Neutrosophic Fuzzy Score Matrices: A Robust Framework for Advancing Medical Diagnostics

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Abstract

In this paper, we introduce the Single-Valued Neutrosophic Fuzzy Matrix (SVNFM), which consists of entries that are all single-valued neutrosophic fuzzy sets (SVNFS). Our objective is to provide a practical tool for dealing with uncertain and indeterminate input. To achieve this, we first define a neutrosophic fuzzy matrix (NFM) and discuss its fundamental properties. The use of various operations in decision making is a notable characteristic of single-valued neutrosophic fuzzy matrices (SVNFMs). In this paper, we propose a multi-criteria group decision-making method that incorporates novel operations on neutrosophic fuzzy matrices. Finally, we present a case study to demonstrate the effectiveness of the proposed strategy.

Keywords: Neutrosophic Fuzzy Sets; Single Valued Neutrosophic Fuzzy Sets;Neutrosophic Matrices;Neutrosophic Fuzzy Matrices

1 Introduction

Fuzzy sets(FS) were first developed by American mathematician and computer scientist Lotfi A. Zadeh,[1] Zadeh published "Fuzzy Sets," an influential article that served as the basis for fuzzy set theory and fuzzy logic. He pioneered the idea of fuzzy sets as a means of handling and resembling uncertainty and vagueness in logical and mathematical systems. In many disciplines, including artificial intelligence, control systems, decision-making, and pattern recognition, fuzzy sets are particularly helpful in addressing ambiguity and uncertainty. Fuzzy sets offer a way to represent and work with ambiguous information in a structured way. They fill the gap between qualitative and quantitative data, enabling more accurate modeling of uncertain real-world issues. Fuzzy sets are an effective mathematical tool to handle uncertainty and imprecision in a variety of applications. They make it possible for a more complex and adaptable representation.
As an extension of fuzzy set theory, Smarandache[2] created the idea of neutrosophy, which includes neutrosophic sets. Fuzzy sets can handle uncertainty, indeterminacy, and ambiguity in a variety of ways. Neutrosophic sets are a generalization of fuzzy sets. An element can belong to a set of neutrosophic sets that have varying degrees of truth, indeterminacy, and falsity. Compared to conventional binary membership functions used in fuzzy sets, the three-valued membership structure enables a more thorough representation of uncertainty. The toolkit for coping with ambiguous and insufficient information in mathematical and logical systems has been widened by Florentin Smarandache's contributions to neutrosophy and neutrosophic sets.

S. Das et al.[3] introduced the concept of a neutrosophic fuzzy set (NFS), which combines the features of Fuzzy Set (FS) and Neutrosophic Set (NS). This fusion resulted in the development of several innovative concepts. However, the NFS faces challenges when dealing with real-world problems due to the nonstandard

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interval of its neutrosophic components. To address this issue, the authors propose the use of single valued neutrosophic fuzzy set (SVNFS).[3] Furthermore, the authors suggest various set-theoretic operations, provide numerical illustrations, and discuss the similarity of SVNFS, along with inferring their properties.[3]

In terms of mathematics, the concept of matrices is expanded to accommodate neutrosophic information through the utilization of neutrosophic matrices.[4] A conventional matrix is composed of rows and columns containing numerical values. In contrast, a neutrosophic matrix allows its entries to possess values derived from both neutrosophic sets and real numbers. The degree of membership, non-membership, and indeterminacy of each element within a neutrosophic set is used to represent the possibility of an element simultaneously belonging to and not belonging to the set.

The structure of the paper is organized in the following manner. The initial section contains essential definitions and findings that are crucial for our study. Subsequent sections focus on defining and discussing the significance of the Single-Valued Neutrosophic Fuzzy Matrix (SVNFM). Additionally, these sections cover various SVNFM arithmetic operations and different types of SVNFM. In the main result section, we have successfully proven that the set of all single valued neutrosophic fuzzy matrices forms a vector space. The subsequent section will delve into the topic of using the NFM score matrix for medical diagnosis. Finally, the concluding section presents some concluding remarks and observations.

2 Related Works

This section presents notable research on Neutrosophic Matrices. Neutrosophic matrices originate from the field of neutrosophic set theory, which was first proposed by Smarandache.[19] Smarandache's fundamental contributions laid the groundwork for the advancement of neutrosophic matrices, providing a comprehensive structure to effectively manage situations including indeterminacy, vagueness, and uncertainty within a wider scope. A considerable study is dedicated to the utilization of neutrosophic matrices in decision-making procedures. Researchers, like Deli et al.,[20] investigated the incorporation of neutrosophic matrices into decision models, emphasizing their efficacy in managing intricate and ambiguous data. Neutrosophic matrices have become prominent in decision-making processes because they can effectively manage unclear and conflicting information. In a recent study conducted by Karaaslan and Faruk,[22] a thorough examination of neutrosophic decision matrices is presented, highlighting their importance in decision support systems. Recently, Rozina Ali[23] authored a work that focuses on the algebraic perspective of neutrosophic matrix theory. It imparts the reader with a comprehensive understanding of these structures. Al-Odhari[24] is investigating the algebraic operations of neutrosophic matrices that are associated with the concept of indeterminacy in real numbers. The ongoing development of neutrosophic matrices encompasses a range of research endeavors that jointly enhance our comprehension of their theoretical foundations and real-world implementations in numerous fields.

3 Preliminaries

Definition 3.1. [5]
Let E be the universal set, Then a **fuzzy set** X over E is defined by $X = \{(x, \mu_X(x))/x \in E$, where $\mu_X : E \to [0,1]$ is called the membership function of X. The value $\mu_X(x)$ for each $x \in E$,reflects the degree to which x is a member of the fuzzy set X.

