



MBJ-Neutrosophic WI Ideals in Lattice Wajsberg Algebra

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Abstract

In this study, we introduce the concepts of MBJ-Neutrosophic WI-ideal and MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebras. We demonstrate that every MBJ-Neutrosophic WI-ideal of lattice Wajsberg algebra is an MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebra. Additionally, we talk about its opposite. Furthermore, we discover that in lattice H-Wajsberg algebra, every MBJ-Neutrosophic lattice ideal is an MBJ-Neutrosophic WI-ideal.

Keywords: Wajsberg algebra (WA); Lattice Wajsberg algebra(LWA); WI-ideal; MBJ-Neutrosophic WI-ideal; MBJ-Neutrosophic lattice ideal.

1. Introduction

Different kinds of uncertainties are faced in a wide variety of real-world circumstances and in many complex systems, including biological, behavioral, and chemical ones. The fuzzy set was first presented by L.A. Zadeh [4] in 1965 to handle uncertainties in various practical applications, and K. Atanassov introduced the intuitionistic fuzzy set on a universe X in 1983 as a generalization of the fuzzy set. By extending the concepts of classic set, (intuitionistic) fuzzy set, and interval valued (intuitionistic) fuzzy set, Smarandache ([14], [15], and [16]) created the concept of neutrosophic set. The membership functions for truth, falsehood, and indeterminacy in the neutrosophic set are fuzzy sets. We use the interval valued fuzzy set as a basis for thinking about a generalization of neutrosophic set.

The idea of Wajsberg algebra was first put out by Mordchaj Wajsberg [1] in 1935. Lattice valued logic is developing as a study area that has a significant impact on the advancement of algebraic logic, computer science, and artificial intelligence technologies. In 1984, Font et al. [3] proposed Wajsberg algebra's lattice structure, examined its features, and expanded Wajsberg algebra as a substitute model for the infinite-valued Lukasiewicz logic.

Lattice Wajsberg algebras are an algebraic structure that are created by combining a lattice with a Wajsberg algebra. Font, Rodriguez, and Torrens [3] presented the idea of lattice Wajsberg algebras in 1984 and analyzed some of its features. Filter theory is crucial for the overall advancement of lattice Wajsberg algebras. In a lattice Wajsberg algebra, they presented the idea of implicative filters and looked into their characteristics. The concepts of fuzzy implicative and anti-fuzzy implicative filters of lattice Wajsberg algebras were proposed by Basheer Ahamed and Ibrahim [7,18], who also established several properties using examples. Another significant advancement in lattice Wajsberg algebras is the theory of ideals. The Wajsberg implicative ideal (WI-ideal) of lattice Wajsberg algebra was presented by the authors [8], who also deduced various features.

In this paper, we introduce the notions of MBJ-Neutrosophic WI-ideal and MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebras. We show that every MBJ-Neutrosophic WI-ideal of lattice Wajsberg algebra is a MBJ-Neutrosophic lattice ideal of lattice Wajsberg algebra. Also, we discuss its converse part. Further, we obtain every MBJ-Neutrosophic lattice ideal is an MBJ-Neutrosophic WI-ideal in lattice H-Wajsberg algebra.

2. Preliminaries

2.1. Definition [3]: Let $(L, \rightarrow, *, 1)$ be an algebra with an unary operation $*$, and a binary operation $\rightarrow$ is called a Wajsberg algebra (W-algebra) if and only if it satisfies the following axioms for all $x, y, z \in L$

$$(W1) \ 1 \rightarrow x = x$$

$$(W2) \ x \rightarrow y \rightarrow y \rightarrow z \rightarrow x \rightarrow z = 1$$

$$(W3) \ x \rightarrow y \rightarrow y = (y \rightarrow x) \rightarrow x$$

$$(W4) \ (x * \rightarrow y *) \rightarrow y \rightarrow x = 1$$

2.2. Proposition [3]: The W-algebra $(L, \rightarrow, *, 1)$ satisfies the following equations and implications for all $x, y, z \in L$

