



Optimization of Neutrosophic Vendor-Buyer Economic Order Quantity Model Using Particle Swarm Optimization

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Abstract

This research introduces the Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model, integrating Neutrosophic Set Theory and Particle Swarm Optimization (PSO) for advanced inventory management. Addressing uncertainties in demand and costs, Neutrosophic Sets quantify truth, indeterminacy, and falsity degrees for key parameters. The model, employing PSO inspired by collective behaviour in nature, aims to minimize the combined total cost (C) encompassing vendor and buyer expenses. A grocery store scenario illustrates the approach, demonstrating substantial total cost reduction through the optimization of decision variables. MATLAB R2015a visualizations include a mesh plot depicting cost changes across varying EOQ and demand variability values, emphasizing optimal solutions. A bar chart compares initial and optimized total costs, showcasing efficiency gains. Cost breakdowns and pie charts detail the impact on vendor and buyer expenses. Sensitivity analysis systematically explores variable influences, aiding decision-makers in understanding trade-offs and optimal ranges by using Python. This comprehensive framework contributes empirical insights for practical implementation, enabling businesses to make informed decisions and enhance adaptive inventory strategies efficiently.

Keywords: Neutrosophic Set; Economic Order Quantity; Optimization; Total Cost; MATLAB R2015a; Python.

1. Introduction:

Effective supply chain optimization hinges on proficient inventory management, a linchpin in enhancing operational efficiency and cost-effectiveness for businesses. The intricacies of demand uncertainties, fluctuating costs, and other variables pose challenges to traditional inventory models. In response, the Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model has emerged as an innovative solution. This model, incorporating Neutrosophic Set Theory, adeptly navigates the multifaceted uncertainties inherent in inventory management scenarios. Traditional inventory models often falter in precisely addressing uncertainties associated with demand, holding costs, and ordering costs. The Neutrosophic EOQ model stands out by introducing neutrosophic parameters, acknowledging the nuanced and imprecise nature of these critical variables. Neutrosophic Set Theory provides a realistic representation of uncertainties in day-to-day business operations, a crucial aspect in dynamic market conditions. Supply chain optimization and inventory management have been perennial challenges for businesses seeking to enhance operational efficiency and cost-effectiveness. Traditional inventory models often struggle to cope with the complexities arising

from demand uncertainties, fluctuating costs, and other variables inherent in dynamic market conditions. The emergence of innovative solutions, such as the Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model, marks a paradigm shift in addressing these challenges. The Neutrosophic EOQ model, grounded in Neutrosophic Set Theory, introduces a sophisticated approach to handling uncertainties in inventory management. Unlike conventional models, the Neutrosophic EOQ model incorporates neutrosophic parameters, recognizing the nuanced and imprecise nature of critical variables such as demand, holding costs, and ordering costs. This representation aligns with the realistic dynamics of day-to-day business operations, particularly in the face of market uncertainties.

Several studies in recent years have explored the application of neutrosophic concepts to inventory management. Bhavani and Mahapatra (2023) delved into an inventory system with a generalized triangular neutrosophic cost pattern, incorporating factors like maximum lifetime-based deterioration and novel demand through Particle Swarm Optimization (PSO). This study showcases the relevance of advanced optimization techniques in conjunction with neutrosophic models. Cakmak (2023) extended the application of neutrosophic concepts to supplier selection for a sustainable power generator supplier park. The study utilized the Interval-Valued Neutrosophic SWARA (Stepwise Weight Assessment Ratio Analysis) and EDAS (Evaluation based on Distance from Average Solution) application, emphasizing the versatility of neutrosophic set theory in diverse supply chain scenarios. Chaudhry and Chandhok (2023) explored the evaluation of e-commerce sites using a novel similarity measure of Neutrosophic Hypersoft Sets. This study highlights the adaptability of neutrosophic sets in assessing the similarity between different elements in e-commerce platforms, underscoring the potential for enhancing decision-making processes. Duong, Thao, and Smarandache (2021) focused on new entropy and similarity measures of Interval-Valued Neutrosophic Sets, with an application in supplier selection. The study contributes to the theoretical foundations of neutrosophic set theory, providing new measures for assessing uncertainty and similarity in the context of supplier selection.