Definition 3.2. [6]
Consider the universal set X and $x \in X$. A **Single Valued Neutrosophic Set (SVNS)** A in X is distinguished by function of truth-membership $\mathcal{T}_A$, function of indeterminacy-membership $\mathcal{I}_A$ and function of falsity-membership $\mathcal{F}_A$. For each point x in X, $\mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x) \in [0,1]$.Thus, a SVNS N is denoted by, $N = \{(x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x))|x \in X\}$

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Definition 3.3. [3]
Let E be the universal set.Then a **Neutrosophic Fuzzy Set(NFS)** A on E is defined by $A = \{(x, \mu_A(x), \mathcal{T}_A(x, \mu), \mathcal{I}_A(x, \mu), \mathcal{F}_A(x$ $E\}$ where each membership value is expressed by a truth, indeterminacy and falsity membership function which are respectively denoted as $\mathcal{T}_A(x, \mu), \mathcal{I}_A(x, \mu)$ and $\mathcal{F}_A(x, \mu)$. Moreover $\mathcal{T}_A, \mathcal{I}_A$ and $\mathcal{F}_A$ are real standard or non-standard subsets of $]0^-, 1^+[$, That is,
$\mathcal{T}_A : E \to]0^-, 1^+[$,
$\mathcal{I}_A : E \to]0^-, 1^+[$,
$\mathcal{F}_A : E \to]0^-, 1^+[$.
Where,$0^- \leq Sup(\mathcal{T}_A) + Sup(\mathcal{I}_A) + Sup(\mathcal{F}_A) \leq 3^+$.

Definition 3.4. [3]
Let E be the universal set. then **Single-valued Neutrosophic Fuzzy Set(SVNFS)** S on E is defined by $S = \{(x, \mu_S(x), \mathcal{T}_S(x, \mu), \mathcal{I}_S(x, \mu), \mathcal{F}_S(x, \mu))/x \in E\}$, where $\mathcal{T}_S(x, \mu), \mathcal{I}_S(x, \mu), \mathcal{F}_S(x, \mu) \in [0, 1]$,and $0 \leq \mathcal{T}_S(x, \mu) + \mathcal{I}_S(x, \mu) + \mathcal{F}_S(x, \mu) \leq 3$.

Definition 3.5. [25]
A Neutrosophic Matrix (NSM) of order $m \times n$ is defined as $A = [< a^T_{ij}, a^I_{ij}, a^F_{ij} >]$ where $a^T_{ij}, a^I_{ij}, a^F_{ij}$ are referred to as truth-membership, indeterminacy-membership, and falsity-membership values of theij-th element in A. These values must satisfy the condition $0 \leq a^T_{ij} + a^I_{ij} + a^F_{ij} \leq 3$ for all i, j. To simplify the notation, we can write $A = [aij]_{m\times n}$ where $a_{ij} =< a^T_{ij}, a^I_{ij}, a^F_{ij} >$.

4 Single-Valued Neutrosophic Fuzzy Matrix (SVNFM)

Definition 4.1. The single-valued neutrophic fuzzy matrix of order $m \times n$ is defined as follows.

$$\kappa = \begin{bmatrix} k_{11} & k_{12} & k_{13}..... & k_{1n} \\ k_{21} & k_{22} & k_{23}..... & k_{2n} \\ . & . & ...... & . \\ . & . & ...... & . \\ k_{m1} & k_{m2} & k_{m3}..... & k_{mn} \end{bmatrix}$$

Where, $(k_{ij}) = (\mu_\kappa(x_{ij}), \mathcal{T}_\kappa(x_{ij}, \mu), \mathcal{I}_\kappa(x_{ij}, \mu), \mathcal{F}_\kappa(x_{ij}, \mu)) \in I^4$, where each membership value is expressed by a truth, indeterminacy and falsity membership function which are respectively denoted as $\mathcal{T}_A(x, \mu), \mathcal{I}_A(x, \mu)$ and $\mathcal{F}_A(x, \mu)$, $0 \leq \mathcal{T}_\kappa(x_{ij}, \mu) + \mathcal{I}_\kappa(x_{ij}, \mu) + \mathcal{F}_\kappa(x_{ij}, \mu) \leq 3$, and $I^4 = [0, 1] \times [0, 1] \times [0, 1] \times [0, 1]$ is the Cartesian product of unit intervals of the real line.
Any matrix $\kappa \in \mathcal{NFM}_{m\times n}(I^4)$ is called a single-valued Neutrosophic Fuzzy Matrix of order $m \times n$

4.1 The Significance of Single-Valued Neutrosophic Fuzzy Matrices

Consider the three largest towns in a district, taking into account their distances from one another. We can use a crisp matrix to represent the distances between the towns because the distance is a crisp quantity.

$$p_{ij} = \begin{cases} 0, & \text{if } i = j \\ d_{ij}, & \text{if } d_{ij} \text{ is the distance between towns i and j} \end{cases}$$

Then the final matrix will be of the following form.

$$P = \begin{bmatrix} 0 & 12 & 13 \\ 12 & 0 & 22 \\ 13 & 22 & 0 \end{bmatrix}$$

The distance between towns can be effectively represented using a traditional matrix since it is a clear and fixed value. However, when it comes to representing the expected travel time between towns, a conventional matrix is not suitable. This is because the travel time is not a precise value and can vary due to various external factors. Additionally, calculating the travel time is a complex process that involves dealing with multiple uncertainties. To address these challenges, the use of Single-Valued Neutrosophic Fuzzy Matrices is a valuable representation technique.