$$(1) \ x \rightarrow x = 1$$

$$(2) \ \text{If } x \rightarrow y = y \rightarrow x = 1, \text{ then } x = y$$

$$(3) \ x \rightarrow 1 = 1$$

$$(4) \ x \rightarrow y \rightarrow x = 1$$

$$(5) \ \text{If } x \rightarrow y = y \rightarrow z = 1, \text{ then } x \rightarrow z = 1$$

$$(6) \ x \rightarrow y \rightarrow z \rightarrow x \rightarrow z \rightarrow y = 1$$

$$(7) \ x \rightarrow y \rightarrow z = y \rightarrow (x \rightarrow z)$$

$$(8) \ x \rightarrow 0 = x \rightarrow 1 * = x *$$

$$(9) \ (x *) * = x$$

$$(10) \ x * \rightarrow y * = y \rightarrow x$$

$$(11) \ \text{If } x \leq y \text{ then } y \rightarrow z \leq x \rightarrow z$$

$$(12) \ (x \vee y) * = (x * \wedge y *)$$

$$(13) \ (x \wedge y) * = (x * \vee y *)$$

$$(14) \ (x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$$

$$(15) \ x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)$$

$$(16) \ (x \rightarrow y) \vee (y \rightarrow x) = 1$$

$$(17) \ x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$$

$$(18) \ (x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$$

$$(19) \ (x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$$

$$(20) \ (x \wedge y) \rightarrow z = (x \rightarrow y) \vee (x \rightarrow z)$$

$$(21) \ x \leq y \rightarrow z \text{ if and only if } y \leq x \rightarrow z$$

$$(22) \ \text{If } x \leq y \text{ then } z \rightarrow x \leq z \rightarrow y$$

2.3. Definition [3]: The W-algebra L is called a Lattice W-algebra if it satisfies the following conditions for all $x, y \in L$,

- (1) A partial ordering “ $\leq$ ” on L such that $x \leq y$ if and only if $x \rightarrow y = 1$
- (2) $x \vee y = (x \rightarrow y) \rightarrow y$
- (3) $x \wedge y = ((x^* \rightarrow y^*) \rightarrow y^*)^*$

Thus $(L, \vee, \wedge, *, \rightarrow, 0, 1)$ is a Lattice W-algebra with lower bound 0 and an upper bound 1.

2.4. Definition [17]: Let X be a non-empty set. A MBJ-neutrosophic set of the form $A = \{(\zeta; M_A(\zeta), \widetilde{B}_A(\zeta), J_A(\zeta) / \zeta \in X)\}$ where M_A and J_A are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively and $\widetilde{B}_A$ is an IVF set in X which is called an indeterminate interval valued membership function. For the sake of simplicity, we shall use the symbol $A = (M_A, \widetilde{B}_A, J_A)$ for the MBJ-Neutrosophic set $A = \{(\zeta; M_A(\zeta), \widetilde{B}_A(\zeta), J_A(\zeta) / \zeta \in X)\}$.

In an MBJ-Neutrosophic set $A = (M_A, \widetilde{B}_A, J_A)$ in X we take $\widetilde{B}_A: X \rightarrow [I], \zeta \rightarrow [B_A^-(\zeta), B_A^+(\zeta)]$ with $B_A^-(\zeta) = B_A^+(\zeta)$ then $A = (M_A, \widetilde{B}_A, J_A)$ is a neutrosophic set in X.

2.5. Definition [3]. Let L be a lattice. An ideal I of L is a nonempty subset of L is called a lattice ideal, if it satisfies the following axioms for all $x, y \in I$

- (i) $x \in I, y \in L$ and $y \leq x$ imply $y \in I$
- (ii) $x, y \in I$ implies $x \vee y \in I$

2.6. Definition [7]. Let A be a lattice Wajsberg algebra. Let I be a nonempty subset of A, then I is called WI ideal of lattice Wajsberg algebra A satisfies,

- (i) $0 \in I$
- (ii) $(x \rightarrow y)^* \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in A$.

