On the Accuracy of a 1U Cube-Sat on Low Orbit Depending on Extended Kalman Filter Algorithm in Mathematical Dual Space Transformation Formulas

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Abstract

The objective of this paper is to study and analyze the accuracy of 1 U cube-sat on low orbit by using the mathematical novel space of dual (non real) representation numbers and with Extended Kalman filter algorithm. Also, we analyze the numerical results and compare its dual representation with the classical representation by using the formulas of projective duality of spaces between dual case and classical case.

Keywords: Cube-sat; Kalman Algorithm; Orbit; Estimation; Dual space; Dual residue

1. Introduction

Scientists' interests in exploring the features of space and reaching an accurate understanding of the explanation of the occurrence of certain phenomena varied, so they began to think about finding mechanisms that allow observation and measurement by working on a set of accurate or approximate mathematical equations, and they had to verify their correctness, so they made spacecraft of different sizes and shapes suitable for the task at hand. With the development of space science, there have become multiple needs aimed at modifying the tasks of spacecraft located within their orbits, and this is not directly possible .Lim had to be manufactured from scratch for another vehicle, and this requires time and high cost, so it was proposed to manufacture small spacecraft with the lowest cost and lowest manufacturing time called cubic satellites with a weight ranging from 100G to 16kg volume units were used expressing the weight and dimensions of these moons of rank 1U and their dimensions 10*10*10 $cm^3$and the standard weight of this one is 1.33 kg [1].

The dual space is a novel mathematical space that helps with the representation of all mathematical and physical equations in with an additional higher dimension. We use this space to present some formulas for generalizing results presented in [11] with a novel approach depending on the concept of projective duality.

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The position of the cubic satellite [11]

The position of a cubic satellite is meant to be the inclination of a body-related sentence and its center is applicable and moving with it from a reference sentence centered on the center of the body and not moving with it as a result of applying rotational movements to it, that is, the amount of change in the angle of inclination of the moon from its fixed set of coordinates as a result of external disturbances during its movement within the orbit. The amount of change in angle and its constancy at a specific angle is one of the most important performance parameters that determine the stability of a cubic satellite, in other words, a cubic satellite is judged to be completely stable if the amount of change in angle is small, does not exceed 0.01 rad and is constant during its movement, and the term positioning of a cubic satellite is used to denote the amount of approach of the three satellite angles (Roll, Pitch, Yaw) during its movement from the required angles after applying position estimation algorithms . [2] To clarify the concept of position relative to a cubic satellite, we assume that the moon has a reference sentence, associated with it, centered with it and fixed (Fixed Frame) and another sentence associated with it and moving with it (Moving Frame), the angle of inclination is meant to be the deviation of the three axes of the moving body sentence (Moving Frame) from the reference sentence (Fixed Frame).

There are two sentences and some of their parameters, where the existence of the moving coordinate sentence represented by the coordinates (x,y,z) the fixed coordinate sentence represented by the coordinates (X,Y, Z) in addition to the position angles angle: Roll (Φ) is the angle of rotation about the X-axis, angle :Pitch(θ)is the angle of rotation about the axial, angle: Yaw (ψ) is the angle of rotation about the axis .the order of the three position angles (Φ, θ, ψ) will be adopted when dealing with the position of the cubic satellite.

Determining the position of a cubic satellite and its algorithms

There is a set of algorithms (methods) that are used in this field and are divided into three classifications: the first classification is position determination algorithms, the second classification is position estimation algorithms and the third classification is nonlinear observations [3-4].

The algorithms for determining the position (Attitude Determination Algorithms) are characterized by a low execution time, this is due to their lack of dependence on the kinematic Dynamic models representing the moon, but they are unable to determine the rest of the system states, such as determining the amount of disturbance in the measurements, and the reason is because their mechanism of action depends entirely on the values of measurements received from they are based on forecasting the values of system States and filtering error values. Among these algorithms are the least squares algorithm, The Quest method, the Q-method and the TRIAD algorithm [5].

