Algebraic properties applied to sin trigonometric complex neutrosophic sets

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Abstract

This article presents a new way of analyzing multiple attribute decision-making (MADM) using $(\flat_1, \flat_2, \flat_3)$ sin trigonometric complex neutrosophic sets (ST-CNS). Complex neutrosophic weighted averaging (ST-CNWA), sin trigonometric complex neutrosophic weighted geometric (ST-CNWG), sin trigonometric complex generalized neutrosophic weighted averaging (ST-CGNWA), and sin trigonometric complex generalized neutrosophic weighted geometric (ST-CGNWG). During our discussion, we presented an algorithm that utilized these operators. There are extensive numerical illustrations of score values. Furthermore, we will discuss commutativity, idempotency, and monotonicity of sin trigonometric complex neutrosophic sets as part of our discussion. It is easier, faster, and more convenient to find the best option this way. Consequently, the sin trigonometric complex $(\flat_1, \flat_2, \flat_3)$ is more closely related to precise conclusions. Also revealed by the study was an intriguing and fascinating observation.

Keywords: ST-CNWA; ST-CNWG; ST-CGNWA and ST-CGNWG.

1 Introduction

Many uncertain theories including fuzzy set (FS),[1] intuitionistic FS (IFS),[2] Pythagorean FS (PFS)[3] and spherical FS (SFS).[4] An FS consists of elements with values ranging from 0 to 1 for their membership value (MV); Later, Atanassov proposed the concept of an IFS that is divided into categories using non-membership value (NMV), which cannot exceed one.[2] In cases where the combined MV and NMV values are greater than 1, a single problem may be conveyed to the DM. The square sum of the MV and NMV of an IFS with a value less than one is classified as PFS, which can be characterized by Yager.[3] Cuong et al.[5] developed the picture FS concept with three pointers: positive MV, neutral MV and negative MV. Moreover, it is more beneficial than both PFS and IFS. An example of a generalization of picture FS with AOs is proposed by Liu et al.[6] Liu et al.[7] they proposed a generalized PFS with AO and applications. An analysis of AOs using PFS and interval values.[8] As part of the DM approach challenge, the sum of the positive, neutral, and negative MVs rarely exceeds one. The concept of SFS is proposed by Ashraf et al.,[4] whereby the square sum of positive, neutral, and negative values is not greater than 1. Fatmaa et al.[9] investigated the notion of SFS using TOPSIS models. Recently palanikumar et al. discussed the different AOs such as[10]-.[12]

A fuzzy spherical Dombi AO was developed by Ashraf et al.[13] Further information about SFSs and T-SFSs can be found at.[14,15] In 2022, Temel et al.[16] discussed its application to MADM based on Muirhead power normal SFS. Peng et al.[17] explore neutrosophic set with MADM using MABAC and TOPSIS

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approaches. A generalization of PFS using TOPSIS was presented by Zhang et al.[18] There are several algebraic structures and aggregation operators that can be applied in many applications as discussed by,[19],[20].[21] An introduction is found in section 1. There was a discussion of PFS and NS in section 2. A definition of complex $(\flat_1, \flat_2, \flat_3)$ and some operations on it are discussed in section 3. In section 4 an interaction between MADM and some AOs is described based on some ST-CNNs for $(\flat_1, \flat_2, \flat_3)$. A numerical example and an algorithm are discussed in section 5. Section 6 concludes with a conclusion. Based on the findings of the study, the following conclusions can be drawn:

1. To introduce score values for ST-CNSs.
2. By using $(\flat_1, \flat_2, \flat_3)$ST-CNNs, $(\flat_1, \flat_2, \flat_3)$ST-CNWA, $(\flat_1, \flat_2, \flat_3)$ST-CNWG, G$(\flat_1, \flat_2, \flat_3)$ST-CNWA and G$(\flat_1, \flat_2, \flat_3)$ST-CNWG operators are developed.
3. On the basis of AOs, MADM is explored using $(\flat_1, \flat_2, \flat_3)$ST-CNSs.
4. We proposed approaches is demonstrated with a few numerical examples.

2 Basic concepts

This section contains a number of important definitions that we must review for our further learning.

Definition 2.1. [8] Let Ξ be an universal. The PIVFS $\Psi = \left\{\partial, \left\langle \widetilde{\Lambda_\Psi^{\mathscr{T}}}(\partial), \widetilde{\Lambda_\Psi^{\mathscr{F}}}(\partial) \right\rangle \middle| \partial \in \Xi \right\}$, where $\widetilde{\Lambda_\Psi^T}, \widetilde{\Lambda_\Psi^F} : \Xi \to Int([0,1])$ denote the MV and NMV of $\partial \in \Xi$ to Ψ, respectively, and $0 \leq (\Lambda_\Psi^{\mathscr{T}}(\partial))^2 + (\Lambda_\Psi^{\mathscr{F}}(\partial))^2 \leq 1$. For convenience, $\Psi = \left\langle \left[\Lambda_\Psi^{\mathscr{T}}, \Lambda_\Psi^{\mathscr{T}}\right], \left[\Lambda_\Psi^{\mathscr{F}}, \Lambda_\Psi^{\mathscr{F}}\right] \right\rangle$ is called a Pythagorean interval-valued fuzzy number (PyIVFN).