3. MBJ-Neutrosophic WI-ideals in Lattice Wajsberg algebra:

3.1. Definition: A MBJ-N set $A = (M_A, \widetilde{B}_A, J_A)$ in a LWA L is called a MBJ-N WI-ideal of ‘L’ if the following attributes are true.

$$(\forall \varsigma_1 \in L) (M_A(0) \geq M_A(\varsigma_1), \widetilde{B}_A(0) \geq \widetilde{B}_A(\varsigma_1), J_A(0) \leq J_A(\varsigma_1)) \quad (1)$$

$$\text{And } (\forall \varsigma_1, \varsigma_2 \in L) \begin{pmatrix} M_A(\varsigma_1) \geq \min\{M_A((\varsigma_1 \rightarrow \varsigma_2)'), M_A(\varsigma_2)\} \\ \widetilde{B}_A(\varsigma_1) \geq \text{rmin}\{\widetilde{B}_A((\varsigma_1 \rightarrow \varsigma_2)'), \widetilde{B}_A(\varsigma_2)\} \\ J_A(\varsigma_1) \leq \max\{J_A((\varsigma_1 \rightarrow \varsigma_2)'), J_A(\varsigma_2)\} \end{pmatrix} \quad (2)$$

The set of all MBJ-NWI-ideals of L is denoted by MBJ-NWI(L)

3.2. Example:

Let $A = \{0, a, b, c, d, r, s, t, 1\}$ be a set with Figure (1) as a partial ordering. Define a quasi-complement “ $*$ ” and a binary operation “ $\rightarrow$ ” on A as in Table (1) and Table(2).

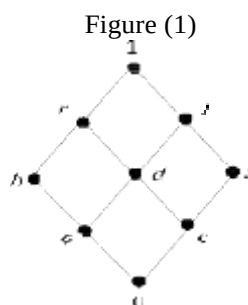


Table 1:

ς_1	$\begin{matrix} x^* \\ \varsigma_1^* \end{matrix}$
0	1
a	t
b	b
c	r
d	d
r	c
s	b
t	a
1	0

Table 2:

x	00	0	A	a	B	C	c	d	r	s	t	1
0	1	1	1	1	1	1	1	1	1	1	1	1
a	t	1	1	1	t	1	1	1	t	1	1	1
b	b	t	1	1	s	t	1	1	s	t	1	1
c	r	r	r	r	1	1	1	1	1	1	1	1
d	d	r	r	r	t	1	1	1	t	1	1	1
r	c	d	r	r	s	t	1	1	s	t	1	1
s	b	b	b	b	r	r	r	1	1	1	1	1
t	a	b	b	b	d	r	r	t	1	1	1	1
1	0	a	b	b	c	d	r	s	t	1	1	1

Define $\vee$ and $\wedge$ operations on A as follows,

$$(\varsigma_1 \vee \varsigma_2) = (\varsigma_1 \rightarrow \varsigma_2) \rightarrow \varsigma_2,$$

$$(\varsigma_1 \wedge \varsigma_2) = ((\varsigma_1^* \rightarrow \varsigma_2^*) \rightarrow \varsigma_2^*)^* \text{ for all } \varsigma_1, \varsigma_2 \in A.$$

Then $(A, \vee, \wedge, *, 0, 1)$ is a lattice Wajsberg algebra.

Consider an MBJ-neutrosophic set $S = (M_A, \widetilde{B}_A, J_A)$ on A as

$$M_A(\varsigma_1) = \begin{cases} 1 & \text{if } \varsigma_1 \in \{0, b\} \text{ for every } \varsigma_1 \in A \\ 0.6 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$\widetilde{B}_A(\varsigma_1) = \begin{cases} [0.5, 0.6] & \text{if } \varsigma_1 \in \{0, b\} \text{ for every } \varsigma_1 \in A \\ [0.3, 0.4] & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$J_A(\varsigma_1) = \begin{cases} 0 & \text{if } \varsigma_1 \in \{0, b\} \text{ for every } \varsigma_1 \in A \\ 0.4 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

Then S is an MBJ-neutrosophic WI-ideal.