Nonlinear observers are one of the methods of estimating the position of a cubic satellite and angular velocity changes, where they estimate the beam of the state depending on the inputs and outputs and the

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mathematical model of the system, and one of the best types of these observations is the SMC sliding mode monitor, which is characterized by the possibility of dividing the motion of any nonlinear system into independent particles with smaller dimensions, and this reduces the complexity of the system in addition to its low sensitivity to noise, but its disadvantages are replacing the original system with a new system close to it according to the desired requirements, and this causes moving away a little from representing all the states of the system, which in turn leads to disturbances in the functioning of the system other than taken into account [7], [6].

Satellite Attitude Estimation Algorithms [11]

Position estimation algorithms are characterized by good performance in terms of accuracy, in addition to estimating all the states of the system using sensors installed on the cubic satellite, but they take a long time to implement because they use kinematic Dynamic models of the moon, from these algorithms: Kalman filter, extended Kalman filter, imperceptible Kalman filter.[8] the mechanism of action of the extended Kalman filter will be presented in detail below.

Extended Kalman Filter [11]

The extended Kalman filter is an update of the Kalman filter used to estimate or predict some parameters or states in linear dynamical systems based on a set of measurements in order to reduce the mean square of the smallest error), it is a recursive state guesser, that is, it performs a set of estimates of the state beam (position and angular velocity estimation) and then makes appropriate corrections iteratively to reach an optimal solution to the values of the state beam, and also solves and addresses the nonlinear problems suffered by the Kalman filter after converting nonlinear models to linear according to the F transformation sequence and then finding the necessary estimates for different system States, [9]the equations for updating the filter times are given by the extended Kalman According to Table 1, the equations for updating the measurements of the extended Kalman Filter are given according to Table (2) , where the values estimated are denoted by an upper (-) sign:

Table 1: equations for updating the Times of the extended Kalman filter (prediction) [9].

Significance of the equation	**The equation**
The equation of State beam estimation	$\hat{x}_{\bar{k}} = f(\hat{x}_{k-1}, u_{k-1}, 0)$ (1)
The equation for estimating the error covariance matrix in the state beam	$P_{\bar{k}} = AP_{k-1}A^T + W_k Q_{k-1} W_k^T$ (2)
Measurement adjustment equation	$z_k = Hx_k + v_k$ (3)

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Table 2: equations for updating measurements of the extended Kalman filter [9].

Significance of the equation	The equation
$K_k = P_k^{-} H_k^T [H_k P_k^{-} H_k^T + V_k R_k V_k^T]^{-1}$ (4)	The formula for updating the candidate's profit
$\hat{x}_k = \hat{x}_k^{-} + K_k(z_k - h(\hat{x}_k^{-}, 0)$ (5)	The state beam update equation
$P_k = [1 - K_k H_k] P_k^{-}$ (6)	The equation for updating the error covariance matrix
$\lim_{P_k \to 0} K_k = H^{-1}$ (7) $\lim_{P_k \to 0} K_k = 0$ (8)	

We note from relation (7)that when the measurement error R ends up to zero, the filter gain is H^{-1} and by compensation in the condition beam update relation (5) it results that the new value of the condition beam is the measurement value, but if the error variance ends up to zero, then the filter gain will be equal to zero and therefore the value of the new condition Beam will be equal to the value of the estimate (prediction), that is, in short, the principle of this algorithm is based on two stages (prediction - correction) to reach solutions for filtering errors from the states of the studied system.

There are six steps shown in order that explain the principle of operation of the extended Kalman filter, namely:

The first step:

In this step, a set of values is set as the system matrices (A,B, H) and the initial values of the input, position and angular velocity are determined, in addition to determining the values of the error matrices for both estimating the system state (Q) and measurement .

The second step:

In this step, the first stage of the Kalman filter's work is carried out, which is to predict the state of the system $\hat{x}_{\bar{k}}$ (position) based on the initial values that have been set and then calculate the error in the state of the system $P_{\bar{k}}$

The third step:

After the prediction, a measurement of the state of the $z_k$system is performed using sensors or by mathematical methods depending on the state of the studied system.

The fourth step:

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After performing both the measurement and the forecast, the Kalman profit is calculated, which has the task of making a harmony between the prediction stage and the measurement stage, that is, to show the effect of the value with the lowest error, that is, if the value of the measurement error is large in this case, the measurement value will be excluded and the prediction value will be adopted, but if the error value is small, the measurement value is the one that adopts and neglects the prediction value.