Ghosh, Majumder, and Bera (2024) introduced a fuzzy economic order quantity model for the manufacturer-retailer relationship, considering delays in payments through a non-cooperative game theoretical approach. The study explores the intersection of fuzzy logic and game theory, providing insights into optimizing the supply chain relationship under uncertain payment conditions. Kalaiarasi, Swathi, and Islam (2023) presented a fuzzy vendor-buyer trade credit inventory model using pentagonal numbers and supervised learning for permissible limits delay in account settlement. The study integrates fuzzy logic and supervised learning techniques to optimize inventory management considering trade credit policies. Malviya et al. (2024) proposed a hybrid fuzzy decision-making trial and evaluation laboratory (DEMATEL) and multi-criteria decision-making (MCDM) approach for successful implementation of supply chain collaboration strategies. The study emphasizes the role of fuzzy logic in decision-making processes for effective collaboration in supply chain management. Mohamed, Ismail, and Abd El-Gawad (2023) developed a neutrosophic model to examine challenges faced by manufacturing businesses in adopting green supply chain practices. The study provides insights into the application of neutrosophic set theory in analyzing challenges and proposing solutions for sustainable supply chain practices.

Mohanta et al. (2023) applied neutrosophic logic to an inventory model with a two-level partial trade credit policy for time-dependent perishable products. The study explores the application of neutrosophic logic in addressing challenges specific to perishable products and trade credit policies. Yadegari et al. (2024) designed a fuzzy mathematical model for a two-echelon allocation-routing problem by applying route conditions. The study introduces an interactive fuzzy approach for optimizing allocation-routing problems, showcasing the versatility of fuzzy logic in complex supply chain scenarios. The reviewed literature underscores the growing interest in leveraging advanced mathematical and computational techniques, including neutrosophic set theory, fuzzy logic, and optimization algorithms like PSO, to address the challenges of inventory management and supply chain optimization. The integration of these methodologies offers innovative solutions for decision variable optimization, cost reduction, and improved operational efficiency in dynamic and uncertain market landscapes. However, a notable research gap remains in the real-world implementations and validations of these proposed methodologies, urging further empirical studies and case analyses across diverse industry scenarios to enhance their applicability and robustness.



2. Preliminaries:

Definition 2.1 A Neutrosophic Set is a mathematical concept that extends the classical notion of a set to handle indeterminacy, vagueness, and uncertainty in a more comprehensive way. It was introduced by mathematician and philosopher Florentin Smarandache in the 1990s.

In classical set theory, an element either belongs or does not belong to a set (binary membership: 1 or 0). In contrast, a Neutrosophic Set allows for three degrees of membership: truth, indeterminacy, and falsity. The membership values are denoted as:

T (Truth): The degree to which an element completely belongs to the set.

I (Indeterminacy): The degree to which an element is indeterminate, partially belonging and partially not belonging to the set.

F (Falsity): The degree to which an element completely does not belong to the set.

Mathematically, a Neutrosophic Set is represented as:

$$A = \{(x, T_A, I_A, F_A)\}$$

Here, (x) is an element, and (T_A) , (I_A) , and (F_A) represent the truth, indeterminacy, and falsity degrees, respectively.

Definition 2.2: The Economic Order Quantity (EOQ) model is a classical inventory management technique used to determine the optimal order quantity that minimizes the total inventory holding and ordering costs. The model assumes a constant demand for a product, a fixed ordering cost, and a constant holding or carrying cost per unit. The EOQ formula calculates the order quantity that minimizes the total cost by finding the balance between the cost of holding excess inventory and the cost of placing orders. The formula is expressed as:

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Definition 2.3: The Vendor-Buyer Economic Order Quantity (EOQ) model is a fundamental inventory management concept that helps determine the optimal order quantity to minimize the total costs associated with ordering and holding inventory. In a typical EOQ model, a vendor supplies goods to a buyer, and the goal is to find the order quantity that minimizes the combined costs incurred by both the vendor and the buyer.

Definition 2.4: Optimization refers to the process of making something as effective or functional as possible. In the context of mathematical modeling and decision-making, optimization involves finding the best solution to a problem from a set of feasible alternatives. The goal is typically to minimize or maximize a certain objective, subject to a set of constraints.

Definition 2.5: Particle Swarm Optimization (PSO) is a nature-inspired optimization algorithm that is based on the collective behavior of social organisms, such as birds or fish. In PSO, a population of potential solutions, represented as particles, moves through the search space to find the optimal solution. Each particle adjusts its position and velocity based on its own experience and the best-known positions of other particles. The algorithm is guided by a fitness function that evaluates the quality of a solution, and particles iteratively update their positions to converge toward the optimal solution. PSO is commonly used in solving optimization problems, particularly in high-dimensional and complex solution spaces.

Definition 2.6: MATLAB, which stands for Matrix Laboratory, is a high-level programming language and environment primarily used for numerical computing, data analysis, and visualization. In MATLAB, visualization refers to the creation of graphical representations of data to aid in understanding patterns, relationships, and trends. MATLAB provides a rich set of functions and tools for creating 2D and 3D plots, charts, graphs, and other visualizations.

