Selection process real-life application for new type complex neutrosophic sets using various aggregation operators

Ibraheem Abu Falahah[1], **Abdallah Al-Husban**[2], **Lejo J. Manavalan**[3], **Jamil J. Jaber**[4,5], **M. Palanikumar**[6,*], **G. Balaji**[7]

[1]Department of Mathematics, Faculty of Science, The Hashemite University,

P.O. Box 330127, Zarqa, 13133, Jordan

[2]Department of Mathematics, Faculty of Science and Technology, Irbid

National University, P.O. Box: 2600 Irbid, Jordan

[3]Department of Mathematics, Little Flower College-680103, Guruvayoor, India

[4]Department of Finance and Banking, Applied Science Private University,

Faculty of Business, Amman. 11937, Jordan;

[5]Department of Finance, School of Business, The University of Jordan, Aqaba, 77110, Jordan.
[6]Department of Mathematics, SRM Valliammai Engineering College, Kattankulathur, 603203,Tamilnadu, India

[7]Department of Mathematics, Al- Ameen Engineering College, Erode.

Emails: iabufalahah@hu.edu.jo; dralhosban@inu.edu.jo; lejo@littleflowercollege.edu.in; j.jaber@ju.edu.jo; palanimaths86@gmail.com; g.balaji@gmail.com

* Corresponding author: M.Palanikumar.

Abstract

A new approach to multiple attribute decision-making (MADM) is presented in this article, which is based on $(\jmath_1, \jmath_2, \jmath_3)$ complex neutrosophic sets (CNS). We are extending the CNS in this way. Complex neutrosophic weighted averaging (CNWA), complex neutrosophic weighted geometric (CNWG), complex generalized neutrosophic weighted averaging (CGNWA), and complex generalized neutrosophic weighted geometric (CGNWG). An algorithm utilizing these operators was presented during our discussion. Extensive score and accuracy values are illustrated numerically. We will also discuss idempotency, boundedness, commutativity, and monotonicity of complex neutrosophic sets as part of this communication. You can find the best option faster, easier, and more conveniently with them. Therefore, complex $(\jmath_1, \jmath_2, \jmath_3)$ is more closely associated with more precise conclusions. A fascinating and intriguing finding was also revealed by the study.

Keywords: CNWA; CNWG; CGNWA and CGNWG.

1 Introduction

Many uncertain theories including fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS)[3] and spherical FS (SFS).[4] An FS consists of elements with values ranging from 0 to 1 for their membership value (MV); Later, Atanassov proposed the concept of an IFS that is divided into categories using non-membership value (NMV), which cannot exceed one.[2] In cases where the combined MV and NMV values are greater than 1, a

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single problem may be conveyed to the DM. The square sum of the MV and NMV of an IFS with a value less than one is classified as PFS, which can be characterized by Yager.[3] According to Cuong et al.[5] developed the picture FS concept with three pointers: positive MV, neutral MV and negative MV. Moreover, it is more beneficial than both PFS and IFS. An example of a generalization of picture FS with AOs is proposed by Liu et al.[6] Liu et al.[7] they proposed a generalized PFS with AO and applications. Recently, many researchers discussed new aggregating operator by[8]-.[11] An analysis of AOs using PFS and interval values.[12] As part of the DM approach challenge, the sum of the positive, neutral, and negative MVs rarely exceeds one. The concept of SFS is proposed by Ashraf et al.,[4] whereby the square sum of positive, neutral, and negative values is not greater than 1. Fatmaa et al.[13] investigated the notion of SFS using TOPSIS models.

A fuzzy spherical Dombi AO was developed by Ashraf et al.[14] Further information about SFSs and T-SFSs can be found at.[15,16] In 2022, Temel et al.[17] discussed its application to MADM based on Muirhead power normal SFS. Peng et al.[18] explore neutrosophic set with MADM using MABAC and TOPSIS approaches. A generalization of PFS using TOPSIS was presented by Lhang et al.[19] There are several algebraic structures and aggregation operators that can be applied in many applications as discussed by,[20],[21].[22] An introduction is found in section 1. There was a discussion of PFS and NS in section 2. A definition of complex $(\jmath_1, \jmath_2, \jmath_3)$ and some operations on it are discussed in section 3. In section 4 an interaction between MADM and some AOs is described based on some CNNs for $(\jmath_1, \jmath_2, \jmath_3)$. A numerical example and an algorithm are discussed in section 5. Section 6 concludes with a conclusion. Based on the findings of the study, the following conclusions can be drawn:

1. To introduce score values and accuracy values for CNSs.
2. By using $(\jmath_1, \jmath_2, \jmath_3)$CNNs, $(\jmath_1, \jmath_2, \jmath_3)$CNWA, $(\jmath_1, \jmath_2, \jmath_3)$CNWG, G$(\jmath_1, \jmath_2, \jmath_3)$CNWA and G$(\jmath_1, \jmath_2, \jmath_3)$CNWG operators are developed.
3. On the basis of AOs, MADM is explored using $(\jmath_1, \jmath_2, \jmath_3)$CNSs.
4. We proposed approaches is demonstrated with a few numerical examples.

2 Background

This section contains a number of important definitions that we must review for our further learning.

Definition 2.1. [12] L Υ be an universal. The PIVFS $\mho = \Big\{\wp, \Big\langle \widetilde{\Gamma^{\mathcal{T}}_{\mho}}(\wp), \widetilde{\Gamma^{\mathcal{F}}_{\mho}}(\wp) \Big\rangle \Big| \wp \in \Upsilon\Big\}$, where $\widetilde{\Gamma^{T}_{\mho}}, \widetilde{\Gamma^{F}_{\mho}} : \Upsilon \to Int([0,1])$ denote the MV and NMV of $\wp \in \Upsilon$ to $\mho$, respectively, and $0 \leq (\Gamma^{\mathcal{T}}_{\mho}(\wp))^2 + (\Gamma^{\mathcal{F}}_{\mho}(\wp))^2 \leq 1$. For convenience, $\mho = \Big\langle \Big[\Gamma^{\mathcal{T}}_{\mho}, \Gamma^{\mathcal{T}}_{\mho}\Big], \Big[\Gamma^{\mathcal{F}}_{\mho}, \Gamma^{\mathcal{F}}_{\mho}\Big] \Big\rangle$ is called a Pythagorean interval-valued fuzzy number (PyIVFN).

Definition 2.2. The NS $\mho = \Big\{x, \langle \Gamma^{\mathcal{T}}_{\mho}(\wp), \Gamma^{\mathcal{I}}_{\mho}(\wp), \Gamma^{\mathcal{F}}_{\mho}(\wp)\rangle \big| \wp \in \Upsilon\Big\}$, where $\Gamma^{\mathcal{T}}_{\mho}, \Gamma^{\mathcal{I}}_{\mho}, \Gamma^{\mathcal{F}}_{\mho} : \Upsilon \to [0,1]$ denote the positive MV, neutral MV and negative MV of $\wp \in \Upsilon$, respectively and $0 \leq (\Gamma^{\mathcal{T}}_{\mho}(\wp)) + (\Gamma^{\mathcal{I}}_{\mho}(\wp)) + (\Gamma^{\mathcal{F}}_{\mho}(\wp)) \leq 2$. For $M = \langle \Gamma^{\mathcal{T}}_{\mho}, \Gamma^{\mathcal{I}}_{\mho}, \Gamma^{\mathcal{F}}_{\mho}\rangle$ is called a neutrosophic number (NN).

Definition 2.3. The NS $\mho = \Big\{\wp, \langle \Gamma^{\mathcal{T}}_{\mho}(\wp), \Gamma^{\mathcal{I}}_{\mho}(\wp), \Gamma^{\mathcal{F}}_{\mho}(\wp)\rangle \big| \wp \in \Upsilon\Big\}$, where $\Gamma^{\mathcal{T}}_{\mho}, \Gamma^{\mathcal{I}}_{\mho}, \Gamma^{\mathcal{F}}_{\mho} : \Upsilon \to [0,1]$ denote the positive MV, neutral MV and negative MV of $\wp \in \Upsilon$ to $\mho$, respectively and $0 \leq (\Gamma^{\mathcal{T}}_{\mho}(\wp))^2 + (\Gamma^{\mathcal{I}}_{\mho}(\wp))^2 + (\Gamma^{\mathcal{F}}_{\mho}(\wp))^2 \leq 2$. For all $\wp \in \Upsilon$, $\sqrt{2 - \big((\Gamma^{\mathcal{T}}_{\mho}(\wp))^2 + (\Gamma^{\mathcal{I}}_{\mho}(\wp))^2 + (\Gamma^{\mathcal{F}}_{\mho}(\wp))^2\big)}$ is called the value of refusal of membership of $\wp$ in $\mho$. For convenience, $\mho = \langle \Gamma^{\mathcal{T}}_{\mho}, \Gamma^{\mathcal{I}}_{\mho}, \Gamma^{\mathcal{F}}_{\mho}\rangle$ is called a Pythagorean neutrosophic number (PyNN).

Definition 2.4. L $\mho_1 = (a_1, b_1) \in N$and $\mho_2 = (a_2, b_2) \in N$. Then the distance between $\mho_1$ and $\mho_2$ is defined as $\mathcal{D}(\mho_1, \mho_2) = \sqrt{(a_1 - a_2)^2 + \frac{1}{2}(b_1 - b_2)^2}$, where N is a natural number.

Definition 2.5. For any PIVFN $\mho = \Big\langle [\mathfrak{X}^{Tl}, \mathfrak{X}^{Tu}], [\mathfrak{X}^{Fl}, \mathfrak{X}^{Fu}] \Big\rangle$, the score function of $\mho$ is defined as

$$S(\mho) = \frac{1}{2}\Big((\mathfrak{X}^{Tl})^2 + (\mathfrak{X}^{Tu})^2 - (\mathfrak{X}^{Fl})^2 - (\mathfrak{X}^{Fu})^2\Big),\ S(\mho) \in [-1, 1]$$

Accuracy function of $\mho$ is

$$H(\mho) = \frac{1}{2}\Big((\mathfrak{X}^{Tl})^2 + (\mathfrak{X}^{Tu})^2 + (\mathfrak{X}^{Fl})^2 + (\mathfrak{X}^{Fu})^2\Big),\ H(\mho) \in [0, 1].$$

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3 Operations for of $(\jmath_1, \jmath_2, \jmath_3)$CNN

We discuss the concept of $(\jmath_1, \jmath_2, \jmath_3)$ complex neutrosophic number (CNN). As a result, the $(\jmath_1, \jmath_2, \jmath_3)$CNN and its operations were defined, , where $\mathfrak{Z}$ refer $\left(\frac{\mathfrak{Z}}{2\pi}\right)$.