The fifth step:

After calculating the value of the Kalman gain, correction and adjustment will be made to the value of the state beam of the system, as well as adjustment of the system state error covariance matrix

The sixth step:

In this step, the work is repeated starting from the second step, but taking into account that the prediction will be made based on the previous values of both the system state and the error estimation Matrix.

Developed position estimation algorithm [11]

The process of adjusting the angular velocity of the cubic satellite and the stability of its angles during its movement within the orbit are the basic parameters relied on to assess its stability, as in this research, the use of the quadratic representation was adopted to achieve the position of the satellite, which includes both the angular velocity and the angles of motion on the three coordinate axes, and in order to accurately estimate these two parameters, the extended Kalman filter was adopted because the nature of the equations of motion of the cubic satellite is not linear from the point of view and for its advantages on the other hand, with an amendment to its mechanism of operation by adding a seventh step to the stages of its operation, it was based on the idea that the two stages are forecasting and measurement and the measurement stage is considered a stage As for the prediction, it is used in order to give a prediction that is likely to be true and false in order to compare with the measurement value, hence the idea came to take advantage of this step to make it non-binding in each repetition, but provided that the amount of error change (the difference between the error of estimating the previous state with the error of estimating the new state) ∓1% in order to increase the accuracy in estimating the position, if it is achieved and the error falls within the mentioned area, then a new value will not be predicted (here the prediction error is eliminated) the values of the system state will depend However, if the error is greater than the domain values, then new values are predicted, perhaps they give an error of a case less than the measurement error, and the rest of the steps of the algorithm are followed, which was called the estimation Decision Algorithm and expressed by the abbreviated code ED, thus becoming the name of the candidate developer EDEKF (Estimation Decision Extended Kalman) it should be noted that this developed algorithm got its name from the fact that it focuses only on reducing estimation decisions and therefore it can then expresses the steps of the developed position estimation algorithm, where the modifications to the EKF filter algorithm were expressed within the scheme according to point

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2. Results and discussion

Table 3: The results that were taken from [11].

Exact value	The parameter
1U	Satellite size
600km	Elevation
$10*10*10cm^3$	Dimensions
1.33kg	Weight
96.57min	Orbit time
$[1.356*10^{-7}\ 2.7*10^{-7}]$N.m	The sum of the values of the momenta of external forces
Circular file 2*2	Operator
(15mm, 45mm, 90mm)	Operator dimensions
(0,0,0) deg	Required position angles
(0,0,0) rad/sec	Required angular velocity
PID	The controller

Table 4: Position accuracy degree

High position accuracy (deg)	Good position accuracy (deg)	Medium position accuracy (deg)	Acceptable position accuracy (deg)	Poor position accuracy (deg)
< 10	$]0.05 \div 0.9]$	$]1 \div 5]$	$]5 \div 10]$	> 10

Table 5: Position estimation algorithm and developed position estimation algorithm

Position estimation algorithm	EKF
The developed position estimation algorithm	EDEKF

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Dual space representation

The dual space is defined as a generalized number system $\{a + bJ;\ J.J = 0\}$. this space contains the space of real equations as a subspace with a higher additional dimension.

The results of the simulation according to dual representation:

The constants $k, g, h, m, n, v, a, b, c$ are real parameters can be chosen according to dual system with isometric projection on the real line.

Table 6: The results of the simulation according to dual representation

Exact value	**The parameter**
1+JU	Satellite size
600+kJ km	Elevation
10*10*10+hJ cm^3	Dimensions
1.33+gJ kg	Weight
96.57+J min	Orbit time
[1.356*10^{-7} 2.7*10^{-7}]N.m	The sum of the values of the momenta of external forces
Circular file 2*2	Operator
(15+lJ mm, 45+lJ mm, 90+lJ mm)	Operator dimensions
(0,0,0)+(a,b,c)J deg	Required position angles
(0,0,0)+(m,n,v)J rad/sec	Required angular velocity
PID	The controller

Table 7: Position accuracy degree in dual formula

High position accuracy in dual formula (deg)	**Good position accuracy in dual formula (deg)**	**Medium position accuracy in dual formula (deg)**	**Acceptable position accuracy in dual formula (deg)**	**Poor position accuracy in dual formula (deg)**
$< 10 + 10J$	$]0.05 + J \div 0.9 + 2J]$	$]1 + J \div 5 + J]$	$]5 + J \div 10 + J]$	$> 10 + J$

Table 8: Position estimation algorithm and developed position estimation algorithm

Position estimation algorithm	EKF
The developed position estimation algorithm	EDEKF

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3. Discussion of the canonical formulas of dual equations

For the diagrams that explain the classical simulation, see [11].

