On 2-Commutative Derivations of Semi-prime Rings

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Abstract

The main purpose of this paper is to study some results concerning the generalized derivation D defined on semi-prime ring R, we obtain a derivation d which commuting and 2-commuting on R. Also, we present many examples to clarify the validity of our work.

Keywords: ring; semi-prime ring; commutative ring; derivation

1. Introduction

This research has been motivated by [1] and [2]. The history of commuting and centralizing mappings goes back to (1955) when Divinsky [3] proved that a simple Artinian ring is commutative if it has a commuting nontrivial automorphism. Posner in [4] has proved that the existence of a non-zero centralizing derivation on a prime ring forces the ring to be commutative (Posner's second theorem).Luch [5] generalized the Divinsky result, we have just mentioned above, to arbitrary prime ring. Mayne in [6] proved that in case there exists a nontrivial centralizing automorphism on a prime ring, then the ring is commutative (Mayne's theorem). In [7], authors have shown that every semi-commuting automorphism of a prime ring is commuting provided that R has either characteristic different from 3 or non- zero center and thus they proved the commutativity of prime ring having nontrivial semi-commuting automorphism except in the indicated cases Generalized derivation of operators on various algebraic structures have been an active area of research since the last fifty years due to their usefulness in various fields of mathematics. Some authors have studied centralizers in the general framework of semi-prime rings (see [12-24]). Muhammad A.C.and Mohammed S.S. [16] proved, let R be a semiprime ring and $d: R \rightarrow R$ a mapping satisfies d(x)y=xd(y) for all $x, y \in R$.Then d is a centralizer. Molnar [15] has proved, let R be a 2-torsion free prime ring and let $d: R \rightarrow R$ be an additive mapping.If d(xyx)=d(x)yx holds for every $x, y \in R$, then d is a left centralizer. Muhammad A.C. and A.B. Thaheem [17] proved, let d and g be a pair of derivations of semiprime ring R semiprime ring R satisfying $d(x)x + xg(x) \in Z(R)$, then cd and $c \in Z(R)$, .J. Vukman [24] proved, let R be a 2-torsion free semiprime ring and let $d: R \rightarrow R$ be an additive centralizing mapping on R, in this case, d is commuting on R .B.Zalar [24] has proved, let R be a 2- torsion free semiprime ring and $d: R \rightarrow R$ an dditive mapping which satisfies $d(x{}^\wedge 2) = d(x)x$ for all $x \in R$ .Then d is a left centralizer. Hvala [11] initiated the algebraic study of generalized derivation and extended some results concerning derivation to generalized derivation..In this paper we study and investigate some results concerning generalized derivation D on semiprime ring R, we give some results about that.

2. Preliminaries

Throughout this paper, R will represent an associative ring without identity and has a cancellation property with the center Z(R). We recall that R is semiprime if xRx= (0) implies x=0 and it is prime if xRy=(0) implies x=0 or y=0 .A prime ring is semiprime but the converse is not true in general. A ring R is 2-torsion free in case 2x=0 implies that x=0 for any $x \in R$. An additive mapping $d: R \rightarrow R$ is called a derivation if d(xy)=d(x)y+xd(y) holds for all $x, y \in R$. A mapping d is called centralizing if $[d(x), x] \in Z(R)$ for all $x \in R$ if $[d(x), x] = 0$ for all $x \in R$, then it is called commuting, and is called central if $d(x) \in Z(R)$ for all $x \in R$. Every central mapping is obviously commuting but not conversely in general. In [10]Q. Deng and H.E. Bell extended the notion of commutativity to one of n-commutativity, where n is an arbitrary positive integer, by defining a mapping d to be n-commuting on U if $[x^n, d(x)] = 0$ for all $x \in U$, where U be a non empty subset of R. Following Bresar [9] an additive mapping $D: R \rightarrow R$ is called a generalized derivation on R if there exists a derivation $d: R \rightarrow R$ such that

