Stable Neutrosophic Crisp Topological Space

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Abstract

The significance and influence of neutrosophic crisp set theory in numerous scientific domains, particularly topology, led us to construct a new definition of topology based on neutrosophic crisp sets, allowing us to regulate its key mathematical ideas. Therefore, we constructed this definition and dubbed it stable neutrosophic crisp topology, where we went over the key idea which is the interior with the vital features.

Keywords: Neutrosophic crisp set; stable neutrosophic topological space; stable neutrosophic crisp open set; stable interior neutrosophic.

1. Introduction

Neutrosophic Science was first introduced by Smarandache[1,2]. It provided the groundwork for new mathematical theories based on principles., Smarandache (Salama) present the concept of neutrosophic sets[9,17,18]. A number of researchers who studied and researched in this field[10 − 16], for example, Al-Swidi, L.A.A. conferment of a fresh conceptualization of sets called neutrosophic axial sets[3,4] and Al-Obaidi, A. H. gave fresh insights into the concepts of crisp open functions and crisp closed functions that are weakly neutrosophic[5 − 8]. Many researchers provided the ideas behind the neutrosophic set. in numerous publications to provide a natural basis for developing new areas of neutrosophic mathematics and dealing mathematically with large scale phenomena of our daily lives such as a fuzzy set that is intuitive.

Salama and Florentin (2012) defined three distinct kinds of the neutrosophic crisp set. The first type requires the condition ($K_1 \cap K_2 = \emptyset$, $K_1 \cap K_3 = \emptyset$, $K_2 \cap K_3 = \emptyset$), to which we give the symbol ($NC_1 - set$), while the second type, symbolized ($NC_2 - set$), requires the condition ($K_1 \cap K_2 = \emptyset$, $K_1 \cap K_3 = \emptyset$, $K_2 \cap K_3 = \emptyset \; and \; K_1 \cup K_2 \cup K_3 = X$), and the third type ($NC_3 - set$) requires the condition ($K_1 \cap K_2 \cap K_3 = \emptyset \; and \; K_1 \cup K_2 \cup K_3 = X$). Moreover, through their research they defined types of empty sets, which are: ($\emptyset_1^N =< \emptyset, \emptyset, X >$,$\emptyset_2^N =< \emptyset, X, \emptyset >$,$\emptyset_3^N =< \emptyset, X, X >$,$\emptyset_4^N =< \emptyset, \emptyset, \emptyset >$) and synonymously, they defined types of comprehensive sets and their forms, these are: ($X_1^N =< X, \emptyset, \emptyset >$, $X_2^N =< X, X, \emptyset >$,$X_3^N =< X, \emptyset, X >$,$X_4^N =< X, X, X >$). As for the complement of any neutrosophic crisp set, there are three types: $(K^N)^{C1} =< K_1^C, K_2^C, K_3^C >$, $(K^N)^{C2} =< K_3, K_2, K_1 >$,$(C^N)^{C3} =< K_3, K_2^C, K_1 >$), meanwhile, we defined equality ($=_1$ and $=_2$) on the basis of partial subsets. Hence, Salama and Florentin defined the first and second subsets as follows: (($\mathbb{L}^N \subseteq_1 K^N \leftrightarrow \mathbb{L}_1 \subseteq K_1, \mathbb{L}_2 \subseteq K_2, \mathbb{L}_3 \supseteq K_3$),($\mathbb{L}^N \subseteq_2 K^N \leftrightarrow \mathbb{L}_1 \subseteq K_1, \mathbb{L}_2 \supseteq K_2, \mathbb{L}_3 \supseteq K_3$)), while the union and intersection were defined as follows:($\mathbb{L}^N \cup_1 K^N =< \mathbb{L}_1 \cup K_1, \mathbb{L}_2 \cup K_2, \mathbb{L}_3 \cap K_3 >$,$L^N \cup_2 K^N =< \mathbb{L}_1 \cup K_1, \mathbb{L}_2 \cap K_2, \mathbb{L}_3 \cap K_3 >$, $\mathbb{L}^N \cap_1 K^N =< \mathbb{L}_1 \cap K_1, \mathbb{L}_2 \cap K_2, \mathbb{L}_3 \cup K_3 >$, $\mathbb{L}^N \cap_2 K^N =< \mathbb{L}_1 \cap K_1, \mathbb{L}_2 \cup K_2, \mathbb{L}_3 \cup K_3 >$).

