Orthogonal distance and similarity for single-valued neutrosophic fuzzy soft expert environment and its application in decision-making

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Abstract

A soft expert set(SES) is a concept that combines elements of soft sets and expert systems. It aims to incorporate expert knowledge and uncertainty-handling capabilities into the analysis and decision-making processes. On the other hand, the idea of single neutrosophic sets (SVNSs) and fuzzy sets (FSs) are imported models for handling the uncertainty data. In this work, the authors combine the critical features of FSs and SVNSs under expert systems in one model. Accordingly, this model worked to provide decision-makers with more flexibility in the process of interpreting uncertain information. From a scientific point of view, the process of evaluating this high-performance SVNFSES disappears. Therefore, in this paper, we initiated a new approach known as single-valued neutrosophic fuzzy soft expert sets (SVNFSESs) as a new development in a fuzzy soft computing environment. We investigate some fundamental operations on SVNFSESS along with their basic properties. Also, we investigate AND and OR operations between two SVNFSESS as well as several numerical examples to clarify the above fundamental operations. Finally, we have given an Orthogonal Distance and Similarity for SVNFSESs to construct a new algorithm to demonstrate the method's effectiveness in handling some real-life applications.

Kewords: Neutrosophic sets; neutrosophic soft sets; Single-valued neutrosophic soft sets; expert soft set; optimization; Decision Making.

1 Introduction

A similarity measure or similarity function is a real-valued a function that evaluates the closeness between two items. Similarity measure takes huge values on comparable items and either zero an incentive for non-comparable items. Similarity measures are inversely proportional to the distance between the sets. Neutrosophic set theory[1] is an extension of classical set theory that aims to handle uncertainty, vagueness, and

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indeterminacy in a more comprehensive way. It was introduced by Florentin Smarandache in 1995 and has since found applications in various fields, including decision-making, expert systems, image processing, and artificial intelligence. A neutrosophic set is characterized by three components: the membership function, the non-membership function, and the indeterminacy function. These functions assign degrees of truth, falsity, and indeterminacy, respectively, to each element in the set. Unlike classical sets, which assign elements either a membership degree of 1 or 0, neutrosophic sets allow for partial membership, non-membership, or indeterminacy. The membership function represents the degree to which an element belongs to the set. It ranges between 0 and 1, where 0 indicates no membership, 1 indicates full membership, and values in between represent degrees of partial membership. The non-membership function represents the degree to which an element does not belong to the set. It also ranges between 0 and 1, where 0 indicates no non-membership, 1 indicates full non-membership, and values in between represent degrees of partial non-membership. The indeterminacy function represents the degree of indeterminacy or uncertainty associated with an element's membership or non-membership. It ranges between 0 and 1, where 0 represents complete determinacy, 1 represents complete indeterminacy, and values in between represent degrees of partial indeterminacy. Neutrosophic sets provide a more nuanced and flexible approach to handling uncertainty and imprecision in various domains. They can capture situations where the degree of membership or non-membership is uncertain or ambiguous, allowing for more realistic modeling of complex and uncertain systems. Neutrosophic set theory has been applied in decision-making processes to handle conflicting and uncertain information. Neutrosophic logic and inference have been developed to reason with neutrosophic sets and make decisions based on the degrees of truth, falsity, and indeterminacy. Moreover, neutrosophic fuzzy sets combine neutrosophic sets and fuzzy sets,[2] incorporating degrees of truth, falsity, and indeterminacy into fuzzy logic systems. This hybrid approach enhances the modeling of uncertainty, vagueness, and ambiguity in decision-making processes. Overall, neutrosophic set theory offers a valuable framework for dealing with uncertainty and indeterminacy, providing a more comprehensive representation and analysis of complex systems and decision problems. Its applications continue to evolve, and researchers are exploring new extensions and variations to address specific challenges in different domains.

Soft set[3] theory is a mathematical framework that provides a flexible and intuitive way to handle uncertainty, vagueness, and imprecision in decision-making and data analysis. It was introduced by Molodtsov in 1999 as a generalization of classical set theory. In soft set theory, a soft set is defined as a collection of objects with a characteristic function that assigns degrees of membership to each object in a set. Unlike classical sets, where an object is either a member or non-member, soft sets allow for partial membership or degrees of uncertainty associated with membership. SSs have received wide attention from researchers around the world and they have introduced a lot of works for example, neutrosophic soft set(NSS),[4,5] interval-NSS[6,7] complex interval-NSS,[8] weighted Similarity Measure on neutrosophic soft set[9,10] and a lot of models employed in solving real-life applications see.[11–16] In other hande, Soft Expert Set[17–19] is a concept that combines elements of soft sets and expert systems. It aims to incorporate expert knowledge and uncertainty handling capabilities into the analysis and decision-making processes.

In traditional expert systems, human expertise is captured and represented using rules or knowledge bases. These systems rely on the knowledge and experience of domain experts to make informed decisions. However, they may not adequately handle uncertainties or imprecise information. With this design, many research works emerged that combine NS and SES like neutrosophic soft expert sets (NSESs),[20]interval-NSESs,[21]Generalized neutrosophic soft expert set,[22] neutrosophic soft expert graphs,[23] possibility neutrosophic soft expert sets[24,25] and a lot of models employed in solving real-life applications see.[26–31] In this work, we will collect a number of properties present in each of fuzzy set, neutrosophic set, and soft expert set under a single value in one model called Single value neutrosophic fuzzy soft expert set (SVNFSES). Based on this model, we present several Orthogonal Distance and Similarity and explain the mechanism for their use in solving a decision-making problem. This article is organized as follows: We review some basic definitions of the associated studies in Section 2. Section 3 presents the formulation of the SVNFSE-set and its operations. While in section 4, we demonstrate the set-theoretic operations of SVNSE-sets together with some propositions and examples. Section 5 discusses the applications of the SVNFSE-set in decision-making problems based on Orthogonal Distance and Similarity on SVNFSE-sets. Finally, conclusions and suggestions for further studies are pointed out in section 6.

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2 Preliminaries

In this section, we recapitulate some of the ideas like FS, NS, SVNS, SS, and SVNSS that are considered beneficial in developing our new concept.

Definition 2.1. [2] Assume that $\hat{\mathbb{U}} = \{u_1, u_2, u_3, ..., u_n\}$ be a reference set. Then the FS formed as following structure:

$$\mathcal{Q} = \left\{ \left(u, \left\langle \ddot{\partial}^t_Q (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$$

where $\ddot{\partial}^t_Q (u_i)$ refer to true membership of object u_i in $\hat{\mathbb{U}}$ and persistent as a mapping:$\ddot{\partial}^t_Q : \hat{\mathbb{U}} \to [0, 1]$.

Definition 2.2. [2] Let $\mathcal{Q}_1 = \left\{ \left(u, \left\langle \ddot{\partial}^t_{Q_1} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ and $\mathcal{Q}_2 = \left\{ \left(\tau, \left\langle \ddot{\partial}^t_{Q_2} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ be two FS on reference set $\hat{\mathbb{U}}$. Then the fundamental operation on FSs defined as following:

1.Union $Q_3 = \left\{ \left(u, \left\langle \max \left(\ddot{\partial}^t_{Q_1} (u_i) , \ddot{\partial}^t_{Q_2} (u_i) \right) \right\rangle \right) \middle| u \in \hat{\mathbb{T}} \right\}$.