Definition 2.2. The NS $\Psi = \left\{x, \langle \Lambda_\Psi^{\mathscr{T}}(\partial), \Lambda_\Psi^{\mathscr{I}}(\partial), \Lambda_\Psi^{\mathscr{F}}(\partial) \rangle \middle| \partial \in \Xi \right\}$, where $\Lambda_\Psi^{\mathscr{T}}, \Lambda_\Psi^{\mathscr{I}}, \Lambda_\Psi^{\mathscr{F}} : \Xi \to [0,1]$ denote the positive MV, neutral MV and negative MV of $\partial \in \Xi$, respectively and $0 \leq (\Lambda_\Psi^{\mathscr{T}}(\partial)) + (\Lambda_\Psi^{\mathscr{I}}(\partial)) + (\Lambda_\Psi^{\mathscr{F}}(\partial)) \leq 2$. For $M = \langle \Lambda_\Psi^{\mathscr{T}}, \Lambda_\Psi^{\mathscr{I}}, \Lambda_\Psi^{\mathscr{F}} \rangle$ is called a neutrosophic number (NN).

Definition 2.3. The NS $\Psi = \left\{\partial, \langle \Lambda_\Psi^{\mathscr{T}}(\partial), \Lambda_\Psi^{\mathscr{I}}(\partial), \Lambda_\Psi^{\mathscr{F}}(\partial) \rangle \middle| \partial \in \Xi \right\}$, where $\Lambda_\Psi^{\mathscr{T}}, \Lambda_\Psi^{\mathscr{I}}, \Lambda_\Psi^{\mathscr{F}} : \Xi \to [0,1]$ denote the positive MV, neutral MV and negative MV of $\partial \in \Xi$ to Ψ, respectively and $0 \leq (\Lambda_\Psi^{\mathscr{T}}(\partial))^2 + (\Lambda_\Psi^{\mathscr{I}}(\partial))^2 + (\Lambda_\Psi^{\mathscr{F}}(\partial))^2 \leq 2$. For all $\partial \in \Xi$, $\sqrt{2 - \left((\Lambda_\Psi^{\mathscr{T}}(\partial))^2 + (\Lambda_\Psi^{\mathscr{I}}(\partial))^2 + (\Lambda_\Psi^{\mathscr{F}}(\partial))^2\right)}$ is called the value of refusal of membership of ∂ in Ψ. For convenience, $\Psi = \langle \Lambda_\Psi^{\mathscr{T}}, \Lambda_\Psi^{\mathscr{I}}, \Lambda_\Psi^{\mathscr{F}} \rangle$ is called a Pythagorean neutrosophic number (PyNN).

Definition 2.4. Let $\Psi_1 = (a_1, b_1) \in N$and $\Psi_2 = (a_2, b_2) \in N$. Then the distance between Ψ_1 and Ψ_2 is defined as $\mathcal{D}(\Psi_1, \Psi_2) = \sqrt{(a_1 - a_2)^2 + \frac{1}{2}(b_1 - b_2)^2}$, where N is a natural number.

Definition 2.5. For any PIVFN $\Psi = \left\langle [\mathcal{R}^{\mathscr{T}l}, \mathcal{R}^{\mathscr{T}u}], [\mathcal{R}^{\mathscr{F}l}, \mathcal{R}^{\mathscr{F}u}] \right\rangle$, the score function of Ψ is defined as

$$S(\Psi) = \frac{1}{2}\Big((\mathcal{R}^{\mathscr{T}l})^2 + (\mathcal{R}^{\mathscr{T}u})^2 - (\mathcal{R}^{\mathscr{F}l})^2 - (\mathcal{R}^{\mathscr{F}u})^2\Big),\ S(\Psi) \in [-1, 1]$$

Accuracy function of Ψ is

$$H(\Psi) = \frac{1}{2}\Big((\mathcal{R}^{\mathscr{T}l})^2 + (\mathcal{R}^{\mathscr{T}u})^2 + (\mathcal{R}^{\mathscr{F}l})^2 + (\mathcal{R}^{\mathscr{F}u})^2\Big),\ H(\Psi) \in [0, 1].$$

3 Operations for $(\flat_1, \flat_2, \flat_3)$ST-CNN

We discuss the concept of $(\flat_1, \flat_2, \flat_3)$ sin trigonometric complex neutrosophic number (ST-CNN). As a result, the $(\flat_1, \flat_2, \flat_3)$ST-CNN and its operations were defined and simply $\pi/2 = \theta$.

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Definition 3.1. The $(\flat_1, \flat_2, \flat_3)$ sin trigonometric complex neutrosophic set $(\flat_1, \flat_2, \flat_3)$ (ST-CNS) $\sin\Psi = \left\{\partial, \left\langle\left(\mathcal{R}^{\mathscr{T}}_{\Psi}(\partial)e^{i2\pi\aleph^{\mathscr{T}}_{\Psi}(\partial)}, \mathcal{R}^{\mathscr{I}}_{\Psi}(\partial)e^{i2\pi\aleph^{\mathscr{I}}_{\Psi}(\partial)}, \mathcal{R}^{\mathscr{F}}_{\Psi}(\partial)e^{i2\pi\aleph^{\mathscr{F}}_{\Psi}(\partial)}\right)\right\rangle \middle| \partial \in \Xi\right\}$, where $\mathcal{R}^{T}_{\Psi}, \mathcal{R}^{I}_{\Psi}, \mathcal{R}^{F}_{\Psi} : \Xi \to [0,1]$ denote the positive MV, neutral MV and negative MV of $\partial \in \Xi$ to Ψ, $\aleph^{T}_{\Psi}, \aleph^{I}_{\Psi}, \aleph^{F}_{\Psi} : \Xi \to [0,1]$ denote the phase positive MV, phase neutral MV and phase negative MV of $\partial \in \Xi$ to Ψ, respectively and $0 \leq (\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{\Psi}(\partial))^{\flat_1} + (\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{\Psi}(\partial))^{\flat_2} + (\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_{\Psi}(\partial))^{\flat_3} \leq 1$ and $0 \leq (\sin^2\theta\cdot\aleph^{\mathscr{T}}_{\Psi}(\partial))^{\flat_1} + (\sin^2\theta\cdot\aleph^{\mathscr{I}}_{\Psi}(\partial))^{\flat_2} + (\sin^2\theta\cdot\aleph^{\mathscr{F}}_{\Psi}(\partial))^{\flat_3} \leq 1$. For convenience, $\sin\Psi = \left\langle\left(\mathcal{R}^{\mathscr{T}}_{\Psi}e^{i2\pi\aleph^{\mathscr{T}}_{\Psi}}, \mathcal{R}^{\mathscr{I}}_{\Psi}e^{i2\pi\aleph^{\mathscr{I}}_{\Psi}}, \mathcal{R}^{\mathscr{F}}_{\Psi}e^{i2\pi\aleph^{\mathscr{F}}_{\Psi}}\right)\right\rangle$ is represent a $(\flat_1, \flat_2, \flat_3)$ST-CNN.