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D(xy)=D(x)y+xd(y) holds for all $x, y \in R$. However, generalized derivation covers the concept of derivation. Also with d=0, a generalized derivation covers the concept of left multiplier (left centralizer) that is, an additive mapping D satisfying D(xy)=D(x)y for all $x, y \in R$. As usual, we write [x,y] for xy-yx and make use of the commutator identities [xy,z]=x[y,z]+[x,z]y and [x,yz]=y[x,z]+[x,y]z, and the symbol xoy stands for the anti-commutator xy+yx. The following Lemmas are necessary for the paper

Lemma:

Any anticommutative semiprime ring R is commutative, where A ring R is said to be anticommutative if xy=-yx (that is, xy+yx=0) for all $x, y \in R$

3. Main results

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non- zero derivation d such that $D([x, y]) = [x, y]$ for all $x, y \in R$. Then d is commuting on R.

Proof: For $x, y \in R$, we have D([x,y])=[x,y] for all $x, y \in R$, which gives $D(x)y + xd(y) - D(y)x - yd(x) - [x, y] = 0$ (1) Replacing y by yz in (1), we obtain $D(x)yz + xd(y)z + xyd(z) - D(y)zx - yd(z)x - yzd(x) - y[x, z] - [x, y]z = 0$ for all $x, y \in R$. (2)

Substituting (1) in (2) gives

$D(y)[x, z] + yd(x)z + xd(y)z + xyd(z) - yd(z)x - yzd(x) - y[x, z] = 0$ for all $x, y \in R$. (3)

Replacing z by x in (3), we obtain $xd(y)x + xyd(x) - yxd(x) = 0$ for all $x, y \in R$ (4)

Replacing y by x in (4), we get xd(x)x=0 for all $x \in R$ (5)

By using the cancellation property on x, from left, we obtain d(x)x=0 for all $x \in R$ (6)

Again by using the cancellation property on x, from right, we get xd(x)=0 for all $x \in R$. (7)

Subtracting (6) and (7), we obtain $[d(x), x] = 0$ for all $x \in R$ .Thus ,d is commuting on R, by this we complete our proof.

A slight modification in the proof of the above theorem yields the following.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $D([x, y]) + [x, y] = 0$ for all $x, y \in R$ .Then d is commuting on R.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that D(xoy)=(xoy) for all $x, y \in R$. Then d is 2- commuting on R.

Proof: For any $x, y \in R$,we have D(xoy)=(xoy) for all $x, y \in R$. This can be written as $D(x)y + xd(y) + D(y)x + yd(x) - (xoy) = 0$ for all $x, y \in R$ (8)

Replacing y by yx in above equation, we obtain $D(x)yx + xd(y)x + xyd(x) + D(y)x^2 + yd(x)x - (xoy)x = 0$ for all $x, y \in R$ (9)

According to (8) the relation above reduced to (xoy)d(x)=0 for all $x, y \in R$.

By using the cancellation property on d(x), we get (xoy)=0 for all $x, y \in R$ (10).

By Lemma 2.1, we get $[x, y] = 0$ for all $x, y \in R$ (11)

Replacing x by x^2 and y by d(x),we get $[d(x), x^2] = 0$ for all $x, y \in R$. Thus, d is 2- commuting on R.

We complete our proof.

A slight modification in the proof of the Theorem (3.3) yields the following

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that D(xoy)+(xoy) =0 for all $x, y \in R$.Then d is 2-commuting on R.

Proposition

Let R be a semiprime ring.If R admits a non-zero ideal generalized derivation D associated with a non-zero derivation d such that $D([x, y]) \pm (xoy) = 0$ for all $x, y \in R$. Then 2- d is commuting on R .