2.Intersection $Q_3 = \left\{ \left(\tau, \left\langle \min \left(\ddot{\partial}^t_{Q_1} (u_i) , \ddot{\partial}^t_{Q_2} (u_i) \right) \right\rangle \right) \middle| \partial \in \hat{\mathbb{U}} \right\}$.

3.Complement $Q_1^c = \left\{ \left(u, \left\langle \left(1 - \ddot{\partial}^t_{Q_1} (u_i) \right) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$.

4.Subset $\mathcal{Q}_1 \subseteq \mathcal{Q}_2$ if $\ddot{\partial}^t_{Q_1} (u_i) \leq \ddot{\partial}^t_{Q_2} (u_i)$.

Definition 2.3. [1] Assume that $\hat{\mathbb{U}} = \{u_1, u_2, \tau_3, ..., u_n\}$ be a reference set $\hat{\mathbb{U}}$. Then the NS formed as following structure:

$$\hat{\hat{\mathcal{A}}}_{NS} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$$

where $\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i)$ refer to true membership,indeterminacy membership and falsehood membership of object u_i in $\hat{\mathbb{U}}$ and persistent as a mapping:$\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) : \hat{\mathbb{U}} \to [0, 1]$.

Definition 2.4. [24] Let $\hat{\hat{\mathcal{A}}}_{SVNS} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ and $\hat{\hat{\mathcal{B}}}_{SVNS} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}} (u_i) \right\rangle \right) \middle| u \in \hat{\mathbb{U}} \right\}$ be two SVNS on reference set $\hat{\mathbb{U}}$ Then the fundamental operation on SVNS defined as following:

1.Union
$\hat{\hat{\mathcal{C}}}_{SVNS} = \left\{ \left(u, \left\langle \max \left[\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}} (u_i) \right] , \min \left[\ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}} (u_i) \right] , \min \left[\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}} (u_i) \right] \right\rangle \right) | u \in U \right\}$.

2.Intersection
$\hat{\hat{\mathcal{C}}}_{SVNS} = \left\{ \left(u, \left\langle \min \left[\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}} (u_i) \right] , \max \left[\ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}} (u_i) \right] , \max \left[\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) , \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}} (u_i) \right] \right\rangle \right) | u \in U \right\}$.

3.Complement
$\hat{\hat{\mathcal{A}}}^c_{SVNS} = \left\{ \left(u, \left\langle \left[\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) \right] , \left[1 - \ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) \right] , \left[\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) \right] \right\rangle \right) | u \in U \right\}$.

4.Subset
$\hat{\hat{\mathcal{A}}}_{SVNS} \subseteq \hat{\hat{\mathcal{B}}}_{sVNS}$ if$\ddot{\partial}^t_{\hat{\hat{\mathcal{A}}}} (u_i) \leq \ddot{\partial}^t_{\hat{\hat{\mathcal{B}}}} (u_i)$,$\ddot{\partial}^i_{\hat{\hat{\mathcal{A}}}} (u_i) \geq \ddot{\partial}^i_{\hat{\hat{\mathcal{B}}}} (u_i)$,$\ddot{\partial}^f_{\hat{\hat{\mathcal{A}}}} (u_i) \geq \ddot{\partial}^f_{\hat{\hat{\mathcal{B}}}} (u_i)$.

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Definition 2.5. [24] Let $\hat{\mathbb{U}} = \{u_1, u_2, u_3, ..., u_n\}$ and $\hat{\mathbb{E}} = \{z_1, z_2, z_3, ..., z_m\}$ be a reference set and attribute set, respectively. Then a SS over $\hat{\mathbb{U}}$ given as structures as follows:

$$\overset{\leftrightarrow}{S} = \left\{ \left(z, \left\langle \overset{\leftrightarrow}{S}(z_i) \right\rangle \right) \middle| z \in \hat{\mathbb{E}} \right\}$$

where the function $\overset{\leftrightarrow}{S}$ given by following mapping:

$$\overset{\leftrightarrow}{S} = E \rightarrow P^{(U)}$$

Here $P^{(U)}$ refer to collection of subsets of reference set $\hat{\mathbb{T}}$.

Definition 2.6. [24] A term $\mathcal{P}^{svnss}$ is said to be SVNSS on soft reference set $\left(\hat{\mathbb{U}}, \hat{\mathbb{E}}\right)$, where $\mathcal{P}^{svnss} : \hat{\mathbb{E}} \rightarrow SVN(P)^{(U)}$, such that $SVN(P)^{(U)}$ is a collection of all SVNS-subsets over $\hat{\mathbb{U}}$.

3 Structuring the concept of single value neutrosophic fuzzy soft expert set (SVNFSESS)

In this section, we introduce the concept of a SVNFES and define some properties of this model, namely, the null of the SVNFES , the absolute of the SVNFES , a subset of the SVNFES , and the equality of the SVNFES . Illustrated examples are also given.

Definition 3.1. The ordered pair (H, O) pointing to single-neutrosophic soft expert set (SNFSESS) on U. If

1. The mapping $\mathrm{H} : \mathrm{O} \rightarrow SVNFN^{\mathrm{U}}$ where $\mathrm{O} \subseteq \mathrm{Z} = \mathrm{M} \times \mathrm{N} \times \mathrm{Y}$, such that for all $z \in \mathrm{Z}$ then $z = (m \times n \times y = 0 \; or \; 1)$

2. Here $\mathrm{U} = \{u_1, u_2, u_3, ..., u_s\}, \mathrm{M} = \{m_1, m_2, m_3, ..., m_s\}, \mathrm{N} = \{n_1, n_2, n_3, ..., n_s\}$ represent to reference set, attribute set, set of experts respectively and $\mathrm{Y} = \{0, 1\}$.

3. A single neutrosophic soft set (PIVNSS) $\mathcal{H}$ on $\hat{\mathbb{U}}$ has the structures as follows:

$$\mathcal{H}^{svnfses} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z \in \hat{\mathbb{Z}} \right\}$$

Where $\ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j)$ represent of the three SVNFSES memberships as a single real number and $\mathcal{P}^{ivnss}$ refer to SVNSESN degree of element $u_i \in \hat{\mathbb{U}}$ to $\mathcal{H}^{svnses}$ who also can denoted by $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$

