An update of our Condensed Matter or Superconductor Model of the Solar System based on Eilenberger equation

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Abstract

In the well-known Aharonov-Bohm effect, a charged particle experiences a phase shift as it moves through a region of space where the magnetic field is zero. The presence of a magnetic flux in the area, which influences the wave function of the particle, results in this phase shift. The Aharonov-type interaction, in which the phase shift is caused by a topological flaw in the system, such as a spin texture or a Berry phase, rather than a magnetic field, has attracted increasing interest in recent years. In this regards, in a recent paper, we argued in favour of Gross-Pitaevskii model as a more complete description of both solar system and spiral galaxies, especially taking into account the nature of chirality and vortices in galaxies (see Prespacetime J, 2021, & SMIC, 2020). In this paper, we will discuss shortly a nonlinear cosmology model inspired by analogy between cosmology phenomena and low temperature physics, especially via superconductor / superfluid vortices dynamics. We described: (a) a nonlinear cosmology model based on Navier-Stokes turbulence equations, which then they are connected to superfluid turbulence, and (b) the superfluid turbulence can lead to superfluid quantized vortices, which can be viewed as large scale version of Bohr's quantization rule, and (c) this superfluid quantized vortice interpretation of Bohr's rule allow us to predict quantization of planetary orbits in solar system including new possible orbits beyond Pluto. In more specific way. we apply the new model based on Bogoliubov-de Gennes equation correspondence with Bohr-Sommerfeld quantization rules. Then we put forth an argument that from Bohr-Sommerfeld quantization rules we can come up with a model of quantized orbits of planets in our solar system, be it for inner planets and also for Jovian planets. In effect we also tried to explain Sedna's orbit in the same scheme.

Keyword: Condensed Matter; Superconductor Model; Solar System; Aharonov-Bohm effect

1. Introduction

Occasionally, cosmology and astronomy revelations have opened our eyes that the universe is substantially more entangled than what it appeared in 100-200 years prior. What's more, regardless of all invading fame of General Relativistic augmentation to Cosmology, considering antiquated Greek rationalists' theories, for example, *hydor* model (*Thales*) and streaming liquid model (*Heracleitus*) it appears to be yet qualified to ask: does it imply that the Ultimate hypothesis that we attempt to discover ought to compare to hydrodynamics or a disturbance hypothesis?[1]
Cosmology models of various kind have been developed in the past decades, with the Lambda-CDM as accepted Standard Model. However, there are known problems with the so-called Lambda-CDM model which forms the basis

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of Big Bang Cosmology, one of these problems is that Lambda-CDM model is based on linear cosmology, while many phenomena in the Universe are mostly nonlinear in processes and nature.
In this paper, we will discuss shortly a nonlinear cosmology model inspired by analogy between cosmology phenomena and low temperature physics, especially superfluid vortex. We described: (a) a nonlinear cosmology model based on Navier-Stokes turbulence equations, which then they are connected to superfluid turbulence, and (b) the superfluid turbulence can lead to superfluid quantized vortices, which can be viewed as large scale version of Bohr's quantization rule, and (c) this superfluid quantized vortices interpretation of Bohr's rule allow us to predict quantization of planetary orbits in solar system including new possible orbits beyond Pluto.
Meanwhile, in a recent article, we presented some new arguments on the theoretical dwarf star thought to be a companion to our Sun, known as the Nemesis, which is postulated to explain a perceived cycle of mass extinctions in Earth's history. Some speculated that such a star could affect the orbit of objects in the far outer solar system, sending them on a collision course with Earth. While recent astronomical surveys failed to find any evidence that such a star exists, we outline in this article some theoretical findings including our own serendipity findings, suggesting that such a dwarf star companion of the Sun remains a possibility[2]. And one good indicator for such a dwarf companion of the Sun is Sedna, a planetoid which has been discovered around 2004 by Mike Brown and his Caltech team. Sedna location and eccentric orbit are such that it is not supposed to be there[1,6,14].
Therefore, a physical explanation of why Sedna is located there can be a good start to begin to search the existence and location of the supposedly dwarf companion of the Sun.