Definition 3.2. The $\sin\Psi = \langle(\mathcal{R}^{\mathscr{T}}e^{i2\pi\aleph^{\mathscr{T}}}, \mathcal{R}^{\mathscr{I}}e^{i2\pi\aleph^{\mathscr{I}}}, \mathcal{R}^{\mathscr{F}}e^{i2\pi\aleph^{\mathscr{F}}})\rangle$, $\sin\Psi_1 = \langle(\mathcal{R}^{\mathscr{T}}_1e^{i2\pi\aleph^{\mathscr{T}}_1}, \mathcal{R}^{\mathscr{I}}_1e^{i2\pi\aleph^{\mathscr{I}}_1}, \mathcal{R}^{\mathscr{F}}_1e^{i2\pi\aleph^{\mathscr{F}}_1})\rangle$ and $\sin\Psi_2 = \langle(\mathcal{R}^{\mathscr{T}}_2e^{i2\pi\aleph^{\mathscr{T}}_2}, \mathcal{R}^{\mathscr{I}}_2e^{i2\pi\aleph^{\mathscr{I}}_2}, \mathcal{R}^{\mathscr{F}}_2e^{i2\pi\aleph^{\mathscr{F}}_2})\rangle$ be any three of $(\flat_1, \flat_2, \flat_3)$ST-CNNs, and $\Xi \geqslant 0$. Then

1. $\sin\Psi_1 \uplus \sin\Psi_2 =$
$$\begin{bmatrix} \sqrt[\flat_1]{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_1)^{\flat_1} + (\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_2)^{\flat_1} - (\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_1)^{\flat_1}\cdot(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_2)^{\flat_1}} \\ e^{i2\pi\sqrt[\flat_1]{(\sin^2\theta\cdot\aleph^{\mathscr{T}}_1)^{\flat_1} + (\sin^2\theta\cdot\aleph^{\mathscr{T}}_2)^{\flat_1} - (\sin^2\theta\cdot\aleph^{\mathscr{T}}_1)^{\flat_1}\cdot(\sin^2\theta\cdot\aleph^{\mathscr{T}}_2)^{\flat_1}}}, \\ \sqrt[\flat_2]{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_1)^{\flat_2} + (\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_2)^{\flat_2} - (\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_1)^{\flat_2}\cdot(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_2)^{\flat_2}} \\ e^{i2\pi\sqrt[\flat_2]{(\sin^2\theta\cdot\aleph^{\mathscr{I}}_1)^{\flat_2} + (\sin^2\theta\cdot\aleph^{\mathscr{I}}_2)^{\flat_2} - (\sin^2\theta\cdot\aleph^{\mathscr{I}}_1)^{\flat_2}\cdot(\sin^2\theta\cdot\aleph^{\mathscr{I}}_2)^{\flat_2}}}, \\ (\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_1)^{1_3}\cdot(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_2)^{1_3} \\ e^{i2\pi(\sin^2\theta\cdot\aleph^{\mathscr{F}}_1)^{1_3}\cdot(\sin^2\theta\cdot\aleph^{\mathscr{F}}_2)^{1_3}} \end{bmatrix},$$

2. $\sin\Psi_1 \ominus \sin\Psi_2 =$
$$\begin{bmatrix} (\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_1)^{l_1}\cdot(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_2)^{l_1}e^{i2\pi(\sin^2\theta\cdot\aleph^{\mathscr{F}}_1)^{l_1}\cdot(\sin^2\theta\cdot\aleph^{\mathscr{F}}_2)^{l_1}}, \\ \sqrt[\flat_2]{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_1)^{\flat_2} + (\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_2)^{\flat_2} - (\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_1)^{\flat_2}\cdot(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_2)^{\flat_2}} \\ e^{i2\pi\sqrt[\flat_2]{(\sin^2\theta\cdot\aleph^{\mathscr{I}}_1)^{\flat_2} + (\sin^2\theta\cdot\aleph^{\mathscr{I}}_2)^{\flat_2} - (\sin^2\theta\cdot\aleph^{\mathscr{I}}_1)^{\flat_2}\cdot(\sin^2\theta\cdot\aleph^{\mathscr{I}}_2)^{\flat_2}}}, \\ \sqrt[\flat_3]{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_1)^{\flat_3} + (\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_2)^{\flat_3} - (\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_1)^{\flat_3}\cdot(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_2)^{\flat_3}} \\ e^{i2\pi\sqrt[\flat_3]{(\sin^2\theta\cdot\aleph^{\mathscr{F}}_1)^{\flat_3} + (\sin^2\theta\cdot\aleph^{\mathscr{F}}_2)^{\flat_3} - (\sin^2\theta\cdot\aleph^{\mathscr{F}}_1)^{\flat_3}\cdot(\sin^2\theta\cdot\aleph^{\mathscr{F}}_2)^{\flat_3}}} \end{bmatrix}$$