Definition 3.1. The $(\jmath_1, \jmath_2, \jmath_3)$ Complex neutrosophic set ($(\jmath_1, \jmath_2, \jmath_3)$CNS) $\mho = \left\{\wp, \left\langle \left(\mathfrak{X}^{\mathcal{T}}_{\mho}(\wp)e^{i2\pi\mathfrak{Z}^{\mathcal{T}}_{\mho}(\wp)}, \mathfrak{X}^{\mathcal{I}}_{\mho}(\wp)e^{i2\pi\mathfrak{Z}^{\mathcal{I}}_{\mho}(\wp)}, \mathfrak{X}^{\mathcal{F}}_{\mho}(\wp)e^{i2\pi\mathfrak{Z}^{\mathcal{F}}_{\mho}(\wp)}\right)\right\rangle \middle| \wp \in \Upsilon\right\}$, where $\mathfrak{X}^{T}_{\mho}, \mathfrak{X}^{I}_{\mho}, \mathfrak{X}^{F}_{\mho} : \Upsilon \to [0, 1]$ denote the positive MV, neutral MV and negative MV of $\wp \in \Upsilon$ to $\mho$, $\mathfrak{Z}^{T}_{\mho}, \mathfrak{Z}^{I}_{\mho}, \mathfrak{Z}^{F}_{\mho} : \Upsilon \to [0, 1]$ denote the phase positive MV, phase neutral MV and phase negative MV of $\wp \in \Upsilon$ to $\mho$, respectively and $0 \leq (\mathfrak{X}^{\mathcal{T}}_{\mho}(\wp))^{\jmath_1} + (\mathfrak{X}^{\mathcal{I}}_{\mho}(\wp))^{\jmath_2} + (\mathfrak{X}^{\mathcal{F}}_{\mho}(\wp))^{\jmath_3} \leq 1$ and $0 \leq (\mathfrak{Z}^{\mathcal{T}}_{\mho}(\wp))^{\jmath_1} + (\mathfrak{Z}^{\mathcal{I}}_{\mho}(\wp))^{\jmath_2} + (\mathfrak{Z}^{\mathcal{F}}_{\mho}(\wp))^{\jmath_3} \leq 1$. For convenience, $\mho = \left\langle \left(\mathfrak{X}^{\mathcal{T}}_{\mho}e^{i2\pi\mathfrak{Z}^{\mathcal{T}}_{\mho}}, \mathfrak{X}^{\mathcal{I}}_{\mho}e^{i2\pi\mathfrak{Z}^{\mathcal{I}}_{\mho}}, \mathfrak{X}^{\mathcal{F}}_{\mho}e^{i2\pi\mathfrak{Z}^{\mathcal{F}}_{\mho}}\right)\right\rangle$ is represent a $(\jmath_1, \jmath_2, \jmath_3)$CNN.

Definition 3.2. The $\mho = \langle(\mathfrak{X}^{\mathcal{T}}e^{i2\pi\mathfrak{Z}^{\mathcal{T}}}, \mathfrak{X}^{\mathcal{I}}e^{i2\pi\mathfrak{Z}^{\mathcal{I}}}, \mathfrak{X}^{\mathcal{F}}e^{i2\pi\mathfrak{Z}^{\mathcal{F}}})\rangle$, $\mho_1 = \langle(\mathfrak{X}^{\mathcal{T}}_1e^{i2\pi\mathfrak{Z}^{\mathcal{T}}_1}, \mathfrak{X}^{\mathcal{I}}_1e^{i2\pi\mathfrak{Z}^{\mathcal{I}}_1}, \mathfrak{X}^{\mathcal{F}}_1e^{i2\pi\mathfrak{Z}^{\mathcal{F}}_1})\rangle$ and $\mho_2 = \langle(\mathfrak{X}^{\mathcal{T}}_2e^{i2\pi\mathfrak{Z}^{\mathcal{T}}_2}, \mathfrak{X}^{\mathcal{I}}_2e^{i2\pi\mathfrak{Z}^{\mathcal{I}}_2}, \mathfrak{X}^{\mathcal{F}}_2e^{i2\pi\mathfrak{Z}^{\mathcal{F}}_2})\rangle$ be any three of $(\jmath_1, \jmath_2, \jmath_3)$CNNs, and $\Xi > 0$. Then

1. $$\mho_1 \bigvee \mho_2 = \begin{bmatrix} \sqrt[\jmath_1]{(\mathfrak{X}^{\mathcal{T}}_1)^{\jmath_1} + (\mathfrak{X}^{\mathcal{T}}_2)^{\jmath_1} - (\mathfrak{X}^{\mathcal{T}}_1)^{\jmath_1} \cdot (\mathfrak{X}^{\mathcal{T}}_2)^{\jmath_1}}\, e^{i2\pi \sqrt[\jmath_1]{(\mathfrak{Z}^{\mathcal{T}}_1)^{\jmath_1} + (\mathfrak{Z}^{\mathcal{T}}_2)^{\jmath_1} - (\mathfrak{Z}^{\mathcal{T}}_1)^{\jmath_1} \cdot (\mathfrak{Z}^{\mathcal{T}}_2)^{\jmath_1}}}, \\ \sqrt[\jmath_2]{(\mathfrak{X}^{\mathcal{I}}_1)^{\jmath_2} + (\mathfrak{X}^{\mathcal{I}}_2)^{\jmath_2} - (\mathfrak{X}^{\mathcal{I}}_1)^{\jmath_2} \cdot (\mathfrak{X}^{\mathcal{I}}_2)^{\jmath_2}}\, e^{i2\pi \sqrt[\jmath_2]{(\mathfrak{Z}^{\mathcal{I}}_1)^{\jmath_2} + (\mathfrak{Z}^{\mathcal{I}}_2)^{\jmath_2} - (\mathfrak{Z}^{\mathcal{I}}_1)^{\jmath_2} \cdot (\mathfrak{Z}^{\mathcal{I}}_2)^{\jmath_2}}}, \\ (\mathfrak{X}^{\mathcal{F}}_1)^{1_3} \cdot (\mathfrak{X}^{\mathcal{F}}_2)^{1_3} e^{i2\pi(\mathfrak{Z}^{\mathcal{F}}_1)^{1_3} \cdot (\mathfrak{Z}^{\mathcal{F}}_2)^{1_3}} \end{bmatrix},$$

2. $$\mho_1 \ominus \mho_2 = \begin{bmatrix} (\mathfrak{X}^{\mathcal{F}}_1)^{l_1} \cdot (\mathfrak{X}^{\mathcal{F}}_2)^{l_1} e^{i2\pi(\mathfrak{Z}^{\mathcal{F}}_1)^{l_1} \cdot (\mathfrak{Z}^{\mathcal{F}}_2)^{l_1}}, \\ \sqrt[\jmath_2]{(\mathfrak{X}^{\mathcal{I}}_1)^{\jmath_2} + (\mathfrak{X}^{\mathcal{I}}_2)^{\jmath_2} - (\mathfrak{X}^{\mathcal{I}}_1)^{\jmath_2} \cdot (\mathfrak{X}^{\mathcal{I}}_2)^{\jmath_2}}\, e^{i2\pi \sqrt[\jmath_2]{(\mathfrak{Z}^{\mathcal{I}}_1)^{\jmath_2} + (\mathfrak{Z}^{\mathcal{I}}_2)^{\jmath_2} - (\mathfrak{Z}^{\mathcal{I}}_1)^{\jmath_2} \cdot (\mathfrak{Z}^{\mathcal{I}}_2)^{\jmath_2}}}, \\ \sqrt[\jmath_3]{(\mathfrak{X}^{\mathcal{F}}_1)^{\jmath_3} + (\mathfrak{X}^{\mathcal{F}}_2)^{\jmath_3} - (\mathfrak{X}^{\mathcal{F}}_1)^{\jmath_3} \cdot (\mathfrak{X}^{\mathcal{F}}_2)^{\jmath_3}}\, e^{i2\pi \sqrt[\jmath_3]{(\mathfrak{Z}^{\mathcal{F}}_1)^{\jmath_3} + (\mathfrak{Z}^{\mathcal{F}}_2)^{\jmath_3} - (\mathfrak{Z}^{\mathcal{F}}_1)^{\jmath_3} \cdot (\mathfrak{Z}^{\mathcal{F}}_2)^{\jmath_3}}} \end{bmatrix}$$

3. $$\Xi \cdot \mho = \begin{bmatrix} \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{X}^{\mathcal{T}})^{\jmath_1}\right)^{\Xi}}\, e^{i2\pi \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{Z}^{\mathcal{T}})^{\jmath_1}\right)^{\Xi}}}, \\ \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{X}^{\mathcal{I}})^{\jmath_2}\right)^{\Xi}}\, e^{i2\pi \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{Z}^{\mathcal{I}})^{\jmath_2}\right)^{\Xi}}}, \\ ((\mathfrak{X}^{\mathcal{F}})^{\jmath_3})^{\Xi} e^{i2\pi((\mathfrak{Z}^{\mathcal{F}})^{\jmath_3})^{\Xi}} \end{bmatrix},$$

4. $$\mho^{\Xi} = \begin{bmatrix} ((\mathfrak{X}^{\mathcal{T}})^{\jmath_1})^{\Xi} e^{i2\pi((\mathfrak{Z}^{\mathcal{T}})^{\jmath_1})^{\Xi}}, \\ \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{X}^{\mathcal{I}})^{\jmath_2}\right)^{\Xi}}\, e^{i2\pi \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{Z}^{\mathcal{I}})^{\jmath_2}\right)^{\Xi}}}, \\ \sqrt[\jmath_3]{1 - \left(1 - (\mathfrak{X}^{\mathcal{F}})^{\jmath_3}\right)^{\Xi}}\, e^{i2\pi \sqrt[\jmath_3]{1 - \left(1 - (\mathfrak{Z}^{\mathcal{F}})^{\jmath_3}\right)^{\Xi}}} \end{bmatrix}.$$

Definition 3.3. For any CNIVN $\mho_i = \langle(\mathfrak{X}^{\mathcal{T}}_ie^{i2\pi\mathfrak{Z}^{\mathcal{T}}_i}, \mathfrak{X}^{\mathcal{I}}_ie^{i2\pi\mathfrak{Z}^{\mathcal{I}}_i}, \mathfrak{X}^{\mathcal{F}}_ie^{i2\pi\mathfrak{Z}^{\mathcal{F}}_i})\rangle$, the score function of $\mho$ $S(\mho) = \frac{C+D}{2}$, where $C = (\mathfrak{X}^T)^2 - (\mathfrak{X}^I)^2 + 1 - (\mathfrak{X}^F)^2$ and $D = (\mathfrak{Z}^T)^2 - (\mathfrak{Z}^I)^2 - (\mathfrak{Z}^F)^2$, where $S(\mho) \in [-1, 1]$. The accuracy function $H(\mho) = \frac{C_1+D_1}{2}$, where $C_1 = (\mathfrak{X}^T)^2 + (\mathfrak{X}^I)^2 + (\mathfrak{X}^F)^2$ and $D_1 = (\mathfrak{Z}^T)^2 + (\mathfrak{Z}^I)^2 + (\mathfrak{Z}^F)^2$, where $H(\mho) \in [0, 1]$.