Proof: For any $x, y \in R$, we have $D([x, y]) - (xoy) = 0$.Then $[D([x, y]), r] - [(xoy), r] = 0$ for all $x, y, r \in R$. Replacing y by x, we obtain $2[x^2, r] = 0$ for all $x, r \in R$. By using the cancellation property with replacing r by d(x), we obtain, 2-d is commuting on R.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)oD(y)=0 for all $x, y \in R$. Then d is commuting on R.

Proof: We have d(x)oD(y)=0 for all $x, y \in R$ (12)

Replacing y by yr, we obtain $(d(x)oy)d(r) - y[d(x), d(r)] + \big(d(x)oD(y)\big)rD(y)[d(x), r] = 0$ for all $x, y \in U$, $r \in R$. (13)

According to (12), then (13) reduced to

$(d(x)oy)d(r) - y[d(x), d(r)]D(y)[d(x), r] = 0$ for all $x, y \in U$, $r \in R$.

Replacing r by d(x), we get

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$(d(x)oy)d^2(x) - y[d(x), d^2(x)] = 0$ for all $x, y \in R$ (14)
Replacing y by zy in (14), with using (14), we obtain $[d(x), z]yd^2(x) = 0$ for all $x, y, z \in R$. (15)
By using the cancellation property on (15), from right, we obtain $[d(x), z]y = 0$ for all $x, y, z \in R$ (16)
Since R is semiprime from above relation, we get $[d(x), z] = 0$ for all $x, y \in R$ (17)
Replacing z by x, we obtain, d is commuting on R.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $[d(x), D(y)] = 0$ for all $x, y \in R$. Then d is 2- commuting on R.

Proof: We have $[d(x), D(y)] = 0$ for all $x, y \in R$. (18)
Replacing y by yz in (18) and using the result with (18),we obtain
$D(y)[d(x), z] + y[d(x), d(z)] + [d(x), y]d(z) = 0$ for all $x, y \in R$. (19)
Replacing z by zd(x) in (19) and using the result with (19), we get $yz[d(x), d^2(x)] + y[d(x), z]d^2(x) + [d(x), y]zd^2(x) = 0$ for all $x, y \in R$. (20)
Again, replacing y by ry in (20) and using the result with (20), we obtain $[d(x), z]yd^2(x) = 0$ for all $x, y, z \in R$.
By using similar arguments as in the proof of Theorem 3.6, we obtain the required result.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)oD(y)=xoy for all $x, y \in R$. Then d is 2-commuting on R.

Proof: For any $x, y \in R$, we have d(x)oD(y)=xoy for all $x, y \in R$. Replacing y by yr, we get
$(d(x)oy)d(r) - y[d(x), d(r)] + \big(d(x)oD(y)\big)r - D(y)[d(x), r] = (xoy)r - y[x, r]$ for all $x, y, r \in R$. Using our relation, we obtain $(d(x)oy)d(r) - y[d(x), d(r)] - D(y)[d(x), r] + y[x, r] = 0$ for all $x, y, r \in R$. (21)
In (21) replacing r by d(x), we obtain
$(d(x)oy)d^2(x) - y[d(x), d^2(x)] + y[x, d(x)] = 0$ for all $x, y \in R$. (22)
Replacing y by zy in (22), we obtain
$(z(d(x)oy) + [d(x), z]y)d^2(x) - zy[d(x), d^2(x)] + zy[x, d(x)] = 0$ for all $x, y \in R$. (23)
According to (22), above relation reduced to $[d(x), z]yd^2(x) = 0$ for all $x, y, z \in R$. (24)
By using similar arguments as in the proof of Theorem 3.6, we obtain the required result.
A slight modification in the proof of the Theorem (3.8), yields the following

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)oD(y)+xoy=0 for all $x, y \in R$. Then d is 2- commuting on R.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $d(x)D(y) - xy \in z(R)$ for all $x, y \in R$. Then d is commuting on R.