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In the present scenario, web mapping platforms utilize GPS information from individual phones to estimate real-time movement and speed of traffic. This data is then used to adjust the calculations of average speeds, taking into account heavy traffic periods or clear conditions. These adjustments significantly contribute to more accurate travel time estimates. Furthermore, web mapping platforms utilize posted speed limits and historical traffic patterns to calculate the Expected Travel Time (ETA). When you input your destination into a web mapping platform, an initial estimate is generated based on the distance between your starting point and destination, posted speed limits, and the current traffic conditions. As you travel, your estimated time of arrival (ETA) is continuously updated to account for changes in traffic flow and your typical speed. In cases where high-quality data from a transit authority is not available, most web mapping platforms rely on aggregated crowd-sourced data to predict public transit crowding. These platforms analyze user feedback voluntarily shared by individuals to estimate the level of crowding in a vehicle. Users who have granted their device's Maps notification and location permissions and indicated that they are using public transit will receive notifications accordingly. Given the inherent uncertainties and ambiguities in crowd-sourcing, the representation of this type of data can benefit from the application of single valued neutrosophic fuzzy set(SVNFS). To achieve a more accurate estimation of the Estimated Time of Arrival (ETA) between three main towns,single valued neutrosophic fuzzy matrix (SVNFM) can be utilized as follows. $P^{'} = \begin{bmatrix} (0.0, 0.0, 0.0, 1.0) & (0.4, 0.6, 0.2, 0.1) & (0.2, 0.6.0.3, 0.1) \\ (0.4, 0.6, 0.2, 0.1) & (0.0, 0.0, 0.0, 1.0) & (0.7, 0.8, 0.0, 0.1) \\ (0.2, 0.6.0.3, 0.1) & (0.7, 0.8, 0.0, 0.1) & (0.0, 0.0, 0.0, 1.0) \end{bmatrix}$

4.2 Some Arithmetic Operations on SVNFM

Definition 4.2. Matrix Addition
Let $P\&Q$ be two Single-Valued Neutrosophic Fuzzy Matrices of order $m \times n$. $P = [p_{ij}]_{m\times n}$ and $Q = [q_{ij}]_{m\times n}$, then
$P + Q = [max(\mu_P(x_{ij}), \mu_Q(x_{ij})), max(\mathcal{T}_P(x_{ij}, \mu), \mathcal{T}_Q(x_{ij}, \mu)), max(\mathcal{I}_P(x_{ij}, \mu), \mathcal{I}_Q(x_{ij}, \mu)),$
$min(\mathcal{F}_P(x_{ij}, \mu), \mathcal{F}_Q(x_{ij}, \mu))]_{m\times n}$

Definition 4.3. Matrix Subtraction
$P - Q = [\mu_{P-Q}(x_{ij}), \mathcal{T}_{P-Q}(x_{ij}, \mu), \mathcal{I}_{P-Q}(x_{ij}, \mu), \mathcal{F}_{P-Q}(x_{ij}, \mu)]_{m\times n}$
Where, $\mu_{P-Q}(x_{ij}) = \begin{cases} \mu_P(x_{ij}), & \text{if } \mu_P(x_{ij}) \geq \mu_Q(x_{ij}) \\ 0, & \text{otherwise} \end{cases}$

$\mathcal{T}_{P-Q}(x_{ij}, \mu) = \begin{cases} \mathcal{T}_P(x_{ij}, \mu), & \text{if } \mathcal{T}_P(x_{ij}, \mu) \geq \mathcal{T}_Q(x_{ij}, \mu) \\ 0, & \text{otherwise} \end{cases}$

$\mathcal{I}_{P-Q}(x_{ij}, \mu) = \begin{cases} \mathcal{I}_P(x_{ij}, \mu), & \text{if } \mathcal{I}_P(x_{ij}, \mu) \geq \mathcal{I}_Q(x_{ij}, \mu) \\ 0, & \text{otherwise} \end{cases}$

$\mathcal{F}_{P-Q}(x_{ij}, \mu) = \begin{cases} \mathcal{F}_P(x_{ij}, \mu), & \text{if } \mathcal{F}_P(x_{ij}, \mu) \leq \mathcal{F}_Q(x_{ij}, \mu) \\ 0, & \text{otherwise} \end{cases}$

Definition 4.4. Component wise Matrix Multiplication
$P \circ Q = [min(\mu_P(x_{ij}), \mu_Q(x_{ij})), min(\mathcal{T}_P(x_{ij}, \mu), \mathcal{T}_Q(x_{ij}, \mu)), min(\mathcal{I}_P(x_{ij}, \mu), \mathcal{I}_Q(x_{ij}, \mu)),$
$max(\mathcal{F}_P(x_{ij}, \mu), \mathcal{F}_Q(x_{ij}, \mu))]_{m\times n}$

Definition 4.5. Scalar Multiplication of SVNFM
Let $P = [\mu_P(x_{ij}), \mathcal{T}_P(x_{ij}, \mu), \mathcal{I}_P(x_{ij}, \mu), \mathcal{F}_P(x_{ij}, \mu)]_{m\times n}$ be a single-valued Neutrosophic Fuzzy Matrix of order $m \times n$, k be any scalar then the scalar product kP is defined as follows
$kP = [min(k, \mu_P(x_{ij})), min(k, \mathcal{T}_P(x_{ij}, \mu)), min(k, \mathcal{I}_P(x_{ij}, \mu)), max(1 - k, \mathcal{F}_P(x_{ij}, \mu))]_{m\times n}$

Definition 4.6. Max-Min product of SVNFM
Let P be a SVNFM of order $m \times n$ and Q be a SVNFM of order $n \times r$ then the matrix product PQ is defined as follows
$P * Q = [max_k\{min\{\mu_P(x_{ik}), \mu_Q(x_{kj})\}\}, max_k\{min\{\mathcal{T}_P(x_{ik}, \mu),)\mathcal{T}_Q(x_{kj}, \mu)\}\},$
$min_k\{max\{\mathcal{I}_P(x_{ik}, \mu),)\mathcal{I}_Q(x_{kj}, \mu)\}\}, min_k\{max\{\mathcal{F}_P(x_{ik}, \mu),)\mathcal{F}_Q(x_{kj}, \mu)\}\}]_{m\times r}$

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where, $k = 1, 2, 3, ..., n.$
$i = 1, 2, 3, ..., m.$
$j = 1, 2, 3..., r.$