3. $$\Xi\cdot\sin\Psi = \begin{bmatrix} \sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}})^{\flat_1}\right)^{\Xi}}e^{i2\pi\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{T}})^{\flat_1}\right)^{\Xi}}}, \\ \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}})^{\flat_2}\right)^{\Xi}}e^{i2\pi\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{I}})^{\flat_2}\right)^{\Xi}}}, \\ ((\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3})^{\Xi}e^{i2\pi((\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3})^{\Xi}} \end{bmatrix},$$

4. $$\sin\Psi^{\Xi} = \begin{bmatrix} ((\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}})^{\flat_1})^{\Xi}e^{i2\pi((\sin^2\theta\cdot\aleph^{\mathscr{T}})^{\flat_1})^{\Xi}}, \\ \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}})^{\flat_2}\right)^{\Xi}}e^{i2\pi\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{I}})^{\flat_2}\right)^{\Xi}}}, \\ \sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3}\right)^{\Xi}}e^{i2\pi\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3}\right)^{\Xi}}} \end{bmatrix}.$$

Definition 3.3. For any ST-CNIVN $\sin\Psi_i = \langle(\mathcal{R}^{T}_ie^{i2\pi\aleph^{T}_i}, \mathcal{R}^{I}_ie^{i2\pi\aleph^{I}_i}, \mathcal{R}^{F}_ie^{i2\pi\aleph^{F}_i})\rangle$, the score function of Ψ $S(\Psi) = \frac{C+D}{2}$, where $C = (\mathcal{R}^{\mathscr{T}})^2 - (\mathcal{R}^{\mathscr{I}})^2 + 1 - (\mathcal{R}^{\mathscr{F}})^2$ and $D = (\aleph^{\mathscr{T}})^2 - (\aleph^{\mathscr{I}})^2 - (\aleph^{\mathscr{F}})^2$, where $S(\Psi) \in [-1, 1]$.

4 AOs based on $(\flat_1, \flat_2, \flat_3)$ST-CNN

Here we describe the AOs using $(\flat_1, \flat_2, \flat_3)$ST-CNWA, $(\flat_1, \flat_2, \flat_3)$ST-CNWG, G$(\flat_1, \flat_2, \flat_3)$ST-CNWA, and G$(\flat_1, \flat_2, \flat_3)$ST-CNWG.

4.1 $(\flat_1, \flat_2, \flat_3)ST-CNWA$

Definition 4.1. Let $\sin\Psi_i = \langle(\mathcal{R}^{\mathscr{T}}_ie^{i2\pi\aleph^{\mathscr{T}}_i}, \mathcal{R}^{\mathscr{I}}_ie^{i2\pi\aleph^{\mathscr{I}}_i}, \mathcal{R}^{\mathscr{F}}_ie^{i2\pi\aleph^{\mathscr{F}}_i})\rangle$ be the $(\flat_1, \flat_2, \flat_3)$ST-CNNs, $W = (\varsigma_1, \varsigma_2, ..., \varsigma_n)$ be the weight of Ψ_i, $\varsigma_i \geq 0$ and $\biguplus_{i=1}^{n}\varsigma_i = 1$. Then $(\flat_1, \flat_2, \flat_3)$ST-CNWA $(\Psi_1, \Psi_2, ..., \Psi_n) = \biguplus_{i=1}^{n}\varsigma_i\sin\Psi_i$.

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Theorem 4.2. *Let* $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1,\flat_2,\flat_3)$*ST-CNNs. Then* $(\flat_1,\flat_2,\flat_3)ST-CNWA(\Psi_1,\Psi_2,...,\Psi_n)$

$$= \begin{bmatrix} \sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi \sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}}, \\ \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}\, e^{i2\pi \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}}, \\ \bigodot_{i=1}^{n}((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i} e^{i2\pi(\bigodot_{i=1}^{n}((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i})} \end{bmatrix}.$$

Proof. If $n=2$, then $(\flat_1,\flat_2,\flat_3)$ST-CNWA$(\Psi_1,\Psi_2) = \varsigma_1\sin\Psi_1 \biguplus \varsigma_2\sin\Psi_2$, where

$$\varsigma_1\sin\Psi_1 = \begin{bmatrix} \sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}}\, e^{i2\pi \sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}}}, \\ \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}}\, e^{i2\pi \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}}}, \\ ((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{F}})^{\flat_3})^{\varsigma_1} e^{i2\pi(((\sin^2\theta\cdot\aleph_1^{\mathscr{F}})^{\flat_3})^{\varsigma_1})} \end{bmatrix}$$

$$\varsigma_2\sin\Psi_2 = \begin{bmatrix} \sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}}\, e^{i2\pi \sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}}}, \\ \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}}\, e^{i2\pi \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}}}, \\ ((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{F}})^{\flat_3})^{\varsigma_2} e^{i2\pi(((\sin^2\theta\cdot\aleph_2^{\mathscr{F}})^{\flat_3})^{\varsigma_2})} \end{bmatrix}$$