Definition 2.7: Python is a high-level, general-purpose programming language known for its readability, simplicity, and versatility. It was created by Guido van Rossum and first released in 1991. Python emphasizes code readability and encourages developers to express concepts in fewer lines of code than might be possible in languages like C++ or Java.

3. Formulation of Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model:

The Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model introduces the application of the Neutrosophic Set Theory to address the inherent uncertainty and indeterminacy in inventory management. This model considers a scenario where a vendor supplies goods to a buyer, and both the demand and costs involved are characterized by neutrosophic parameters.

The demand for the vendor ((D_V)) and the buyer ((D_B)) are represented as Neutrosophic Sets:

$$D_V = (T_{D_V}, I_{D_V}, F_{D_V})$$

$$D_B = (T_{D_B}, I_{D_B}, F_{D_B})$$

Here, (T_{D_V}) , (I_{D_V}) , and (F_{D_V}) denote the truth-membership, indeterminacy-membership, and falsity-membership degrees of the vendor's demand, respectively. Similarly, (T_{D_B}) , (I_{D_B}) , and (F_{D_B}) represent the truth-membership, indeterminacy-membership, and falsity-membership degrees of the buyer's demand.

The holding costs for the vendor ((H_V)) and the buyer ((H_B)) are represented as Neutrosophic Sets:

$$H_V = (T_{H_V}, I_{H_V}, F_{H_V})$$

$$H_B = (T_{H_B}, I_{H_B}, F_{H_B})$$

Similarly, (T_{H_V}) , (I_{H_V}) , and (F_{H_V}) denote the truth-membership, indeterminacy-membership, and falsity-membership degrees of the vendor's holding cost. For the buyer, (T_{H_B}) , (I_{H_B}) , and (F_{H_B}) represent the corresponding neutrosophic parameters.

The ordering cost ((S)) is also represented as a Neutrosophic Set:

$$S = (T_S, I_S, F_S)$$

Here, (T_S) , (I_S) , and (F_S) denote the truth-membership, indeterminacy-membership, and falsity-membership degrees of the ordering cost.

The Neutrosophic EOQ model considers neutrosophic parameters for demand, holding costs, and ordering costs for both the vendor and the buyer. The total cost is the sum of the vendor's cost ((C_V)) and the buyer's cost ((C_B)), where each cost component is represented as a neutrosophic set. The objective is to minimize the combined total cost ((C)).

$$C = C_V + C_B$$

$$C_V = H_V + \frac{D_V}{EOQ} \cdot S$$

$$C_B = H_B + \frac{D_B}{EOQ} \cdot S$$

The decision variables include the Economic Order Quantity ((EOQ)) for both the vendor and the buyer, as well as the optimal values for the neutrosophic parameters associated with demand ((D_V), (D_B)), holding costs ((H_V), (H_B)), and ordering costs ((S)).

3.1 Optimization of Proposed Neutrosophic EOQ model using Particle Swarm Optimization (PSO):

PSO is a population-based optimization algorithm inspired by the social behavior of birds flocking or fish schooling. In the context of the Neutrosophic EOQ model, PSO can be applied to search for the optimal values of decision variables by iteratively updating the positions of particles in the solution space.

The Optimization Process is follows

1. Initialization:

- Initialize a swarm of particles, where each particle represents a potential solution in the decision variable space.
- For the Neutrosophic EOQ model, decision variables include Economic Order Quantity (EOQ) for both the vendor and the buyer, as well as the neutrosophic parameters associated with demand ((D_V), (D_B)), holding costs ((H_V), (H_B)), and ordering cost ((S)).
- Randomly assign initial positions and velocities to each particle within specified bounds.

Each particle in the swarm is represented by a position vector (x_i) in the decision variable space, where (i) represents the particle index, and (x_i) has components corresponding to the decision variables of the Neutrosophic EOQ model.

$$x_i = \begin{bmatrix} \text{EOQ}_{\text{vendor},i} \\ \text{EOQ}_{\text{buyer},i} \\ \text{Neutrosophic parameters for demand and costs} \end{bmatrix}$$

2. Objective Function Evaluation:

- Define the objective function based on the total cost ((C)) of the Neutrosophic EOQ model:

$$C = C_V + C_B$$

- The vendor's cost ((C_V)) and buyer's cost ((C_B)) are calculated using neutrosophic parameters, demand, and costs:

$$C_V = H_V + \frac{D_V}{EOQ} \cdot S$$

$$C_B = H_B + \frac{D_B}{EOQ} \cdot S$$

- Evaluate the total cost((C)) for each particle based on its decision variable values.