Finally, they defined three types of points that are: ($P^{N1} =< \{P_1\}, \{P_2\}, \{P_3\} >, \{P_1\} \neq \{P_2\} \neq \{P_3\}$ where $P_1, P_2, P_3 \in X$,$P^{N2} =< \{P\}, \emptyset, \{P\}^C >$,$P^{N3} =< \emptyset, \{P\}, \{P\}^C >$), but they do not cover a complete area in relation to space. Therefore, we decided to add two additional definitions, namely $P^{N4} =< \{P\}, \emptyset, \emptyset >$,$P^{N5} =< \emptyset, \{P\}, \emptyset >$, $\{P\}$,is singleton [12] and we defined an additional sixth point as follows: ($P^{N6} =< \mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3 >, \mathbb{A}_i \neq \emptyset, i$=1 or 2 or 3). As for the belonging of points to sets, they were used as follows:(($P^{N1} \in_1 \mathbb{L}^N \leftrightarrow \{P_1\} \subseteq \mathbb{L}_1, \{P_2\} \subseteq \mathbb{L}_2, \{P_3\} \supseteq \mathbb{L}_3$ or $P^{N1} \in'_1 \mathbb{L}^N \leftrightarrow \{P_1\} \subseteq \mathbb{L}_1, \{P_2\} \supseteq \mathbb{L}_2, \{P_3\} \supseteq$

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$\mathbb{L}_3, P^{N2} \in_2 \mathbb{L}^N \leftrightarrow p \in \mathbb{L}_1, P^{N3} \in_3 \mathbb{L}^N \leftrightarrow p \in \mathbb{L}_2, P^{N4} \in_4 \mathbb{L}^N \leftrightarrow p \in \mathbb{L}_1, P^{N3} \in_3 \mathbb{L}^N \leftrightarrow p \in \mathbb{L}_1, P^{N6} \in_6 \mathbb{L}^N \leftrightarrow \mathbb{A}_1 \subseteq \mathbb{L}_1, \mathbb{A}_2 \subseteq \mathbb{L}_2, \mathbb{A}_3 \supseteq \mathbb{L}_3$).

A new type of topological space is defined based on neutrosophic crisp sets depending on an intersection of the first type and a union of the second type. Thus, the belonging of sets to families will be normal, and equality (=) is also considered a normal equality.

2. Stable Neutrosophic Crisp Topological Space (SNCT-space)

Definition 2.1: Let X be a fixed set that is not empty, a (SNCT-space) is a family $\wr$ satisfies the following condition:

1. $\emptyset_1^N, X_1^N \in \wr$
2. $\forall \mathbb{A}^N, \mathbb{B}^N \in \wr, \exists\ \mathbb{K}^N \in \wr, \ni\ \mathbb{K}^N \subseteq_1 \mathbb{A}^N \cap_1 \mathbb{B}^N$
3. $\forall\ \mathbb{A}_i^N \in \wr, \exists\ \mathbb{F}^N \in \wr\ \ni\ \mathbb{F}^N \subseteq_1 \cup_2 \mathbb{A}_i^N \quad i = 1,2, \dots, n$

Then$(X, \wr)$is a (SNCT-space). For any $\mathbb{A}^N \in \wr$ is a stable neutrosophic crisp open set and its denoted by $(SNCO - set)$, the complement of type 2 for $(SNCO - set)$ is stable neutrosophic crisp closed set and denoted by $(SNCC - set)$.