2. Bohr-Sommerfeld Quantization Rules And Quantised Vortices Approach

In this section, we will review the work which was carried out by me and with kind help of Prof. Florentin Smarandache from UNM, during the past ten years or so. The basic assumption here is that the Solar System's planetary orbits are quantized. But how do their orbits behave? Do they follow Titius-Bode's law? Our answer can be summarized as follows:

Navier-Stokes equations superfluid quantized vortices Bohr's quantization rule

Our predictive model based on that scheme has yielded some interesting results which may be comparable with the observed orbits of planetoids beyond Pluto, including what is dubbed as Sedna. And it seems that the proposed model is slightly better compared to Nottale-Schumacher's gravitational Schrödinger model and Titius-Bode's empirical law. See table 1.
Here we present Bohr-Sommerfeld quantization rules for planetary orbit distances, which results in a good quantitative description of planetary orbit distance in the Solar system[1].
First of all, let us point out some motivations for utilization of Bohr-Sommerfeld quantization rules: (a) the neat correspondence between Bohr-Sommerfeld quantization rules and topological quantization as found in superfluidity, and (b) there is neat correspondence between Bogoliubov de Gennes and generalized Bohr-Sommerfeld quantization can be applied to large scale systems like Solar system. (c) In the next section, we suggest another alternative approach, i.e. Eilenberger equation, which reduces to scalar model of Riccati equation[20]. As we have discussed how Riccati equation can be neatly linked to Newton equation, then it seems possible to utilize this approach too.
Sonin's preface in his book can be paraphrased as follows[4]:
"The movement of vortices has been a region of study for over a century. During the old-style time of vortex elements, from the late 1800s, many fascinating properties of vortices were found, starting with the outstanding Kelvin waves engendering along a disconnected vortex line (Thompson, 1880). … The circumstance changed after crafted by Onsanger (1949) and Feynman (1955) who uncovered that turning superfluids are strung by a variety of vortex lines with quantized dissemination. With this revelation, the quantum time of vortex elements started."
The quantization of circulation for nonrelativistic superfluid is given by[3] :

$$\oint v dr = N \frac{\hbar}{m_s} \qquad (1)$$

where $N, \hbar, m_s$ represent winding number, reduced Planck constant, and superfluid particle's mass, respectively. And the total number of vortices is given by[3]:

$$N = \frac{\omega\, 2\pi\, r^2 m}{\hbar} \qquad (2)$$

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And based on the above equation (2), Sivaram & Arun[16] are able to give an estimate of the number of galaxies in the universe, along with an estimate of the number stars in a galaxy. However, they do not give explanation between the quantization of circulation (3) and the quantization of angular momentum. According to Fischer[17], the quantization of angular momentum is a relativistic extension of quantization of circulation, and therefore it yileds Bohr-Sommerfeld quantization rules.
Furthermore, it was suggested that Bohr-Sommerfeld quantization rules can yield an explanation of planetary orbit distances of the Solar system and exoplanets[1,15]. Here, we begin with Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p\, dx = 2\pi\, n\hbar, \quad (3)$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is:

$$\int_0^T v^2\, d\tau = \omega^2 T = 2\pi\omega, \quad (4)$$

where $T = \frac{2\pi}{\omega}$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum) $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}. \quad (5)$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (4), a new constant g was introduced:

$$mvr = \frac{ng}{2\pi}. \quad (6)$$

Just like in the elementary Bohr theory (just before Schrodinger), this pair of equations yields known simple solution for the orbit radius for any quantum number of the form:

$$r = \frac{n^2 g^2}{4\pi^2 GMm^2}, \quad (7a)$$

or

$$r = \frac{n^2 GM}{v_0^2}, \quad (7b)$$

where r, n, G, M, v_0 represent orbit radii (semimajor axes), quantum number ($n = 1,2,3,\ldots$), Newton gravitation constant, mass of the nucleus of orbit, and specific velocity, respectively. In equation (10), we denote:

$$v_0 = \frac{2\pi}{g} GMm. \quad (8)$$

The value of m, g in equation (8) are adjustable parameters.
Interestingly, we can remark here that equation (7b) is exactly the same with what is obtained by Nottale using his Schrodinger-Newton formula[16]. Therefore here we can verify that the result is the same, either one uses Bohr-Sommerfeld quantization rules of Schrodinger-Newton equation. The applicability of equation (7b) includes that one can predict new exoplanets (i.e., extrasolar planets) with remarkable result.
Therefore, one can find a neat correspondence between Bohr-Sommerfeld quantization rules and motion of quantized vortices in condensed-matter systems, especially in superfluid helium[1,21]. Here we propose a conjecture that superfluid vortices quantization rules also provide a good description for the planetary orbits in our Sollar System. An idea that given the chemistry composition of Jovian planets are different from inner olanets began around 15 years ago, therefore it is likely both series of planets have different origin. By assuming inner planets orbits have different quantum number from Jovian planets, here by using "least square difference" method in order to seek the most optimal straight line for Jovian planets orbits in a different quantum number. Then it came out that such a straight line can only be modelled if we assume that the Jovian planets were originated from a twin star system: the Sun and its companion, using the notion of $\mu = \frac{m_1 m_2}{m_c}$ is the reduces mass. Although based on statistical optimization[21,22], it yields new prediction of 3 planetoids in the outer orbits beyond Pluto, from which prediction, Sedna was discovered later by Mike Brown et al. (2004).

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Table 1: Comparison between Laurent Nottale's results, Titius-Bode law, and CSV

Object	No.	Titius	Nottale	CSV	Observ.	Δ, %
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	−6.95
Mars	6	16	15.4	15.4	15.24	−1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	−10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	−7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	−3.72
2003EL61	7			557.7	520	−7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

3. Eilenberger Equation Reduces To Scalar Riccati Equation

In this section, we suggest another alternative approach, i.e. Eilenberger equation, which reduces to scalar model of Riccati equation[20]. As we have discussed how Riccati equation can be neatly linked to Newton equation, then it seems possible to utilize this approach too[20]. In his paper, Schopohl wrote:

"A new parametrization of the Eilenberger equations of superconductivity in terms of the solutions to a scalar differential equation of the Riccati type is introduced. It is shown that the quasiclassical propagator, and in particular the local density of states, may be reconstructed, without explicit knowledge of any eigenfunctions and eigenvalues, by solving a simple initial value problem for the linearized Bogoliubov-de Gennes equations. The Riccati parametrization of the quasiclassical propagator leads to a stable and fast numerical method to solve the Eilenberger equations.[19]"

Therefore, it appears that we can utilize Eilenberger equation which is an alternative to Bogoliubov-De Gennes equation for description of superconductors. According to Schopol, the Eilenberger reduces to Riccati equation:

$$h_{vF}\frac{\partial}{\partial x}b_x + [2\bar{\varepsilon}_n + \Delta(x)b_x] + \Delta^{\dagger}(x) = 0, \qquad (9)$$

which after some steps it will yield a system of coupled Riccati ODEs. Interestingly it can be shown that angular momentum conservation combined with power law potentials can be recast into a Riccati ODE:

$$\frac{1}{2}m\dot{r}^2 + \left(\frac{1}{2}+\frac{1}{\epsilon}\right)\frac{1}{mr^2} - \frac{m\ddot{r}r}{\epsilon} - E = 0. \qquad (10)$$

Therefore, our hypothesis is that such a Riccati ODE (10) may be linked to scalar Riccati ODE as a reduction to Eilenberger equation. Numerical solution of equation (10) can be done with Mathematica or other computer algebra software.

In retrospect, we can recall the fact that there is a known Pioneer anomaly, which can be interpreted as an anomalous (scalar) acceleration after the Pioneer spacecraft enters the Jupiter's orbit and on. Therefore, it can be interpreted as a possible indicator of the existence of scalar effect of Riccati ODE.

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4. Proof Of Concept: Derivation Of Paired Bohr-Sommerfeld Quantization From Eilenberger Equation And Bogoliubov-De Gennes Equations

In the aforementioned sections in a preceding paper, we argued in favor of reasoning that Bohr quantization rule can be derived from Eilenberger equations as well as Bogoliubov-de Gennes equations (cf. Umniyati, Christianto, Smarandache, SMIC 2020). More than that, provided that Bogoliubov-de Gennes equations are a model for superconductors / superfluidity, then it follows that such a derivation suggests that BdG equations can offer physical explanation of origin of Bohr radius and Bohr quantization, beyond merely Schrodinger equation. To put that argument in other way, we may hypothesize a 3D space composed of quantum superconductor crystal,[i] such as in arguments by G. Gremaud, then the effect includes superconductor's effect, for instance spin supercurrent is likely to be measureable. To provide an outline of proof of this concept, we outlined here derivation of paired Bohr-Sommerfeld quantization from EILENBERGER EQUATION and BdG equations.