Example 3.2. Consider that a tourism company would like to evaluate a group of hotels it owns to see who is suitable. This evaluation was based on two experts working for the company. Now we assume that $\hat{\mathbb{U}}$ includes three hotels$\{u_1, u_2, u_3\}$, and the object is evaluated by two experts$\mathrm{N} = \{n_1, n_2\}$, and for the criteria that were adopted in this evaluation process, it can be represented by $\mathrm{M} = \{m_1, m_2, m_3\}$ such that $m_1 = Food\ services m_2 = Staff, m_3 = Number\ of\ rooms$. Now for $\mathrm{O} \subseteq \mathrm{Z} = \mathrm{M} \times \mathrm{N} \times \mathrm{Y}$, Now our concept presents the opinions of the two experts as follows:

$$\mathcal{H}(z_1 = (m_1, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.3 \rangle} \right), \left(\frac{u_2}{\langle 0.3, 0.6, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.8, 0.1, 0.6 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_2 = (m_1, n_2, 1)) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.4, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.1, 0.5, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.2, 0, 0.8 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_3 = (m_2, n_1, 1)) = \left\{ \left(\frac{u_1}{\langle 0.7, 0.6, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.6, 0.3, 0.1 \rangle} \right), \left(\frac{u_3}{\langle 0.2, 0.3, 0.5 \rangle} \right) \right\}.$$
$$\mathcal{H}(z_4 = (m_2, n_2, 1)) = \left\{ \left(\frac{u_1}{\langle 0.5, 0.3, 0.2 \rangle} \right), \left(\frac{u_2}{\langle 0.6, 0.4, 0.7 \rangle} \right), \left(\frac{u_3}{\langle 0.5, 0.4, 0.3 \rangle} \right) \right\}.$$

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$\mathcal{H}\left(z_5=\left(m_3, n_1, 1\right)\right)=\left\{\left(\frac{u_1}{\langle 0.7,0.5,0.9\rangle}\right),\left(\frac{u_2}{\langle 0.2,0.6,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.3,0.4,0.6\rangle}\right)\right\}.$
$\mathcal{H}\left(z_6=\left(m_3, n_2, 1\right)\right)=\left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right),\left(\frac{u_2}{\langle 0.3,0.6,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.8,0.1,0.6\rangle}\right)\right\}.$
$\mathcal{H}\left(z_7=\left(m_1, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right),\left(\frac{u_2}{\langle 0.3,0.6,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.8,0.1,0.6\rangle}\right)\right\}.$
$\mathcal{H}\left(z_8=\left(m_1, n_2, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.6,0.2\rangle}\right),\left(\frac{u_2}{\langle 0.7,0.2,0.5\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.1,0.3\rangle}\right)\right\}.$
$\mathcal{H}\left(z_9=\left(m_2, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.6,0.9\rangle}\right)\right\}.$
$\mathcal{H}\left(z_{10}=\left(m_2, n_2, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.3,0.4,0.7\rangle}\right),\left(\frac{u_2}{\langle 0.3,0.4,0.2\rangle}\right),\ \left(\frac{u_3}{\langle 0.6,0.6,0.2\rangle}\right)\right\}.$
$\mathcal{H}\left(z_{11}=\left(m_3, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.6\rangle}\right),\left(\frac{u_2}{\langle 0.1,0.2,0.8\rangle}\right),\ \left(\frac{u_3}{\langle 0.2,0.4,0.7\rangle}\right)\right\}.$
$\mathcal{H}\left(z_{12}=\left(m_3, n_2, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.6,0.8,0.6\rangle}\right),\left(\frac{u_2}{\langle 0.2,0.5,0.6\rangle}\right),\ \left(\frac{u_3}{\langle 0.4,0.9,0.9\rangle}\right)\right\}.$

Also $\mathcal{H}\left(z_i\right)$ can represent as a matrix as a following form:
$\mathcal{H}\left(z_i\right)=$

$$\begin{pmatrix}
((0.2,0.5,0.3)) & ((0.3,0.6,0.7)) & ((0.8,0.1,0.6)) \\
((0.5,0.4,0.2)) & ((0.1,0.5,0.7)) & ((0.2,0.0,0.8)) \\
((0.7,0.6,0.2)) & ((0.6,0.3,0.1)) & ((0.2,0.3,0.5)) \\
((0.5,0.3,0.2)) & ((0.6,0.4,0.7)) & ((0.5,0.4,0.3)) \\
((0.7,0.5,0.9)) & ((0.2,0.6,0.7)) & ((0.3,0.4,0.6)) \\
((0.2,0.5,0.3)) & ((0.3,0.6,0.7)) & ((0.8,0.1,0.6)) \\
((0.4,0.6,0.2)) & ((0.7,0.2,0.5)) & ((0.1,0.1,0.3)) \\
((0.4,0.5,0.8)) & ((0.9,0.8,0.7)) & ((0.1,0.6,0.9)) \\
((0.3,0.4,0.7)) & ((0.3,0.4,0.2)) & ((0.6,0.6,0.2)) \\
((0.4,0.5,0.6)) & ((0.1,0.2,0.8)) & ((0.2,0.4,0.7)) \\
((0.6,0.8,0.6)) & ((0.2,0.5,0.6)) & ((0.4,0.9,0.9))
\end{pmatrix}$$

Definition 3.3. (Agree SVNFSES): Agree SVNFSES $\mathcal{H}_1$ represents agreement of all expert's opinions and is defined as follows:

$$\mathcal{H}_{\mathcal{O}}=\left\{H_O\left(z_i\right): z_i \in M \times N \times\{1\}\right\}$$

Example 3.4. Take the terms $\mathcal{H}_{\mathcal{O}}\left(z_1\right)$ in Example 3.2. above

$\mathcal{K}\left(z_1=\left(m_1, n_1, 1\right)\right)=\left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right),\left(\frac{u_2}{\langle 0.3,0.6,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.8,0.1,0.6\rangle}\right)\right\}.$

Definition 3.5. (Disagree SVNFSES): disagree SVNFSES $\mathcal{H}_0$ represents disagreement of all expert's opinions and is defined as follows:

$$\mathcal{H}_{\mathcal{O}}=\left\{H_O\left(z_i\right): z_i \in M \times N \times\{0\}\right\}$$

Example 3.6. Take the terms $\mathcal{H}_{\mathcal{O}}\left(z_9\right)$ in Example 3.2. above

$\mathcal{K}\left(z_9=\left(m_2, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.6,0.9\rangle}\right)\right\}.$

Definition 3.7. (SVNFSE-subset): Let $\mathcal{H}_{\mathrm{O}}=(\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ and $\mathcal{K}_{\mathrm{P}}=(\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then $\mathcal{H}_{\mathrm{O}}$ is said SVNFSE-subset of $\mathcal{K}_{\mathrm{P}}$ and denoted by $\mathcal{H}_{\mathrm{O}} \subseteq \mathcal{K}_{\mathrm{P}}$ if:

1. $\mathcal{H}_{\mathrm{O}}\left(u_i\right)$ is SVNFSE-subset of $\mathcal{K}_{\mathrm{P}}\left(u_i\right), \forall u_i \in \hat{\mathbb{U}}$.

2.O $\subseteq$ P.

Example 3.8. Take the terms $\mathcal{H}_{\mathcal{O}}\left(z_i\right)$ in Example 3.2. above and take

$\mathcal{K}\left(z_1=\left(m_1, n_1, 1\right)\right)=\left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right),\left(\frac{u_2}{\langle 0.3,0.6,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.8,0.1,0.6\rangle}\right)\right\}.$
$\mathcal{K}\left(z_9=\left(m_2, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.6,0.9\rangle}\right)\right\}.$

Now, its clear the two terms are $\subseteq \mathcal{H}_{\mathcal{O}}$.