Definition 4.7. Score Matrix
The score matrix of κ_1 and κ_2 is denoted by $S_c(\kappa_1, \kappa_2)$ and is defined as
$S_c(\kappa_1, \kappa_2) = [U - V]$
where $U = [u_{ij}], u_{ij} = \mu_{\kappa_1}(x_{ij}) + \mathcal{T}_{\kappa_1}(x_{ij}, \mu) + \mathcal{I}_{\kappa_1}(x_{ij}, \mu) - \mathcal{F}_{\kappa_1}(x_{ij}, \mu)$ and
$V = [v_{ij}], v_{ij} = \mu_{\kappa_2}(x_{ij}) + \mathcal{T}_{\kappa_2}(x_{ij}, \mu) + \mathcal{I}_{\kappa_2}(x_{ij}, \mu) - \mathcal{F}_{\kappa_2}(x_{ij}, \mu)$

Definition 4.8. Transpose of a SVNFM
Transpose of the matrix P is given by P^T and is defined as $P^T = [\mu_P(x_{ji}), \mathcal{T}_P(x_{ji}, \mu), \mathcal{I}_P(x_{ji}, \mu), \mathcal{F}_P(x_{ji}, \mu)]$

Definition 4.9. Complement of a SVNFM Complement of the matrix P is defines as
$\overline{P} = [\mathcal{F}_P(x_{ij}, \mu), 1 - \mathcal{I}_P(x_{ij}, \mu), \mathcal{T}_P(x_{ij}, \mu), \mu_P(x_{ij})]$

Definition 4.10. Trace of a SVNFM
Let P be a SVNFM of order $n \times n$ then trace of P is denoted by $tr(P)$ and is defined as follows
$tr(P) = \sum_{i=1}^{n} p_{ii}$

Remark:Python toolkit enables effortless execution of all arithmetic operations.

4.3 Types of Matrices

Definition 4.11. Zero Neutrosophic Fuzzy Matrix
Let P be a SVNFM of order $m \times n$, if all the entries of P are $(0, 0, 0, 1)$ then P is called a Zero Neutrosophic Fuzzy Matrix. Denoted By **0**.

Definition 4.12. Universal Neutrosophic Fuzzy Matrix
Let P be a SVNFM of order $m \times n$, if all the entries of P are $(1, 1, 1, 0)$ then P is called a Universal Neutrosophic Fuzzy Matrix. Denoted By **I**.

Definition 4.13. Symmetric Neutrosophic Fuzzy Matrix
Let P be a SVNFM of order $n \times n$,then P is said to be Symmetric Neutrosophic Fuzzy Matrix if $P_{ij} = P_{ji}$, $i, j = 1, 2, 3...., n$

Definition 4.14. Upper Triangular Neutrosophic Fuzzy Matrix
Let P be a SVNFM of order $n \times n$,then P is said to be Upper Triangular Neutrosophic Fuzzy Matrix if $P_{ij} = (0, 0, 0, 1)$ for all $i > j$, where $i, j = 1, 2, 3...., n$

Definition 4.15. Lower Triangular Neutrosophic Fuzzy Matrix
Let P be a SVNFM of order $n \times n$,then P is said to be Lower Triangular Neutrosophic Fuzzy Matrix if $P_{ij} = (0, 0, 0, 1)$ for all $i < j$, where $i, j = 1, 2, 3...., n$

4.4 Main Result

In this section, we illustrate the distributive and associative properties of matrix multiplication. We show that when considering component-wise addition, multiplication, and scalar multiplication, the set of all Neutrosophic Fuzzy Matrices $\mathcal{NFM}_{n\times n}$ forms a vector space..

Theorem
The set $\mathcal{NFM}_{n\times n}$ forms a vector space under the operations Neutrosophic fuzzy matrix addition and scalar multiplication.

Proof
To verify that a given space is indeed a vector space, we must examine each and every one of its axioms.

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A_1 : Let $\mathcal{M} = [< \mu_{\mathcal{M}}(x_{ij}), \mathcal{T}_{\mathcal{M}}(x_{ij},\mu), \mathcal{I}_{\mathcal{M}}(x_{ij},\mu), \mathcal{F}_{\mathcal{M}}(x_{ij},\mu) >] \in \mathcal{NFM}_{n\times n}$ and $\mathcal{N} = [< \mu_{\mathcal{N}}(x_{ij}), \mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{N}}(x_{ij},\mu) >] \in \mathcal{NFM}_{n\times n}$ from the definition of matrix addition, it is clear that $\mathcal{M}+\mathcal{N} \in \mathcal{NFM}_{n\times n}$

A_2 :$\mathcal{M} = [< \mu_{\mathcal{M}}(x_{ij}), \mathcal{T}_{\mathcal{M}}(x_{ij},\mu), \mathcal{I}_{\mathcal{M}}(x_{ij},\mu), \mathcal{F}_{\mathcal{M}}(x_{ij},\mu) >]$,
$\mathcal{N} = [< \mu_{\mathcal{N}}(x_{ij}), \mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{N}}(x_{ij},\mu) >]$,
$\mathcal{L} = [< \mu_{\mathcal{L}}(x_{ij}), \mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{N}}(x_{ij},\mu) >] \in \mathcal{NFM}_{n\times n}$

$\mathcal{M}+(\mathcal{N}+\mathcal{L}) = [< \mu_{\mathcal{M}}(x_{ij}), \mathcal{T}_{\mathcal{M}}(x_{ij},\mu), \mathcal{I}_{\mathcal{M}}(x_{ij},\mu), \mathcal{F}_{\mathcal{M}}(x_{ij},\mu) >]$+$[< max(\mu_{\mathcal{N}}(x_{ij}), \mu_{\mathcal{L}}(x_{ij}))$,
$max(\mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{T}_{\mathcal{L}}(x_{ij},\mu)), max(\mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{L}}(x_{ij},\mu)), min(\mathcal{F}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{L}}(x_{ij},\mu)) >]$
= $[< max(\mu_{\mathcal{M}}(x_{ij})), \mu_{\mathcal{N}}(x_{ij}), \mu_{\mathcal{L}}(x_{ij}), max(\mathcal{T}_{\mathcal{M}}(x_{ij},\mu), \mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{T}_{\mathcal{L}}(x_{ij},\mu))$,
$max(\mathcal{I}_{\mathcal{M}}(x_{ij},\mu), \mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{L}}(x_{ij},\mu)), min(\mathcal{F}_{\mathcal{M}}(x_{ij},\mu), \mathcal{F}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{L}}(x_{ij},\mu)) >]$