Now,

$$\varsigma_1\sin\Psi_1 \biguplus \varsigma_2\sin\Psi_2 = \begin{bmatrix} \sqrt[\flat_1]{\begin{matrix}\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}\right)+\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}\right)\\ -\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}\right)\end{matrix}} \\ e^{i2\pi\sqrt[\flat_1]{\begin{matrix}\left(1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}\right)+\left(1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}\right)\\ -\left(1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}\right)\end{matrix}}}, \\ \sqrt[\flat_2]{\begin{matrix}\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}\right)+\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}\right)\\ -\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}\right)\end{matrix}} \\ e^{i2\pi\sqrt[\flat_2]{\begin{matrix}\left(1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}\right)+\left(1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}\right)\\ -\left(1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}\right)\end{matrix}}}, \\ ((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{F}})^{\flat_3})^{\varsigma_1}\cdot((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{F}})^{\flat_3})^{\varsigma_2} e^{i2\pi(((\sin^2\theta\cdot\aleph_1^{\mathscr{F}})^{\flat_3})^{\varsigma_1}\cdot((\sin^2\theta\cdot\aleph_2^{\mathscr{F}})^{\flat_3})^{\varsigma_2})} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}} \\ e^{i2\pi\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_1}\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_2}}}, \\ \sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}} \\ e^{i2\pi\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_1}\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_2}}}, \\ ((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{F}})^{\flat_3})^{\varsigma_1}\cdot((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{F}})^{\flat_3})^{\varsigma_2} \\ e^{i2\pi(((\sin^2\theta\cdot\aleph_1^{\mathscr{F}})^{\flat_3})^{\varsigma_1}\cdot((\sin^2\theta\cdot\aleph_2^{\mathscr{F}})^{\flat_3})^{\varsigma_2})} \end{bmatrix}$$

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Hence, $(\flat_1, \flat_2, \flat_3)ST-CNWA(\Psi_1, \Psi_2)$

$$= \begin{bmatrix} \sqrt[\flat_1]{1-\bigodot_{i=1}^{2}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{2}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}}, \\ \sqrt[\flat_2]{1-\bigodot_{i=1}^{2}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}, e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{2}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}} \\ \bigodot_{i=1}^{2}((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i} e^{i2\pi(\bigodot_{i=1}^{2}((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i})} \end{bmatrix}.$$

It valid for $n \geq 3$,
Thus, $(\flat_1, \flat_2, \flat_3)ST-CNWA(\Psi_1, \Psi_2, ..., \Psi_l)$

$$= \begin{bmatrix} \sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}}, \\ \sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}, e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}} \\ \bigodot_{i=1}^{l}((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i} e^{i2\pi(\bigodot_{i=1}^{l}((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i})} \end{bmatrix}.$$

If $n = l + 1$, then $(\flat_1, \flat_2, \flat_3)$ST-CNWA $(\Psi_1, \Psi_2, ..., \Psi_l, \Psi_{l+1})$

$$= \begin{bmatrix} \sqrt[\flat_1]{\begin{array}{l} \biguplus_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}\right)+\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_{l+1}}\right) \\ -\bigodot_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_{l+1}}\right) \end{array}} \\ e^{i2\pi\sqrt[\flat_1]{\begin{array}{l} \biguplus_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}\right)+\left(1-\left(1-(\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_{l+1}}\right) \\ -\bigodot_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_{l+1}}\right) \end{array}}}, \\ \sqrt[\flat_2]{\begin{array}{l} \biguplus_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}\right)+\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_{l+1}}\right) \\ -\bigodot_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_{l+1}}\right) \end{array}} \\ e^{i2\pi\sqrt[\flat_2]{\begin{array}{l} \biguplus_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}\right)+\left(1-\left(1-(\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_{l+1}}\right) \\ -\bigodot_{i=1}^{l}\left(1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}\right)\cdot\left(1-\left(1-(\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_{l+1}}\right) \end{array}}}, \\ \bigodot_{i=1}^{l}((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i}\cdot((\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{F}})^{\flat_3})^{\varsigma_{l+1}} e^{i2\pi(\bigodot_{i=1}^{l}((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i}\cdot((\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{F}})^{\flat_3})^{\varsigma_{l+1}})} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt[\flat_1]{1-\bigodot_{i=1}^{l+1}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{l+1}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}}, \\ \sqrt[\flat_2]{1-\bigodot_{i=1}^{l+1}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}, e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{l+1}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}} \\ \bigodot_{i=1}^{l+1}((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i} e^{i2\pi(\bigodot_{i=1}^{l+1}((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i})} \end{bmatrix}.$$

Theorem 4.3. *Let* $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1, \flat_2, \flat_3)$*ST-CNNs. Then* $(\flat_1, \flat_2, \flat_3)$*ST-CNWA* $(\Psi_1, \Psi_2, ..., \Psi_n) = \sin\Psi$ *(idempotency property).*

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Proof. Since $\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}}=\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}$, $\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}}=\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}$, $\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}}=\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}$, $\sin^2\theta\cdot\aleph_i^{\mathscr{T}}=\sin^2\theta\cdot\aleph^{\mathscr{T}}$, $\sin^2\theta\cdot\aleph_i^{\mathscr{I}}=\sin^2\theta\cdot\aleph^{\mathscr{I}}$, $\sin^2\theta\cdot\aleph_i^{\mathscr{F}}=\sin^2\theta\cdot\aleph^{\mathscr{F}}$ and $\biguplus_{i=1}^n\varsigma_i=1$. Now, $(\flat_1,\flat_2,\flat_3)ST-CNWA(\Psi_1,\Psi_2,...,\Psi_n)$

$$
\begin{aligned}
&= \begin{bmatrix}
\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}},\\
\sqrt[\flat_2]{1-\bigodot_{i=1}^n\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^n\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}},\\
\bigodot_{i=1}^n((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i} e^{i2\pi(\bigodot_{i=1}^n((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3})^{\varsigma_i})}
\end{bmatrix}\\
&= \begin{bmatrix}
\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}})^{\flat_1}\right)^{\biguplus_{i=1}^n\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{T}})^{\flat_1}\right)^{\biguplus_{i=1}^n\varsigma_i}}},\\
\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}})^{\flat_2}\right)^{\biguplus_{i=1}^n\varsigma_i}}\, e^{i2\pi\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{I}})^{\flat_2}\right)^{\biguplus_{i=1}^n\varsigma_i}}},\\
((\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3})^{\biguplus_{i=1}^n\varsigma_i} e^{i2\pi((\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3})^{\biguplus_{i=1}^n\varsigma_i}}
\end{bmatrix}\\
&= \begin{bmatrix}
\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}})^{\flat_1}\right)}\, e^{i2\pi\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{T}})^{\flat_1}\right)}},\\
\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}})^{\flat_2}\right)}\, e^{i2\pi\sqrt[\flat_2]{1-\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{I}})^{\flat_2}\right)}},\\
(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3} e^{i2\pi(\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3}}
\end{bmatrix}\\
&= \Psi.
\end{aligned}
$$

