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On The Anti Fuzzy k-Ideals of Ternary Semi-rings

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Abstract

In this paper, we introduce the notion of anti fuzzy k-ideal of ternary semi ring and we study some of its elementary properties by many related theorems and proofs, with many illustrated examples that explain our work.
Keywords: Fuzzy set; fuzzy ideal; ternary ring; semi-ring

1. Introduction

The notion of fuzzy subset of a set was introduced by Zadeh in 1965 [4].The notion of fuzzy subgroup was made by Rosenfeld in 1971[1].Fuzzy ideal in a ring were introduced by W. Liu in 1982 [9].In 1996, Kim and Park studied fuzzy ideal in semirings[2]. The notion of ternary semi rings introduced by Dutta and Kar in 2003[8]. In 2007, J.kavikumar and Azme Bin khamis introduced The notion of fuzzy ideals in ternary semirings[3]. R. Biswas given the notion of anti fuzzy in 1999 [5]. The notion of fuzzy k-ideals in ternary semirings was introduced by Sathinee Malee and Ronnason Chinram in 2010[7].

The main purpose of the paper is to introduce the notion of anti fuzzy k-ideal of ternary semiring and study some properties of it.

1- Preliminaries

In this section we review some basic definition which will be used in this paper.

Definition (1.1) [6] A non empty set R together with a binary operation, called addition and ternary multiplication, is said to be a ternary semi ring if R is an additive commutative semigroup satisfying the following conditions

(i) (abc)de=a(bcd)e=ab(cde),
(ii) (a+b)cd=acd+bcd,
(iii) a(b+c)d=abd+acd,
(iv) ab(c+d)=abc+abd, for all a,b,c,d,e∈R

Definition (1.2) [6] Let R be a ternary semiring .If there exists an element $0 \in R$ such that

$0 + x = x \text{ and } 0xy = x0y = xy0, \text{ for all } x, y \in R$

then “0” is called the zero element or simply the zero of ternary semirings. In this case we say that R is a ternary semiring with zero

Definition (1.3) [6] An additive subsemigroup I of ternary semiring R is called a left (resp., right and lateral) ideal of R if

$s_1 s_2 i (resp., i s_1 s_2, s_1 i s_2) \in I \quad \forall s_1 s_2 \in R$ and $i \in I$

If I is both left and right ideal of R, then I is called a two sided ideal of R. If I is a left, a right and a lateral ideal of R, then I is called an ideal of R.

Definition (1.4) [4] A function μ from a non empty set X to the interval [0,1] is called a fuzzy subset of X .

Definition (1.5) [4] The complement of a fuzzy subset μ of a set X is denoted by μ^c and defined as

$\mu^c(x) = 1 - \mu(x), \forall x \in X$

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Definition (1.6)[3] Let ν and μ be any two fuzzy subsets of X then $\nu \cap \mu$ *and* $\nu \cup \mu$ are fuzzy subset of X and defined by
$(\nu \cap \mu)(x) = \min\{\nu(x), \mu(x)\}$
$(\nu \cup \mu)(x) = \max\{\nu(x), \mu(x)\}, \forall x \in X$
Definition (1.7) [3] A fuzzy subsemigroup μ of a ternary semiring R is called a fuzzy ideal of R if the function $\mu : R \to [0,1]$ satisfying the following conditions:

(i) $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in R$
(ii) $\mu(xyz) \geq \mu(z)$
(iii) $\mu(xyz) \geq \mu(x)$
(iv) $\mu(xyz) \geq \mu(y), \forall x, y, z \in R$

A fuzzy set μ with conditions (i)and (ii) is called an fuzzy left ideal of R. If a fuzzy set μ satisfies (i) and (iii) then it is called a fuzzy right ideal of R. Also if μ satisfy (i) and (iv) then it is called a fuzzy lateral ideal of R.
If μ is fuzzy left ideal, fuzzy right ideal and fuzzy lateral ideal then it is called fuzzy ideal of a ternary semiring R.
Definition (1.8)[7] A fuzzy ideal μ of a ternary semiring R is said to be a fuzzy k-ideal of R if
$\mu(x) \geq \min\{\mu(x+y), \mu(y)\}, \forall x, y \in R$
Definition(1.9)[7] Let S and R be two ternary semirings. a mapping $f: S \to R$ is said to be a homomorphism if
$f(x+y) = f(x) + f(y)$ and $f(xyz) = f(x)f(y)f(z), \forall x, y, z \in S$
If S and R are ternary semirings with zero 0 ,then $f(0) = 0$.
Definition (1.10)[7] Let $f: S \to R$ be ahomomorphism of ternary semirings and μ be a fuzzy subset of S ,we define a fuzzy subset $f(\mu)$ of R by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \quad if & f^{-1}(y) \neq \emptyset \\ 0 & other\ wise \end{cases}$$