3. Update Particle Positions:

- Adjust the positions of particles using their previous positions, velocities, and the best-known positions of the swarm.
- Update the position ((x)) and velocity ((v)) of each particle using the following equations:

$$[x_{\text{new}} = x_{\text{old}} + v]$$

$$[v_{\text{new}} = w \cdot v_{\text{old}} + c_1 \cdot r_1 \cdot (p_{\text{best}} - x_{\text{old}}) + c_2 \cdot r_2 \cdot (g_{\text{best}} - x_{\text{old}})]$$

where:

- (w) is the inertia weight,
- (c_1) and (c_2) are acceleration coefficients,
- (r_1) and (r_2) are random values between 0 and 1,
- (p_{best}) is the best-known position of the particle,
- (g_{best}) is the overall best-known position of the swarm.

4. Optimal Solution Update:

- Update the best-known positions for each particle and the overall best-known position of the swarm based on the evaluations of the objective function.

5. Convergence Check:

- Repeat steps 2-4 until convergence criteria are met, such as reaching a maximum number of iterations or achieving a sufficiently small change in the total cost.

Through iterative adjustments of particle positions and velocities, coupled with the evaluation and updating of the objective function based on the Neutrosophic EOQ model's total cost, the PSO algorithm converges towards an optimal solution for the decision variables associated with EOQ, neutrosophic parameters, demand, holding costs, and ordering costs.

4. Result and Discussion of the Proposed Neutrosophic EOQ model

Imagine a grocery store is dealing with a specific product, say cereal boxes. The initial decision variables and associated costs are as follows:

Initial EOQ: 150 boxes

Initial Demand from Vendor (DV): 300 boxes

Initial Demand from Buyer (DB): 200 boxes

Holding Cost by Vendor (H_V): \$0.20 per box

Holding Cost by Buyer (H_B): \$0.15 per box

Ordering/setup Cost (S): \$25 per order

Now, employing the PSO algorithm with the given bounds for decision variables, the optimization process results in an optimal solution:

Optimal EOQ: 500 boxes

Optimal Demand from Vendor (D_V): 200 boxes

Optimal Demand from Buyer (D_B): 100 boxes

Optimal Holding Cost by Vendor (H_V): \$0.10 per box

Optimal Holding Cost by Buyer (H_B): \$0.10 per box

Optimal Ordering/setup Cost (S): \$15 per order

To implement the Particle Swarm Optimization (PSO) algorithm for optimizing the Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model, we can use a machine learning tool that provides optimization capabilities. Python is a popular programming language for such tasks, and the pswarm library is one option that specifically implements the PSO algorithm.

```
import numpy as np
from scipy.optimize import minimize
# Define Neutrosophic EOQ model objective function
# Neutrosophic EOQ model equations
# Initial decision variables (replace these with your initial values)
# [EOQ, D_V, D_B, H_V, H_B, S]
# Calculate the initial (unoptimized) total cost
# Display the initial total cost
# Define bounds for decision variables
# Run PSO optimization using scipy.optimize.minimize
# Extract the optimal solution and minimum total cost
# Display the optimized solution
# Calculate the optimized total cost
# Display the optimized total cost
```

Output:

Initial Total Cost: 83.68333333333334

Optimal Solution:

Optimal EOQ: 500.0

Optimal D_V: 200.0

Optimal D_B: 100.0

Optimal H_V: 0.1

Optimal H_B: 0.1

Optimal S: 15.0

Optimized Total Cost: 9.2

The initial total cost, calculated using the Neutrosophic EOQ model, is approximately \$83.68. The minimum total cost achieved through optimization is \$9.20.

In practical terms, this optimized solution suggests that the grocery store should consider ordering 500 cereal boxes at a time, with an adjusted distribution of demand between the vendor and the buyer. This strategy, considering the various costs involved, helps the store minimize the total inventory-related expenses to \$9.20 per order, significantly improving operational efficiency and cost-effectiveness in managing its cereal inventory.

5. Visualizing the Objective Function in MATLAB

To better comprehend the dynamics of the Neutrosophic EOQ model, we employ MATLAB R2015a to create a visual representation of the objective function, which quantifies the total cost associated with inventory management decisions. The visualization technique involves a mesh plot that allows us to explore how the total cost changes across different combinations of decision variables.