Example 2.2: Let X={$\mathbb{a}$, $\mathbb{b}$, $\mathbb{c}$} be a nonempty fixed set , let $\mathbb{A}^N, \mathbb{B}^N, \mathbb{C}^N, \mathbb{D}^N, \mathbb{E}^N, \mathbb{F}^N$are a $NC - sets$, such that:

$\mathbb{A}^N =< \{\mathbb{a}\}, \{\mathbb{b}, \mathbb{c}, \mathbb{d}\}, \{\mathbb{e}\} >$ $\qquad \mathbb{D}^N =< \{\mathbb{a}\}, \emptyset, \emptyset >$

$\mathbb{B}^N =< \{\mathbb{a}\}, \{\mathbb{e}, \mathbb{f}\}, \{\mathbb{c}\} >$ $\qquad \mathbb{E}^N =< \{\mathbb{a}\}, \emptyset, \{\mathbb{e}\} >$

$\mathbb{C}^N =< \{\mathbb{a}\}, \emptyset, \{\mathbb{e}, \mathbb{c}, \mathbb{d}\} >$ $\qquad \mathbb{F}^N =< \{\mathbb{a}\}, \emptyset, \{\mathbb{c}\} >$

$\wr = \{\mathbb{A}^N, \mathbb{B}^N, \mathbb{C}^N, \mathbb{D}^N, \mathbb{E}^N, \mathbb{F}^N, \emptyset_1^N, X_1^N\}$

Then $(X, \wr)$ is (SNCT-space).

Example 2.3: Consider X={$\mathbb{m}$, $\mathbb{mm}$, $\mathbb{k}$, $\mathbb{d}$, $\mathbb{h}$} be a nonempty fixed set, let $\mathbb{A}^N, \mathbb{B}^N, \mathbb{C}^N, \mathbb{D}^N, \mathbb{E}^N, \mathbb{F}^N, \mathbb{G}^N, \mathbb{H}^N$ are a $NC - sets$, such that:

$\mathbb{A}^N =< \{\mathbb{m}\}, \{\mathbb{mm}, \mathbb{k}\}, \{\mathbb{h}\} >$ $\qquad \mathbb{D}^N =< \{\mathbb{m}\}, \{\mathbb{mm}\}, \{\mathbb{h}, \mathbb{k}\} >$

$\mathbb{B}^N =< \{\mathbb{m}\}, \{\mathbb{mm}, \mathbb{d}, \mathbb{h}\}, \{\mathbb{k}\} >$ $\qquad \mathbb{E}^N =< \{\mathbb{m}\}, \emptyset, \{\mathbb{k}\} >$

$\mathbb{C}^N =< \{\mathbb{m}\}, \emptyset, \emptyset >$ $\qquad \mathbb{F}^N =< \{\mathbb{m}\}, \emptyset, \{\mathbb{h}\} >$

$\mathbb{G}^N =< \{\mathbb{m}\}, \{\mathbb{mm}\}, \{\mathbb{k}\} >$ $\qquad \mathbb{H}^N =< \{\mathbb{m}\}, \emptyset, \{\mathbb{h}, \mathbb{k}\} >$

$\mathbb{S} = \{\mathbb{A}^N, \mathbb{B}^N, \mathbb{C}^N, \mathbb{D}^N, \mathbb{E}^N, \mathbb{F}^N, \mathbb{G}^N, \mathbb{H}^N, \emptyset_1^N, X_1^N\}$

Then $(X, \wr)$ is (SNCT-space).

We will illustrate the definition of subset relation of both kinds and the inteior points of any NC-set with respect to the (SNCT-space), which depends on the union of both types. The properties that are accomplished and those that are not, which are equivalent to the properties of the internal points, will also be covered, along with all the properties in connection to the first and second equality. Through examples, we shall show that in classical topological spaces. For binary operations $(\cap, \cup, =, \subseteq, \in)$, when the index is not mentioned, it refer to the classical relation.

Definition 2.4: Let$(X, \wr)$ be a (SNCT-space), $\mathbb{A}^N$is a NC- set, then the stable interior of $\mathbb{A}^N$ denoted by $Si_{ij}(\mathbb{A}^N)$ and define as: $Si_{ij}(\mathbb{A}^N) = \cup_i \left\{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_j \mathbb{A}^N\right\}$ $\quad i, j = 1,2$. It can be noted that the index i is an indication of the type of union and the index j is an indication of the type of the subsets.