In the same Example 3.2, let us consider an MBJ-neutrosophic set $S = (M_A, \widetilde{B}_A, J_A)$ on A as

$$M_A(\varsigma_1) = \begin{cases} 1 & \text{if } \varsigma_1 \in \{a, b\} \text{ for every } \varsigma_1 \in A \\ 0.42 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$\widetilde{B}_A(\varsigma_1) = \begin{cases} [0.8, 0.9] & \text{if } \varsigma_1 \in \{a, b\} \text{ for every } \varsigma_1 \in A \\ [0.3, 0.4] & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$J_A(s_1) = \begin{cases} 0 & \text{if } s_1 \in \{a, b\} \text{ for every } s_1 \in A \\ 0.46 & \text{otherwise for all } s_1 \in A \end{cases}$$

Then S is not a MBJ-neutrosophic WI-ideal of A for

$$M_A(t) < \min\{M_A((t \rightarrow b)^*), M_A(b)\}$$

$$\widetilde{B}_A(t) < \text{rmin}\{\widetilde{B}_A((t \rightarrow b)^*), \widetilde{B}_A(b)\}$$

$$J_A(t) > \max\{J_A((t \rightarrow b)^*), J_A(b)\}$$

3.3. Proposition:

Every MBJ-NWI-ideal $A=(M_A, \widetilde{B}_A, J_A)$ of L accomplish the following assertions.

$$(\forall s_1, s_2 \in L) \left(s_1 \leq s_2 \Rightarrow \begin{cases} M_A(s_1) \geq M_A(s_2) \\ \widetilde{B}_A(s_1) \geq \widetilde{B}_A(s_2) \\ J_A(s_1) \leq J_A(s_2) \end{cases} \right) \quad (3)$$

Proof: Let $A \in \text{NWI}(L)$ and $s_1, s_2 \in L$ such that $s_1 \leq s_2$. Since $(s_1 \rightarrow s_2)' = 0$,

We have, $M_A(s_1) \geq \min\{M_A((s_1 \rightarrow s_2)'), M_A(s_2)\} = \min\{M_A(0), M_A(s_2)\} = M_A(s_2)$,

$$\widetilde{B}_A(s_1) \geq \text{rmin}\{\widetilde{B}_A((s_1 \rightarrow s_2)'), \widetilde{B}_A(s_2)\} = \text{rmin}\{\widetilde{B}_A(0), \widetilde{B}_A(s_2)\} = \widetilde{B}_A(s_2),$$

$$J_A(s_1) \leq \max\{J_A((s_1 \rightarrow s_2)'), J_A(s_2)\} = \max\{J_A(0), J_A(s_2)\} = J_A(s_2).$$

3.4. Proposition: Every MBJ-N WI-ideal $A = (M_A, \widetilde{B}_A, J_A)$ of L accomplish the following assertions.

$$(\forall s_1, s_2, s_3 \in L) \left(s_1 \leq s_2' \rightarrow s_3 \Rightarrow \begin{cases} M_A(s_1) \geq \min\{M_A(s_2), M_A(s_3)\} \\ \widetilde{B}_A(s_1) \geq \text{rmin}\{\widetilde{B}_A(s_2), \widetilde{B}_A(s_3)\} \\ J_A(s_1) \leq \max\{J_A(s_2), J_A(s_3)\} \end{cases} \right) \quad (4)$$

Proof: Let $A \in \text{NWI}(L)$ such that for all $s_1, s_2, s_3 \in L, s_1 \leq s_2' \rightarrow s_3$.