4 AOs based on $(\jmath_1, \jmath_2, \jmath_3)$CNN

Here we describe the AOs using $(\jmath_1, \jmath_2, \jmath_3)$CNWA, $(\jmath_1, \jmath_2, \jmath_3)$CNWG, G$(\jmath_1, \jmath_2, \jmath_3)$CNWA, and G$(\jmath_1, \jmath_2, \jmath_3)$CNWG.

4.1 $(\jmath_1, \jmath_2, \jmath_3)CNWA$

Definition 4.1. A $\mho_i = \langle(\mathfrak{X}^{\mathcal{T}}_ie^{i2\pi\mathfrak{Z}^{\mathcal{T}}_i}, \mathfrak{X}^{\mathcal{I}}_ie^{i2\pi\mathfrak{Z}^{\mathcal{I}}_i}, \mathfrak{X}^{\mathcal{F}}_ie^{i2\pi\mathfrak{Z}^{\mathcal{F}}_i})\rangle$ be the $(\jmath_1, \jmath_2, \jmath_3)$CNNs, $W = (\nu_1, \nu_2, ..., \nu_n)$ be the weight of $\mho_i$, $\nu_i \geq 0$ and $\bigvee_{i=1}^{n} \nu_i = 1$. Then $(\jmath_1, \jmath_2, \jmath_3)$CNWA $(\mho_1, \mho_2, ..., \mho_n) = \bigvee_{i=1}^{n} \nu_i\mho_i$.

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Theorem 4.2. *A* $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi\mathfrak{Z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi\mathfrak{Z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi\mathfrak{Z}_i^{\mathcal{F}}})\rangle$ *be the* $(\jmath_1, \jmath_2, \jmath_3)$*CNNs. Then* $(\jmath_1, \jmath_2, \jmath_3)CNWA(\mho_1, \mho_2, ..., \mho_n)$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^n \left(1 - (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}} e^{i2\pi \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^n \left(1 - (\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}}, \\ \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^n \left(1 - (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}} e^{i2\pi \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^n \left(1 - (\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}}, \\ \bigwedge_{i=1}^n ((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} e^{i2\pi(\bigwedge_{i=1}^n ((\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i})} \end{bmatrix}.$$

Proof. If $n = 2$, then $(\jmath_1, \jmath_2, \jmath_3)$CNWA$(\mho_1, \mho_2) = \nu_1\mho_1 \bigvee \nu_2\mho_2$, where

$$\nu_1\mho_1 = \begin{bmatrix} \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}} e^{i2\pi \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}}}, \\ \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}} e^{i2\pi \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}}}, \\ ((\mathfrak{X}_1^{\mathcal{F}})^{\jmath_3})^{\nu_1} e^{2\pi(((\mathfrak{Z}_1^{\mathcal{F}})^{\jmath_3})^{\nu_1})} \end{bmatrix}$$

$$\nu_2\mho_2 = \begin{bmatrix} \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{X}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}} e^{i2\pi \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{Z}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}}}, \\ \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{X}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}} e^{i2\pi \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{Z}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}}}, \\ ((\mathfrak{X}_2^{\mathcal{F}})^{\jmath_3})^{\nu_2} e^{2\pi(((\mathfrak{Z}_2^{\mathcal{F}})^{\jmath_3})^{\nu_2})} \end{bmatrix}$$

Now,

$$\nu_1\mho_1 \bigvee \nu_2\mho_2 = \begin{bmatrix} \sqrt[\jmath_1]{\begin{array}{c}\left(1 - \left(1 - (\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}\right) + \left(1 - \left(1 - (\mathfrak{X}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}\right) \\ -\left(1 - \left(1 - (\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}\right) \cdot \left(1 - \left(1 - (\mathfrak{X}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}\right)\end{array}} \\ e^{i2\pi \sqrt[\jmath_1]{\begin{array}{c}\left(1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}\right) + \left(1 - \left(1 - (\mathfrak{Z}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}\right) \\ -\left(1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}\right) \cdot \left(1 - \left(1 - (\mathfrak{Z}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}\right)\end{array}}}, \\ \sqrt[\jmath_2]{\begin{array}{c}\left(1 - \left(1 - (\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}\right) + \left(1 - \left(1 - (\mathfrak{X}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}\right) \\ -\left(1 - \left(1 - (\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}\right) \cdot \left(1 - \left(1 - (\mathfrak{X}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}\right)\end{array}} \\ e^{i2\pi \sqrt[\jmath_2]{\begin{array}{c}\left(1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}\right) + \left(1 - \left(1 - (\mathfrak{Z}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}\right) \\ -\left(1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}\right) \cdot \left(1 - \left(1 - (\mathfrak{Z}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}\right)\end{array}}}, \\ ((\mathfrak{X}_1^{\mathcal{F}})^{\jmath_3})^{\nu_1} \cdot ((\mathfrak{X}_2^{\mathcal{F}})^{\jmath_3})^{\nu_2} e^{i2\pi(((\mathfrak{Z}_1^{\mathcal{F}})^{\jmath_3})^{\nu_1} \cdot ((\mathfrak{Z}_2^{\mathcal{F}})^{\jmath_3})^{\nu_2})} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}\left(1 - (\mathfrak{X}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}} e^{i2\pi \sqrt[\jmath_1]{1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\nu_1}\left(1 - (\mathfrak{Z}_2^{\mathcal{T}})^{\jmath_1}\right)^{\nu_2}}}, \\ \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}\left(1 - (\mathfrak{X}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}} e^{2\pi \sqrt[\jmath_2]{1 - \left(1 - (\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\nu_1}\left(1 - (\mathfrak{Z}_2^{\mathcal{I}})^{\jmath_2}\right)^{\nu_2}}}, \\ ((\mathfrak{X}_1^{\mathcal{F}})^{\jmath_3})^{\nu_1} \cdot ((\mathfrak{X}_2^{\mathcal{F}})^{\jmath_3})^{\nu_2} e^{i2\pi(((\mathfrak{Z}_1^{\mathcal{F}})^{\jmath_3})^{\nu_1} \cdot ((\mathfrak{Z}_2^{\mathcal{F}})^{\jmath_3})^{\nu_2})} \end{bmatrix}$$

Hence, $(\jmath_1, \jmath_2, \jmath_3)CNWA(\mho_1, \mho_2)$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^2 \left(1 - (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}} e^{i2\pi \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^2 \left(1 - (\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}}, \\ \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^2 \left(1 - (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}, e^{i2\pi \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^2 \left(1 - (\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}} \\ \bigwedge_{i=1}^2 ((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} e^{i2\pi(\bigwedge_{i=1}^2 ((\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i})} \end{bmatrix}.$$

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It valid for $n \geq 3$,
Thus, $(\jmath_1, \jmath_2, \jmath_3)CNWA(\mho_1, \mho_2, ..., \mho_l)$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{l} \left(1 - (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}} e^{i2\pi \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{l} \left(1 - (\mathfrak{z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}}, \\ \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{l} \left(1 - (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}, e^{i2\pi \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{l} \left(1 - (\mathfrak{z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}} \\ \bigwedge_{i=1}^{l} ((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} e^{i2\pi (\bigwedge_{i=1}^{l} ((\mathfrak{z}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i})} \end{bmatrix}.$$

If $n = l + 1$, then $(\jmath_1, \jmath_2, \jmath_3)$CNWA $(\mho_1, \mho_2, ..., \mho_l, \mho_{l+1})$

$$= \begin{bmatrix} \sqrt[\jmath_1]{\begin{array}{l} \bigvee_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}\right) + \left(1 - \left(1 - (\mathfrak{X}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_{l+1}}\right) \\ - \bigwedge_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}\right) \cdot \left(1 - \left(1 - (\mathfrak{X}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_{l+1}}\right) \end{array}} \\ e^{i2\pi \sqrt[\jmath_1]{\begin{array}{l} \bigvee_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}\right) + \left(1 - \left(1 - (\mathfrak{z}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_{l+1}}\right) \\ - \bigwedge_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}\right) \cdot \left(1 - \left(1 - (\mathfrak{z}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_{l+1}}\right), \end{array}}} \\ \sqrt[\jmath_2]{\begin{array}{l} \bigvee_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}\right) + \left(1 - \left(1 - (\mathfrak{X}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_{l+1}}\right) \\ - \bigwedge_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}\right) \cdot \left(1 - \left(1 - (\mathfrak{X}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_{l+1}}\right) \end{array}} \\ e^{i2\pi \sqrt[\jmath_2]{\begin{array}{l} \bigvee_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}\right) + \left(1 - \left(1 - (\mathfrak{z}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_{l+1}}\right) \\ - \bigwedge_{i=1}^{l} \left(1 - \left(1 - (\mathfrak{z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}\right) \cdot \left(1 - \left(1 - (\mathfrak{z}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_{l+1}}\right), \end{array}}} \\ \bigwedge_{i=1}^{l} ((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} \cdot ((\mathfrak{X}_{l+1}^{\mathcal{F}})^{\jmath_3})^{\nu_{l+1}} e^{i2\pi (\bigwedge_{i=1}^{l} ((\mathfrak{z}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} \cdot ((\mathfrak{z}_{l+1}^{\mathcal{F}})^{\jmath_3})^{\nu_{l+1}})} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{l+1} \left(1 - (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}} e^{i2\pi \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{l+1} \left(1 - (\mathfrak{z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}}, \\ \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{l+1} \left(1 - (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}, e^{i2\pi \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{l+1} \left(1 - (\mathfrak{z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}} \\ \bigwedge_{i=1}^{l+1} ((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} e^{i2\pi (\bigwedge_{i=1}^{l+1} ((\mathfrak{z}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i})} \end{bmatrix}.$$

Theorem 4.3. *A $\mho_i = \langle (\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi \mathfrak{z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi \mathfrak{z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi \mathfrak{z}_i^{\mathcal{F}}}) \rangle$ be the $(\jmath_1, \jmath_2, \jmath_3)$CNNs. Then $(\jmath_1, \jmath_2, \jmath_3)$CNWA $(\mho_1, \mho_2, ..., \mho_n) = \mho$ (idempotency property).*