Example 2.5: In example (2.3) Let $\mathbb{Z}^N =< \{\mathbb{m}\}, \{\mathbb{mm}\}, \{\mathbb{h}\} >$

$Si_{11}(\mathbb{Z}^N) = \cup_1 \{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_1 \mathbb{Z}^N\} =< \{\mathbb{m}\}, \{\mathbb{mm}\}, \{\mathbb{h}\} >$

$Si_{12}(\mathbb{Z}^N) = \cup_1 \{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_2 \mathbb{Z}^N\} =< \{\mathbb{m}\}, \{\mathbb{mm}, \mathbb{k}\}, \{\mathbb{h}\} >$

$Si_{22}(\mathbb{Z}^N) = \cup_2 \{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_2 \mathbb{Z}^N\} =< \{\mathbb{m}\}, \{\mathbb{mm}\}, \{\mathbb{h}\} >$

$Si_{21}(\mathbb{Z}^N) = \cup_2 \{\mathbb{S}^N \in \wr, \mathbb{S}^N \subseteq_1 \mathbb{Z}^N\} =< \{\mathbb{m}\}, \emptyset, \{\mathbb{h}\} >$

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According to Definition (2.4), there are multiple properties of stable interior and their proof is simple and direct, relying on the properties of NC-sets and some simple mathematical techniques. Therefore, the researchers will prove some of them.

Preposition 2.6: Let (X, $\mathcal{Z}$) be a SNCT-space, $\mathbb{L}^N, \mathbb{K}^N are\ a\ NC-set$ of any type, Then the properties hold for i=1,2:

a) $Si_{21}(\emptyset_1{}^N) =_i \emptyset_1{}^N, Si_{21}\left(X_1{}^N\right) =_i X_1{}^N$
b) $Si_{21}(\mathbb{K}^N) \subseteq_1 \mathbb{K}^N$
c) $\mathbb{L}^N \subseteq_i \mathbb{K}^N, Then\ Si_{21}(\mathbb{L}^N) \subseteq_i Si_{21}(\mathbb{K}^N)$
d) $Si_{21}(\mathbb{L}^N \cap_i \mathbb{K}^N) =_i Si_{21}(\mathbb{L}^N) \cap_i Si_{21}(\mathbb{K}^N)$
e) $Si_{21}(\mathbb{L}^N) \cup_i Si_{21}(\mathbb{K}^N) \subseteq_i Si_{21}(\mathbb{L}^N \cup_i \mathbb{K}^N)$

Proof: We will prove this part in light of the NC- point $p^{N1}$and belonging of first type ($\in_1$) and first type of subset ($\subseteq_1$).

b) Let $\mathbb{K}^N =< \mathbb{K}_1, \mathbb{K}_2, \mathbb{K}_3 >$ be a NC- set of any type, and $p^{N1} =< \{p_1\}, \{p_2\}, \{p_3\} >$ be a NC- point

Let $p^{N1} \in_1\ Si_{21}(\mathbb{K}^N)$ →there exist a $SNCO-set$ $\mathfrak{l}^N =< \mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3 >$, such that $p^{N1} \in_1\ \mathfrak{l}^N \subseteq_1\ \mathbb{K}^N$

Which is mean $\{p_1\} \subseteq \mathfrak{l}_1 \subseteq \mathbb{K}_1, \{p_2\} \subseteq \mathfrak{l}_1 \subseteq \mathbb{K}_1, \{p_3\} \supseteq \mathfrak{l}_1 \supseteq \mathbb{K}_1. \{p_1\} \subseteq \mathbb{K}_1, \{p_2\} \subseteq \mathbb{K}_1, \{p_3\} \supseteq \mathbb{K}_1$

So we get $p^{N1} \in_1 \mathbb{K}^N$. Thus $Si_{21}(\mathbb{K}^N) \subseteq_1 \mathbb{K}^N$

c) We will prove this part in light of the NC- point $(p^{N1}, \in_1, \subseteq_1)$and $(p^{N2}, \in_2, \subseteq_2)$

Let$\mathbb{K}^N =< \mathbb{K}_1, \mathbb{K}_2, \mathbb{K}_3 >$, $\mathbb{L}.^N =< \mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3 >$ are a NC-set of any type,

Let $p^{N1} \in_1 Si_{21}(L^N), p^{N1} =< \{p_1\}, \{p_2\}, \{p_3\} >$

$\rightarrow there\ exist\ \mathfrak{l}^N =< \mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3 >\in \mathcal{Z}, such\ that\ p^{N1} \in_1\ \mathfrak{l}^N \subseteq_1\ \mathbb{L}^N$