$$\text{Then } 1 = s_1 \rightarrow (s_2' \rightarrow s_3) = s_3' \rightarrow (s_1 \rightarrow s_2) = (s_1 \rightarrow s_2)' \rightarrow s_3,$$

$$\text{And so } (s_1 \rightarrow s_2)' \rightarrow s_3 = 0.$$

By (2), we get that

$$M_A(s_1) \geq \min\{M_A((s_1 \rightarrow s_2)'), M_A(s_2)\}$$

$$\geq \min\{\min\{M_A(((s_1 \rightarrow s_2)' \rightarrow s_3)'), M_A(s_3)\}, M_A(s_2)\}$$

$$= \min\{\min\{M_A(0), M_A(s_3)\}, M_A(s_2)\}$$

$$= \min\{M_A(s_3), M_A(s_2)\}$$

$$\widetilde{B}_A(s_1) \geq \text{rmin}\{\widetilde{B}_A((s_1 \rightarrow s_2)'), \widetilde{B}_A(s_2)\}$$

$$\geq \text{rmin}\{\text{rmin}\{\widetilde{B}_A(((s_1 \rightarrow s_2)' \rightarrow s_3)'), \widetilde{B}_A(s_3)\}, \widetilde{B}_A(s_2)\}$$

$$= \text{rmin}\{\text{rmin}\{\widetilde{B}_A(0), \widetilde{B}_A(s_3)\}, \widetilde{B}_A(s_2)\}$$

$$= \text{rmin}\{\widetilde{B}_A(s_3), \widetilde{B}_A(s_2)\}, \text{ and}$$

$$J_A(s_1) \leq \max\{J_A((s_1 \rightarrow s_2)'), J_A(s_2)\}$$

$$\leq \max\{\max\{J_A(((s_1 \rightarrow s_2)' \rightarrow s_3)'), J_A(s_3)\}, J_A(s_2)\}$$

$$= \max\{\max\{J_A(0), J_A(s_3)\}, J_A(s_2)\}$$

$$= \max\{J_A(s_3), J_A(s_2)\}.$$

Hence the proof.

3.5. Definition: A MBJ-Nset $A=(M_A, \widetilde{B}_A, J_A)$ in L is called a MBJ-Lattice ideal of L if it satisfies(3)

$$\text{and } (\forall \varsigma_1, \varsigma_2 \in L) \left(\begin{array}{l} M_A(\varsigma_1 \vee \varsigma_2) \geq \min\{M_A(\varsigma_1), M_A(\varsigma_2)\} \\ \widetilde{B}_A(\varsigma_1 \vee \varsigma_2) \geq \text{rmin}\{\widetilde{B}_A(\varsigma_1), \widetilde{B}_A(\varsigma_2)\} \\ J_A(\varsigma_1 \vee \varsigma_2) \leq \max\{J_A(\varsigma_1), J_A(\varsigma_2)\} \end{array} \right) \quad (5)$$

3.6. Example: Let L be the Lattice implication algebra as in Example 3.2 and $A=(M_A, \widetilde{B}_A, J_A)$ be a MBJ-N set in L which is defined by

$$M_A(\varsigma_1) = \begin{cases} 1 & \text{if } \varsigma_1 \in \{0, d\} \text{ for every } \varsigma_1 \in A \\ 0.6 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$\widetilde{B}_A(\varsigma_1) = \begin{cases} [0.5, 0.6] & \text{if } \varsigma_1 \in \{0, d\} \text{ for every } \varsigma_1 \in A \\ [0.3, 0.4] & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$J_A(\varsigma_1) = \begin{cases} 0 & \text{if } \varsigma_1 \in \{0, d\} \text{ for every } \varsigma_1 \in A \\ 0.4 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

Then $A = (M_A, \widetilde{B}_A, J_A)$ is an MBJ-neutrosophic lattice ideal of A .

We discuss the relationship between a MBJ-NLI-ideal and a MBJ-N Lattice ideal.