Proof. Since $\mathfrak{X}_i^{\mathcal{T}} = \mathfrak{X}^{\mathcal{T}}$, $\mathfrak{X}_i^{\mathcal{I}} = \mathfrak{X}^{\mathcal{I}}$, $\mathfrak{X}_i^{\mathcal{F}} = \mathfrak{X}^{\mathcal{F}}$, $\mathfrak{z}_i^{\mathcal{T}} = \mathfrak{z}^{\mathcal{T}}$, $\mathfrak{z}_i^{\mathcal{I}} = \mathfrak{z}^{\mathcal{I}}$, $\mathfrak{z}_i^{\mathcal{F}} = \mathfrak{z}^{\mathcal{F}}$ and $\bigvee_{i=1}^{n} \nu_i = 1$. Now, $(\jmath_1, \jmath_2, \jmath_3)CNWA(\mho_1, \mho_2, ..., \mho_n)$

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$$= \begin{bmatrix} \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}}, \\ \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}}, \\ \bigwedge_{i=1}^{n}((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i} e^{i2\pi(\bigwedge_{i=1}^{n}((\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3})^{\nu_i})} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1-\left(1-(\mathfrak{X}^{\mathcal{T}})^{\jmath_1}\right)^{\bigvee_{i=1}^{n}\nu_i}}\, e^{i2\pi \sqrt[\jmath_1]{1-\left(1-(\mathfrak{Z}^{\mathcal{T}})^{\jmath_1}\right)^{\bigvee_{i=1}^{n}\nu_i}}}, \\ \sqrt[\jmath_2]{1-\left(1-(\mathfrak{X}^{\mathcal{I}})^{\jmath_2}\right)^{\bigvee_{i=1}^{n}\nu_i}}\, e^{i2\pi \sqrt[\jmath_2]{1-\left(1-(\mathfrak{Z}^{\mathcal{I}})^{\jmath_2}\right)^{\bigvee_{i=1}^{n}\nu_i}}}, \\ ((\mathfrak{X}^{\mathcal{F}})^{\jmath_3})^{\bigvee_{i=1}^{n}\nu_i} e^{i2\pi((\mathfrak{Z}^{\mathcal{F}})^{\jmath_3})^{\bigvee_{i=1}^{n}\nu_i}} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\jmath_1]{1-\left(1-(\mathfrak{X}^{\mathcal{T}})^{\jmath_1}\right)}\, e^{i2\pi \sqrt[\jmath_1]{1-\left(1-(\mathfrak{Z}^{\mathcal{T}})^{\jmath_1}\right)}}, \\ \sqrt[\jmath_2]{1-\left(1-(\mathfrak{X}^{\mathcal{I}})^{\jmath_2}\right)}\, e^{i2\pi \sqrt[\jmath_2]{1-\left(1-(\mathfrak{Z}^{\mathcal{I}})^{\jmath_2}\right)}}, \\ (\mathfrak{X}^{\mathcal{F}})^{\jmath_3} e^{i2\pi(\mathfrak{Z}^{\mathcal{F}})^{\jmath_3}} \end{bmatrix}$$

$$= \mho.$$

Theorem 4.4. *A* $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi\mathfrak{Z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi\mathfrak{Z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi\mathfrak{Z}_i^{\mathcal{F}}})\rangle$ *be the* $(\jmath_1,\jmath_2,\jmath_3)$*CNNs. Then* $(\jmath_1,\jmath_2,\jmath_3)$*CNWA*$(\mho_1,\mho_2,...,\mho_n)$ *where* $\overleftarrow{\mathfrak{X}^{\mathcal{T}}} = \min \mathfrak{X}_{ij}^{\mathcal{T}}$, $\overrightarrow{\mathfrak{X}^{\mathcal{T}}} = \max \mathfrak{X}_{ij}^{\mathcal{T}}$, $\overleftarrow{\mathfrak{X}^{\mathcal{I}}} = \min \mathfrak{X}_{ij}^{\mathcal{I}}$, $\overrightarrow{\mathfrak{X}^{\mathcal{I}}} = \max \mathfrak{X}_{ij}^{\mathcal{I}}$, $\overleftarrow{\mathfrak{X}^{\mathcal{F}}} = \min \mathfrak{X}_{ij}^{\mathcal{F}}$, $\overrightarrow{\mathfrak{X}^{\mathcal{F}}} = \max \mathfrak{X}_{ij}^{\mathcal{F}}$, $\overleftarrow{\mathfrak{Z}^{\mathcal{T}}} = \min \mathfrak{Z}_{ij}^{\mathcal{T}}$, $\overrightarrow{\mathfrak{Z}^{\mathcal{T}}} = \max \mathfrak{Z}_{ij}^{\mathcal{T}}$, $\overleftarrow{\mathfrak{Z}^{\mathcal{I}}} = \min \mathfrak{Z}_{ij}^{\mathcal{I}}$, $\overrightarrow{\mathfrak{Z}^{\mathcal{I}}} = \max \mathfrak{Z}_{ij}^{\mathcal{I}}$, $\overleftarrow{\mathfrak{Z}^{\mathcal{F}}} = \min \mathfrak{Z}_{ij}^{\mathcal{F}}$, $\overrightarrow{\mathfrak{Z}^{\mathcal{F}}} = \max \mathfrak{Z}_{ij}^{\mathcal{F}}$ *and where* $1 \le i \le n,\ j = 1,2,...,i_j$*. Then,* $\left\langle \overleftarrow{\mathfrak{X}^{\mathcal{T}} e^{i2\pi\mathfrak{Z}^{\mathcal{T}}}}, \overleftarrow{\mathfrak{X}^{\mathcal{I}} e^{i2\pi\mathfrak{Z}^{\mathcal{I}}}}, \overrightarrow{\mathfrak{X}^{\mathcal{F}} e^{i2\pi\mathfrak{Z}^{\mathcal{F}}}} \right\rangle \le (\jmath_1,\jmath_2,\jmath_3)CNWA(\mho_1,\mho_2,...,\mho_n) \le \left\langle \overrightarrow{\mathfrak{X}^{\mathcal{T}} e^{i2\pi\mathfrak{Z}^{\mathcal{T}}}}, \overrightarrow{\mathfrak{X}^{\mathcal{I}} e^{i2\pi\mathfrak{Z}^{\mathcal{I}}}}, \overleftarrow{\mathfrak{X}^{\mathcal{F}} e^{i2\pi\mathfrak{Z}^{\mathcal{F}}}} \right\rangle$ *(Boundedness property).*

Proof. Since, $\overleftarrow{\mathfrak{X}^{\mathcal{T}}} = \min \mathfrak{X}_{ij}^{\mathcal{T}}$, $\overrightarrow{\mathfrak{X}^{\mathcal{T}}} = \max \mathfrak{X}_{ij}^{\mathcal{T}}$ and $\overleftarrow{\mathfrak{X}^{\mathcal{T}}} \le \mathfrak{X}_{ij}^{\mathcal{T}} \le \overrightarrow{\mathfrak{X}^{\mathcal{T}}}$ and $\overleftarrow{\mathfrak{Z}^{\mathcal{T}}} = \min \mathfrak{Z}_{ij}^{\mathcal{T}}$, $\overrightarrow{\mathfrak{Z}^{\mathcal{T}}} = \max \mathfrak{Z}_{ij}^{\mathcal{T}}$ and $\overleftarrow{\mathfrak{Z}^{\mathcal{T}}} \le \mathfrak{Z}_{ij}^{\mathcal{T}} \le \overrightarrow{\mathfrak{Z}^{\mathcal{T}}}$. Now,

$$\begin{aligned} \overleftarrow{\mathfrak{X}^{\mathcal{T}} e^{i2\pi\mathfrak{Z}^{\mathcal{T}}}} &= \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{X}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{Z}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}} \\ &\le \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_{ij}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_{ij}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}} \\ &\le \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{X}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{Z}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}} \\ &= \overrightarrow{\mathfrak{X}^{\mathcal{T}} e^{i2\pi\mathfrak{Z}^{\mathcal{T}}}}. \end{aligned}$$

Since, $\overleftarrow{\mathfrak{X}^{\mathcal{I}}} = \min \mathfrak{X}_{ij}^{\mathcal{I}}$, $\overrightarrow{\mathfrak{X}^{\mathcal{I}}} = \max \mathfrak{X}_{ij}^{\mathcal{I}}$ and $\overleftarrow{\mathfrak{X}^{\mathcal{I}}} \le \mathfrak{X}_{ij}^{\mathcal{I}} \le \overrightarrow{\mathfrak{X}^{\mathcal{I}}}$ and $\overleftarrow{\mathfrak{Z}^{\mathcal{I}}} = \min \mathfrak{Z}_{ij}^{\mathcal{I}}$, $\overrightarrow{\mathfrak{Z}^{\mathcal{I}}} = \max \mathfrak{Z}_{ij}^{\mathcal{I}}$ and $\overleftarrow{\mathfrak{Z}^{\mathcal{I}}} \le \mathfrak{Z}_{ij}^{\mathcal{I}} \le \overrightarrow{\mathfrak{Z}^{\mathcal{I}}}$. Now,

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$$\begin{aligned}
\overleftarrow{\mathfrak{x}^{\mathcal{I}} e^{i2\pi \mathfrak{z}^{\mathcal{I}}}} &= \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{x}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{z}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}} \\
&\le \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{x}_{ij}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{z}_{ij}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}} \\
&\le \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{x}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{z}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}} \\
&= \overrightarrow{\mathfrak{x}^{\mathcal{I}} e^{i2\pi \mathfrak{z}^{\mathcal{I}}}}.
\end{aligned}$$