The Eilenberger equation is a partial differential equation that describes the behavior of quasiparticles in a superconductor. The Bogoliubov-de Gennes equations can be derived from the Eilenberger equation by taking the limit of the equation in the low-temperature and weak-coupling limit.

To derive the Bogoliubov-de Gennes equations from the Eilenberger equation and plot the solutions, we can use the following code in Mathematica:

```
(* Define constants *)
hbar = QuantityMagnitude[Quantity["Reduced Planck Constant"]];
kb = QuantityMagnitude[Quantity["Boltzmann Constant"]];
vF = 1.5*10^6;
DebyeTemp = 300;
Delta0 = 1.76*kb*DebyeTemp;

(* Define parameters *)
xi = 1.0;
gamma = 0.1;
T = 0.1*Delta0/kb;
eta = hbar/(2*T);
D = hbar*vF/Pi/DebyeTemp;
l = 10*xi;

(* Define Eilenberger equation *)
eilenberger = I eta D[x]^2 f[x, t] == (2 Delta[x, t] + gamma f[x, t]) g[x, t] - gamma Conjugate[f[x, t]] h[x, t] - 2 I xi g[x, t];

(* Derive BdG equations *)
bdg = {I D[x] u[x, t] == Delta[x, t] v[x, t] + gamma u[x, t],
    -I D[x] v[x, t] == Delta[x, t] Conjugate[u[x, t]] + gamma v[x, t],
    I D[x] g[x, t] == Delta[x, t] Conjugate[f[x, t]] + gamma g[x, t],
    -I D[x] h[x, t] == Delta[x, t] f[x, t] + gamma h[x, t]};

(* Define boundary conditions *)
bc = {u[-l, t] == Exp[-I Delta0 t/hbar], v[-l, t] == 0, g[-l, t] == 0, h[-l, t] == Conjugate[Exp[-I Delta0 t/hbar]],
   u[l, t] == 0, v[l, t] == 0, g[l, t] == 0, h[l, t] == 0};

(* Solve equations numerically *)
{uSol, vSol, gSol, hSol} = NDSolveValue[{bdg, eilenberger, bc}, {u, v, g, h}, {x, -l, l}, {t, 0, 1}];

(* Plot solutions *)
Plot[{Abs[uSol[x, 0]], Abs[vSol[x, 0]], Abs[gSol[x, 0]], Abs[hSol[x, 0]]},
```


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```
 {x, -l, l}, PlotRange -> All,
 PlotStyle -> {Blue, Green, Red, Purple},
 AxesLabel -> {"x", "Wavefunction"},
 PlotLegends -> {"u", "v", "g", "h"}]
```

Mathematica code 1.[1]

In the above code, we first define the relevant constants, including Planck's constant hbar, the Boltzmann constant kb, the Fermi velocity vF, the Debye temperature DebyeTemp, and the superconducting gap Delta0. We then define the parameters of the system, including the coherence length xi, the impurity scattering rate gamma, the temperature T, the quasiparticle energy.

The Bogoliubov-de Gennes equations describe the behavior of superconductors and superfluids, including the formation of Cooper pairs. In the context of a hydrogen-antihydrogen molecule in water, these equations can be used to describe the behavior of the electron and positron pairs that form the molecule.

To derive the Bohr radius of a hydrogen-antihydrogen molecule of water from the Bogoliubov-de Gennes equations, we can use the following code in Mathematica:

```
(* Define constants *)
m = QuantityMagnitude[Quantity["Electron Mass"]];
q = QuantityMagnitude[Quantity["Elementary Charge"]];
hbar = QuantityMagnitude[Quantity["Reduced Planck Constant"]];
eps0 = QuantityMagnitude[Quantity["Vacuum Electric Permittivity"]];
k = QuantityMagnitude[Quantity["Boltzmann Constant"]];
T = 300;

(* Define parameters *)
a = QuantityMagnitude[Quantity["Bohr Radius"]];
v0 = 1000 q / (4 Pi eps0 a);
delta = 1.76 k T / hbar;
mu = m / 2 (1 + v0 / delta);

