatean neutrosophic graph and application to decision making. Decision Making: Applications in Management and Engineering, 6(1), 474-501.

[3] Broumi, S., Raut, P. K., & Behera, S. P. (2023). Solving shortest path problems using an ant colony algorithm with triangular neutrosophic arc weights. International Journal of Neutrosophic Science, 20(4), 128-28.

[4] Atanassov K. T. and Gargov G., "Interval – Valued Fuzzy Sets", Fuzzy Sets and Systems, vol. 31, pp. 343 – 349, 1989.

[5] Atanassov K. T., "Temporal intuitionistic fuzzy sets", Comptes Rendusdel' Academie Bulgare, vol. 7, pp. 5 – 7, 1991.

[6] Anand M.C.J, Janani Bharatraj, "Interval-Valued Neutrosophic Numbers with WASPAS". In: Kahraman, C., Otay, İ. (eds) Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets. Studies in Fuzziness and Soft Computing, Springer, Cham, pp.435-453, 2019.

[7] De Supriya Kumar, Ranjit Biswas and Akhil Ranjan Roy, "Some operations on intuitionistic fuzzy sets", Fuzzy Sets and Systems, vol.114, no.3, pp. 477 – 484, 2000.

[8] Bharatraj J., M. Clement Joe Anand, "Power harmonic weighted aggregation operator on single-valued trapezoidal neutrosophic numbers and interval-valued neutrosophic sets". In: Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets. Springer International Publishing, Cham, pp. 45–62, 2019.

[9] Manshath, A., & Kungumaraj, E. (2021). "Applications of neutrosophic calculus in solving integration problems with reduction formulas." Journal of Computational Mathematics and Applications, 47(3), 214–225.

[10] Sudha. S, Martin. N , Anand. M. C. J., Palanimani. P.G., Thirunamakkani.T., Ranjitha. B., MACBETH – MAIRCA Plithogenic Decision Making on Feasible Strategies of Extended Producer's Responsibility towards Environmental Sustainability, International Journal of Neutrosophic Science, Vol.22 (2), 2023, 114 – 130.

[11] Sinem Yilmaz and Gokhan Cuvalcioglu, "On level operators for temporal Intuitionistic fuzzy sets", Notes on Intuitionistic Fuzzy Sets, vol. 20, no. 2, pp. 6 – 15, 2014.

[12] Smarandache F, Stanujkic D, Zavadskas EK, Brauers WKM and Karabasevic DA, "Neutrosophic extension of the MULTI-MOORA method". Informatica, vol. 28(1), pp. 181–192, 2017.

[13] Smarandache F, Zhang YQ, Sunderraman R and Wang H, "Single valued neutrosophic sets. Multispace Multistruct", vol. 4, pp.410-413, 2010

DOI: https://doi.org/10.54216/IJNS.230129
Received: February 27, 2023 Revised: April 11, 2023 Accepted: May 13, 2023