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Definition 3.9. (Equality of SVNFSE-sets): Let $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ and $\mathcal{K}_{\mathrm{P}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then$\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ is called equal of $\mathcal{K}_{\mathrm{P}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ and denoted by $\mathcal{H}_{\mathcal{O}} = \mathcal{K}_{\mathcal{P}}$ if:

1. $\mathcal{H}(u_i)$ is SVNFSE-subset of $\mathcal{K}(u_i)$ and $\mathcal{K}(u_i)$ is SVNFSE-subset of $\mathcal{H}(u_i)$, $\forall u_i \in \hat{\mathbb{U}}$.

2. $\mathcal{O}$ is subset of $\mathcal{P}$ and $\mathcal{P}$ is subset of $\mathcal{O}$, $\forall u_i \in \hat{\mathbb{U}}$.

Example 3.10. Conseder $\mathcal{H}_{\mathcal{O}}(z_i)$ in Example 3.2. above and take $\mathcal{H}_{\mathcal{O}} =$

$$\begin{pmatrix} (0.2, 0.50.3) & (0.3, 0.6, 0.8) & (0.8, 0.1, 0.6) \\ (0.5, 0.80.2) & (0.2, 0.4, 0.8) & (0.8, 0, 0.2) \\ (0.4, 0.50.3) & (0.9, 0.6, 0.8) & (0.8, 0.7, 0.7) \end{pmatrix} \text{ and}$$

$\mathcal{G}_{\mathcal{C}} =$

$$\begin{pmatrix} (0.1, 0.30.3) & (0.6, 0.8, 0.8) & (0.8, 0.1, 0.1) \\ (0.5, 0.80.2) & (0.2, 0.4, 0.8) & (0.8, 0, 0.2) \\ (0.4, 0.7, 0.8) & (0.4, 0.6, 0.8) & (0.8, 0.5, 0.4) \end{pmatrix} \text{ and}$$

$\mathcal{K}_{\mathcal{P}} =$

$$\begin{pmatrix} (0.2, 0.50.3) & (0.3, 0.6, 0.8) & (0.8, 0.1, 0.6) \\ (0.5, 0.80.2) & (0.2, 0.4, 0.8) & (0.8, 0, 0.2) \\ (0.4, 0.50.3) & (0.9, 0.6, 0.8) & (0.8, 0.7, 0.7) \end{pmatrix}$$

Then, its clear $\mathcal{H}_{\mathcal{O}} = \mathcal{K}_{\mathcal{P}}$ and $\mathcal{H}_{\mathcal{O}} \neq \mathcal{G}_{\mathcal{C}}$.

Definition 3.11. (null SVNFSE-set): Let $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be SVNFSE-set on reference set $\hat{\mathbb{U}}$. Then we say that $\mathcal{H}_{\mathrm{O}}$ is " null SVNFSE-set" and denoted by $\widehat{\Phi}_{(0)}$ if $\mathcal{H}(u_i) = (0, 1, 1)$ and $\Theta(z_i) = 0$,$\forall u_i \in \hat{\mathbb{U}}$.

Example 3.12. Taking into account the matrix notation of $\mathcal{H}_{\mathcal{O}}$ as an Example 3.2, it can be observed that we possess.

$$\widehat{\Phi}_{(0)} = \begin{pmatrix} ((0,1,1)) & ((0,1,1)) & ((0,1,1)) \\ ((0,1,1)) & ((0,1,1)) & ((0,1,1)) \\ ((0,1,1)) & ((0,1,1)) & ((0,1,1)) \end{pmatrix}$$

Definition 3.13. (absolute SVNFES-set): Let $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be SVNFSE-set on reference set $\hat{\mathbb{U}}$. Then we say that $\mathcal{H}_{\mathrm{O}}$ is " absolute SVNFES-set" and denoted by $\widehat{\Omega}_{(1)}$ if $\mathcal{H}(u_i) = (1, 0, 0)$ and $\Theta(z_i) = 1$,$\forall u_i \in \hat{\mathbb{U}}$.

Example 3.14. Taking into account the matrix notation of $\mathcal{H}_{\mathrm{O}}$ as an Example 3.2, it can be observed that we possess.

$$\widehat{\Phi}_{(0)} = \begin{pmatrix} ((1,0,0)) & ((1,0,0)) & ((1,0,0)) \\ ((1,0,0)) & ((1,0,0)) & ((1,0,0)) \\ ((1,0,0)) & ((1,0,0)) & ((1,0,0)) \end{pmatrix}$$

Definition 3.15. (Complement operation of SVNFSE-set): Let $\mathcal{H}_{\mathrm{O}} = (\mathcal{H}, \mathcal{O} \subseteq \mathcal{Z})$ be SVNFSE-set on reference set $\hat{\mathbb{U}}$ and defied as follows

$$\mathcal{H}_{\mathrm{O}} = \left\{ \left(u, \left\langle \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z \in \hat{\mathbb{Z}} \right\}$$

Then,complement operation of PIVNS-set defined as follows

$$\mathcal{H}_{\mathrm{O}}{}^c = \left\{ \left(u, \left\langle \ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j), 1 - \ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z \in \hat{\mathbb{Z}} \right\}$$

Here: we follow the complement role of SVNS-complement.

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Example 3.16. Take the terms $\mathcal{H}_{\mathcal{O}}\left(z_{i=1,9}\right)$ in Example 3.2. above and take

$$\mathcal{H}\left(z_1=(m_1,n_1,1)\right)=\left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right),\left(\frac{u_2}{\langle 0.3,0.6,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.8,0.1,0.6\rangle}\right)\right\}.$$
$$\mathcal{H}\left(z_9=(m_2,n_1,0)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.6,0.9\rangle}\right)\right\}.$$

Then the complement of them given as following:

$$\mathcal{H}^c\left(z_1=(m_1,n_1,1)\right)=\left\{\left(\frac{u_1}{\langle 0.3,0.5,0.2\rangle}\right),\left(\frac{u_2}{\langle 0.7,0.4,0.3\rangle}\right),\ \left(\frac{u_3}{\langle 0.6,0.9,0.8\rangle}\right)\right\}.$$
$$\mathcal{H}^c\left(z_9=(m_2,n_1,0)\right)=\left\{\left(\frac{u_1}{\langle 0.8,0.5,0.4\rangle}\right),\left(\frac{u_2}{\langle 0.7,0.2,0.9\rangle}\right),\ \left(\frac{u_3}{\langle 0.9,0.4,0.1\rangle}\right)\right\}.$$

4 The set-theoretic operations pertaining to SVNSE-sets

Now, in this section, we introduce the set-theoretic operations pertaining to SVNSE-sets as well as some properties and numerical examples that illustrate how these tools work in algebraic environments.

Definition 4.1. (Union of SVNFSE-sets) Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}^t_{\mathcal{H}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^i_{\mathcal{H}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^f_{\mathcal{H}}\left(z_i\right)\left(u_j\right)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$
and
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}^t_{\mathcal{K}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^i_{\mathcal{K}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^f_{\mathcal{K}}\left(z_i\right)\left(u_j\right)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$
be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then,the union of SVNFSE-sets denoted by $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{K}_{\mathrm{P}}$ and defiend as following: $\mathcal{D}_{\mathrm{S}}=\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{K}_{\mathrm{P}}$, where $\hat{\cup}$ denotes SVNFSE-sets-union.