3.7. Theorem: Every MBJ-NLI-ideal is a MBJ-N Lattice ideal.

Proof: Let $A = (M_A, \widetilde{B}_A, J_A) \in \text{NLI}(L)$. The condition (3) is valid.

$$\text{Since } ((\varsigma_1 \vee \varsigma_2) \rightarrow \varsigma_2)'$$

$$= (((\varsigma_1 \rightarrow \varsigma_2) \rightarrow \varsigma_2) \rightarrow \varsigma_2)'$$

$$= (\varsigma_1 \rightarrow \varsigma_2)'$$

$$\leq (\varsigma_1)' \text{ for all } \varsigma_1, \varsigma_2 \in L,$$

by(3) and(2),we have

$$M_A(\varsigma_1 \vee \varsigma_2) \geq \min \{M_A(((\varsigma_1 \vee \varsigma_2) \rightarrow \varsigma_2)'), M_A(\varsigma_2)\} \geq \min\{M_A(\varsigma_1), M_A(\varsigma_2)\},$$

$$\widetilde{B}_A(\varsigma_1 \vee \varsigma_2) \geq \text{rmin}\{\widetilde{B}_A(((\varsigma_1 \vee \varsigma_2) \rightarrow \varsigma_2)'), \widetilde{B}_A(\varsigma_2)\} \geq \text{rmin}\{\widetilde{B}_A(\varsigma_1), \widetilde{B}_A(\varsigma_2)\},$$

$$J_A(\varsigma_1 \vee \varsigma_2) \leq \max \{J_A(((\varsigma_1 \vee \varsigma_2) \rightarrow \varsigma_2)'), J_A(\varsigma_2)\} \leq \max\{J_A(\varsigma_1), J_A(\varsigma_2)\}$$

Hence, $A=(M_A, \widetilde{B}_A, J_A)$ is a MBJ-N Latticeideal.

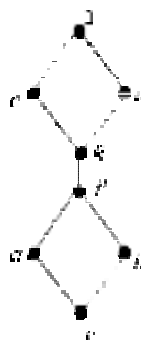
By the example given below, we observed the falsity of the converse of the Theorem 3.7.

3.8. Example:

Let $A=\{0, a, b, p, q, c, d, 1\}$ be a set with Figure(2) as a partial ordering. Define a quasi complement “ $*$ ” and a binary operation “ $\rightarrow$ ” on A as in Table (3) and Table (4).

Figure (2)

Define $\vee$ and $\wedge$ operations on A as follows



$$(\varsigma_1 \vee \varsigma_2) = (\varsigma_1 \rightarrow \varsigma_2) \rightarrow \varsigma_2 ,$$

$$(\varsigma_1 \wedge \varsigma_2) = ((\varsigma_1^* \rightarrow \varsigma_2^*) \rightarrow \varsigma_2^*)^* \text{ for all } \varsigma_1, \varsigma_2 \in A$$

Table 1:

ς_1	$\mathbf{x}^*$ ς_1^*
0	1
a	b
b	a
p	0
q	0
c	0
d	0
1	0

Table 2:

ς_1	00	0	A	a	b	C	p	q	C	d	1
0	1		1	1	1	1	1	1	1	1	1
a		b	1		b	1	1	1	1	1	1
b		a	a	1	1	1	1	1	1	1	1
p		0	a	b	1	1	1	1	1	1	1
q		0	a	b	p	1	1	1	1	1	1
c		0	a	b	p	d	1	d	1	1	1
d		0	a	b	p	c	C	1	1	1	1
1		0	a	b	p	q	C	d	1	1	1

Let $A=(M_A, \tilde{B}_A, J_A)$ be a MBJ-N set in L which is defined by

$$M_A(\varsigma_1) = \begin{cases} 1 & \text{if } \varsigma_1 \in \{0, b, d\} \text{ for every } \varsigma_1 \in A \\ 0.7 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$\tilde{B}_A(\varsigma_1) = \begin{cases} [0.5, 0.6] & \text{if } \varsigma_1 \in \{0, b, d\} \text{ for every } \varsigma_1 \in A \\ [0.3, 0.4] & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

$$J_A(\varsigma_1) = \begin{cases} 0 & \text{if } \varsigma_1 \in \{0, b, d\} \text{ for every } \varsigma_1 \in A \\ 0.3 & \text{otherwise for all } \varsigma_1 \in A \end{cases}$$