Proof: For any $x, y \in R$, we have $d(x)D(y) - xy \in z(R)$, replacing y by yr, we obtain $(d(x)D(Y) - xy)r + d(x)yd(r) \in z(R)$ for all $x, y, r \in R$ (25).
This implies that $[d(x)yd(r), r] = 0$ for all $x, y, r \in R$. (26)
Hence it follows that
$d(x)[yd(r), r] + [d(x), r]yd(r) = 0$ for all $x, y, r \in R$. (27)
In (27) replacing y by d(x)y, we obtain
$[d(x), r]d(x)yd(r) = 0$ for all $x, y, r \in R$. (28)
By using the cancellation property on d(x)yd(r), we obtain
$[d(x), r] = 0$ for all $x, r \in R$. (29)
Replacing r by x in above relation, we obtain $[d(x), x] = 0$ for all $x \in R$. (30)
Then according to (30), we obtain d is commuting on R.
By same method in above theorem, we can prove the following

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $d(x)D(y) + xy \in Z(R)$ for all $x, y \in R$.Then d is commuting on R.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $[d(x)D(y)] = [x, y]$ for all $x, y \in R$.Then d is 2- commuting on R.

Proof: For any $x, y \in R$, we have
$[d(x)D(y)] = [x, y]$ for all $x, y \in R$. (31).
Replacing y by yz in (31), with using the result with (31), we obtain
$D(y)[d(x), z] + y[d(x), d(z)] + [d(x), y]d(z) = y[x, z]$ for all $x, y \in R$. (32).
Again, replacing z by zd(x) in (32) with using the result with (32), we obtain
$y[d(x), z]d^2(x) + yz[d(x), d^2(x)] + [d(x), y]zd^2(x) = yz[x, d(x)]$ for all $x, y \in R$. (33).
Replacing y by ry in (33), we obtain

$\mathrm{ryz}[\mathrm{d}(\mathrm{x}),\mathrm{d}^2(\mathrm{x})]+\mathrm{ry}[\mathrm{d}(\mathrm{x}),\mathrm{z}]\mathrm{d}^2(\mathrm{x})+\mathrm{r}[\mathrm{d}(\mathrm{x}),\mathrm{y}]\mathrm{zd}^2(\mathrm{x})+[\mathrm{d}(\mathrm{x}),\mathrm{r}]\mathrm{yzd}^2(\mathrm{x})=\mathrm{ryz}[\mathrm{x},\mathrm{d}(\mathrm{x})]$ for all $\mathrm{x},\mathrm{y},\mathrm{r}\in\mathrm{R}$. (34).
According to (33), the relation (34) reduced to $[\mathrm{d}(\mathrm{x}),\mathrm{r}]\mathrm{yzd}^2(\mathrm{x})=0$ for all $\mathrm{x},\mathrm{y},\mathrm{r}\in\mathrm{R}$. (35)
Thus, by same method in Theorem 3.6, we complete our proof.
Proceeding on the same lines with necessary variations, we can prove the following.

Theorem

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $[\mathrm{d}(\mathrm{x}),\mathrm{D}(\mathrm{y})]+[\mathrm{x},\mathrm{y}]=0$ for all $\mathrm{x},\mathrm{y}\in\mathrm{R}$. Then d is 2- commuting on R.

Remark

In general we cannot obtain that d is non commuting where non satisfy the conditions which appear in our theorems, the following example explain that.

Example

Let

$R=\left\{\begin{pmatrix}a&0\\0&b\end{pmatrix}/a^2=b+a,a,b\in Z,the\ set\ of\ integers\right\}$

be a ring with cancellation property, and the additive map D define as the following

$D\left(\begin{pmatrix}a&0\\0&b\end{pmatrix}\right)=\begin{pmatrix}a&0\\0&0\end{pmatrix}$ and $d(x)=[a,x]$, where

$a=\begin{pmatrix}0&0\\1&0\end{pmatrix}$ then, let

$x=\begin{pmatrix}a&0\\0&b\end{pmatrix}$ and $y=\begin{pmatrix}g&0\\0&h\end{pmatrix}$. Then