$(\mathcal{M}+\mathcal{N})+\mathcal{L} = [< max(\mu_{\mathcal{M}}(x_{ij}), \mu_{\mathcal{N}}(x_{ij})), max(\mathcal{T}_{\mathcal{M}}(x_{ij},\mu), \mathcal{T}_{\mathcal{N}}(x_{ij},\mu)), max(\mathcal{I}_{\mathcal{M}}(x_{ij},\mu), \mathcal{I}_{\mathcal{N}}(x_{ij},\mu))$,
$min(\mathcal{F}_{\mathcal{M}}(x_{ij},\mu), \mathcal{F}_{\mathcal{N}}(x_{ij},\mu)) >] + [< \mu_{\mathcal{L}}(x_{ij}), \mathcal{T}_{\mathcal{L}}(x_{ij},\mu), \mathcal{I}_{\mathcal{L}}(x_{ij},\mu), \mathcal{F}_{\mathcal{L}}(x_{ij},\mu) >]$
$= [< max(\mu_{\mathcal{M}}(x_{ij})), \mu_{\mathcal{N}}(x_{ij}), \mu_{\mathcal{L}}(x_{ij}), max(\mathcal{T}_{\mathcal{M}}(x_{ij},\mu), \mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{T}_{\mathcal{L}}(x_{ij},\mu))$,
$max(\mathcal{I}_{\mathcal{M}}(x_{ij},\mu), \mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{L}}(x_{ij},\mu)), min(\mathcal{F}_{\mathcal{M}}(x_{ij},\mu), \mathcal{F}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{L}}(x_{ij},\mu)) >]$

Hence $\mathcal{M}+(\mathcal{N}+\mathcal{L}) = (\mathcal{M}+\mathcal{N})+\mathcal{L}$

Similarly, we can prove that
$\mathcal{M}\circ(\mathcal{N}\circ\mathcal{L}) = (\mathcal{M}\circ\mathcal{N})\circ\mathcal{L}$

A_3 : $\mathcal{M}+\mathcal{N} = [< max(\mu_M(x_{ij}), \mu_N(x_{ij})), max(\mathcal{T}_M(x_{ij},\mu), \mathcal{T}_N(x_{ij},\mu)), max(\mathcal{I}_M(x_{ij},\mu), \mathcal{I}_N(x_{ij},\mu))$,
$min(\mathcal{F}_M(x_{ij},\mu), \mathcal{F}_N(x_{ij},\mu)) >]$
$= [< max(\mu_N(x_{ij}), \mu_M(x_{ij})), max(\mathcal{T}_N(x_{ij},\mu), \mathcal{T}_M(x_{ij},\mu)), max(\mathcal{I}_N(x_{ij},\mu), \mathcal{I}_M(x_{ij},\mu))$,
$min(\mathcal{F}_N(x_{ij},\mu), \mathcal{F}_M(x_{ij},\mu)) >]$
$= \mathcal{N}+\mathcal{M}$

A_4 : For any matrix$\mathcal{M} \in \mathcal{NFM}_{n\times n}$ we have $\mathcal{M}+\mathbf{0} = \mathcal{M}$ That is, the Zero Neutrosophic Matrix $\mathbf{0}$ is the additive identity.
For any matrix$\mathcal{M} \in \mathcal{NFM}_{n\times n}$, we have $\mathcal{M}\circ\mathbf{I} = \mathcal{M}$, that is,the Universal Neutrosophic Fuzzy Matrix $\mathbf{I}$ serves as the multiplicative identity.

M_1: Let k be a scalar and $\mathcal{M} \in \mathcal{NFM}_{n\times n}$, then by the definition of scalar multiplication for Neutrosophic Fuzzy Matrices we have
$k\mathcal{M} \in \mathcal{NFM}_{n\times n}$

M_2: First we prove distributive law.
Suppose that $\mathcal{M} \leq \mathcal{N}, \mathcal{L}$
$\mathcal{M}\circ(\mathcal{N}+\mathcal{L}) = min(\mu_{\mathcal{M}}(x_{ij}), max(\mu_{\mathcal{N}}(x_{ij}), \mu_{\mathcal{L}}(x_{ij}))), min(\mathcal{T}_{\mathcal{M}}(x_{ij},\mu), max(\mathcal{T}_{\mathcal{N}}(x_{ij},\mu), \mathcal{T}_{\mathcal{L}}(x_{ij},\mu)))$,
$min(\mathcal{I}_{\mathcal{M}}(x_{ij},\mu), max(\mathcal{I}_{\mathcal{N}}(x_{ij},\mu), \mathcal{I}_{\mathcal{L}}(x_{ij},\mu))), max(\mathcal{F}_{\mathcal{M}}(x_{ij},\mu), min(\mathcal{F}_{\mathcal{N}}(x_{ij},\mu), \mathcal{F}_{\mathcal{L}}(x_{ij},\mu)))$
= $\mathcal{M}$