Since, $\overleftarrow{(\mathfrak{x}^{\mathcal{F}})^{\jmath_3}} = \min(\mathfrak{x}_{ij}^{\mathcal{F}})^{\jmath_3}$, $\overrightarrow{(\mathfrak{x}^{\mathcal{F}})^{\jmath_3}} = \max(\mathfrak{x}_{ij}^{\mathcal{F}})^{\jmath_3}$ and $\overleftarrow{(\mathfrak{x}^{\mathcal{F}})^{\jmath_3}} \le (\mathfrak{x}_{ij}^{\mathcal{F}})^{\jmath_3} \le \overrightarrow{(\mathfrak{x}^{\mathcal{F}})^{\jmath_3}}$, $\overleftarrow{(\mathfrak{z}^{\mathcal{F}})^{\jmath_3}} = \min(\mathfrak{z}_{ij}^{\mathcal{F}})^{\jmath_3}$, $\overrightarrow{(\mathfrak{z}^{\mathcal{F}})^{\jmath_3}} = \max(\mathfrak{z}_{ij}^{\mathcal{F}})^{\jmath_3}$ and $\overleftarrow{(\mathfrak{z}^{\mathcal{F}})^{\jmath_3}} \le (\mathfrak{z}_{ij}^{\mathcal{F}})^{\jmath_3} \le \overrightarrow{(\mathfrak{z}^{\mathcal{F}})^{\jmath_3}}$. We have, $\overleftarrow{(\mathfrak{x}^{\mathcal{F}})^{\jmath_3}} = \bigwedge_{i=1}^{n}((\overleftarrow{\mathfrak{x}^{\mathcal{F}}})^{\jmath_3})^{\nu_i} \le \bigwedge_{i=1}^{n}((\mathfrak{x}_{ij}^{\mathcal{F}})^{\jmath_3})^{\nu_i} \le \bigwedge_{i=1}^{n}((\overrightarrow{\mathfrak{x}^{\mathcal{F}})^{\jmath_3}})^{\nu_i} = \overrightarrow{(\mathfrak{x}^{\mathcal{F}})^{\jmath_3}}$ and $\overleftarrow{(\mathfrak{z}^{\mathcal{F}})^{\jmath_3}} = \bigwedge_{i=1}^{n}((\overleftarrow{\mathfrak{z}^{\mathcal{F}})^{\jmath_3}})^{\nu_i} \le \bigwedge_{i=1}^{n}((\mathfrak{z}_{ij}^{\mathcal{F}})^{\jmath_3})^{\nu_i} \le \bigwedge_{i=1}^{n}((\overrightarrow{\mathfrak{z}^{\mathcal{F}})^{\jmath_3}})^{\nu_i} = \overrightarrow{(\mathfrak{z}^{\mathcal{F}})^{\jmath_3}}$.
Therefore,

$$\begin{aligned}
&\frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{x}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}\right)^2-\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{x}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}\right)^2\\ +1-\left(\bigwedge_{i=1}^{n}((\overrightarrow{\mathfrak{x}^{\mathcal{F}}})^{\jmath_3})^{\nu_i}\right)^2\end{array}\right]\\
+\left[\begin{array}{c}e^{i2\pi\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{z}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}\right)^2}-e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overleftarrow{\mathfrak{z}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}\right)^2}\\ -e^{i2\pi\left(\bigwedge_{i=1}^{n}((\overrightarrow{\mathfrak{z}^{\mathcal{F}}})^{\jmath_3})^{\nu_i}\right)^2}\end{array}\right]\end{array}\right]\\
\le\ &\frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{x}_{ij}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\right)^2-\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{x}_{ij}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\right)^2\\ +1-\left(\bigwedge_{i=1}^{n}((\mathfrak{x}_{ij}^{\mathcal{F}})^{\jmath_3})^{\nu_i}\right)^2\end{array}\right]\\
+\left[\begin{array}{c}e^{i2\pi\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{z}_{ij}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\right)^2}-e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{z}_{ij}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\right)^2}\\ -e^{i2\pi\left(\bigwedge_{i=1}^{n}((\mathfrak{z}_{ij}^{\mathcal{F}})^{\jmath_3})^{\nu_i}\right)^2}\end{array}\right]\end{array}\right]\\
\le\ &\frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{x}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}\right)^2-\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{x}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}\right)^2\\ +1-\left(\bigwedge_{i=1}^{n}((\overleftarrow{\mathfrak{x}^{\mathcal{F}}})^{\jmath_3})^{\nu_i}\right)^2\end{array}\right]\\
+\left[\begin{array}{c}e^{i2\pi\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{z}^{\mathcal{T}}})^{\jmath_1}\right)^{\nu_i}}\right)^2}-e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\overrightarrow{\mathfrak{z}^{\mathcal{I}}})^{\jmath_2}\right)^{\nu_i}}\right)^2}\\ -e^{i2\pi\left(\bigwedge_{i=1}^{n}((\overleftarrow{\mathfrak{z}^{\mathcal{F}}})^{\jmath_3})^{\nu_i}\right)^2}\end{array}\right]\end{array}\right]
\end{aligned}$$

Hence, $\left\langle \overleftarrow{\mathfrak{x}^{\mathcal{T}} e^{i2\pi \mathfrak{z}^{\mathcal{T}}}}, \overleftarrow{\mathfrak{x}^{\mathcal{I}} e^{i2\pi \mathfrak{z}^{\mathcal{I}}}}, \overrightarrow{\mathfrak{x}^{\mathcal{F}} e^{i2\pi \mathfrak{z}^{\mathcal{F}}}} \right\rangle \le (\jmath_1, \jmath_2, \jmath_3) CNWA(\mho_1, \mho_2, ..., \mho_n)$
$\le \left\langle \overrightarrow{\mathfrak{x}^{\mathcal{T}} e^{i2\pi \mathfrak{z}^{\mathcal{T}}}}, \overrightarrow{\mathfrak{x}^{\mathcal{I}} e^{i2\pi \mathfrak{z}^{\mathcal{I}}}}, \overleftarrow{\mathfrak{x}^{\mathcal{F}} e^{i2\pi \mathfrak{z}^{\mathcal{F}}}} \right\rangle$

Theorem 4.5. *A* $\mho_i = \langle (\mathfrak{x}_{t_{ij}}^{\mathcal{T}} e^{i2\pi \mathfrak{z}_{t_{ij}}^{\mathcal{T}}}, \mathfrak{x}_{t_{ij}}^{\mathcal{I}} e^{i2\pi \mathfrak{z}_{t_{ij}}^{\mathcal{I}}}, \mathfrak{x}_{t_{ij}}^{\mathcal{F}} e^{i2\pi \mathfrak{z}_{t_{ij}}^{\mathcal{F}}}) \rangle$ *and*
$W_i = \langle (\mathfrak{x}_{h_{ij}}^{\mathcal{T}} e^{i2\pi \mathfrak{z}_{h_{ij}}^{\mathcal{T}}}, \mathfrak{x}_{h_{ij}}^{\mathcal{I}} e^{i2\pi \mathfrak{z}_{h_{ij}}^{\mathcal{I}}}, \mathfrak{x}_{h_{ij}}^{\mathcal{F}} e^{i2\pi \mathfrak{z}_{h_{ij}}^{\mathcal{F}}}) \rangle$ *CNWAs. For any i, if there is* $(\mathfrak{x}_{t_{ij}}^{\mathcal{T}})^2 \le (\mathfrak{x}_{h_{ij}}^{\mathcal{T}})^2$ *and*

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$(\mathfrak{X}^{\mathcal{I}}_{t_{ij}})^2 \leq (\mathfrak{X}^{\mathcal{I}}_{h_{ij}})^2$ *and* $(\mathfrak{X}^{\mathcal{F}}_{t_{ij}})^2 \geq (\mathfrak{X}^{\mathcal{F}}_{h_{ij}})^2$ *and* $(\mathfrak{Z}^{\mathcal{T}}_{t_{ij}})^2 \leq (\mathfrak{Z}^{\mathcal{T}}_{h_{ij}})^2$ *and* $(\mathfrak{Z}^{\mathcal{I}}_{t_{ij}})^2 \leq (\mathfrak{Z}^{\mathcal{I}}_{h_{ij}})^2$ *and* $(\mathfrak{Z}^{\mathcal{F}}_{t_{ij}})^2 \geq (\mathfrak{Z}^{\mathcal{F}}_{h_{ij}})^2$ *or* $\mho_i \leq W_i$. *Prove that* $(\jmath_1, \jmath_2, \jmath_3)CNWA(\mho_1, \mho_2, ..., \mho_n) \leq (\jmath_1, \jmath_2, \jmath_3)CNWA(W_1, W_2, ..., W_n)$*, where* $(i = 1, 2, ..., n); (j = 1, 2, ..., i_j)$ *(monotonicity property).*

Proof. For any i, $(\mathfrak{X}^{\mathcal{T}}_{t_{ij}})^2 \leq \left(\mathfrak{X}^{\mathcal{T}}_{h_{ij}}\right)^2$.
Therefore, $1 - (\mathfrak{X}^{\mathcal{T}}_{t_i})^2 \geq 1 - (\mathfrak{X}^{\mathcal{T}}_{h_i})^2$.
Hence, $\bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{T}}_{t_i}\right)^2\right)^{\nu_i} \geq \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{T}}_{h_i}\right)^2\right)^{\nu_i}$
and $\sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{T}}_{t_i}\right)^{\jmath_1}\right)^{\nu_i}} \leq \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{T}}_{h_i}\right)^{\jmath_1}\right)^{\nu_i}}$.
For any i, $(\mathfrak{Z}^{\mathcal{T}}_{t_{ij}})^2 \leq \left(\mathfrak{Z}^{\mathcal{T}}_{h_{ij}}\right)^2$.
Therefore, $1 - (\mathfrak{Z}^{\mathcal{T}}_{t_i})^2 \geq 1 - (\mathfrak{Z}^{\mathcal{T}}_{h_i})^2$.
Hence, $\bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{T}}_{t_i}\right)^2\right)^{\nu_i} \geq \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{T}}_{h_i}\right)^2\right)^{\nu_i}$
and $\sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{T}}_{t_i}\right)^{\jmath_1}\right)^{\nu_i}} \leq \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{T}}_{h_i}\right)^{\jmath_1}\right)^{\nu_i}}$.
Hence, $e^{i2\pi \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{T}}_{t_i}\right)^{\jmath_1}\right)^{\nu_i}}} \leq e^{i2\pi \sqrt[\jmath_1]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{T}}_{h_i}\right)^{\jmath_1}\right)^{\nu_i}}}$.
For any i, $(\mathfrak{X}^{\mathcal{I}}_{t_{ij}})^{\jmath_2} \leq \left(\mathfrak{X}^{\mathcal{I}}_{h_{ij}}\right)^{\jmath_2}$.
Therefore, $1 - \left(\mathfrak{X}^{\mathcal{I}}_{t_i}\right)^{\jmath_2} \geq 1 - \left(\mathfrak{X}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}$.
Hence, $\bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{I}}_{t_i}\right)^{\jmath_2}\right)^{\nu_i} \geq \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}\right)^{\nu_i}$.
This implies that $\sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{I}}_{t_i}\right)^{\jmath_2}\right)^{\nu_i}} \leq \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{X}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}\right)^{\nu_i}}$.
For any i, $(\mathfrak{Z}^{\mathcal{I}}_{t_{ij}})^{\jmath_2} \leq \left(\mathfrak{Z}^{\mathcal{I}}_{h_{ij}}\right)^{\jmath_2}$.
Therefore, $1 - \left(\mathfrak{Z}^{\mathcal{I}}_{t_i}\right)^{\jmath_2} \geq 1 - \left(\mathfrak{Z}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}$.
Hence, $\bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{I}}_{t_i}\right)^{\jmath_2}\right)^{\nu_i} \geq \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}\right)^{\nu_i}$.
This implies that $\sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{I}}_{t_i}\right)^{\jmath_2}\right)^{\nu_i}} \leq \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}\right)^{\nu_i}}$.
Hence, $e^{i2\pi \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{I}}_{t_i}\right)^{\jmath_2}\right)^{\nu_i}}} \leq e^{i2\pi \sqrt[\jmath_2]{1 - \bigwedge_{i=1}^{n} \left(1 - \left(\mathfrak{Z}^{\mathcal{I}}_{h_i}\right)^{\jmath_2}\right)^{\nu_i}}}$.
For any i, $\left(\mathfrak{X}^{\mathcal{F}}_{t_{ij}}\right)^2 \geq \left(\mathfrak{X}^{\mathcal{F}}_{h_{ij}}\right)^2$ and $\left(\mathfrak{X}^{\mathcal{F}}_{t_{ij}}\right)^{\jmath_3} \geq \left(\mathfrak{X}^{\mathcal{F}}_{h_{ij}}\right)^{\jmath_3}$.
Therefore, $1 - \left(\bigwedge_{i=1}^{n} \mathfrak{X}^{\mathcal{F}}_{t_{ij}}\right)^{\jmath_3} \leq 1 - \left(\bigwedge_{i=1}^{n} \mathfrak{X}^{\mathcal{F}}_{h_{ij}}\right)^{\jmath_3}$.
For any i, $\left(\mathfrak{Z}^{\mathcal{F}}_{t_{ij}}\right)^2 \geq \left(\mathfrak{Z}^{\mathcal{F}}_{h_{ij}}\right)^2$ and $\left(\mathfrak{Z}^{\mathcal{F}}_{t_{ij}}\right)^{\jmath_3} \geq \left(\mathfrak{Z}^{\mathcal{F}}_{h_{ij}}\right)^{\jmath_3}$.
Therefore, $-e^{i2\pi\left(\bigwedge_{i=1}^{n} \mathfrak{Z}^{\mathcal{F}}_{t_{ij}}\right)^{\jmath_3}} \leq -e^{i2\pi\left(\bigwedge_{i=1}^{n} \mathfrak{Z}^{\mathcal{F}}_{h_{ij}}\right)^{\jmath_3}}$.