Thus we have $A=(M_A, \tilde{B}_A, J_A)$ is a MBJ-neutrosophic lattice ideal of A , but not a MBJ-neutrosophic WI-ideal of A for

$$M_A(p) < \min\{M_A((p \rightarrow d)^*), M_A(d)\}$$

$$\tilde{B}_A(p) < \min\{\tilde{B}_A((p \rightarrow d)^*), \tilde{B}_A(d)\}$$

$$J_A(p) > \max\{J_A((p \rightarrow d)^*), J_A(d)\}$$

Now we investigate that under which condition, a MBJ-N Lattice ideal can be a MBJ-NLI-ideal.

3.9. Theorem:

In a Lattice H-implication algebra L , every MBJ-N Lattice ideal is a MBJ-NLI-ideal.

Proof: Let $A=(M_A, \tilde{B}_A, J_A)$ be a MBJ-N Lattice ideal of a Lattice H-implication algebra L .

Moreover, since $0 \leq \varsigma_1$ for all $\varsigma_1 \in L$, it follows from (1) that

$$M_A(0) \geq M_A(\varsigma_1), \tilde{B}_A(0) \geq \tilde{B}_A(\varsigma_1) \text{ and } J_A(0) \leq J_A(\varsigma_1).$$

Also, from $\varsigma_1 \leq \varsigma_1 \vee \varsigma_2$ for all $\varsigma_1, \varsigma_2 \in L$, by (3) and (5) we get that,

$$\begin{aligned} M_A(\varsigma_1) &\geq M_A(\varsigma_1 \vee \varsigma_2) = M_A(\varsigma_2 \vee (\varsigma_1 \vee \varsigma_2)) \\ &= M_A(\varsigma_2 \vee (\varsigma_1 \rightarrow \varsigma_2)) \\ &\geq \min\{M_A(\varsigma_2), M_A((\varsigma_1 \rightarrow \varsigma_2)')\}, \\ \widetilde{B}_A(\varsigma_1) &\geq \widetilde{B}_A(\varsigma_1 \vee \varsigma_2) = \widetilde{B}_A(\varsigma_2 \vee (\varsigma_1 \vee \varsigma_2)) \\ &= \widetilde{B}_A(\varsigma_2 \vee (\varsigma_1 \rightarrow \varsigma_2)) \\ &\geq \min\{\widetilde{B}_A(\varsigma_2), \widetilde{B}_A((\varsigma_1 \rightarrow \varsigma_2)')\}, \\ \text{And } J_A(\varsigma_1) &\leq J_A(\varsigma_1 \vee \varsigma_2) = J_A(\varsigma_2 \vee (\varsigma_1 \vee \varsigma_2)) \\ &= J_A(\varsigma_2 \vee (\varsigma_1 \rightarrow \varsigma_2)) \\ &\leq \max\{J_A(\varsigma_2), J_A((\varsigma_1 \rightarrow \varsigma_2)')\}. \end{aligned}$$

Therefore, $A = (M_A, \widetilde{B}_A, J_A) \in \text{MBJ-NLI}(L)$.

4. Conclusion:

In this paper, we have introduced the definitions of MBJ-neutrosophic WI-ideal and MBJ-neutrosophic lattice ideal of lattice Wajsberg algebra. We have discussed some of their properties with illustrations. Also, we have shown that every MBJ-neutrosophic WI-ideal of lattice Wajsberg algebra is an MBJ-neutrosophic lattice ideal of lattice Wajsberg algebra. But the converse part is true only in the lattice H-Wajsberg algebras. We hope that more links of logics emerge by the stipulating of this work.

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