$\mathcal{M}\circ\mathcal{N}+\mathcal{M}\circ\mathcal{L} = [< min(\mu_M(x_{ij}), \mu_N(x_{ij})), min(\mathcal{T}_M(x_{ij},\mu), \mathcal{T}_N(x_{ij},\mu))$,
$min(\mathcal{I}_M(x_{ij},\mu), \mathcal{I}_N(x_{ij},\mu)), max(\mathcal{F}_M(x_{ij},\mu), \mathcal{F}_N(x_{ij},\mu)) >] + [< min(\mu_M(x_{ij}), \mu_L(x_{ij}))$,
$min(\mathcal{T}_M(x_{ij},\mu), \mathcal{T}_L(x_{ij},\mu)), min(\mathcal{I}_M(x_{ij},\mu), \mathcal{I}_L(x_{ij},\mu)), max(\mathcal{F}_M(x_{ij},\mu), \mathcal{F}_L(x_{ij},\mu)) >]$
=$max(min(\mu_M(x_{ij}), \mu_N(x_{ij})), min(\mu_M(x_{ij}), \mu_L(x_{ij}))), max(min(\mathcal{T}_M(x_{ij},\mu), \mathcal{T}_N(x_{ij},\mu))$,
$min(\mathcal{T}_M(x_{ij},\mu), \mathcal{T}_L(x_{ij},\mu))), max(min(\mathcal{I}_M(x_{ij},\mu), \mathcal{I}_N(x_{ij},\mu)), min(\mathcal{I}_M(x_{ij},\mu), \mathcal{I}_L(x_{ij},\mu)))$,
$min(max(\mathcal{F}_M(x_{ij},\mu), \mathcal{F}_N(x_{ij},\mu)), max(\mathcal{F}_M(x_{ij},\mu), \mathcal{F}_L(x_{ij},\mu)))$
=$\mathcal{M}$
Similarly for $\mathcal{M} \geq \mathcal{N}, \mathcal{L}$ we can prove the same identity.
Therefore, the distributive law holds.
Let k be a scalar and $\mathcal{M}, \mathcal{N} \in \mathcal{NFM}_{n\times n}$
$k(\mathcal{M}+\mathcal{N}) = k\mathbf{I}\circ(\mathcal{M}+\mathcal{N}) = k\mathbf{I}\circ\mathcal{M}+k\mathbf{I}\circ\mathcal{N} = k\mathcal{M}+k\mathcal{N}.$

M_3: Let k_1, k_2 be any two scalars, then
$(k_1+k_2)\mathcal{M} = (k_1+k_2)\mathbf{I}\circ\mathcal{M} = (k_1\mathbf{I}+k_2\mathbf{I})\circ\mathcal{M} = k_1\mathbf{I}\circ\mathcal{M}+k_2\mathbf{I}\circ\mathcal{M} = k_1\mathcal{M}+k_2\mathcal{M}$

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Therefore,$\mathcal{NFM}_{n\times n}$ forms a vector space.

4.5 Medical Diagnosis Based on Score Matrix for Neutrosophic Fuzzy Matrix

A person suffering from a disease will display a variety of symptoms, including fever, coughing, fatigue, sore throat, headaches, sneezing, etc. Additionally, each viral illness will show multiple symptoms. For instance, dengue, typhoid, and chikungunya can cause flu, chronic fatigue, sore throat, coughing, and other symptoms. The symptoms of the coronavirus infection include fever, cough, nasal congestion, chronic fatigue, and others. Typically, a medical diagnosis is made based on the persistent symptoms because we are unable to draw any conclusions from the transient symptoms. The ambiguity and uncertainties in the available medical data must be taken into account when considering prolonged symptoms, but when we use NFSs to represent these data, we can greatly reduce these types of distractions.

Algorithm
For our case analysis, let us select five patients $\mathcal{P} = \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, \mathcal{P}_5\}$ each patient has multiple symptoms such as
$\mathbb{S}$ ={Prolonged Temperature,Prolonged Headache,Prolonged Sore Throat,Prolonged Diarrhoea,Prolonged Shortness of Breath}.
Now employing the medical data represented by Neutrosophic Fuzzy Sets,we have to deduce the type of disease affecting the person such as
$\mathcal{D}$ ={Dengue, Covid-19, Viral Fever,Rat Fever, Chikungunya}

Stage-1
The typical symptoms observed in each patientare represented in **Table-I**[9] as Single Valued Neutrosophic Fuzzy Set data and we are representing these data using a Single Valued Neutrosophic Fuzzy Matrix as follows

$$P = \begin{bmatrix} (0.8, 0.7, 0.2, 0.1) & (0.7, 0.6, 0.2, 0.2) & (0.5, 0.6.0.1, 0.1) & (0.2, 0.2, 0.0, 0.1) & (0.7, 0.3, 0.6, 0.4) \\ (0.4, 0.6, 0.2, 0.2) & (0.3, 0.6, 0.2, 0.0) & (0.5, 0.7.0.0, 0.1) & (0.5, 0.2, 0.4, 0.3) & (0.7, 0.6, 0.2, 0.2) \\ (0.2, 0.6, 0.1, 0.1) & (0.4, 0.6, 0.1, 0.0) & (0.8, 0.6.0.1, 0.1) & (0.3, 0.2, 0.4, 0.4) & (0.2, 0.3, 0.4, 0.2) \\ (0.4, 0.6, 0.2, 0.1) & (0.6, 0.8, 0.1, 0.0) & (0.5, 0.7.0.1, 0.0) & (0.3, 0.2, 0.5, 0.6) & (0.5, 0.3, 0.4, 0.4) \\ (0.9, 0.6, 0.5, 0.1) & (0.8, 0.9, 0.1, 0.0) & (0.2, 0.6.0.1, 0.3) & (0.6, 0.2, 0.5, 0.5) & (0.5, 0.6, 0.1, 0.1) \end{bmatrix}$$

Stage-2
The type of symptoms that are typically found for each disease is represented in **Table-II**,[9] and we are representing the symptom-disease relationship using the following single valued neutrosophic fuzzy matrix Q.