Example 4.2. Taking into account one part of SVNFSES $\mathcal{H}_{\mathrm{O}}$ as an Example 3.2, and $\mathcal{K}_{\mathrm{P}}$ given in an Example 3.4,

$$\mathcal{H}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_1}{\langle 0.5,0.4,0.2\rangle}\right),\left(\frac{u_2}{\langle 0.1,0.5,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.2,0,0.8\rangle}\right)\right\}.$$

and

$$\mathcal{K}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.6,0.9\rangle}\right)\right\}.$$

then,the union of SVNFSE-sets can be possess as following :

$$\mathcal{D}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_1}{\langle 0.5,0.4,0.2\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.5,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.2,0,0.9\rangle}\right)\right\}.$$

Also $\mathcal{D}\left(u_i\right)$ can represent as a matrix as a following form:
$\mathcal{D}_{\Psi}=$

$$\left(((0.5,0.4,0.2))\quad((0.9,0.5,0.7))\quad((0.2,0,0.9))_{1\times 3}\right)$$

Definition 4.3. (Intersection of SVNFSE-sets) Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}^t_{\mathcal{H}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^i_{\mathcal{H}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^f_{\mathcal{H}}\left(z_i\right)\left(u_j\right)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$
and
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}^t_{\mathcal{K}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^i_{\mathcal{K}}\left(z_i\right)\left(u_j\right),\ddot{\partial}^f_{\mathcal{K}}\left(z_i\right)\left(u_j\right)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$
be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then,the intersection of SVNFSE-sets denoted by $\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{K}_{\mathrm{P}}$ and defiend as following: $\mathcal{C}_{\mathrm{L}}=\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{K}_{\mathrm{P}}$, where $\hat{\cap}$ denotes SVNFSE-sets-intersection.

Example 4.4. Taking into account one part of SVNFSES $\mathcal{H}_{\mathrm{O}}$ as an Example 3.2, and $\mathcal{K}_{\mathrm{P}}$ given in an Example 3.4,

$$\mathcal{H}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_1}{\langle 0.5,0.4,0.2\rangle}\right),\left(\frac{u_2}{\langle 0.1,0.5,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.2,0,0.8\rangle}\right)\right\}.$$

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and

$\mathcal{K}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.9,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0.6,0.9\rangle}\right)\right\}.$

then,the intersection of SVNFSE-sets can be possess as following :

$\mathcal{C}\left(z_2=(m_1,n_2,1)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right),\left(\frac{u_2}{\langle 0.1,0.8,0.7\rangle}\right),\ \left(\frac{u_3}{\langle 0.1,0,0.8\rangle}\right)\right\}.$

Also $\mathcal{C}\left(u_i\right)$ can represent as a matrix as a following form:
$\mathcal{C}_{\mathcal{L}}=$

$\left(((0.4,0.5,0.8)) \quad ((0.1,0.8,0.7)) \quad ((0.1,0,0.8))_{1\times 3}\right)$

Proposition 4.5. *Let*
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j),\ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j),\ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$
be a SVNFSE-set on reference set $\hat{\mathbb{U}}$. *Then,the following statements hold:*

1. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{H}_{\mathrm{O}}=\mathcal{H}_{\mathrm{O}}$.
2. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{H}_{\mathrm{O}}=\mathcal{H}_{\mathrm{O}}$.
3. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\widehat{\Phi}_{(0)}=\mathcal{H}_{\mathrm{O}}$.
4. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\widehat{\Phi}_{(0)}=\widehat{\Phi}_{(0)}$.
5. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\widehat{\Omega}_{\ (1)}=\widehat{\Omega}_{\ (1)}$.
6. $\mathcal{H}_{\Theta}\hat{\cap}\widehat{\Omega}_{(1)}=\mathcal{H}_{\mathrm{O}}$.

Proposition 4.6. *Let*
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle \ddot{\partial}^t_{\mathcal{H}}(z_i)(u_j),\ddot{\partial}^i_{\mathcal{H}}(z_i)(u_j),\ddot{\partial}^f_{\mathcal{H}}(z_i)(u_j)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$,
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle \ddot{\partial}^t_{\mathcal{K}}(z_i)(u_j),\ddot{\partial}^i_{\mathcal{K}}(z_i)(u_j),\ddot{\partial}^f_{\mathcal{K}}(z_i)(u_j)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$ *and*
$\mathcal{G}_{\mathrm{C}}=\left\{\left(u,\left\langle \ddot{\partial}^t_{\mathcal{G}}(z_i)(u_j),\ddot{\partial}^i_{\mathcal{G}}(z_i)(u_j),\ddot{\partial}^f_{\mathcal{G}}(z_i)(u_j)\right\rangle|u\in\hat{\mathbb{U}},z_i\in\hat{\mathbb{Z}}\right\}$ *be three SVNFSE-sets on reference set* $\hat{\mathbb{U}}$. *Then,the following statements hold:*

1. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}=\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{H}_{\mathrm{O}}$.
2. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{G}_{\mathrm{C}}=\mathcal{G}_{\mathrm{C}}\hat{\cap}\mathcal{H}_{\mathrm{O}}$.
3. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\left(\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}\right)\hat{\cup}\mathcal{K}_{\mathrm{P}}$.
4. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\left(\mathcal{G}_{\mathrm{C}}\hat{\cap}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{G}_{\mathrm{C}}\right)\hat{\cap}\mathcal{K}_{\mathrm{P}}$.
5. $\mathcal{H}_{\mathrm{O}}\hat{\cup}\left(\mathcal{G}_{\mathrm{C}}\hat{\cap}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}\right)\hat{\cap}\left(\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{K}_{\mathrm{C}}\right)$.
6. $\mathcal{H}_{\mathrm{O}}\hat{\cap}\left(\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{K}_{\mathrm{P}}\right)=\left(\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{G}_{\mathrm{C}}\right)\hat{\cup}\left(\mathcal{H}_{\mathrm{O}}\hat{\cap}\mathcal{K}_{\mathrm{P}}\right)$.