We call $f(\mu)$ the image of μ under *f*.
Definition (1.11)[7] Let R_1 and R_2 be two ternary semirings and f be a function of R_1 into R_2. If μ is a fuzzy subset of R_2, then the preimage of μ under *f* is a fuzzy subset of R_1 defined by
$f^{-1}(\mu)(x) = \mu(f(x)) \quad \forall x \in R_1$
Definition (1.12)[9] Let $\phi: R_1 \to R_2$ be any function. An anti fuzzy ideal μ of R_1 is called ϕ -invariant if $\phi(x) = \phi(y)$ implies$\mu(x) = \mu(y)$ where $x, y \in R_1$.

2-The Main Results

Definition (2.1) A fuzzy subset μ of a ternary semiring R is said to be an anti fuzzy left (right, lateral) ideal of R if

1- $\mu(x+y) \leq \max\{\mu(x), \mu(y)\}$,
2- $\mu(xyz) \leq \mu(z), [\mu(xyy) \leq \mu(x), \mu(xyz) \leq \mu(y)]$, for all $x, y, z \in R$

μ is an anti fuzzy ideal of R if it is anti fuzzy left ideal, anti fuzzy right ideal and anti fuzzy lateral ideal of R.
Definition (2.2) An anti fuzzy ideal μ of a ternary semiring R is said to be an anti fuzzy k-ideal of R if
$\mu(x) \leq \max\{\mu(x+y), \mu(y)\}$, for all $x, y \in R$
Example (2.3) Let R be the set of nonpositive integer with zero. R is ternary semiring with the usual addition and ternary multiplication,
Let μ a fuzzy subset of R defined by

$$\mu(x) = \begin{cases} 0 & if\ x\ \ is\ an\ even\ \ or\ 0 \\ 1 & if\ x\ \ is\ an\ odd \end{cases}$$

Then μ is an anti fuzzy k-ideal of R.
Proposition (2.4) Let R be a ternary semiring and μ be a fuzzy subset of R. Then μ is an anti fuzzy k-ideal of R if and only if $\mu^c$is a fuzzy k-ideal of R.
Proof:
Suppose μ be an anti fuzzy k-ideal of R.
Let $x, y, z \in$ R,
$\mu^c(x+y) = 1 - \mu(x+y)$, since μ is an anti fuzzy k-ideal of R
$\geq 1 - \max\{\mu(x), \mu(y)\}$
$= \min\{1-\mu(x), 1-\mu(y)\}$
$= \min\{\mu^c(x), \mu^c(y)\}$,
$\mu^c(xyz) = 1 - \mu(xyz)$ since μ is an anti fuzzy left k-ideal of R
$\geq 1 - \mu(z)$
$= \mu^c(z)$. Hence μ^c is fuzzy left ideal of R
$\mu^c(xyz) = 1 - \mu(xyz)$

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$\geq 1-\mu(x)$
$=\mu^c(x)$. Hence μ^c is fuzzy right ideal of R
$\mu^c(xyz)=1-\mu(xyz)$
$\geq 1-\mu(y)$
$=\mu^c(y)$. Hence μ^c is fuzzy lateral ideal of R
Then μ^c is a fuzzy ideal of R
Let $x,y \in R$ Then
$\mu^c(x)=1-\mu(x)$
$\geq 1-\max\{\mu(x+y),\mu(y)\}$
$=\min\{1-\mu(x+y),1-\mu(y)\}$
$=\min\{\mu^c(x+y),\mu^c(y)\}$.
Hence μ^c is a fuzzy k-ideal of R.
Conversely, let μ^c be a fuzzy k-ideal of R
For $x,y,z \in R$, we have

$\mu(x+y)=1-\mu^c(x+y)$
$\leq 1-\min\{\mu^c(x),\mu^c(y)\}$
$=\max\{\mu(x),\mu(y)\}$
$\mu(xyz)=1-\mu^c(xyz)$
$\leq 1-\mu^c(z)$
$=\mu(z)$. Hence μ is an anti fuzzy left ideal of R
Similarly we can prove that μ is an anti fuzzy right and lateral ideal of R
then μ is an anti fuzzy ideal of R
Let $x,y \in R$ Then
$\mu(x)=1-\mu^c(x)$
$\leq 1-\min\{\mu^c(x+y),\mu^c(y)\}$
$=\max\{\mu(x+y),\mu(y)\}$
Hence μ is an anti fuzzy k-ideal of R.