(* Define BdG equations *)
BdG = {-hbar^2/(2m) D[u[x], {x, 2}] + v0/(4 Pi eps0 (x - a/2)) u[x] - mu u[x] + delta v[x],
      -hbar^2/(2m) D[v[x], {x, 2}] + v0/(4 Pi eps0 (x - a/2)) v[x] + mu v[x] - delta u[x]};

(* Solve equations numerically *)
{xmin, xmax} = {-10^-9, 10^-9};
bc = {u[xmin] == 0, v[xmin] == 0, u[xmax] == 0, v[xmax] == 0};
{uSol, vSol} = NDSolveValue[{BdG == {0, 0}, bc}, {u, v}, {x, xmin, xmax}];

(* Compute Bohr radius *)
rho[x_] := Abs[uSol[x]]^2 - Abs[vSol[x]]^2;
r = NIntegrate[x rho[x], {x, xmin, xmax}]/NIntegrate[rho[x], {x, xmin, xmax}];

(* Plot solutions *)
Plot[{Abs[uSol[x]], Abs[vSol[x]], rho[x]}, {x, xmin, xmax},
 PlotStyle -> {Blue, Green, Red},
 AxesLabel -> {"x", "Wavefunction"},
 PlotLegends -> {"u", "v", "rho"}]
```

Mathematica code 2[2]

[1] The above code were generated by ChatGPT/OpenAI.
[2] The above code were generated by ChatGPT/OpenAI.

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In the above code, we first define the relevant constants using QuantityMagnitude to obtain their numerical values in SI units. We then define the parameters of the system, including the Bohr radius a of the hydrogen-antihydrogen molecule, the potential energy v0, the superconducting gap delta, and the chemical potential mu. Next, we define the Bogoliubov-de Gennes equations as a set of coupled differential equations, BdG. We then solve these equations numerically using NDSolveValue, with boundary conditions set such that the wavefunctions u and v vanish at the edges of the domain. We then compute the Bohr radius r by integrating the density rho of the electron-positron pair, which is given by the difference between the squared magnitudes of the wavefunctions u and v. Finally, we plot the solutions for the wavefunctions and density using Plot, with the x-axis.

5. Further Evidences: Superfluidity Of Solar Interior And Pairing Of Tno Objects

In the sections, we put forth an argument in favor of low temperature physics model of solar system, in particular using Bogoliubov-de Gennes equations which are normally utilized for superconductors. While this makes the model a bit simpler and comprehensible, one may ask what are other evidences available to justify the BdG model for the Solar system. In this regard, allow us to submit two evidences which seem correspond to the model we suggested above.

First, the BdG model can be related to pairing of electrons, and as it has been discussed for instance in [29], when it is stated:

It is shown that the Bogoliubov-de Gennes equations pair the electrons in states which are linear combinations of the normal states… For a homogeneous system, we point out that the kernel of the self-consistency equation derived from the Bogoliubov-de Gennes equations needs to be constrained by the BCS pairing condition.

In this regard, we can point out that Pluto and Charon seem like evidence related to this pairing condition. Furthermore, Sedna also has a pair planetoid. We can expect that planetoids found around Kuiper Belt (or may be dubbed as TNOs) can take place in pairs.

Second, we can point out the Solar interior which has superfluid inner structure as another evidence. This fact has been observed and reported by Prof. Oliver K. Manuel: [23-24]

"Measurements are reviewed showing that the interior of the Sun, the inner planets, and ordinary meteorites consist mostly of the same elements: iron, oxygen, nickel, silicon, magnesium, sulfur and calcium. These results do not support the standard Solar model."

And in more recent paper Manuel *et al.* wrote in favor of superfluid model of solar interior:

"The present magnetic fields are probably deep-seated remnants of very ancient origin. These could have been generated from two mechanisms. These are: a) Bose-Einstein condensation of iron-rich, zero-spin material into a rotating, superfluid, superconductor surrounding the Solar core and/or b) superfluidity and quantized vortices in nucleon-paired Fermions at the core."[25]. Other hint for physical evidence of superconductor/superfluidity nature of solar system may be found in icy dwarf nature of some planetoids and other TNOs objects and other objects beyond Kuiper Belt.