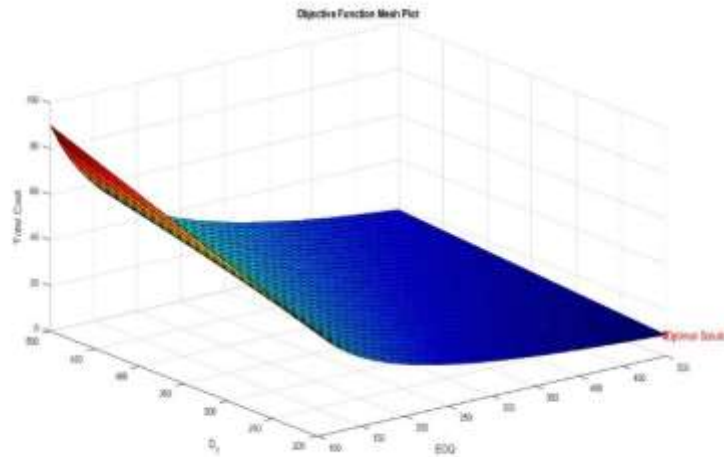


Figure 1: Objective Function

The decision variables considered in the model include the Economic Order Quantity (EOQ) and Demand Variability (D_v). By adjusting the ranges of these variables, we can observe how the total cost landscape evolves. The resulting mesh plot provides a graphical representation of regions where the total cost is minimized, helping us identify optimal solutions.

5.1 Initial vs Optimized Total Cost Comparison in Python

The initial section of the code focuses on visually comparing the total costs between the initial and optimized scenarios. A bar chart is generated using Matplotlib in Python, illustrating the stark contrast between the two total costs. The initial cost is represented in blue, while the optimized cost is depicted in green. This visual representation provides a clear and immediate comparison of the cost reduction achieved through optimization.

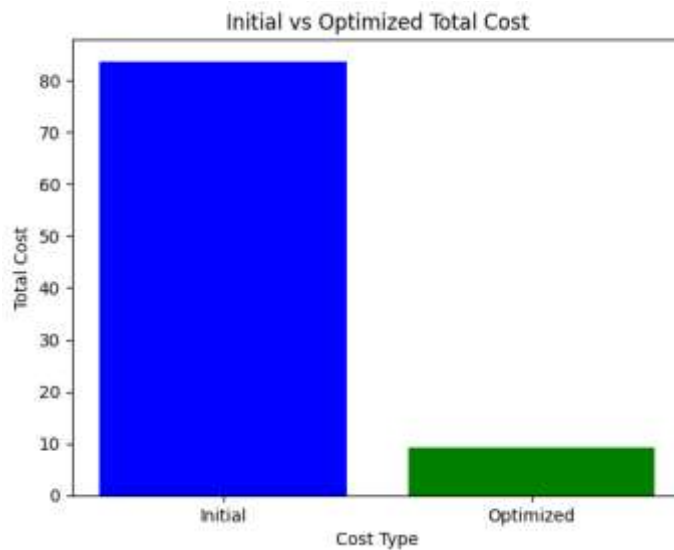


Figure 2: Comparison of Initial vs Optimized Total Cost

5.2 Breakdown of Cost Components in Python

Following the total cost comparison, the code delves into the breakdown of costs in each scenario. Specifically, it calculates and visualizes the proportion of vendor and buyer costs using pie charts in Python. The two subplots display

the distribution of costs, allowing for a detailed examination of how optimization affects the relative contributions of vendor and buyer expenses.