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$$= \frac{1}{2}\times \left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_{ti}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\right)^2 - \left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_{ti}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\right)^2 \\ +1-\left(\bigwedge_{i=1}^{n}(\mathfrak{X}_{tij}^{\mathcal{F}})^{\jmath_3}\right)^2\end{array}\right] \\ \left[\begin{array}{c} e^{i2\pi\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_{ti}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\right)^2} - e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_{ti}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\right)^2} \\ -e^{i2\pi\left(\bigwedge_{i=1}^{n}(\mathfrak{Z}_{tij}^{\mathcal{F}})^{\jmath_3}\right)^2}\end{array}\right]\end{array}\right]$$

$$\leq \frac{1}{2}\times \left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_{hi}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\right)^2 - \left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_{hi}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\right)^2 \\ +1-\left(\bigwedge_{i=1}^{n}(\mathfrak{X}_{hij}^{\mathcal{F}})^{\jmath_3}\right)^2\end{array}\right] \\ \left[\begin{array}{c} e^{i2\pi\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_{hi}^{\mathcal{T}})^{\jmath_1}\right)^{\nu_i}}\right)^2} - e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_{hi}^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\right)^2} \\ -e^{i2\pi\left(\bigwedge_{i=1}^{n}(\mathfrak{Z}_{hij}^{\mathcal{F}})^{\jmath_3}\right)^2}\end{array}\right]\end{array}\right]$$

Hence, $(\jmath_1,\jmath_2,\jmath_3)CNWA\,(\mho_1,\mho_2,...,\mho_n) \leq (\jmath_1,\jmath_2,\jmath_3)CNWA\,(W_1,W_2,...,W_n)$.

4.2 $(\jmath_1,\jmath_2,\jmath_3)$CNWG

Definition 4.6. A $\mho_i\langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{F}}})\rangle$ be the $(\jmath_1,\jmath_2,\jmath_3)$CNNs. Then $(\jmath_1,\jmath_2,\jmath_3)$CNWG$(\mho_1,\mho_2,...,\mho_n) = \bigwedge_{i=1}^{n}\mho_i^{\nu_i}$.

Theorem 4.7. *A $\mho_i\langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{F}}})\rangle$ be the $(\jmath_1,\jmath_2,\jmath_3)$CNNs. Then $(\jmath_1,\jmath_2,\jmath_3)$CNWG$(\mho_1,\mho_2,...,\mho_n)$*

$$= \left[\begin{array}{c}\bigwedge_{i=1}^{n}((\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1})^{\nu_i}e^{i2\pi(\bigwedge_{i=1}^{n}((\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1})^{\nu_i})}, \\ \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}\,e^{i2\pi\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\nu_i}}}, \\ \sqrt[\jmath_3]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\nu_i}}\,e^{i2\pi\sqrt[\jmath_3]{1-\bigwedge_{i=1}^{n}\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\nu_i}}}\end{array}\right].$$

Proof. It follows from Theorem 4.2.

Theorem 4.8. *A $\mho_i\langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{F}}})\rangle$ be the $(\jmath_1,\jmath_2,\jmath_3)$CNNs and all are equal. Then $(\jmath_1,\jmath_2,\jmath_3)$CNWG$(\mho_1,\mho_2,...,\mho_n) = \mho$.*

Proof. It follows from Theorem 4.3.

Remark 4.9. It has other properties, including boundedness and monotonicity, as well as having $(\jmath_1,\jmath_2,\jmath_3)$CNWG.

Proof. It follows from Theorem 4.4 and Theorem 4.5.

4.3 Generalized $(\jmath_1,\jmath_2,\jmath_3)$CNWA (G$(\jmath_1,\jmath_2,\jmath_3)$CNWA)

Definition 4.10. A $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{Z}_i^{\mathcal{F}}})\rangle$ be the $(\jmath_1,\jmath_2,\jmath_3)$CNN. Then G$(\jmath_1,\jmath_2,\jmath_3)$CNWA $(\mho_1,\mho_2,...,\mho_n) = \left(\bigvee_{i=1}^{n}\nu_i\mho_i^{\Xi}\right)^{1/\Xi}$.

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Theorem 4.11. *A* $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{F}}})\rangle$ *be the* $(\jmath_1, \jmath_2, \jmath_3)$*CNNs. Then* $G(\jmath_1, \jmath_2, \jmath_3)$*CNWA* $(\mho_1, \mho_2, ..., \mho_n)$

$$= \begin{bmatrix} \left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{1/\jmath_1} e^{i2\pi\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{1/\jmath_1}}, \\ \left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{1/\jmath_2} e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{1/\jmath_2}}, \\ \sqrt[\jmath_3]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}\right)^{\jmath_3}\right)^{1/\jmath_3}} \\ e^{i2\pi\sqrt[\jmath_3]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}\right)^{\jmath_3}\right)^{1/\jmath_3}}} \end{bmatrix}.$$

Proof. We can prove this first by demonstrating that,

$$\bigvee_{i=1}^{n} \nu_i \mho_i^{\jmath_1} = \begin{bmatrix} \sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}}, \\ \sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}}, \\ \bigwedge_{i=1}^{n}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i} e^{i2\pi\bigwedge_{i=1}^{n}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}} \end{bmatrix}.$$

Put $n = 2$,

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$\nu_1\mho_1 \bigvee \nu_2\mho_2$

$$
= \left[\begin{array}{c}
\sqrt[\jmath_1]{\begin{array}{c}\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}+\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{X}_2^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\\ -\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\cdot\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{X}_2^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\end{array}}
e^{i2\pi\sqrt[\jmath_1]{\begin{array}{c}\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}+\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{Z}_2^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\\ -\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\cdot\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{Z}_2^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\end{array}}},\\
\sqrt[\jmath_2]{\begin{array}{c}\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}+\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{X}_2^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\\ -\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\cdot\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{X}_2^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\end{array}}
e^{i2\pi\sqrt[\jmath_2]{\begin{array}{c}\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}+\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{Z}_2^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\\ -\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\cdot\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{Z}_2^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\end{array}}}\\
\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_1^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_1}\cdot\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_2^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_1}
e^{i2\pi\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_1^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_1}\cdot\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_2^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_1}}
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{2}\left(1-\left((\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{2}\left(1-\left((\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}},\\
\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{2}\left(1-\left((\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{2}\left(1-\left((\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}},\\
\bigwedge_{i=1}^{2}\left(\sqrt[\jmath_2]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i} e^{i2\pi\bigwedge_{i=1}^{2}\left(\sqrt[\jmath_2]{1-\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}}
\end{array}\right].
$$

Hence,

$$
\bigvee_{i=1}^{l}\nu_i\mho_i^{\Xi} = \left[\begin{array}{c}
\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\, e^{i2\pi\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}},\\
\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\, e^{i2\pi\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}},\\
\bigwedge_{i=1}^{l}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i} e^{i2\pi\bigwedge_{i=1}^{l}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}}
\end{array}\right].
$$

If $n = l+1$, then $\bigvee_{i=1}^{l}\nu_i\mho_i^{\Xi} + \nu_{l+1}\mho_{l+1}^{\Xi} = \bigvee_{i=1}^{l+1}\nu_i\mho_i^{\Xi}$.