Proof. **1.** We take the left side $\mathcal{H}_{\mathrm{O}}\hat{\cup}\mathcal{G}_{\mathrm{C}}=$
$H_O=\left\{\left(u,\left\langle \ddot{\partial}^t_H(z_i)(u_j),\ddot{\partial}^i_H(z_i)(u_j),\ddot{\partial}^f_H(z_i)(u_j)\right\rangle\right)|u\in U,z\in Z\right\}\bigcup$
$G_C=\left\{\left(u,\left\langle \ddot{\partial}^t_G(z_i)(u_j),\ddot{\partial}^i_G(z_i)(u_j),\ddot{\partial}^f_G(z_i)(u_j)\right\rangle\right)|u\in U,z\in Z\right\}$
$=\left\{\left(u,\left\langle\max\left(\ddot{\partial}^t_H(z_i)(u_j),\ddot{\partial}^t_G(z_i)(u_j)\right),\min\left(\ddot{\partial}^i_H(z_i)(u_j),\ddot{\partial}^i_G(z_i)(u_j)\right),\min\left(\ddot{\partial}^f_H(z_i)(u_j),\ddot{\partial}^f_G(z_i)(u_j)\right)\right\rangle\right)\right\}$
$=\left\{\left(u,\left\langle\max\left(\ddot{\partial}^t_G(z_i)(u_j),\ddot{\partial}^t_H(z_i)(u_j)\right),\min\left(\ddot{\partial}^i_G(z_i)(u_j),\ddot{\partial}^i_H(z_i)(u_j)\right),\min\left(\ddot{\partial}^f_G(z_i)(u_j),\ddot{\partial}^f_H(z_i)(u_j)\right)\right\rangle\right)\right\}$
$=G_C=\left\{\left(u,\left\langle \ddot{\partial}^t_G(z_i)(u_j),\ddot{\partial}^i_G(z_i)(u_j),\ddot{\partial}^f_G(z_i)(u_j)\right\rangle\right)|u\in U,z\in Z\right\}\bigcup$
$H_O=\left\{\left(u,\left\langle \ddot{\partial}^t_H(z_i)(u_j),\ddot{\partial}^i_H(z_i)(u_j),\ddot{\partial}^f_H(z_i)(u_j)\right\rangle\right)|u\in U,z\in Z\right\}.$
$=\mathcal{G}_{\mathrm{C}}\hat{\cup}\mathcal{H}_{\mathrm{O}}$.

□

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Note: *The rest of the proof is similar to proof method 1*

Proposition 4.7. *Let*
$\mathcal{H}_{\mathrm{O}} = \left\{ \left(u, \left\langle \ddot{\partial}^{t}_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^{i}_{\mathcal{H}}(z_i)(u_j), \ddot{\partial}^{f}_{\mathcal{H}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z_i \in \hat{\mathbb{Z}} \right\}$,
$\mathcal{K}_{\mathrm{P}} = \left\{ \left(u, \left\langle \ddot{\partial}^{t}_{\mathcal{K}}(z_i)(u_j), \ddot{\partial}^{i}_{\mathcal{K}}(z_i)(u_j), \ddot{\partial}^{f}_{\mathcal{K}}(z_i)(u_j) \right\rangle | u \in \hat{\mathbb{U}}, z_i \in \hat{\mathbb{Z}} \right\}$ *be two SVNFSE-sets on reference set* $\hat{\mathbb{U}}$. *Then,the following statements hold:*

1. $(\mathcal{H}_{\mathrm{O}}{}^{c})^{c} = \mathcal{H}_{\mathrm{O}}$.
2. $\left(\mathcal{H}_{\mathrm{O}} \hat{\cup} \mathcal{K}_{\mathrm{P}}\right)^{c} = \mathcal{H}_{\mathrm{O}}{}^{c} \hat{\cap} \mathcal{K}_{\mathrm{P}}{}^{c}$.
3. $\left(\mathcal{H}_{\mathrm{O}} \hat{\cap} \mathcal{K}_{\mathrm{P}}\right)^{c} = \mathcal{H}_{\mathrm{O}}{}^{c} \hat{\cup} \mathcal{K}_{\mathrm{P}}{}^{c}$.

Here, paragraphs 2 and 3 refer to De Morgan's law.

Proof. **1.** $\mathcal{H}_{\mathrm{O}} = \left\{ u_j \left(\ddot{\partial}^{t}_{\mathcal{H}}(z_i)(u_j)\ , \ddot{\partial}^{i}_{\mathcal{H}}(z_i)(u_j),\ \ddot{\partial}^{f}_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$H^{c}_{O} = \left\{ u_j \left(\ddot{\partial}^{f}_{\mathcal{H}}(z_i)(u_j)\ , 1 - \ddot{\partial}^{i}_{\mathcal{H}}(z_i)(u_j),\ \ddot{\partial}^{t}_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$(\mathcal{H}^{c}_{O})^{c} = \left\{ u_j \left(\ddot{\partial}^{t}_{\mathcal{H}}(z_i)(u_j)\ , 1 - \left(1 - \ddot{\partial}^{i}_{\mathcal{H}}(z_i)(u_j)\right),\ \ddot{\partial}^{f}_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$= \left\{ u_j \left(\ddot{\partial}^{t}_{\mathcal{H}}(z_i)(u_j)\ , \ddot{\partial}^{i}_{\mathcal{H}}(z_i)(u_j),\ \ddot{\partial}^{f}_{\mathcal{H}}(z_i)(u_j) \right) | z_i \in Z, u_j \in U \right\}$
$= \mathcal{H}_{\mathcal{O}}$

Note: The rest of proof is similar to proof method 1 □

Definition 4.8. **(*AND* of SVNFSE-sets):** Let $\mathcal{H}_{\mathcal{O}}and\mathcal{G}_{\mathcal{C}}$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then the "*AND*" operation of both $\mathcal{H}_{\mathcal{O}}$ and $\mathcal{G}_{\mathcal{C}}$ defined as $\mathcal{H}_{\mathcal{O}} \wedge \mathcal{G}_{\mathcal{C}} = \mathcal{R}_{\mathcal{L}}$ such that $\mathcal{R}_{\mathcal{L}}(u_i, u_j)(z_i) = \mathcal{H}_{\mathcal{O}}(u_j)(z_i) \cap \mathcal{G}_{\mathcal{C}}(u_j)(z_i)$ Here $\cap$ refer to the intersection of SVNS.

Example 4.9. Taking into account the SVNFSE $\mathcal{H}_{\mathcal{O}}$ as an Example 3.2, and $\mathcal{G}_{\mathcal{C}}$ given in an Example 3.4, then, **(*AND* of SVNFSE-sets):** can be possess as following :

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right),\ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right) \right\}.$$

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_2) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right),\ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right) \right\}.$$

Definition 4.10. **(*OR* of SVNFSE-sets):** Let $\mathcal{H}_{\mathcal{O}}and\mathcal{G}_{\mathcal{C}}$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then the "*OR*" operation of both $\mathcal{H}_{\mathcal{O}}$ and $\mathcal{G}_{\mathcal{C}}$ defined as $\mathcal{H}_{\mathcal{O}} \vee \mathcal{G}_{\mathcal{C}} = \mathcal{F}_{\mathcal{V}}$ such that $\mathcal{F}_{\mathcal{V}}(u_i, u_j)(z_i) = \mathcal{H}_{\mathcal{O}}(u_j)(z_i) \cap \mathcal{G}_{\mathcal{C}}(u_j)(z_i)$ Here $\cup$ refer to the union of SVNS.