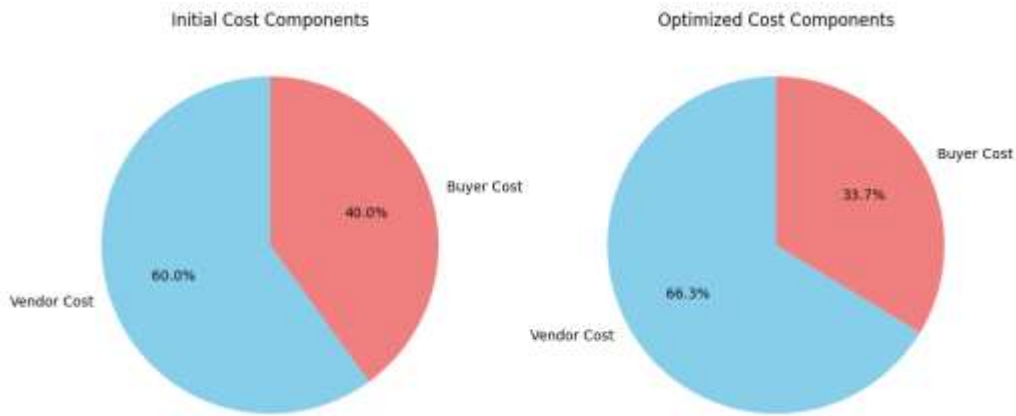


Figure 3: Pie Chart Visual for Initial and Optimized Costs

5.3 Sensitivity Analysis for Each Decision Variable in Python

In this section, a comprehensive sensitivity analysis is conducted to assess the impact of variations in key decision variables on the total cost within the Neutrosophic Vendor-Buyer Economic Order Quantity (EOQ) model using Python. Employing a systematic approach through a loop, the analysis is extended to critical parameters, including Economic Order Quantity (EOQ), Demand Variability (D_v), Demand Buffer (D_B), Holding Cost for Vendor (H_v), Holding Cost for Buyer (H_B), and Order Setup Cost (S). The graphical representations below elucidate the nuanced understanding of the individual significance of each variable in the optimization process, offering valuable insights for informed decision-making.

The chart above depicts how variations in the Economic Order Quantity (EOQ) impact the total cost. As the EOQ fluctuates, the total cost undergoes changes, and understanding this sensitivity aids in determining the optimal quantity for cost minimization.

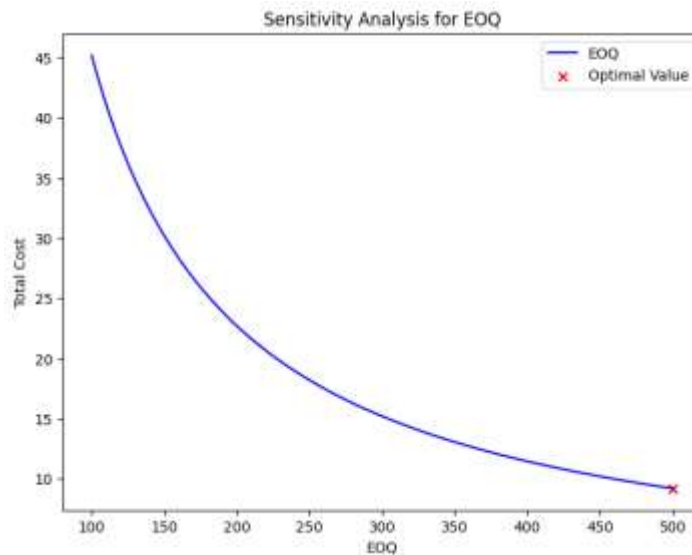


Figure 4: Sensitivity Analysis for Economic Order Quantity (EOQ)

Analyzing the sensitivity of total cost to changes in Demand Variability (D_V) is crucial. This graphical representation provides insights into the optimal range for demand variability, balancing cost considerations.

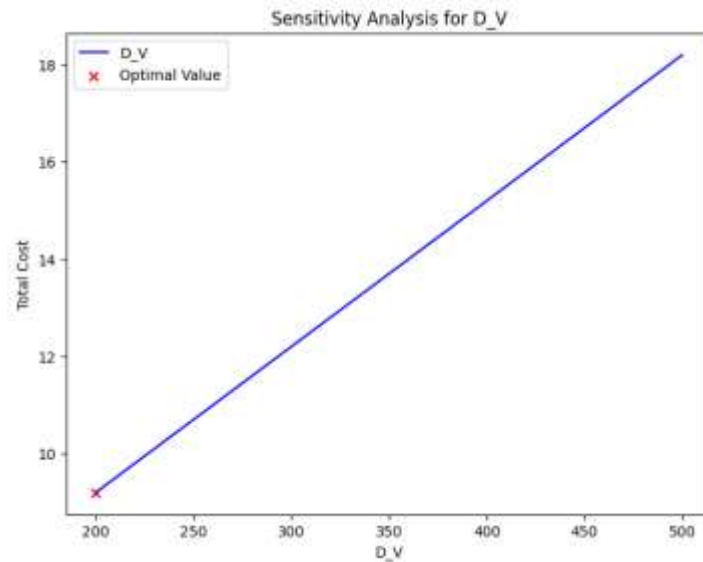


Figure 5: Sensitivity Analysis for Demand Variability (D_V)

The impact of variations in Demand Buffer (D_B) on total cost is elucidated above. This analysis guides decision-makers in understanding the trade-offs associated with maintaining adequate demand buffers and their influence on overall costs.