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Now, $\bigvee_{i=1}^{l}\nu_i\mho_i^{\Xi}+\nu_{l+1}\mho_{l+1}^{\Xi}=\nu_1\mho_1^{\Xi}\bigvee\nu_2\mho_2^{\Xi}\bigvee...\bigvee\nu_l\mho_l^{\Xi}\bigvee\nu_{l+1}\mho_{l+1}^{\Xi}$

$$
=\left[\begin{array}{c}
\sqrt[\jmath_1]{\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{\jmath_1}+\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{X}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\\-\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{\jmath_1}\cdot\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{X}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\end{array}}
e^{i2\pi\sqrt[\jmath_1]{\begin{array}{c}\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{\jmath_1}+\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{Z}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\\-\left(\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{\jmath_1}\cdot\left(\sqrt[\jmath_1]{1-\left(1-\left((\mathfrak{Z}_{l+1}^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_1}}\right)^{\jmath_1}\end{array}}},\\
\sqrt[\jmath_2]{\begin{array}{c}\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{\jmath_2}+\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{X}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2},\\-\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{\jmath_2}\cdot\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{X}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\end{array}}
e^{i2\pi\sqrt[\jmath_2]{\begin{array}{c}\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{\jmath_2}+\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{Z}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2},\\-\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l}\left(1-\left((\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{\jmath_2}\cdot\left(\sqrt[\jmath_2]{1-\left(1-\left((\mathfrak{Z}_{l+1}^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_1}}\right)^{\jmath_2}\end{array}}}\\
\bigwedge_{i=1}^{l}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}\cdot\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_{l+1}^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_1}
e^{i2\pi\bigwedge_{i=1}^{l}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}\cdot\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_{l+1}^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_1}}
\end{array}\right]
$$

$$
\bigvee_{i=1}^{l+1}\nu_i\mho_i^{\jmath_1}=\left[\begin{array}{c}
\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{X}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\,e^{i2\pi\sqrt[\jmath_1]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{Z}_1^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}},\\
\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{X}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\,e^{i2\pi\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{Z}_1^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}},\\
\bigwedge_{i=1}^{l+1}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}e^{i2\pi\bigwedge_{i=1}^{l+1}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}}
\end{array}\right].
$$

$\left(\bigvee_{i=1}^{l+1}\nu_i\mho_i^{\Xi}\right)^{1/\Xi}=$

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$$\left[\begin{array}{c}\left(\sqrt[\Xi]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{1/\jmath_1} e^{i2\pi\left(\sqrt[\Xi]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}\right)^{\nu_i}}\right)^{1/\jmath_1}},\\ \left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{1/\jmath_2} e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{l+1}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{1/\jmath_2}},\\ \sqrt[\jmath_3]{1-\left(1-\left(\bigwedge_{i=1}^{l+1}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}\right)^2\right)^{1/\jmath_3}}\\ e^{i2\pi\sqrt[\jmath_3]{1-\left(1-\left(\bigwedge_{i=1}^{l+1}\left(\sqrt[\jmath_3]{1-\left(1-(\mathfrak{z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}}\right)^{\nu_i}\right)^2\right)^{1/\jmath_3}}}\end{array}\right]$$

Remark 4.12. An operator modified from the G$(\jmath_1,\jmath_2,\jmath_3)$CNWA operator to the $(\jmath_1,\jmath_2,\jmath_3)$CNWA operator is performed if $\Xi=1$.

Theorem 4.13. *If all $\mho_i=\langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{z}_i^{\mathcal{F}}})\rangle$ and all are equal. Then G$(\jmath_1,\jmath_2,\jmath_3)$CNWA$(\mho_1,\mho_2,...,\mho_n)=\mho$.*

Proof. There is a proof based on the Theorem 4.3.

Remark 4.14. In the G$(\jmath_1,\jmath_2,\jmath_3)$CNWA operator, boundedness and monotonicity are satisfied.

Proof. There is a proof based on the Theorem 4.4 and Theorem 4.5.

4.4 Generalized $(\jmath_1,\jmath_2,\jmath_3)$CNWG(G$(\jmath_1,\jmath_2,\jmath_3)$CNWG)

Definition 4.15. A $\mho_i=\langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{z}_i^{\mathcal{F}}})\rangle$ be the $(\jmath_1,\jmath_2,\jmath_3)$CNNs. Then G$(\jmath_1,\jmath_2,\jmath_3)$CNWG$(\mho_1,\mho_2,...,\mho_n)=\frac{1}{\Xi}\left(\bigwedge_{i=1}^{n}(\Xi\mho_i)^{\nu_i}\right)$.

Theorem 4.16. *A $\mho_i=\langle(\mathfrak{X}_i^{\mathcal{T}}e^{i2\pi\mathfrak{z}_i^{\mathcal{T}}},\mathfrak{X}_i^{\mathcal{I}}e^{i2\pi\mathfrak{z}_i^{\mathcal{I}}},\mathfrak{X}_i^{\mathcal{F}}e^{i2\pi\mathfrak{z}_i^{\mathcal{F}}})\rangle$ be the $(\jmath_1,\jmath_2,\jmath_3)$CNNs. Then G$(\jmath_1,\jmath_2,\jmath_3)$CNWG$(\mho_1,\mho_2,...,\mho_n)$*

$$=\left[\begin{array}{c}\sqrt[\jmath_1]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[\jmath_1]{1-\left(1-(\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}}\right)^{\nu_i}\right)^{\jmath_1}\right)^{1/\jmath_1}}\\ e^{i2\pi\sqrt[\jmath_1]{1-\left(1-\left(\bigwedge_{i=1}^{n}\left(\sqrt[\jmath_1]{1-\left(1-(\mathfrak{z}_i^{\mathcal{T}})^{\jmath_1}\right)^{\jmath_1}}\right)^{\nu_i}\right)^{\jmath_1}\right)^{1/\jmath_1}}},\\ \left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{1/\jmath_2} e^{i2\pi\left(\sqrt[\jmath_2]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{z}_i^{\mathcal{I}})^{\jmath_2}\right)^{\jmath_2}\right)^{\nu_i}}\right)^{1/\jmath_2}},\\ \left(\sqrt[\jmath_3]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}\right)^{\nu_i}}\right)^{1/\jmath_3} e^{i2\pi\left(\sqrt[\jmath_3]{1-\bigwedge_{i=1}^{n}\left(1-\left((\mathfrak{z}_i^{\mathcal{F}})^{\jmath_3}\right)^{\jmath_3}\right)^{\nu_i}}\right)^{1/\jmath_3}}\end{array}\right].$$

Proof. As a conclusion, we can say that Theorem 4.11 is based on this proof.

Remark 4.17. There is a conversion that takes place when $\Xi=1$, which converts the G$(\jmath_1,\jmath_2,\jmath_3)$CNWGinto the $(\jmath_1,\jmath_2,\jmath_3)$CNWG.

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Remark 4.18. Boundness and monotonicity properties that are satisfied by G$(\jmath_1, \jmath_2, \jmath_3)$CNWG operators.

Proof. The following proof builds on Theorem 4.4 and Theorem 4.5.

Theorem 4.19. *If all* $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{F}}})\rangle$ *are equal. Then* *G*$(\jmath_1, \jmath_2, \jmath_3)$*CNWG*$(\mho_1, \mho_2, ..., \mho_n) = \mho$.

5 MADM approach based on $(\jmath_1, \jmath_2, \jmath_3)$CNN

L $\mho = \{\mho_1, \mho_2, ..., \mho_n\}$ be the set of n-alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the set of m-attributes, $w = \{w_1, w_2, ..., w_m\}$ be the weights of attributes, for $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$
$\mho_{ij} = \langle(\mathfrak{X}_{ij}^{\mathcal{T}} e^{i2\pi \mathfrak{Z}_{ij}^{\mathcal{T}}}, \mathfrak{X}_{ij}^{\mathcal{I}} e^{i2\pi \mathfrak{Z}_{ij}^{\mathcal{I}}}, \mathfrak{X}_{ij}^{\mathcal{F}} e^{i2\pi \mathfrak{Z}_{ij}^{\mathcal{F}}})\rangle$ denote $(\jmath_1, \jmath_2, \jmath_3)$CNN of alternative $\mho_i$ in attribute C_j. Since $\mathfrak{X}_i^{\mathcal{T}}, \mathfrak{X}_i^{\mathcal{I}}, \mathfrak{X}_i^{\mathcal{F}}, \mathfrak{Z}_i^{\mathcal{T}}, \mathfrak{Z}_i^{\mathcal{I}}, \mathfrak{Z}_i^{\mathcal{F}} \in [0,1]$ and $0 \leq (\mathfrak{X}_i^{\mathcal{T}})^{\jmath_1} + (\mathfrak{X}_i^{\mathcal{I}})^{\jmath_2} + (\mathfrak{X}_i^{\mathcal{F}})^{\jmath_3} \leq 1$ and $0 \leq (\mathfrak{Z}_i^{\mathcal{T}})^{\jmath_1} + (\mathfrak{Z}_i^{\mathcal{I}})^{\jmath_2} + (\mathfrak{Z}_i^{\mathcal{F}})^{\jmath_3} \leq 1$, where $\jmath_1, \jmath_2, \jmath_3$ are a positive integers. By combining n-alternatives with m-attributes, we get $\mathscr{D} = (\widehat{\mho}_{ij})_{n \times m}$. A decision is reached using the following algorithm.

5.1 Algorithm for $(\jmath_1, \jmath_2, \jmath_3)$CNN

Step-1: The decision values for $(\jmath_1, \jmath_2, \jmath_3)$CNN should be entered.
Step-2: The aggregate value of each alternative should be found. On the basis of $(\jmath_1, \jmath_2, \jmath_3)$CNN information aggregation operators, attribute C_j in $\widehat{\mho}_i$, $\mho_{ij} = \langle(\mathfrak{X}_{ij}^{\mathcal{T}} e^{i2\pi \mathfrak{Z}_{ij}^{\mathcal{T}}}, \mathfrak{X}_{ij}^{\mathcal{I}} e^{i2\pi \mathfrak{Z}_{ij}^{\mathcal{I}}}, \mathfrak{X}_{ij}^{\mathcal{F}} e^{i2\pi \mathfrak{Z}_{ij}^{\mathcal{F}}})\rangle$ is aggregated into $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{F}}})\rangle$.
Step-3: For any CNIVN $\mho_i = \langle(\mathfrak{X}_i^{\mathcal{T}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{T}}}, \mathfrak{X}_i^{\mathcal{I}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{I}}}, \mathfrak{X}_i^{\mathcal{F}} e^{i2\pi \mathfrak{Z}_i^{\mathcal{F}}})\rangle$, the score function of $\mho$ $S(\mho) = \frac{C+D}{2}$, where $C = (\mathfrak{X}^T)^2 - (\mathfrak{X}^I)^2 + 1 - (\mathfrak{X}^F)^2$ and $D = (\mathfrak{Z}^T)^2 - (\mathfrak{Z}^I)^2 - (\mathfrak{Z}^F)^2$, where $S(\mho) \in [-1, 1]$.
The accuracy function $H(\mho) = \frac{C_1 + D_1}{2}$, where $C_1 = (\mathfrak{X}^T)^2 + (\mathfrak{X}^I)^2 + (\mathfrak{X}^F)^2$ and $D_1 = (\mathfrak{Z}^T)^2 + (\mathfrak{Z}^I)^2 + (\mathfrak{Z}^F)^2$, where $H(\mho) \in [0, 1]$.
Step-4: Based on the output, we can determine that a value of $\max S_i$ is optimal, and therefore we should choose that value as our optimal solution.