So $\{p_1\} \subseteq \mathbb{L}_{.1} \subseteq \mathbb{K}_{.1}, \{p_2\} \subseteq \mathbb{L}_{.2} \subseteq \mathbb{K}_{.2}, \{p_3\} \supseteq \mathbb{L}_1 \supseteq \mathbb{K}_1$

$\rightarrow\ \{p_{.1}\} \subseteq \mathbb{K}_1, \{p_{.2}\} \subseteq \mathbb{K}_2, \{p_3\} \supseteq \mathbb{K}_1$

$\rightarrow p^{N1} \in_1 Si_{21}(\mathbb{K}^N)$

If we take $p^{N2} =<, \{p\}, \emptyset, \{p\}^C >$ be NC- point

Now let $p^{N2} \in_2 Si_{21}(\mathbb{L}^N)$

there exist $SNCO-set$, $\mathfrak{l}^N =< \mathfrak{l}_1, \mathfrak{l}_2, \mathfrak{l}_3 >$ such that $p^{N2} \in_2\ \mathfrak{l}^N \subseteq_2\ \mathbb{L}^N \subseteq_2 \mathbb{K}^N$,

This mean $p \in \mathfrak{l}_1 \subseteq \mathbb{K}_1 \rightarrow p \in \mathbb{K}_1$, So we get $p^{N2} \in_2 Si_{21}(\mathbb{K}^N)$

Thus $Si_{21}(\mathbb{L}^N) \subseteq_2 Si_{21}(\ \mathbb{K}^N)$.

Remark 2.7: If $\mathbb{A}^N$ is a $SNCO-set$, then not necessary the following:

i.$Si_{21}\ (\mathbb{A}^N) =_i \mathbb{A}^N \quad i = 1,2$

In example 2.3 $Si_{21}(\mathbb{D}^N) =_i Si_{21}(< \{\mathbb{m}\}, \{\mathbb{m}\}, \{\mathbb{h}, \mathbb{k}\} >) =_i < \{\mathbb{m}\}, \emptyset, \{\mathbb{h}, \mathbb{k}\} >$ this show that

$D^N$is $SNCO-set$ but $Si_{21}(\mathbb{D}^N) \neq_i \mathbb{D}^N$ for $i = 1$ or 2

ii. $Si_{21}\left(Si_{21}(\mathbb{A}^N)\right) =_i Si_{21}(\mathbb{A}^N)$

Remark 2.8: a) All the parts of the preposition (2.6) as well approved with respect to the definition of interior's dependence on the union of the first type and the subset relation of the first type.

$(Si_{11}\ (\mathbb{A}^N) =\cup_1 \{\mathbb{S}^N \in\ \mathcal{Z}, \mathbb{S}^N \subseteq_1 \mathbb{A}^N\})$. In this case, we also found that if $\mathbb{A}\ \ is\ SNCO-set,$ then $Si_{11}(\mathbb{A}^N) =_i \mathbb{A}^N$ and $Si_{11}\left(Si_{11}(\mathbb{A}^N)\right) =_i Si_{11}(\mathbb{A}^N)$

b) Only p^{N1}, p^{N2}, and p^{N4} are satisfied when the definition of the stable interior depends on the second type's subset relation. p^{N3} and p^{N5} have not been validated.

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c) All parts of the preposition (2.6) is confirmed with the points that operate with the fractional of the second type when the definition of interior relies on the union of the second kind and the fractional of the second type(Si_{22} $(\mathbb{A}^N) =\cup_2 \{\mathbb{S}^N \in \mathcal{Z}, \mathbb{S}^N \subseteq_2 \mathbb{A}^N\}$). Additionally ,we also found that if $\mathbb{A}^N$ is $SNCO-set$,then $Si_{22}(\mathbb{A}^N) =_i \mathbb{A}^N$ and $Si_{22}(Si_{22}(\mathbb{A}^N)) =_i Si_{22}(\mathbb{A}^N)$.

d) All parts of the preposition (2.6) are approved with the points that operate with the fractional of the second kind when the definition of interior relies on the union of the first kind and the fractional of the second kind

$$(Si_{12} (\mathbb{A}^N) =\cup_1 \{\mathbb{S}^N \in \mathcal{Z}, \mathbb{S}^N \subseteq_2 \mathbb{A}^N\})$$

This remark is directed to all the following theorems via their proof.