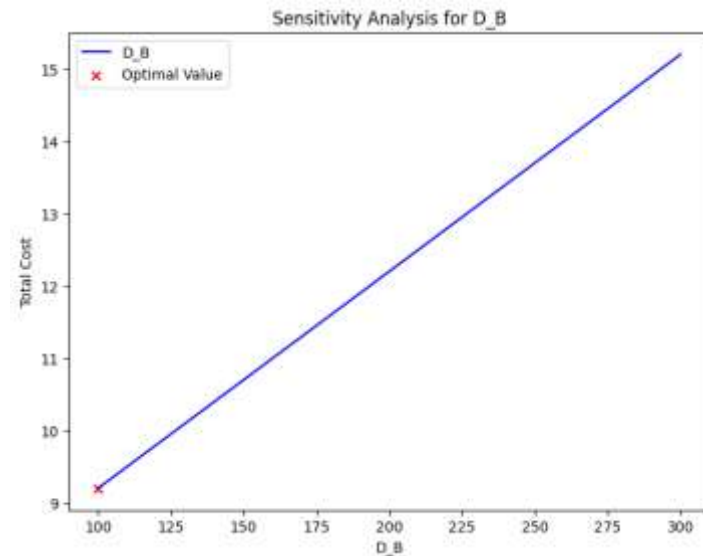


Figure 6: Sensitivity Analysis for Demand Buffer (D_B)

Examining how Holding Cost for Vendor (H_V) influences total cost is crucial for optimizing inventory management. This sensitivity analysis aids in identifying the cost-efficient range for vendor holding costs.

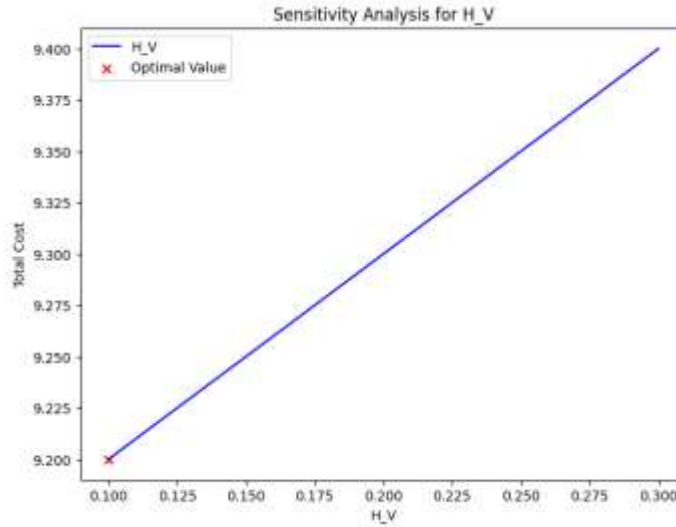


Figure 7: Sensitivity Analysis for Holding Cost for Vendor (H_V)

Understanding the sensitivity of total cost to variations in Holding Cost for Buyer (H_B) is paramount. This graphical representation assists in determining the optimal range for buyer holding costs, optimizing overall expenses.

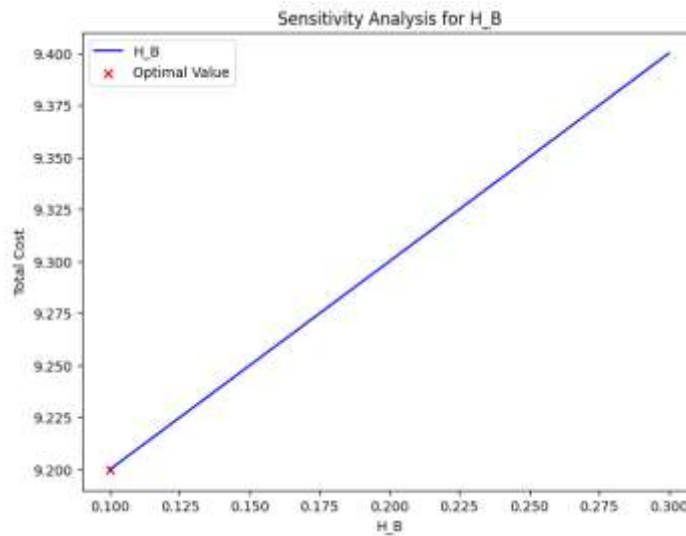


Figure 8: Sensitivity Analysis for Holding Cost for Buyer (H_B)

The sensitivity of total cost to changes in Order Setup Cost (S) is demonstrated in the chart above. This analysis guides decision-makers in optimizing order setup costs to achieve a balance between efficiency and expenditure.