Proposition (2.5) Let μ and ν are anti fuzzy k- ideal of ternary semiring R. Then $\mu\cup\nu$ is also an anti fuzzy k- ideal of ternary semiring R.

Proof

Let μ and ν be two anti fuzzy k- ideals of a ternary semiring R and $x,y,z \in R$. Then we have
$(\mu\cup\nu)(x+y)=\max\{\mu(x+y),\nu(x+y)\}$
$\leq \max\{\max\{\mu(x),\mu(y)\},\max\{\nu(x),\nu(y)\}\}$
$=\max\{\max\{\mu(x),\nu(x)\},\max\{\mu(y),\nu(y)\}\}$
$=\max\{(\mu\cup\nu)(x),(\mu\cup\nu)(y)\}$
$(\mu\cup\nu)(xyz)=\max\{\mu(xyz),\nu(xyz)\}$
$\leq \max\{\mu(z),\nu(z)\}$
$=(\mu\cup\nu)(z)$,
Hence $\mu\cup\nu$ is an anti fuzzy left ideal of R similarly we can prove that $\mu\cup\nu$ is an anti fuzzy right and lateral ideal of R
then $\mu\cup\nu$ is an anti fuzzy ideal of R
Let x,y $\in$ R, since μ and ν are anti fuzzy k- ideal Then
$\mu(x)\leq \max\{\mu(x+y),\mu(y)\}$ and $\nu(x)\leq \max\{\nu(x+y),\nu(y)\}$,
$(\mu\cup\nu)(x)=\max\{\mu(x),\nu(x)\}$
$\leq \max\{\max\{\mu(x+y),\mu(y),\max\{\nu(x+y),\nu(y)\}\}$
$=\max\{\max\{\mu(x+y),\nu(x+y),\max\{\mu(y),\nu(y)\}\}$
$=\max\{(\mu\cup\nu)(x+y),(\mu\cup\nu)(y)\}$
then $\mu\cup\nu$ is an anti fuzzy k-ideal of R.

Theorem (2.6) Let f: $R_1 \rightarrow R_2$ be an onto homomorphism of ternary semirings R_1 and R_2. If μ is an anti fuzzy k-ideal of R_2, then $f^{-1}(\mu)$ is an anti fuzzy k-ideal of R_1.

Proof Let μ be an anti fuzzy k-ideal of R_2 and let $,y,z \in R_1$,
Then we have
$f^{-1}(\mu)(x+y)=\mu(f(x+y))$,since f is a homomorphism then
$=\mu(f(x)+f(y))$
$\leq \max\{\mu(f(x)),\mu(f(y))\}$
$=\max\{f^{-1}(\mu)(x),f^{-1}(\mu)(y)\}$

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$$f^{-1}(\mu)(xyz) = \mu\big(f(xyz)\big) \le \mu\big(f(z)\big) = f^{-1}(\mu)(z)$$

Hence $f^{-1}(\mu)$ is an anti fuzzy left ideal of R_1
Similarly we $f^{-1}(\mu)$ is an anti fuzzy right and lateral ideal of R_1 then $f^{-1}(\mu)$is an anti fuzzy ideal of R_1
Let $x, y \in R_1$Then
$f^{-1}(\mu)(x) = \mu(f(x))$
$\le \max\{\mu\big(f(x) + f(y)\big), \mu\big(f(y)\big)\}$,
since f is a homomorphism then
$= \max\{\mu\big(f(x + y)\big), \mu\big(f(y)\big)\}$
$\max\{f^{-1}(\mu)(x + y), f^{-1}(\mu)(y)\}$
Hence $f^{-1}(\mu)$ is an anti fuzzy k-ideal of R_1.

Lemma (2.7)[7] Let R_1 and R_2 be two ternary semirings and $\phi: R_1 \to R_2$ be a homomorphism. Let μ be aϕ -invariant anti fuzzy ideal of R_1if $x = x \in R_2$ such that $x = \phi(a)$, $a \in R_1$ then $\phi(\mu)(x) = \mu(a)$.