Theorem 4.4. *Let* $\sin\Psi_i=\langle(\mathcal{R}_i^{\mathscr{T}}e^{i2\pi\aleph_i^{\mathscr{T}}},\mathcal{R}_i^{\mathscr{I}}e^{i2\pi\aleph_i^{\mathscr{I}}},\mathcal{R}_i^{\mathscr{F}}e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1,\flat_2,\flat_3)$*ST-CNNs. Then* $(\flat_1,\flat_2,\flat_3)$*ST-CNWA*$(\Psi_1,\Psi_2,...,\Psi_n)$ *where* $\underbrace{\mathcal{R}^{\mathscr{T}}}=\min\mathcal{R}_{ij}^{\mathscr{T}}$, $\overbrace{\mathcal{R}^{\mathscr{T}}}=\max\mathcal{R}_{ij}^{\mathscr{T}}$, $\underbrace{\mathcal{R}^{\mathscr{I}}}=\min\mathcal{R}_{ij}^{\mathscr{I}}$, $\overbrace{\mathcal{R}^{\mathscr{I}}}=\max\mathcal{R}_{ij}^{\mathscr{I}}$, $\underbrace{\mathcal{R}^{\mathscr{F}}}=\min\mathcal{R}_{ij}^{\mathscr{F}}$, $\overbrace{\mathcal{R}^{\mathscr{F}}}=\max\mathcal{R}_{ij}^{\mathscr{F}}$, $\underbrace{\aleph^{\mathscr{T}}}=\min\aleph_{ij}^{\mathscr{T}}$, $\overbrace{\aleph^{\mathscr{T}}}=\max\aleph_{ij}^{\mathscr{T}}$, $\underbrace{\aleph^{\mathscr{I}}}=\min\aleph_{ij}^{\mathscr{I}}$, $\overbrace{\aleph^{\mathscr{I}}}=\max\aleph_{ij}^{\mathscr{I}}$, $\underbrace{\aleph^{\mathscr{F}}}=\min\aleph_{ij}^{\mathscr{F}}$, $\overbrace{\aleph^{\mathscr{F}}}=\max\aleph_{ij}^{\mathscr{F}}$ *and where* $1\le i\le n,\ j=1,2,...,i_j$.

Then, $\left\langle\underbrace{\mathcal{R}^{\mathscr{T}}e^{i2\pi\aleph^{\mathscr{T}}}},\underbrace{\mathcal{R}^{\mathscr{I}}e^{i2\pi\aleph^{\mathscr{I}}}},\overbrace{\mathcal{R}^{\mathscr{F}}e^{i2\pi\aleph^{\mathscr{F}}}}\right\rangle\le(\flat_1,\flat_2,\flat_3)ST-CNWA(\Psi_1,\Psi_2,...,\Psi_n)$
$\le\left\langle\overbrace{\mathcal{R}^{\mathscr{T}}e^{i2\pi\aleph^{\mathscr{T}}}},\overbrace{\mathcal{R}^{\mathscr{I}}e^{i2\pi\aleph^{\mathscr{I}}}},\underbrace{\mathcal{R}^{\mathscr{F}}e^{i2\pi\aleph^{\mathscr{F}}}}\right\rangle$ *(Boundedness property).*

Proof. Since, $\underbrace{\mathcal{R}^{\mathscr{T}}}=\min\mathcal{R}_{ij}^{\mathscr{T}}$, $\overbrace{\mathcal{R}^{\mathscr{T}}}=\max\mathcal{R}_{ij}^{\mathscr{T}}$ and $\underbrace{\mathcal{R}^{\mathscr{T}}}\le\mathcal{R}_{ij}^{\mathscr{T}}\le\overbrace{\mathcal{R}^{\mathscr{T}}}$ and $\underbrace{\aleph^{\mathscr{T}}}=\min\aleph_{ij}^{\mathscr{T}}$, $\overbrace{\aleph^{\mathscr{T}}}=\max\aleph_{ij}^{\mathscr{T}}$ and $\underbrace{\aleph^{\mathscr{T}}}\le\aleph_{ij}^{\mathscr{T}}\le\overbrace{\aleph^{\mathscr{T}}}$. Now,

$$
\begin{aligned}
\underbrace{\mathcal{R}^{\mathscr{T}}e^{i2\pi\aleph^{\mathscr{T}}}} &= \sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\underbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\underbrace{\sin^2\theta\cdot\aleph^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}}\\
&\le \sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\sin^2\theta\cdot\mathcal{R}_{ij}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\sin^2\theta\cdot\aleph_{ij}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}}\\
&\le \sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-(\overbrace{\sin^2\theta\cdot\aleph^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}}\\
&= \overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{T}}}}.
\end{aligned}
$$