$D(x,y)=D(x)y+xd(y)$

$D\left(\begin{pmatrix}a&0\\0&b\end{pmatrix}\begin{pmatrix}g&0\\0&h\end{pmatrix}\right)=D\begin{pmatrix}a&0\\0&b\end{pmatrix}\begin{pmatrix}g&0\\0&h\end{pmatrix}+\begin{pmatrix}a&0\\0&b\end{pmatrix}d\begin{pmatrix}g&0\\0&h\end{pmatrix}$

$D\begin{pmatrix}ag&0\\0&bh\end{pmatrix}=\begin{pmatrix}a&0\\0&0\end{pmatrix}\begin{pmatrix}g&0\\0&h\end{pmatrix}+\begin{pmatrix}a&0\\0&b\end{pmatrix}\begin{pmatrix}0&0\\h-g&0\end{pmatrix}$

$\begin{pmatrix}ag&0\\0&0\end{pmatrix}=\begin{pmatrix}ag&0\\b(h-g)&0\end{pmatrix}$. Then

$\begin{pmatrix}0&0\\b(h-g)&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$. (∗)

$b.\begin{pmatrix}0&0\\h-g&0\end{pmatrix}=b.\begin{pmatrix}0&0\\0&0\end{pmatrix}$.

Then by using the cancellation property on b, we get

$\begin{pmatrix}0&0\\h-g&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$, which implies h=g

with substituting this result in equation (∗), we obtaln D is generalized derivation. Then we have

$d(x)x=xd(x),where\ x=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

$d(x)=\begin{pmatrix}0&0\\b-a&0\end{pmatrix}$, then

$\begin{pmatrix}0&0\\b-a&0\end{pmatrix}\begin{pmatrix}a&0\\0&b\end{pmatrix}=\begin{pmatrix}a&0\\0&b\end{pmatrix}\begin{pmatrix}0&0\\b-a&0\end{pmatrix}\begin{pmatrix}0&0\\(b-a)a&0\end{pmatrix}-\begin{pmatrix}0&0\\b(b-a)&0\end{pmatrix}$. It is easy that d is non commuting on R.

Remark

In our theorems we cannot exclude the condition cancellation property, the following example explain that .

Example

$Let\ R-\left\{\begin{pmatrix}x&y\\0&z\end{pmatrix}/x,y,z\in Z,the\ set\ of\ integers\right\}$.

be a ring with cancellation property, and let m be fixed element of Z and the additive map D define as the following

$D\left(\begin{pmatrix}x&y\\0&z\end{pmatrix}\right)=\begin{pmatrix}0&mx+mz\\0&0\end{pmatrix}$, *and the derivation d def in as*

$d\left(\begin{pmatrix}x&y\\0&z\end{pmatrix}\right)=\begin{pmatrix}0&mx+mz\\0&0\end{pmatrix}$. *Then it is easy that D is generalized derivation on R. then*

Then it is easy that D is generalized derivation

$d(x)x=xd(x),where\ x=\begin{pmatrix}a&b\\0&c\end{pmatrix}$ *then*

$\begin{pmatrix}0&ma-mc\\0&0\end{pmatrix}\begin{pmatrix}a&b\\0&c\end{pmatrix}=\begin{pmatrix}a&b\\0&c\end{pmatrix}\begin{pmatrix}0&ma-mc\\0&0\end{pmatrix}$

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$$\begin{pmatrix} 0 & cma - mc^2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & ama - amc \\ 0 & 0 \end{pmatrix} \quad (*)$$

$$\begin{pmatrix} 0 & m(ca - c^2 - a^2 + ac) \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \text{ then}$$

$$m\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$m\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix}, \text{ then}$$

by using the cancellation property on $\begin{pmatrix} 0 & 2ac \;\; c^2 \;\; a^2 \\ 0 & 0 \end{pmatrix}$ we obtain $m = 0$, therefore, by substituting this result in $(*)$ give d is commuting.

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