Preposition 2.9: Let$(X,\mathcal{Z})$ be SNCT-space , $\mathbb{L}^N, \mathbb{K}^N$are NC- sets, then the following two conditions are equivalent

a) $Si_{ij}((\mathbb{L}^N)^{C_2} \cup_k \mathbb{K}^N) =_k Si_{ij}((\mathbb{L}^N)^{C_2}) \cup_k Si_{ij}(\mathbb{K}^N) \quad i,j,k = 1,2$

b) $(Si_{ij}((\mathbb{L}^N)^{C_2}))^{C_2} \cap_k \left(Si_{ij}(\mathbb{K}^N)\right)^{C_2} =_k (Si_{ij}((\mathbb{L}^N \cap_k (\mathbb{K}^N)^{C_2})^{C_2}))^{C_2} \quad i,j,k = 1,2$

Proof: we will prove this preposition for $i = 2, j = 1$,and $k = 1$ .First, we assume that $\mathbb{L}^N$ and $\mathbb{K}^N$ satisfy the condition (a), $(Si_{21}((\mathbb{L}^N)^{C_2})^{C_2} \cap_1 \left(Si_{21}(\mathbb{K}^N)\right)^{C_2} =_1 (Si_{21}((\mathbb{L}^N)^{C_2}) \cup_2 Si_{21}(\mathbb{K}^N))^{C_2}$

$$=_1 (Si_{21}((\mathbb{L}^N)^{C_2} \cup_2 \mathbb{K}^N))^{C_2} =_1 (Si_{21}((\mathbb{L}^N \cap_1 (\mathbb{K}^N)^{C_2})^{C_2}))^{C_2}$$

Now suppose that $\mathbb{L}^N$ and$\mathbb{K}^N$ satisfy the condition (b)

$$Si_{21}((\mathbb{L}^N)^{C_2} \cup_1 \mathbb{K}^N) =_1 \left(Si_{21}\left(\left(\mathbb{L}^N \cap_2 (\mathbb{K}^N)^{C_2}\right)^{C_2}\right)\right)^{{C_2}^{C_2}}$$

$$=_1 \left(\left(Si_{21}((\mathbb{L}^N)^{C_2})\right)^{C_2} \cap_2 \left(Si_{21}(\mathbb{K}^N)\right)^{C_2}\right)^{C_2}$$

$$=_1 Si_1((\mathbb{L}^N)^{C_2}) \cup_1 Si_{21}(\mathbb{K}^N)$$

Preposition 2.10: Let$(X,\mathcal{Z})$ be SNCT-space, $\mathbb{L}^N$ be NC- set,

then $Si_{ij}((\mathbb{L}^N)^{C_2}) =_k {\emptyset_1}^N$ if $\forall \mathbb{K}^N \in \mathcal{Z} \ni \mathbb{L}^N \cap_i \mathbb{K}^N \neq_k {\emptyset_1}^N$ for $i,j,k = 1,2$

Proof: we will prove this preposition in light of the first type of equal and the definition of $(Si_{11}(L^N))$

Let $Si_{11}((\mathbb{L}^N)^{C_2}) =_1 {\emptyset_1}^N$ if $\exists {\emptyset_1}^N \neq \mathbb{K}^N \in \mathcal{Z} \ni \mathbb{L}^N \cap_1 \mathbb{K}^N =_1 {\emptyset_1}^N \to \mathbb{K}^N \subseteq_1 (\mathbb{L}^N)^{C_2}$

and we have $Si_{11}(\mathbb{K}^N) = \mathbb{K}^N \subseteq_1 Si_{11}((\mathbb{L}^N)^{C_2}) =_1 {\emptyset_1}^N$

We get $\mathbb{K}^N =_1 {\emptyset_1}^N$which is contradiction

Conversely, if possible $Si_{11}((\mathbb{L}^N)^{C_2}) \neq_1 {\emptyset_1}^N$,This mean $\exists p^{Nk} \in_k Si_{11}((\mathbb{L}^N)^{C_2})$ for k=1…6,

and $\mathbb{K}^N$is $(SNCO-set)$ containing $p^{Nk} \ni \mathbb{K}^N \subseteq_1 (\mathbb{L}^N)^{C_2}$,Thus $\mathbb{L}^N \cap_i \mathbb{K}^N =_1 {\emptyset_1}^N$ which is contradiction.