Now, we provide the mathematical transformation formulas that will be used for projective duality, and to build future simulations:

The first formula:

$$\int_{a+bJ}^{c+dJ} (f(x)+Jy)du = \sum_{1}^{n} f(x_i du_i)_i + \int^{c+d} \sqrt{f_i{}^\circ y}$$
$$du \in D(R)$$

$$\sum_{i}^{n} f(x_i du_i)_i = \sum_{J}^{n+J} \int f_i du_i$$

$$f_i{}^\circ y = f_i(y(x))$$

The second projective duality formula:

$$\begin{pmatrix} \sin t & -\cos t & \sec t \\ \cos t & \sin t & -\sec t \\ -\sec t & \cos t & \sin t \end{pmatrix} \frac{dy+J}{dx+J} = \|asint + bcost + dsect\| \begin{pmatrix} \frac{\Delta y+J}{\Delta x} & \frac{\Delta y}{\Delta x+J} & -\frac{\Delta y}{\Delta x} \\ \frac{\Delta y}{\Delta x} & \frac{\Delta y}{\Delta x+J} & \frac{\Delta y+J}{\Delta x} \\ \frac{\Delta y+J}{\Delta x+J} & \frac{\Delta y+J}{\Delta x} & -\frac{\Delta y}{\Delta x} \end{pmatrix}$$

$$\begin{pmatrix} \cos t & -\cos t & -\sec t \\ \sin t & -\sin t & \sec t \\ \sec t & \cos t & \cos t \end{pmatrix} \frac{dy+J}{dx+J} = \|(a+bt)sint - (a-bt)cost + (d+t)sect\| \begin{pmatrix} \frac{\Delta y}{\Delta x} & -\frac{\Delta y}{\Delta x+J} & -\frac{\Delta y}{\Delta x+J} \\ \frac{\Delta y}{\Delta x+J} & \frac{\Delta y}{\Delta x+J} & \frac{\Delta y+J}{\Delta x+J} \\ \frac{\Delta y+J}{\Delta x} & -\frac{\Delta y+J}{\Delta x+J} & -J\frac{\Delta y}{\Delta x} \end{pmatrix}$$

The third projective formula:

$$\lim_{n\to e} x_u + Jy_v = \sum_{1}^{e} x_e \int y_e + Jy_e \int x_e + JRes\,(x_e, y_e, e)$$

The approximated residue formula:

$$JRes\,(x_e, y_e, e) = Res\,(x_1, y_1, 1+J) + JRes\,(x_2, y_2, 2+2J) + Res\,(x_3, y_3, 3+3J) + \cdots + Res\,(x_e, y_e, e+eJ)$$

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Table 9: A comparison between real and dual representation of results

Classical case constant	Classical residue	Dual case constant representation	Approximated Dual residue	Approximated Expected error (order of error)
10	0	10+10J	0.001+0.06J	0.00001J
20	0	20+20J	0.0002+0.006J	0.00001+0.01J
100	0	100+100J	0.000001+0.0006J	0.000001+0.001J
200	0	200+200J	0.0000001+0.00006J	0.0000001+0.0001J
1000	0	1000+1000J	0.00000001+0.000006J	0.00000001+0.00001J
2000	0	2000+2000J	0.000000002+0.0000006J	0.00000001+0.J

These results need to be examined by a novel simulation system depended on mathematical projection model of dual space.

4. Conclusion and recommendations

In this paper is we studied the accuracy of 1 U cube-sat on low orbit by using the mathematical novel space of dual (non real) representation numbers and with Extended Kalman filter algorithm. Also, we analyzed the numerical results and compare its dual representation with the classical representation.

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