5.2 Real life applications

The process of decision making can be applied to a variety of fields, such as selecting the best washing machine, laptop, engineer, or two-wheel motorbike, as well as deciding on a college for education. The evaluation of teacher education is carried out in the selection of colleges for undergoing teacher education. Experts use a variety of standards. A variety of studies have been conducted that examine the reasons why parents choose a particular college based on their fears and aspirations for their college student. In terms of parental decision making, we identify the following factors: The academic factor can be further divided into five components, namely the campus environment, overall costs, academic quality, student-faculty relationships, and career development. Using experts' assessments against the criteria, we want to select the best alternative out of a wide range of alternatives. Choosing a college education that meets a child's needs is an important decision for a parent. There will be five colleges chosen here $\mho = \{\mho_a, \mho_b, \mho_c, \mho_d, \mho_e\}$ that are nominated. The score of the college education evaluated by the experts is represented by $\{C_1$: campus environment, C_2: overall cost, C_3 : academic quality, C_4 : student/faculty relationship$\}$ and their corresponding weights are $w = \{0.4, 0.3, 0.2, 0.1\}$. A list of criteria is used to assess experts in order to determine which option is best. Several factors are considered when making a decision:

	C_1	C_2
$\mho_a$	$(0.45e^{i2\pi(0.35)}, 0.65e^{i2\pi(0.55)}, 0.55e^{i2\pi(0.5)})$	$(0.6e^{i2\pi(0.35)}, 0.6e^{i2\pi(0.55)}, 0.65e^{i2\pi(0.7)})$
$\mho_b$	$(0.5e^{i2\pi(0.3)}, 0.55e^{i2\pi(0.6)}, 0.65e^{i2\pi(0.7)})$	$(0.35e^{i2\pi(0.3)}, 0.75e^{i2\pi(0.45)}, 0.65e^{i2\pi(0.55)})$
$\mho_c$	$(0.65e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.55)}, 0.65e^{i2\pi(0.8)})$	$(0.45e^{i2\pi(0.25)}, 0.7e^{i2\pi(0.55)}, 0.6e^{i2\pi(0.5)})$
$\mho_d$	$(0.55e^{i2\pi(0.45)}, 0.55e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.55)})$	$(0.5e^{i2\pi(0.4)}, 0.55e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.65)})$
$\mho_e$	$(0.45e^{i2\pi(0.5)}, 0.45e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.6)})$	$(0.55e^{i2\pi(0.45)}, 0.65e^{i2\pi(0.5)}, 0.7e^{i2\pi(0.55)})$

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	C_3	C_4
$\mho_a$	$(0.45e^{i2\pi(0.4)}, 0.55e^{i2\pi(0.55)}, 0.6e^{i2\pi(0.6)})$	$(0.45e^{i2\pi(0.45)}, 0.65e^{i2\pi(0.45)}, 0.55e^{i2\pi(0.55)})$
$\mho_b$	$(0.4e^{i2\pi(0.45)}, 0.6e^{i2\pi(0.45)}, 0.55e^{i2\pi(0.55)})$	$(0.35e^{i2\pi(0.5)}, 0.45e^{i2\pi(0.55)}, 0.7e^{i2\pi(0.45)})$
$\mho_c$	$(0.35e^{i2\pi(0.5)}, 0.55e^{i2\pi(0.55)}, 0.45e^{i2\pi(0.45)})$	$(0.4e^{i2\pi(0.225)}, 0.5e^{i2\pi(0.75)}, 0.6e^{i2\pi(0.35)})$
$\mho_d$	$(0.45e^{i2\pi(0.35)}, 0.65e^{i2\pi(0.6)}, 0.55e^{i2\pi(0.55)})$	$(0.35e^{i2\pi(0.55)}, 0.55e^{i2\pi(0.35)}, 0.75e^{i2\pi(0.6)})$
$\mho_e$	$(0.5e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.7)}, 0.65e^{i2\pi(0.5)})$	$(0.5e^{i2\pi(0.45)}, 0.6e^{i2\pi(0.45)}, 0.8e^{i2\pi(0.55)})$

Aggregate information with $(\jmath_1, \jmath_2, \jmath_3)$CNWA operators are as follows:

	$(\jmath_1, \jmath_2, \jmath_3)$CNWA operator $(1, 1, 1)$
$\widehat{\mho_a}$	$0.5001e^{i2\pi(0.3709)}), 0.6169e^{i2\pi(0.5409)}), 0.5884e^{i2\pi(0.5791)})$
$\widehat{\mho_b}$	$0.4241e^{i2\pi(0.3550)}), 0.6241e^{i2\pi(0.5254)}), 0.6333e^{i2\pi(0.5937)})$
$\widehat{\mho_c}$	$0.5212e^{i2\pi(0.3675)}), 0.6159e^{i2\pi(0.5757)}), 0.5849e^{i2\pi(0.5701)})$
$\widehat{\mho_d}$	$0.4984e^{i2\pi(0.4279)}), 0.5721e^{i2\pi(0.5091)}), 0.5709e^{i2\pi(0.5833)})$
$\widehat{\mho_e}$	$0.4967e^{i2\pi(0.4613)}), 0.5436e^{i2\pi(0.5832)}), 0.611e^{i2\pi(0.5587)})$

The score values asre

S_1	S_2	S_3	S_4	S_5
0.0165	-0.057	0.0144	0.0895	0.0693

The following alternative rankings are provided:

$$\mho_d > \mho_e > \mho_a > \mho_c > \mho_b.$$

Consequently, $\mho_d$ is the best college.

5.3 Analysis and discussion

As a result of the above information, we propose to apply the following strategies: CNWA, CNWG, CGNWA and CGNWGbased on score values and accuracy values. Distances can be categorized as follows:

$(\jmath_1 = 1, \jmath_2 = 1, \jmath_3 = 1)$	$CNWA$	$CNWG$	$CGNWA$	$CGNWG$
$Score-\ values$ $(proposed)$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$
$Accuracy-\ values$ $(proposed)$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$	$\mho_d > \mho_e > \mho_a$ $\mho_c > \mho_b$

The Figure-1 shows that score values for the CNWA operator.

The Figure-2 shows that accuracy values for the CNWA operator.

Change the $(\jmath_1, \jmath_2, \jmath_3)$ values from CNWA approach. As a result, we obtain the following closeness values and orders:

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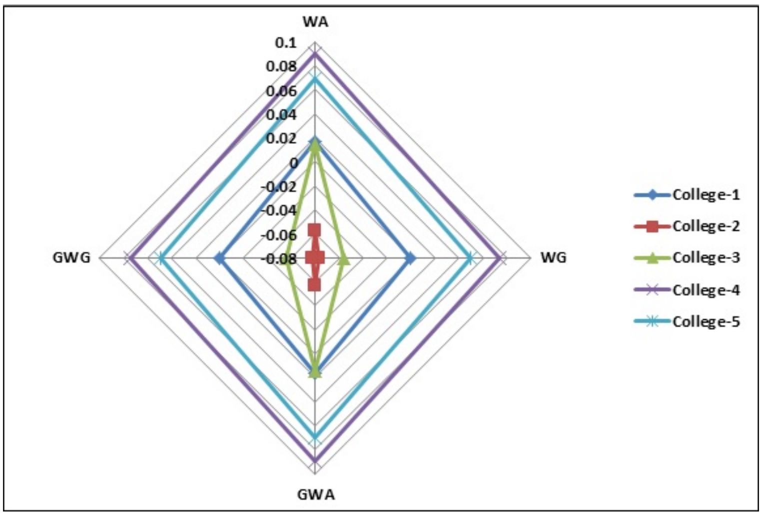

Figure-1. Score values for CNWA operator.

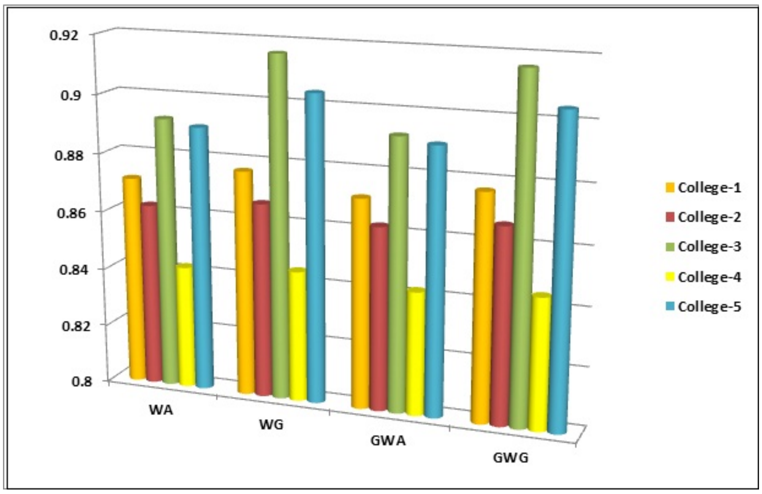

Figure-2. Accuracy values for CNWA operator.

	Relative closeness values					
$(\jmath_1, \jmath_2, \jmath_3)$	S_1	S_2	S_3	S_4	S_5	$Order$
$(1, 1, 1)$	0.0165	-0.057	0.0144	0.0895	0.0693	$\mho_d > \mho_e > \mho_a > \mho_c > \mho_b$
$(1, 2, 1)$	0.0157	-0.061	0.0116	0.0875	0.0646	$\mho_d > \mho_e > \mho_a > \mho_c > \mho_b$
$(1, 1, 2)$	0.2411	0.1776	0.2367	0.3115	0.2936	$\mho_d > \mho_e > \mho_a > \mho_c > \mho_b$
$(2, 1, 2)$	0.2432	0.1816	0.2452	0.3138	0.2947	$\mho_d > \mho_e > \mho_c > \mho_a > \mho_b$

Therefore, $\mho_a$ should be changed to $\mho_c$ as the optimal alternative. As with the NWG operator, GNWA operator and GNWG operator, alternative rankings are determined according to $(\jmath_1, \jmath_2, \jmath_3)$.

6 Conclusion:

The aim of this study was to establish score values and accuracy values for $(\jmath_1, \jmath_2, \jmath_3)$CNSs, which have the advantage of being mathematically simple. Utilizing appropriate methods results in superior score values and accuracy values. For $(\jmath_1, \jmath_2, \jmath_3)$CNWA, $(\jmath_1, \jmath_2, \jmath_3)$CNWG, G$(\jmath_1, \jmath_2, \jmath_3)$CNWA and G$(\jmath_1, \jmath_2, \jmath_3)$CNWG, we have proposed improved AO rules. A few algebraic operations and aspects have also been discussed to create these operators. This article provides an excellent foundation for future research in this field, which is still in its very early stages of development. The ideas presented here will be helpful to future academics who have an interest in this field. The following topics will be discussed in more detail:

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(1) There are three types of normal vague set, normal vague spherical set, and normal vague NS using complex interaction aggregation operators.
(2) Complex cubic NWAs, NWGs, GNWAs and GNWGs may be used to solve the problem.

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