Preposition 2.11: Let $\mathbb{L}^N$ be NC- set of SNCT-space$(X,\mathcal{Z})$, p^{Nk} is NC- point,

then $p^{Nk} \notin_k Si_{ij}((\mathbb{L}^N)^{C_2})$ iff $\forall \mathbb{K}^N \in \mathcal{Z} \ni p^{Nk} \notin_k \mathbb{K}^N$and $\mathbb{L}^N \cap_i \mathbb{K}^N =_i {\emptyset_1}^N$ for $i,j = 1,2$ and $k = 1 \ldots 6$

Proof: we will prove this preposition in light of the second type of equal and the definition of $(Si_{22}(L^N))$and the point p^{N4}

Let $p^{N4} \notin_4 Si_{22}((\mathbb{L}^N)^{C_2}) =_2 \cup_2 \{\mathbb{K}^N \in \mathcal{Z}, \mathbb{K}^N \subseteq_2 (\mathbb{L}^N)^{C_2}\}$

If and only if $p^{N4} \notin_4 \cup_2 \{\mathbb{K}^N \in \mathcal{Z}, \mathbb{K}^N \cap_2 \mathbb{L}^N =_2 {\emptyset_1}^N\}$

If and only if $p^{N4} \notin_4 \mathbb{K}^N, \forall \mathbb{K}^N \in \mathcal{Z}, \mathbb{K}^N \cap_2 \mathbb{L}^N =_2 {\emptyset_1}^N$.

Corollary 2.12: Let L^N be NC- set of SNCT-space$(X,\mathcal{Z})$, then the following statement are equivalent :

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1) $Si_{ij}((\mathbb{L}^N)^{C_2}) =_k \emptyset_1{}^N \quad i,j,k = 1,2$
2) The only $SNCC - set$ containing $\mathbb{L}^N$ is $X_1{}^N$
3) The only SNC$O - set$ disjoint from L^N is $\emptyset_1{}^N$
4) $\forall\ \mathbb{K}^N \in \mathcal{Z},\mathbb{K}^N \neq_i \emptyset_1{}^N$ we have $\mathbb{L}^N \cap_i \mathbb{K}^N =_i \emptyset_1{}^N \quad i,j = 1,2$

Preposition 2.13: Let $\mathbb{L}^N$ be a NC- set of a SNCT-space $(X,\mathcal{Z})$, such that $Si_{ij}((\mathbb{L}^N)^{C_2}) =_i \emptyset_1{}^N$

then for any $\mathbb{K}^N \in \mathcal{Z}$, $\mathbb{K}^N \cap_i Si_{ij}((\mathbb{L}^N \cap_i \mathbb{K}^N)^{C_2}) =_i \emptyset_1{}^N$ for i, j = 1,2

Proof: we will prove this preposition in light of the second type of equal and the definition of $(Si_{12}(L^N))$and the point p^{N2}

Let $Si_{12}((\mathbb{L}^N)^{C_2}) =_2 \emptyset_1{}^N$

and if possible

$\exists\ p^{N2} \in_2 \mathbb{K}^N$and $p^{N2} \in_2 Si_{12}((\mathbb{L}^N \cap_2 \mathbb{K}^N)^{C_2})$, so $\exists\ \mathbb{V}^N \in \mathcal{Z} \ni \mathbb{V}^N \cap_2 \mathbb{L}^N \cap_2 \mathbb{K}^N =_2 \emptyset_1{}^N$

This implies that $Si_{12}((L^N)^{C_2}) \neq_2 \emptyset_1{}^N$,which is contradiction

Corollary 2.14: Let $\mathbb{L}^N$ be NC- set of SNCT-space $(X,\mathcal{Z})$,such that $Si_{11}((\mathbb{L}^N)^{C_2}) =_i \emptyset_1{}^N$

then for any $\mathbb{K}^N \in \mathcal{Z}$, $Si_{11}((\mathbb{L}^N \cap_i \mathbb{K}^N)^{C_2}) =_i Si_{11}((\mathbb{K}^N)^{C_2}) \quad i = 1,2$