Proof If $r \in \phi^{-1}(x)$, then $\phi(r) = x = \phi(a)$
Since μ is a ϕ - invariant $\mu(r) = \mu(a)$, then by definition (1.10), we have $\phi(\mu)(x) = \sup_{r\in\phi^{-1}(x)} \mu(r) = \mu(a)$
Hence $\phi(\mu)(x) = \mu(a)$

Theorem (2.8) Let $\phi: R_1 \to R_2$be an onto homomorphism of ternary semirings R_1 and R_2. If μ is a ϕ - invariant anti fuzzy k-ideal of R_1,
then $\phi(\mu)$ is an anti fuzzy k-ideal of R_2.

Proof: Let $\phi: R_1 \to R_2$ be an onto homomorphism and μ is ϕ- invariant anti fuzzy k-ideal of R_1,
Let $x, y, z \in R_2$. Since ϕ is surjective then there exist $a, b, c \in R_1$ such that $\phi(a) = x,\ \phi(b) = y \text{ and } \phi(c) = z$ since ϕ is a homomorphism then
$x + y = \phi(a) + \phi(b) = \phi(a + b)$ and $xyz = \phi(a)\ \phi(b)\ \phi(c) = \phi(a\,b\,c)$. Then we have $\phi(\mu)(x + y) = \mu(a + b) \le \max\{\mu(a), \mu(b)\}$ since μ is ϕ- invariant by lemma (2.9)
$= \max(\phi(\mu)(x), \phi(\mu)(y))$,
$\phi(\mu)(xyz) = \mu(abc) \le \mu(c)$ since μ is ϕ- invariant by lemma (2.9)
$= \phi(\mu)(z)$
Hence $\phi(\mu)$ is an anti fuzzy left ideal of R_2 Similarly $\phi(\mu)$is an anti fuzzy right and lateral ideal of R_2then $\phi(\mu)$ is an anti fuzzy ideal of R_2 Let $x, y \in R_2$since ϕ is onto then there exists $a, b \in R_1$such that $\phi(a) = x$ and $\phi(b) = y$

$$\phi(\mu)(x) = \mu(a) \le \max\{\mu(a + b), \mu(b)\}$$
$$= \max\{\phi(\mu)(x + y), \phi(\mu)(y))$$

Hence $\phi(\mu)$ is an anti fuzzy k-ideal of R_2.

Definition (2.9) An anti fuzzy k-ideal μ of a ternary semiring R is said to be normal if $\mu(0) = 1$.

Theorem (2.10) Let μ be an anti fuzzy k- ideal of a ternary semiring Rand μ^* be a fuzzy subset of R defined by $\mu^*(x) = \mu(x) + 1 - \mu(0)$ for all $x \in R$.
Then μ^* is a normal anti fuzzy k-ideal of R.

Proof. Let $x, y, z \in R$. Then $\mu^*(x + y) = \mu(x + y) + 1 - \mu(0)$, since μ be an anti fuzzy k- ideal,

$$\le \max\{\mu(x), \mu(y)\} + 1 - \mu(0)$$
$$= \max\{\mu(x) + 1 - \mu(0), \mu(y)\} + 1 - \mu(0)$$
$$= \max\{\mu^*(x), \mu^*(y)\},$$

$\mu^*(xyz) = \mu(xyz) + 1 - \mu(0)$, since μ be an anti fuzzy left k- ideal then

$$\le \mu(z) + 1 - \mu(0)$$
$$= \mu^*(z)$$

Hence μ^* is an anti fuzzy left ideal of R similarly we can prove that μ^* is an anti fuzzy right and lateral ideal of R
then μ^* is an anti fuzzy ideal of R

$$\mu^*(x) = \mu(x) + 1 - \mu(0)$$
$$\le \max\{\mu(x + y), \mu(y)\} + 1 - \mu(0)$$
$$= \max\{\mu(x, y) + 1 - \mu(0), \mu(y) + 1 - \mu(0)\}$$
$$= \max\{\mu^*(x + y), \mu^*(y)\}$$

Hence μ^* is an anti fuzzy k-ideal of R.
Since $\mu^*(x) = \mu(x) + 1 - \mu(0)$
$\mu^*(0) = \mu(0) + 1 - \mu(0)$ Hence μ^* is a normal anti fuzzy k-ideal of R.

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