As with potential location to find the dwarf companion of the Sun, we can mention briefly here that since 2017, there is an object dubbed as G1.9 which was observed around 60-66 AU (around Pluto/Kuiper Belt). We can also note here: while some literatures argue that G1.9 is remnant of supernovae [27-29], others argue that G1.9 cannot be a supernova, instead it is more plausible to argue that G1.9 is brown dwarf star. Therefore, it can be a good start to find out whether the G1.9 is indeed the dwarf companion that we're looking for all along. See also [27a].

6. A few remarks on superfluid model of the solar system

1. Remark on Ceres and lessons on missing Maldek

As we know from history of astronomy, there were Germany researchers such as J. Titius and J. Bode who have discovered planetary orbits following progression of integers. One interesting piece of such an approach is the missing body between Mars and Jupiter. Around 1801, an Italian astronomer named Giuseppe Piazzi found *Ceres*, the first

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asteroid around the same place of missing body between Mars and Jupiter. What is interesting here, from ancient cosmology in the past, we know that Ceres and other asteroids found around the same orbit postulated by Bode, were likely to be remnants of a missing old Planet called *Maldek*. Maldek was supposed to be inhabited by ancient civilization in the past, whom were caught in nuclear wars and suddenly the planet was exploded. This story apparently tells us something to our generation:
(a) technology advancement has distinct characteristics, that is : they can lead to self-destruction path of a civilization (see existential risk findings by Nick Bostrom etc)
(b) world leaders need to learn from that lesson of Maldek extinction before starting next global wars..

2. Remark on plausibility of twin suns

We know that there is already an Archeoastronomy working group. Associated with twin suns (twin sun models) in aboriginal tribe cosmogony and the Mayans. By coincidence in the last few years, we developed a symmetric model of the solar system, thus hypothesizing a solar twin in the form of a dwarf star (We call it a negative mass star/NMS).... And negative mass is generally difficult for physicists to digest unless they are concerned with solid state physics. But just yesterday it was discovered that it turns out that the twin stars hypothesis is also known by the ancient aboriginal tribe and the Mayan tribe.

3. Just to give a glance look at definition of negative mass in Wannier theorem, or known as Wannier-Slater Effective Mass Theory
In the approach to effective-mass theory proposed by Wannier and expounded by Slater, the microscopic wavefunction is expanded as a linear combination of localized Wannier functions, each of which is centered within a different unit cell. The expansion coefficients can be regarded as values of a discrete lattice function which is interpolated between lattice points by a continuous function , for which an effective-mass Schroedinger equation is derived.[31]
"The effective mass is defined to relate an external force with the group velocity of the wave packages. Electrons in an almost empty band, have an effective mass normal, but different from the mass of electron in vacuum. Electrons in an almost filled band, have a NEGATIVE EFFECTIVE MASS. This feature allows us to work them as positive particles with positive effective mass, that is, as holes. To change a negative particle moving to left for a positive particle moving to right is a very common trick. It is used when defining conventional current. However, nobody talks about holes in this situation. If you want to keep the illusion that conventional current is made from positive particles, you would be obliged to accept a negative mass. That is what is special about holes. Since electrons in the top of the valence band has a negative effective mass, when we change the sign of charge, direction and mass, we end with a perfect positive particle, with positive charge, moving in the opposite direction as the original electron and a positive effective mass." [30]

7. Discussion And Concluding Remark

In this paper, we present an argument that Bohr-Sommerfeld quantization condition can be linked to Bogoliubov-de Gennes equations, and in turn it can be shown that such a Bohr-Sommerfeld quantization can be linked to large scale structure quantization such as our solar system.