$$Q = \begin{bmatrix} (0.4, 0.6, 0.2, 0.1) & (0.8, 0.9, 0.1, 0.1) & (0.2, 0.6.0.3, 0.1) & (0.7, 0.8, 0.0, 0.1) & (0.2, 0.8, 0.3, 0.2) \\ (0.7, 0.6, 0.2, 0.2) & (0.5, 0.6, 0.2, 0.2) & (0.8, 0.7.0.1, 0.1) & (0.5, 0.3, 0.4, 0.2) & (0.8, 0.6, 0.3, 0.2) \\ (0.8, 0.7, 0.2, 0.2) & (0.6, 0.6, 0.1, 0.1) & (0.7, 0.5.0.3, 0.3) & (0.4, 0.2, 0.3, 0.4) & (0.2, 0.3, 0.4, 0.3) \\ (0.8, 0.6, 0.2, 0.2) & (0.7, 0.8, 0.2, 0.1) & (0.2, 0.7.0.1, 0.2) & (0.3, 0.2, 0.4, 0.5) & (0.2, 0.3, 0.4, 0.4) \\ (0.9, 0.6, 0.5, 0.1) & (0.6, 0.9, 0.1, 0.0) & (0.2, 0.6.0.1, 0.3) & (0.3, 0.3, 0.5, 0.4) & (0.3, 0.6, 0.2, 0.2) \end{bmatrix}$$

Stage-3
Compute the complement of the matrix Q.
Stage-4
Compute the matrices $P * Q$ and $P * Q^c$
Stage-5
Compute U and V of the score matrix.
Stage-6
Compute the score matrix $S_c(P * Q, P * Q^c)$
Stage-7
Determine the patient $\mathcal{P}'s$ maximum score and come to the conclusion that the patient has the disease $\mathcal{D}_i$.

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Table 1

	Temperature	Headache	Sore Throat	Diarrhoea	Shortness of Breath
$\mathcal{P}_1$	(0.8,0.7,0.2,0.1)	(0.7,0.6,0.2,0.2)	(0.5,0.6.0.1,0.1)	(0.2,0.2,0.0,0.1)	(0.7,0.3,0.6,0.4)
$\mathcal{P}_2$	(0.4,0.6,0.2,0.2)	(0.3,0.6,0.2,0.0)	(0.5,0.7.0.0,0.1)	(0.5,0.2,0.4,0.3)	(0.7,0.6,0.2,0.2)
$\mathcal{P}_3$	(0.2,0.6,0.1,0.1)	(0.4,0.6,0.1,0.0)	(0.8,0.6.0.1,0.1)	(0.3,0.2,0.4,0.4)	(0.2,0.3,0.4,0.2)
$\mathcal{P}_4$	(0.4,0.6,0.2,0.1)	(0.6,0.8,0.1,0.0)	(0.5,0.7.0.1,0.0)	(0.3,0.2,0.5,0.6)	(0.5,0.3,0.4,0.4)
$\mathcal{P}_5$	(0.9,0.6,0.5,0.1)	(0.8,0.9,0.1,0.0)	(0.2,0.6.0.1,0.3)	(0.6,0.2,0.5,0.5)	(0.5,0.6,0.1,0.1)

Table 2

	Temperature	Headache	Sore Throat	Diarrhoea	Shortness of Breath
Dengue Fever	(0.4,0.6,0.2,0.1)	(0.8,0.9,0.1,0.1)	(0.2,0.6.0.3,0.1)	(0.7,0.8,0.0,0.1)	(0.2,0.8,0.3,0.2)
Covid-19	(0.7,0.6,0.2,0.2)	(0.5,0.6,0.2,0.2)	(0.8,0.7.0.1,0.1)	(0.5,0.3,0.4,0.2)	(0.8,0.6,0.3,0.2)
Viral Fever	(0.8,0.7,0.2,0.2)	(0.6,0.6,0.1,0.1)	(0.7,0.5.0.3,0.3)	(0.4,0.2,0.3,0.4)	(0.2,0.3,0.4,0.3)
Rat Fever	(0.8,0.6,0.2,0.2)	(0.7,0.8,0.2,0.1)	(0.2,0.7.0.1,0.2)	(0.3,0.2,0.4,0.5)	(0.2,0.3,0.4,0.4)
Chikungunya	(0.9,0.6,0.5,0.1)	(0.6,0.9,0.1,0.0)	(0.2,0.6.0.1,0.3)	(0.3,0.3,0.5,0.4)	(0.3,0.6,0.2,0.2)

Implementation of the Algorithm

While analyzing a patient's symptoms, there are numerous uncertainties, inconsistencies, and ambiguities that arise. A healthcare professional will draw a conclusion about a patient's condition based on their symptoms. They always ask us about the prolonged symptoms because they cannot draw any conclusions based on one-time symptoms. However, there are many unknowns associated with prolonged symptoms when taking into account available information. It can be challenging for decision makers to make a decision when patients are unconscious because they are sometimes unable to communicate the correct information about their prolonged symptoms due to mental stress. In order to overcome these types of distraction caused by protracted symptoms, we are using single-valued neutrosophic fuzzy sets in this situation. Based on the initial consultation and the fuzzy set theory, each patient will have a membership value that corresponds to their prolonged symptoms. By linking the Neutrosophic Components to each of these fuzzy membership values, we can now convert this into SVNFS.**The algorithm can be simply implemented using the Python toolbox.**

$$Q^c = \begin{bmatrix} (0.1,0.8,0.6,0.4) & (0.1,0.9,0.9,0.8) & (0.1,0.7.0.6,0.2) & (0.1,1,0.8,0.7) & (0.2,0.7,0.8,0.2) \\ (0.2,0.8,0.6,0.7) & (0.2,0.8,0.6,0.5) & (0.1,0.9.0.7,0.8) & (0.2,0.6,0.3,0.5) & (0.2,0.7,0.6,0.8) \\ (0.2,0.8,0.7,0.8) & (0.1,0.9,0.6,0.6) & (0.3,0.7.0.5,0.7) & (0.4,0.7,0.2,0.4) & (0.3,0.6,0.3,0.2) \\ (0.2,0.8,0.6,0.8) & (0.1,0.8,0.8,0.7) & (0.2,0.9,0.7,0.2) & (0.5,0.6,0.2,0.3) & (0.4,0.6,0.3,0.2) \\ (0.1,0.5,0.6,0.9) & (0.0,0.9,0.9,0.6) & (0.3,0.9.0.6,0.2) & (0.4,0.5,0.3,0.3) & (0.2,0.8,0.6,0.3) \end{bmatrix}$$