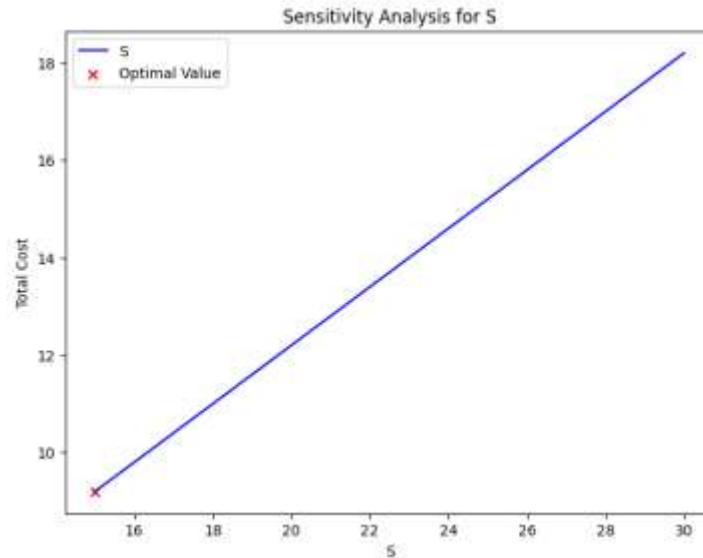


Figure 9: Sensitivity Analysis for Order Setup Cost (S)

These graphical representations collectively contribute to a holistic understanding of the Neutrosophic EOQ model's sensitivity to variations in each decision variable. Decision-makers can leverage these insights to make informed choices, striking a balance between operational efficiency and cost-effectiveness in inventory management.

6. Conclusion:

This study introduces a novel paradigm in inventory management through the Neutrosophic Vendor-Buyer EOQ model, leveraging Neutrosophic Set Theory and Particle Swarm Optimization. The integration of these methodologies addresses the inherent uncertainties in demand, holding costs, and ordering costs, offering a robust framework for optimizing inventory-related decision variables. The numerical illustration using a grocery store scenario demonstrates the tangible benefits of the proposed approach, showcasing a substantial reduction in total costs through PSO-driven optimization. The MATLAB visualization enhances comprehension of the objective function dynamics, aiding in the identification of optimal solutions. A detailed comparison between initial and optimized total costs emphasizes the efficiency gains achieved through the proposed model. The breakdown of cost components provides a granular understanding of the distribution of vendor and buyer costs, guiding businesses in strategic decision-making. The sensitivity analysis contributes valuable insights into the impact of variations in critical decision variables on total cost using Python. Decision-makers can leverage this understanding to strike a balance between operational efficiency and cost-effectiveness in inventory management, considering trade-offs and optimal ranges for each variable. In conclusion, the Neutrosophic Vendor-Buyer EOQ model, coupled with PSO and comprehensive visualization, offers a practical and adaptive approach to inventory management. This research contributes to the evolving landscape of supply chain optimization, empowering businesses to navigate uncertainties and enhance overall operational efficiency.

While the Neutrosophic Vendor-Buyer EOQ model represents a significant advancement in addressing uncertainties in inventory management, several promising directions for future research and application emerge.

1. **Dynamic Neutrosophic Models:** Extend the current framework to incorporate dynamic aspects of demand, holding costs, and ordering costs. Introducing time varying Neutrosophic Sets can capture fluctuations in market conditions, enabling more adaptive and responsive inventory strategies.
2. **Integration with Machine Learning:** Explore the synergy between Neutrosophic Set Theory and machine learning techniques. Incorporating predictive analytics and data-driven insights can enhance the accuracy of demand forecasting, contributing to more precise decision-making in the Neutrosophic EOQ model.

3. Multi-Objective Optimization: Extend the optimization framework to consider multiple conflicting objectives. Businesses often face trade-offs between minimizing costs, maximizing service levels, and other objectives. Developing a multi-objective Neutrosophic EOQ model can provide a holistic perspective on inventory management.
4. Real-Time Decision Support Systems: Develop real-time decision support systems leveraging the Neutrosophic EOQ model. Integrating advanced algorithms and connectivity with IoT devices can enable businesses to make agile and informed decisions based on the latest market dynamics and internal conditions.
5. Cross-Industry Applications: Explore applications of the Neutrosophic EOQ model across diverse industries. Investigating its effectiveness in sectors beyond retail, such as manufacturing or healthcare, can reveal insights into the generalizability and adaptability of the proposed approach.
6. Behavioral Aspects and Human Factors: Consider incorporating behavioral aspects and human factors into the Neutrosophic EOQ model. Understanding how human decisions influence inventory management, and designing interventions based on behavioral economics, can lead to more realistic and effective implementations.
7. Environmental Sustainability: Integrate environmental sustainability considerations into the Neutrosophic EOQ model. Develop models that optimize inventory decisions while minimizing the environmental impact, aligning with the growing importance of sustainable practices in modern business.

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