Example 4.11. Taking into account the SVNFSE $\mathcal{H}_{\mathcal{O}}$ as an Example 3.2, and $\mathcal{G}_{\mathcal{C}}$ given in an Example 3.4, then, **(*OR* of SVNFSE-sets):** can be possess as following :

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_1) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right),\ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right) \right\}.$$

$$\mathcal{R}_{\mathcal{L}}(z_1 \times z_2) = \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right), \left\{ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right),\ \left(\frac{u_1}{\langle 0.2, 0.5, 0.1 \rangle} \right) \right\}.$$

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5 Orthogonal Distance and Similarity Between Two SVNFSESs

A similarity measure or similarity function is a real-valued a function that evaluates the closeness between two items. Similarity measure takes huge values on comparable items and either zero an incentive for non-comparable items. In this part, the similarity measure between two SVNFSES is defined using the normalized orthogonal distance. Similarity measures are inversely proportional to the distance between the sets. In addition, we will provide a practical application in solving a decision-making problem to try these tools

5.1 Normalized Orthogonal Distance Between Two SVNFSESs

Definition 5.1. Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle \mid u \in \hat{\mathbb{U}}, z_{i} \in \hat{\mathbb{Z}}\right\}$,
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle \mid u \in \hat{\mathbb{U}}, z_{i} \in \hat{\mathbb{Z}}\right\}$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then, the normalized orthogonal distance between $\mathcal{H}_{\mathrm{O}}$ and $\mathcal{K}_{\mathrm{P}}$ defined as the following:

$$D^{\perp}\left(\mathcal{H}_{\mathrm{O}}, \mathrm{K}_{\mathrm{P}}\right)=\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\sqrt{\left(\ddot{\partial}_{\mathcal{D}}^{t}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}+\left(\ddot{\partial}_{\mathrm{D}}^{i}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}+\left(\ddot{\partial}_{\mathcal{D}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}}}{\max \left(\left|\mathcal{H}_{\mathrm{O}}\left(z_{i}\right)\right|,\left|\mathrm{K}_{\mathrm{P}}\left(z_{i}\right)\right|\right)}$$

Where

$$\ddot{\partial}_{D}^{t}\left(z_{i}\right)\left(u_{j}\right)=\left[\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right) . \ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right)-\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right) . \ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right)\right]$$

$$\ddot{\partial}_{D}^{i}\left(z_{i}\right)\left(u_{j}\right)=\left[\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right) . \ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)-\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right) . \ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right]$$

$$\ddot{\partial}_{D}^{f}\left(z_{i}\right)\left(u_{j}\right)=\left[\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right) . \ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right)-\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right) . \ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right)\right]$$

and

$$\left|\mathcal{H}_{\mathcal{O}}\left(z_{i}\right)\right|=\sqrt{\left(\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}+\left(\ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}+\left(\ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}}$$

$$\left|\mathcal{K}_{\mathcal{P}}\left(z_{i}\right)\right|=\sqrt{\left(\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}+\left(\ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}+\left(\ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right)^{2}}$$

5.2 Normalized Orthogonal Similarity Between Two SVNFSESs

In this part, we introduce and study the Normalized Orthogonal Similarity Measures (NOSM) of SVNFSESs in order to calculate the ratio of NOSM between two SVNFSESs. After that, we will employ these SMs in one application in DM problem by proposing an algorithm shown in Figure1.

Definition 5.2. Let
$\mathcal{H}_{\mathrm{O}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{H}}^{t}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{H}}^{i}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{H}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle \mid u \in \hat{\mathbb{U}}, z_{i} \in \hat{\mathbb{Z}}\right\}$,
$\mathcal{K}_{\mathrm{P}}=\left\{\left(u,\left\langle\ddot{\partial}_{\mathcal{K}}^{t}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{K}}^{i}\left(z_{i}\right)\left(u_{j}\right), \ddot{\partial}_{\mathcal{K}}^{f}\left(z_{i}\right)\left(u_{j}\right)\right\rangle \mid u \in \hat{\mathbb{U}}, z_{i} \in \hat{\mathbb{Z}}\right\}$ be two SVNFSE-sets on reference set $\hat{\mathbb{U}}$. Then, a real-valued function $\mathcal{D}^{\perp}\left(\mathcal{H}_{\mathcal{O}}, \mathcal{K}_{\mathcal{P}}\right): SVNFSES \times SVNFSES \Rightarrow [0,1]$ is a similarity measure . and its defined as the following function:

$$S^{\perp}\left(\mathcal{H}_{\mathcal{O}}, \mathcal{K}_{\mathcal{P}}\right)=\frac{1}{1+D^{\perp}\left(\mathcal{H}_{\mathcal{O}}, \mathcal{K}_{\mathcal{P}}\right)}$$

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5.3 Application in real-life situations using similarity measure of SVNFSE-sets

In this part of the article, we will discuss how our proposed model works in dealing with a real-life application based on a proposed algorithm that relies on the similarity measure of SVNFSE-sets. Therefore, first, we will explain the mechanism of the similarity measure through the following definition.

In this section of the current research, we will create a new algorithm based on the tools presented in this work to solve one of the decision-making problems (to help a couple choose a new home in one of the residential complexes). This algorithm will present its steps in Figure 1 as following:

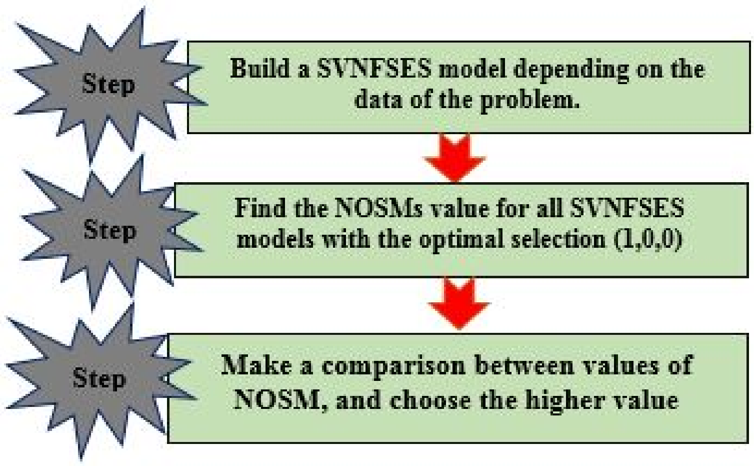

Figure 1: Algorithm

5.4 Statement of the problem

Assume that the married couple, Mr. Xu and Mrs. Xu wants to purchase a house in one of the low-cost residential complexes. In the low-cost residential complexes, there are two houses that represent by reference set $\hat{\mathbb{U}} = \{u_1, u_2, u_3\}$.The two couples in their selection focus on observing the attributes that can be represented by the following attribute set $\hat{\mathbb{M}} = \{m_1, m_2, m_3, \}$ such that m_1 =House area,m_2 =House price, and $m_3 =$ Materials used in building the house .In this scenario, the couple resorts to two experts $\{n_1, n_2\}$ in real estate issues for the purpose of consultation. Here we will analyze the expert's opinions by building two models $(SVNFSES - memberships)$ from our proposed evaluation of this evaluation as a follows:

Step 1. Build three SVNFSES models $\mathcal{H}_{\mathcal{O}}$,$\mathcal{G}_{\mathcal{C}}$ and $\mathcal{Y}_{\mathcal{P}}$ represents the opinion of the two experts (n_1) and (n_2) for three houses (u_1), (u_2) and (u_3) :