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Since, $\underbrace{\mathcal{R}^{\mathscr{I}}} = \min \mathcal{R}^{\mathscr{I}}_{ij}$, $\overbrace{\mathcal{R}^{\mathscr{I}}} = \max \mathcal{R}^{\mathscr{I}}_{ij}$ and $\underbrace{\mathcal{R}^{\mathscr{I}}} \leq \mathcal{R}^{\mathscr{I}}_{ij} \leq \overbrace{\mathcal{R}^{\mathscr{I}}}$ and $\underbrace{\aleph^{\mathscr{I}}} = \min \aleph^{\mathscr{I}}_{ij}$, $\overbrace{\aleph^{\mathscr{I}}} = \max \aleph^{\mathscr{I}}_{ij}$ and $\underbrace{\aleph^{\mathscr{I}}} \leq \aleph^{\mathscr{I}}_{ij} \leq \overbrace{\aleph^{\mathscr{I}}}$. Now,

$$\begin{aligned}
\underbrace{\mathcal{R}^{\mathscr{I}} e^{i2\pi\aleph^{\mathscr{I}}}} &= \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\underbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}\, e^{i2\pi \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\underbrace{\sin^2\theta\cdot\aleph^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}} \\
&\leq \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{ij})^{\flat_2}\right)^{\varsigma_i}}\, e^{i2\pi \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{ij})^{\flat_2}\right)^{\varsigma_i}}} \\
&\leq \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}\, e^{i2\pi \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{\sin^2\theta\cdot\aleph^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}} \\
&= \overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}} e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{I}}}}.
\end{aligned}$$

Since, $\underbrace{(\mathcal{R}^{\mathscr{F}})^{\flat_3}} = \min(\mathcal{R}^{\mathscr{F}}_{ij})^{\flat_3}$, $\overbrace{(\mathcal{R}^{\mathscr{F}})^{\flat_3}} = \max(\mathcal{R}^{\mathscr{F}}_{ij})^{\flat_3}$ and $\underbrace{(\mathcal{R}^{\mathscr{F}})^{\flat_3}} \leq (\mathcal{R}^{\mathscr{F}}_{ij})^{\flat_3} \leq \overbrace{(\mathcal{R}^{\mathscr{F}})^{\flat_3}}$, $\underbrace{(\aleph^{\mathscr{F}})^{\flat_3}} = \min(\aleph^{\mathscr{F}}_{ij})^{\flat_3}$, $\overbrace{(\aleph^{\mathscr{F}})^{\flat_3}} = \max(\aleph^{\mathscr{F}}_{ij})^{\flat_3}$ and $\underbrace{(\aleph^{\mathscr{F}})^{\flat_3}} \leq (\aleph^{\mathscr{F}}_{ij})^{\flat_3} \leq \overbrace{(\aleph^{\mathscr{F}})^{\flat_3}}$.

We have, $\underbrace{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3}} = \bigodot_{i=1}^{n}((\underbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3}})^{\varsigma_i} \leq \bigodot_{i=1}^{n}((\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_{ij})^{\flat_3})^{\varsigma_i} \leq \bigodot_{i=1}^{n}(\overbrace{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3}})^{\varsigma_i} = \overbrace{(\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}})^{\flat_3}}$ and $\underbrace{(\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3}} = \bigodot_{i=1}^{n}((\underbrace{\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3}})^{\varsigma_i} \leq \bigodot_{i=1}^{n}((\sin^2\theta\cdot\aleph^{\mathscr{F}}_{ij})^{\flat_3})^{\varsigma_i}$
$\leq \bigodot_{i=1}^{n}(\overbrace{(\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3}})^{\varsigma_i} = \overbrace{(\sin^2\theta\cdot\aleph^{\mathscr{F}})^{\flat_3}}$.
Therefore,

$$\begin{aligned}
&\frac{1}{2}\times\left[\begin{array}{c}
\left[\begin{array}{c}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\underbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}\right)^2 - \left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\underbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}\right)^2 \\ +1-\left(\bigodot_{i=1}^{n}((\overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}})^{\flat_3})^{\varsigma_i}\right)^2\end{array}\right] \\
+\left[\begin{array}{c}e^{i2\pi\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\underbrace{\sin^2\theta\cdot\aleph^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}\right)^2} - e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\underbrace{\sin^2\theta\cdot\aleph^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}\right)^2} \\ -e^{i2\pi\left(\bigodot_{i=1}^{n}((\overbrace{\sin^2\theta\cdot\aleph^{\mathscr{F}}})^{\flat_3})^{\varsigma_i}\right)^2}\end{array}\right]
\end{array}\right] \\
&\leq \frac{1}{2}\times\left[\begin{array}{c}
\left[\begin{array}{c}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{ij})^{\flat_1}\right)^{\varsigma_i}}\right)^2 - \left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{ij})^{\flat_2}\right)^{\varsigma_i}}\right)^2 \\ +1-\left(\bigodot_{i=1}^{n}((\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}_{ij})^{\flat_3})^{\varsigma_i}\right)^2\end{array}\right] \\
+\left[\begin{array}{c}e^{i2\pi\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{ij})^{\flat_1}\right)^{\varsigma_i}}\right)^2} - e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{ij})^{\flat_2}\right)^{\varsigma_i}}\right)^2} \\ -e^{i2\pi\left(\bigodot_{i=1}^{n}((\sin^2\theta\cdot\aleph^{\mathscr{F}}_{ij})^{\flat_3})^{\varsigma_i}\right)^2}\end{array}\right]
\end{array}\right]
\end{aligned}$$