As with potential location to find the dwarf companion of the Sun, we can mention briefly here that since 2017, there is an object dubbed as G1.9 which was observed around 60-66 AU (around Pluto/Kuiper Belt). Therefore it can be a good start to find out whether the G1.9 is indeed the dwarf companion that we're looking for all along.

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References

[1] Christianto V. A Cantorian superfluid vortex and the quantization of planetary motion. *Apeiron*. 2004;11(1):112-52.
[2] Christianto V, Smarandache F. On the possibility of binary companion of the Sun. *Prespacetime J*. 2020;11(1):

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[3] Christianto V, Smarandache F, Umniyati Y. Towards Gross-Pitaevskiian Description of Solar System and Galaxies. *Prespacetime J*. 2020;11(3):
[4] Sonin E B. Dynamics of quantized vortices in superfluids. Cambridge: Cambridge University Press; 2016.
[5] Huang K. A superfluid Universe. Singapore: World Scientific Publishing Co. Pte. Ltd.; 2016.
[6] Christianto V, Smarandache F, Umniyati Y. A Derivation of Fluidic Maxwell-Proca Equations for Electrodynamics of Superconductors and Its Implication to Chiral Cosmology Model. *Prespacetime* J. 2018;9(7):
[7] Smarandache F, Christianto V. Schrodinger Equation and the Quantization of Celestial Systems. *Prog.In Phys*. 2006;2:5-12.
[8] Christianto V, Rapoport D L, Smarandache F. Numerical Solution of Time-Dependent Gravitational Schrodinger Equation. *Prog. In Phys*. 2007;2:9-11.
[9] Christianto V, Smarandache F. An Exact Navier Stokes equation to Schrodinger equation via Riccati equation. *Prog. In Phys*. 2008;1:
[10] Christianto V. An Exact Solution of Riccati Form of Navier-Stokes Equations with Mathematica. Prespacetime J. 2015;6(7):
[11] Enns R H, McGuire G C. *Nonlinear Physics with Mathematica for Scientists and Engineers.* Berlin: Birkhauser; 2001.
[12] Hassani S. *Mathematical Methods using Mathematica: For Students of Physics and Related Fields*. New York: Springer-Verlag New York Inc.; 2003.
[13] Nurgaliev I S. Singularities are averted by vortices. *Gravitation and Cosmology*. 2010;16(4):313-15.
[14] Christianto V. On the origin of macroquantization in astrophysics and celestial motion. *Annales de la Fondation Louis de Broglie*. 2006;31(1):
[15] Nottale L. Astron. Astrophys. 1997;327:867-89.
[16] Sivaram C, Arun K. Primordial rotation of the Universe, Hydrodynamics, Vortices and angular momenta of celestial objects. *The Open Astronomy J.* 2012;5:7-11.
[17] Fischer U. Motion of quantized vortices as elementary objects. *Ann Phys*. 1999;278:62-85.
[18] Nowakowski M, Rosu H C. Newton's Laws of Motion in Form of Riccati Equation. Phys. Rev. E. 2002;65:
[19] Least square optimization. Available from: https://www.cns.nyu.edu/~eero/NOTES/leastSquares.pdf
[20] Manuel O, Friberg S. Composition of the Solar Interior: Information from isotope ratios. *Proc. 2002 SOHO 12, Local and Global Helioseismology: The Present and Future.* 2003:345-48.
[21] Manuel O. The Standard Solar Model vs Experimental Observations. *Proc. The Third Int. Conf. on Beyond Standard Model Phys*. 2003:307-16.
[22] Manuel O K, Ninham B W, Friberg S E. Superfluidity in the Solar Interior: Implications for Solar Eruptions and Climate. *J. Fusion Energy*. 2002;21:193-98.
[23] Discovery! Possible dwarf planet found far beyond Pluto orbit. Available from: https://www.universetoday.com/tag/oort-cloud/page/3/
[24] Stephen Reynolds, et al. THE YOUNGEST GALACTIC SUPERNOVA REMNANT: G1.9+0.3, url: https://arxiv.org/abs/0803.1487v2; see also [27a] Nemesis, brown dwarf start has been discovered. url: https://www.youtube.com/watch?v=Ev7YhmKv_80
[25] K.J. Borkowski et al. SUPERNOVA EJECTA IN THE YOUNGEST GALACTIC SUPERNOVA REMNANT G1.9+0.3. THE ASTROPHYSICAL JOURNAL LETTERS. url: https://arxiv.org/abs/1305.7399v1
[26] M.Z. Pavlovic. Hydrodynamical and radio evolution of young supernova remnant G1.9+0.3 based on the model of diffusive shock acceleration. MNRAS 000, 1–16 (2017). Url: https://arxiv.org/abs/1702.07418v2
[27] https://www.physicsforums.com/threads/what-is-a-hole-in-the-context-of-solid-state.230447/
[28] https://personal.utdallas.edu/~frensley/technical/currden/node3.html

[i] See Y. Umniyati, Christianto, Smarandache. A New Hypothesis of Spin Supercurrent as Plausible Mechanism of Biological Nonlocal Interaction, Synchronicity, Quantum Communication. Chapter at Intech Open book, Solar Wind, 2021. url: https://www.intechopen.com/chapters/81418

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