Preposition 2.15: For any NC-set $\mathbb{L}^N$ of SNCT-space

and for any $\mathbb{K}^N \in \mathcal{Z}$, $\mathbb{K}^N \cap_i (Si_{ij}(\mathbb{L}^N)^{C_2})^{C_2} \cap_i Si_{ij}((\mathbb{L}^N \cap_i \mathbb{K}^N)^{C_2}) =_i \emptyset_1{}^N \quad i,j = 1,2$

Proof: Let $\mathbb{K}^N \cap_1 (Si_{11}(\mathbb{L}^N)^{C_2})^{C_2} \cap_1 Si_{11}(\mathbb{L}^N \cap_i \mathbb{K}^N)^{C_2} \neq_1 \emptyset_1{}^N$

This mean $\exists\ p^{Nk} \in_k \mathbb{K}^N andp^{Nk} \in_k (Si_{11}((\mathbb{L}^N)^{C_2}))^{C_2}, p^{N1} \in_1 Si_{11}((\mathbb{L}^N \cap_1 \mathbb{K}^N)^{C_2})$,k=1…6

So $\forall V^N \in \mathcal{Z} \ni V^N \cap_1 \mathbb{L}^N \neq_1 \emptyset_1{}^N$ (1)

But $p^{Nk} \in_k Si_{11}(\mathbb{L}^N \cap_1 \mathbb{K}^N)^{C_2}$

If $\exists \mathbb{H}^N \in \mathcal{Z} \ni \mathbb{L}^N \cap_1 \mathbb{K}^N \cap_1 \mathbb{H}^N =_1 \emptyset_1{}^N$,Then $\mathbb{K}^N \cap_1 (Si_{11}((\mathbb{K}^N)^{C_2}))^{C_2} \cap_1 Si_{11}((\mathbb{L}^N \cap_1 \mathbb{K}^N)^{C_2}) =_1 \emptyset_1{}^N$

Which contradiction with (1)

Thus $\mathbb{K}^N \cap_1 (Si_{11}(\mathbb{K}^N)^{C_2})^{C_2} \cap_1 Si_{11}(\mathbb{L}^N \cap_1 \mathbb{K}^N)^{C_2} =_1 \emptyset_1{}^N$

Corollary 2.16: $Si_{ij}((Si_{ij}((\mathbb{L}^N)^{C_2}))^{C_2}) \neq_i \emptyset_1{}^N$

If and only if $\exists \mathbb{K}^N \in \mathcal{Z}, \mathbb{K}^N \neq_i \emptyset_1{}^N \ Si_{ij}((\mathbb{L}^N \cap_i \mathbb{K}^N)^{C_2}) \subseteq_i \mathbb{K}^N$ i, j = 1,2

Corollary 2.17: For any NC- set $\mathbb{L}^N$ of SNCT-space $(X,\mathcal{Z})$,the following statement equivalent:

1) $Si_{ij}((Si_{ij}((\mathbb{L}^N)^{C_2}))^{C_2}) \neq_i \emptyset_1{}^N$
2) $(Si_{11}((\mathbb{L}^N)^{C_2}))^{C_2}$contain no SNCO- set.
3) $Si_{11}((\mathbb{L}^N)^{C_2}) =_1 X_1{}^N$

3. Conclusion

Considering the new topology(SNCT-space), we defined the interior, which is synonymous with Salama's definition from 2012. However, it is evident from the discussions above that, while the stable interior of the SNCO- set is equal to the same set in the case of Si_{11} and Si_{22}, it is not always equal to the same set in the case of Si_{12} and Si_{21}. We changed the definition of stable interior to the one below:

$$Sn(\mathbb{A}^N) = \cup_j \{\mathbb{S}^N \in \mathcal{Z}, \mathbb{S}_1 \subseteq \mathbb{A}_1, \mathbb{S}_2 \subseteq \mathbb{A}_2, \mathbb{S}_1 \cap \mathbb{A}_1 \neq \emptyset\} \quad j = 1,2$$

and saw that this property was also not met.

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