$\mathcal{H}_{\mathcal{O}} =$
$\Big\{\mathcal{H}\left(z_1 = (m_1, n_1, 1)\right) = \left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right)\right. .$
$\mathcal{H}\left(z_2 = (m_1, n_2, 1)\right) = \left\{\left(\frac{u_1}{\langle 0.5,0.4,0.2\rangle}\right)\right\} .$
$\mathcal{H}\left(z_3 = (m_2, n_1, 1)\right) = \left\{\left(\frac{u_1}{\langle 0.7,0.6,0.2\rangle}\right)\right\} .$
$\mathcal{H}\left(z_4 = (m_2, n_2, 1)\right) = \left\{\left(\frac{u_1}{\langle 0.5,0.3,0.2\rangle}\right)\right\} .$
$\mathcal{H}\left(z_5 = (m_3, n_1, 1)\right) = \left\{\left(\frac{u_1}{\langle 0.7,0.5,0.9\rangle}\right)\right\} .$
$\mathcal{H}\left(z_6 = (m_3, n_2, 1)\right) = \left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right)\right\} .$
$\mathcal{H}\left(z_7 = (m_1, n_1, 0)\right) = \left\{\left(\frac{u_1}{\langle 0.2,0.5,0.3\rangle}\right)\right\} .$

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$\mathcal{H}\left(z_8=\left(m_1, n_2, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.6,0.2\rangle}\right)\right\}$.
$\mathcal{H}\left(z_9=\left(m_2, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.8\rangle}\right)\right\}$.
$\mathcal{H}\left(z_{10}=\left(m_2, n_2, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.3,0.4,0.7\rangle}\right)\right\}$.
$\mathcal{H}\left(z_{11}=\left(m_3, n_1, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.4,0.5,0.6\rangle}\right)\right\}$.
$\mathcal{H}\left(z_{12}=\left(m_3, n_2, 0\right)\right)=\left\{\left(\frac{u_1}{\langle 0.6,0.8,0.6\rangle}\right)\right\}\}$

$\mathcal{G}_{\mathcal{C}}=$
$\left\{\mathcal{G}\left(z_1=\left(m_1, n_1, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.1,0.4,0.2\rangle}\right)\right\}\right.$.
$\mathcal{G}\left(z_2=\left(m_1, n_2, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.8,0.7,0.1\rangle}\right)\right\}$.
$\mathcal{G}\left(z_3=\left(m_2, n_1, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.9,0.2,0.1\rangle}\right)\right\}$.
$\mathcal{G}\left(z_4=\left(m_2, n_2, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.6,0.3,0.4\rangle}\right)\right\}$.
$\mathcal{G}\left(z_5=\left(m_3, n_1, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.4,0.5,0.9\rangle}\right)\right\}$.
$\mathcal{G}\left(z_6=\left(m_3, n_2, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.3,0.5,0.6\rangle}\right)\right\}$.
$\mathcal{G}\left(z_7=\left(m_1, n_1, 0\right)\right)=\left\{\left(\frac{u_2}{\langle 0.2,0.5,0.3\rangle}\right)\right\}$.
$\mathcal{G}\left(z_8=\left(m_1, n_2, 0\right)\right)=\left\{\left(\frac{u_2}{\langle 0.9,0.1,0.2\rangle}\right)\right\}$.
$\mathcal{G}\left(z_9=\left(m_2, n_1, 0\right)\right)=\left\{\left(\frac{u_2}{\langle 0.6,0.4,0.8\rangle}\right)\right\}$.
$\mathcal{G}\left(z_{10}=\left(m_2, n_2, 0\right)\right)=\left\{\left(\frac{u_2}{\langle 0.6,0.4,0.7\rangle}\right)\right\}$.
$\mathcal{G}\left(z_{11}=\left(m_3, n_1, 0\right)\right)=\left\{\left(\frac{u_2}{\langle 0.5,0.6,0.2\rangle}\right)\right\}$.
$\mathcal{G}\left(z_{12}=\left(m_3, n_2, 0\right)\right)=\left\{\left(\frac{u_2}{\langle 0.7,0.4,0.2\rangle}\right)\right\}\}$

$\mathcal{Y}_{\mathcal{P}}=$
$\left\{\mathcal{Y}\left(z_1=\left(m_1, n_1, 1\right)\right)=\left\{\left(\frac{u_2}{\langle 0.4,0.2,0.5\rangle}\right)\right\}\right.$.
$\mathcal{Y}\left(z_2=\left(m_1, n_2, 1\right)\right)=\left\{\left(\frac{u_3}{\langle 0.2,0.3,0.3\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_3=\left(m_2, n_1, 1\right)\right)=\left\{\left(\frac{u_3}{\langle 0.3,0.5,0.4\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_4=\left(m_2, n_2, 1\right)\right)=\left\{\left(\frac{u_3}{\langle 0.1,0.3,0.6\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_5=\left(m_3, n_1, 1\right)\right)=\left\{\left(\frac{u_3}{\langle 0.4,0.5,0.4\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_6=\left(m_3, n_2, 1\right)\right)=\left\{\left(\frac{u_3}{\langle 0.6,0.5,0.6\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_7=\left(m_1, n_1, 0\right)\right)=\left\{\left(\frac{u_3}{\langle 0.8,0.5,0.4\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_8=\left(m_1, n_2, 0\right)\right)=\left\{\left(\frac{u_3}{\langle 0.2,0.1,0.9\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_9=\left(m_2, n_1, 0\right)\right)=\left\{\left(\frac{u_3}{\langle 0.9,0.4,0.6\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_{10}=\left(m_2, n_2, 0\right)\right)=\left\{\left(\frac{u_3}{\langle 0.3,0.4,0.3\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_{11}=\left(m_3, n_1, 0\right)\right)=\left\{\left(\frac{u_3}{\langle 0.5,0.6,0.2\rangle}\right)\right\}$.
$\mathcal{Y}\left(z_{12}=\left(m_3, n_2, 0\right)\right)=\left\{\left(\frac{u_3}{\langle 0.2,0.4,0.5\rangle}\right)\right\}\}$

Step 2. Find the NOSM for all $\mathcal{H}_{\mathcal{O}}$,$\mathcal{G}_{\mathcal{C}}$ and $\mathcal{Y}_{\mathcal{P}}$ with the optimal evaluation $(1,0,0)$ according definition 5.1 and 5.2 in part 5, **(the NOSM for all $\mathcal{H}_{\mathcal{O}}$,$\mathcal{G}_{\mathcal{C}}$ and $\mathcal{Y}_{\mathcal{P}}$ presented in Tables 1.)** .

Step 3. The value of NOSM for $\mathcal{H}_{\mathcal{O}}$ is 0.784, therefor the two couple will choose house u_1.

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Table 1: Valudes of NOSM for all $\mathcal{H}_{\mathcal{O}}$,$\mathcal{G}_{\mathcal{C}}$ and $\mathcal{Y}_{\mathcal{P}}$

Category	Degree of NOSM
NOSM for $\mathcal{H}_{\mathcal{O}}$	0.784
NOSM for $\mathcal{G}_{\mathcal{C}}$	0.588
NOSM for $\mathcal{Y}_{\mathcal{P}}$	0.683

6 Conclusion

In this work, the notion of a SVNFSES as combining the critical features of FSs and SVNSs under expert systems in one model or as an extension to SES is introduced. The basic properties of this model namely null, absolute, subset, equality, and complement are presented. Also, the basic set theory like union, intersection, OR, and AND operations as well as some properties on SVNFSESs are discussed. Finally, we presented a decision-making method based orthogonal distance and similarity on SVNFSES and gave an application of this method to solve a decision-making problem. For future research work, users can combine these tools with other fuzzy algebraic tools.

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