$$P{*}Q = \begin{bmatrix} (0.7,0.6,0.2,0.1) & (0.8,0.7,0.1,0.1) & (0.7,0.6.0.1,0.1) & (0.7,0.7,0.2,0.1) & (0.7,0.7,0.3,0.2) \\ (0.7,0.7,0.2,0.2) & (0.6,0.6,0.1,0.1) & (0.5,0.6.0.1,0.1) & (0.4,0.6,0.2,0.2) & (0.3,0.6,0.2,0.2) \\ (0.8,0.6,0.2,0.1) & (0.6,0.6,0.1,0.1) & (0.7,0.6.0.1,0.1) & (0.4,0.6,0.1,0.1) & (0.4,0.6,0.3,0.2) \\ (0.6,0.7,0.2,0.1) & (0.5,0.6,0.1,0.1) & (0.5,0.7,0.3,0.1) & (0.5,0.6,0.2,0.1) & (0.6,0.6,0.3,0.2) \\ (0.7,0.6,0.2,0.1) & (0.9,0.9,0.1,0.1) & (0.8,0.7.0.1,0.1) & (0.7,0.6,0.3,0.1) & (0.8,0.6,0.2,0.2) \end{bmatrix}$$

$$P{*}Q^c = \begin{bmatrix} (0.2,0.7,0.6,0.4) & (0.2,0.7,0.6,0.5) & (0.3,0.7.0.5,0.2) & (0.4,0.7,0.2,0.3) & (0.3,0.7,0.3,0.2) \\ (0.2,0.7,0.6,0.4) & (0.2,0.7,0.6,0.5) & (0.3,0.7.0.5,0.2) & (0.5,0.7,0.2,0.3) & (0.4,0.6,0.3,0.2) \\ (0.2,0.6,0.6,0.4) & (0.2,0.6,0.6,0.5) & (0.3,0.6.0.5,0.2) & (0.4,0.6,0.2,0.3) & (0.3,0.6,0.2,0.2) \\ (0.2,0.8,0.6,0.4) & (0.2,0.8,0.6,0.3) & (0.3,0.8,0.5,0.2) & (0.4,0.7,0.2,0.4) & (0.3,0.7,0.3,0.2) \\ (0.2,0.8,0.6,0.4) & (0.2,0.8,0.6,0.5) & (0.3,0.9.0.5,0.2) & (0.5,0.6,0.2,0.3) & (0.4,0.7,0.3,0.2) \end{bmatrix}$$

Then the value matrices U and V are given by

$$U = \begin{bmatrix} 1.4 & 1.5 & 1.3 & 1.5 & 1.5 \\ 1.4 & 1.2 & 1.1 & 1 & 0.9 \\ 1.5 & 1.2 & 1.3 & 1 & 1.1 \\ 1.4 & 1.1 & 1.4 & 1.2 & 1.3 \\ 1.4 & 1.8 & 1.5 & 1.5 & 1.4 \end{bmatrix}$$

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$$V = \begin{bmatrix} 1.1 & 1.1 & 1.3 & 1 & 1.1 \\ 1.1 & 1 & 1.3 & 1.1 & 1.1 \\ 1 & 0.9 & 1.2 & 0.9 & 0.9 \\ 1.2 & 1.3 & 1.4 & 0.9 & 1.1 \\ 1.2 & 1.18 & 1.5 & 1 & 1.2 \end{bmatrix}$$

Finally, the score matrix is given by

$$S_c(P * Q, P * Q^c) = \begin{bmatrix} 0.3 & 0.4 & 0 & 0.5 & 0.4 \\ 0.3 & 0.2 & -0.2 & -0.1 & -0.2 \\ 0.5 & 0.3 & 0.1 & 0.1 & 0.2 \\ 0.2 & -0.2 & 0 & 0.3 & 0.2 \\ 0.2 & 0. & 0 & 0.5 & 0.2 \end{bmatrix}$$

From the score matrix we can conclude that patients $\mathcal{P}_1$ and $\mathcal{P}_4$ are affected by Rat Fever, patients $\mathcal{P}_2$ and $\mathcal{P}_3$ are affected by Dengue Fever, and patient $\mathcal{P}_5$ is affected by Covid-19.

5 Conclusion

The concept of a neutrosophic matrix was introduced by Varol et al.[4] This concept led us to introduce Neutrosophic Fuzzy Matrices, an extension of the Neutrosophic Matrix, to handle computer science problems involving neutrosophic inputs.

The idea of neutrosophic fuzzy matrices and how they are used to handle uncertainty, indeterminacy, and ambiguity in various fields have been examined in this research paper. To lay the groundwork for the use of neutrosophic fuzzy matrices in real-world applications, we have investigated the characteristics and algebraic operations related with them. In particular, when dealing with data that may simultaneously exhibit truth, falsehood, and indeterminacy, our study has shown that neutrosophic fuzzy matrices offer a useful tool for representing and analyzing complex information. We have increased the number of mathematical tools available for decision-making, image processing, artificial intelligence, and other applications by applying the concepts of neutrosophy and fuzzy logic to matrices.Thispaper is anticipated to stimulate future research on various algorithms for distinct decision-making challenges.

Declarations

Author contribution: Priya Mathews studied and wrote the manuscript, and Lovelymol and Baiju Thankachan reviewed the article.

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