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$$\leq \frac{1}{2}\times \left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}\right)^2-\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}\right)^2\\ +1-\left(\bigodot_{i=1}^{n}((\underbrace{\sin^2\theta\cdot\mathcal{R}^{\mathscr{F}}})^{\flat_3})^{\varsigma_i}\right)^2\end{array}\right]\\ +\left[\begin{array}{c}e^{i2\pi\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{\sin^2\theta\cdot\aleph^{\mathscr{T}}})^{\flat_1}\right)^{\varsigma_i}}\right)^2}-e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\overbrace{\sin^2\theta\cdot\aleph^{\mathscr{I}}})^{\flat_2}\right)^{\varsigma_i}}\right)^2}\\ -e^{i2\pi\left(\bigodot_{i=1}^{n}((\underbrace{\sin^2\theta\cdot\aleph^{\mathscr{F}}})^{\flat_3})^{\varsigma_i}\right)^2}\end{array}\right]\end{array}\right]$$

Hence, $\left\langle \underbrace{\mathcal{R}^{\mathscr{T}}e^{i2\pi\aleph^{\mathscr{T}}}},\underbrace{\mathcal{R}^{\mathscr{I}}e^{i2\pi\aleph^{\mathscr{I}}}},\overbrace{\mathcal{R}^{\mathscr{F}}e^{i2\pi\aleph^{\mathscr{F}}}}\right\rangle \leq (\flat_1,\flat_2,\flat_3)ST-CNWA(\Psi_1,\Psi_2,...,\Psi_n)$
$\leq \left\langle \overbrace{\mathcal{R}^{\mathscr{T}}e^{i2\pi\aleph^{\mathscr{T}}}},\overbrace{\mathcal{R}^{\mathscr{I}}e^{i2\pi\aleph^{\mathscr{I}}}},\underbrace{\mathcal{R}^{\mathscr{F}}e^{i2\pi\aleph^{\mathscr{F}}}}\right\rangle$

Theorem 4.5. *Let* $\sin\Psi_i=\langle(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{t_{ij}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_{ij}}},\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{t_{ij}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_{ij}}},\mathcal{R}^{\mathscr{F}}_{t_{ij}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{F}}_{t_{ij}}})\rangle$ *and* $W_i=\langle(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{h_{ij}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_{ij}}},\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{h_{ij}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_{ij}}},\mathcal{R}^{\mathscr{F}}_{h_{ij}}e^{i2\pi\sin^2\theta\cdot\aleph^{\mathscr{F}}_{h_{ij}}})\rangle$ *ST-CNWAs. For any i, if there is* $(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{t_{ij}})^2\leq(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{h_{ij}})^2$ *and* $(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{t_{ij}})^2\leq(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{h_{ij}})^2$ *and* $(\mathcal{R}^{\mathscr{F}}_{t_{ij}})^2\geq(\mathcal{R}^{\mathscr{F}}_{h_{ij}})^2$ *and* $(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_{ij}})^2\leq(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_{ij}})^2$ *and* $(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_{ij}})^2\leq(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_{ij}})^2$ *and* $(\sin^2\theta\cdot\aleph^{\mathscr{F}}_{t_{ij}})^2\geq(\sin^2\theta\cdot\aleph^{\mathscr{F}}_{h_{ij}})^2$ *or* $\Psi_i\leq W_i$. *Prove that* $(\flat_1,\flat_2,\flat_3)ST-CNWA(\Psi_1,\Psi_2,...,\Psi_n)\leq(\flat_1,\flat_2,\flat_3)ST-CNWA(W_1,W_2,...,W_n)$, *where* $(i=1,2,...,n);(j=1,2,...,i_j)$ *(monotonicity property).*

Proof. For any i, $(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{t_{ij}})^2\leq\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{h_{ij}}\right)^2$.
Therefore, $1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{t_i})^2\geq1-(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{h_i})^2$.
Hence, $\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{t_i}\right)^2\right)^{\varsigma_i}\geq\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{h_i}\right)^2\right)^{\varsigma_i}$
and $\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{t_i}\right)^{\flat_1}\right)^{\varsigma_i}}\leq\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{T}}_{h_i}\right)^{\flat_1}\right)^{\varsigma_i}}$.
For any i, $(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_{ij}})^2\leq\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_{ij}}\right)^2$.
Therefore, $1-(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_i})^2\geq1-(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_i})^2$.
Hence, $\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_i}\right)^2\right)^{\varsigma_i}\geq\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_i}\right)^2\right)^{\varsigma_i}$
and $\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_i}\right)^{\flat_1}\right)^{\varsigma_i}}\leq\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_i}\right)^{\flat_1}\right)^{\varsigma_i}}$.
Hence, $e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{t_i}\right)^{\flat_1}\right)^{\varsigma_i}}}\leq e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{T}}_{h_i}\right)^{\flat_1}\right)^{\varsigma_i}}}$.
For any i, $(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{t_{ij}})^{\flat_2}\leq\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{h_{ij}}\right)^{\flat_2}$.
Therefore, $1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{t_i}\right)^{\flat_2}\geq1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{h_i}\right)^{\flat_2}$.
Hence, $\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{t_i}\right)^{\flat_2}\right)^{\varsigma_i}\geq\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{h_i}\right)^{\flat_2}\right)^{\varsigma_i}$.
This implies that $\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{t_i}\right)^{\flat_2}\right)^{\varsigma_i}}\leq\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\mathcal{R}^{\mathscr{I}}_{h_i}\right)^{\flat_2}\right)^{\varsigma_i}}$.
For any i, $(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_{ij}})^{\flat_2}\leq\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_{ij}}\right)^{\flat_2}$.
Therefore, $1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_i}\right)^{\flat_2}\geq1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_i}\right)^{\flat_2}$.
Hence, $\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_i}\right)^{\flat_2}\right)^{\varsigma_i}\geq\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_i}\right)^{\flat_2}\right)^{\varsigma_i}$.
This implies that $\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_i}\right)^{\flat_2}\right)^{\varsigma_i}}\leq\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_i}\right)^{\flat_2}\right)^{\varsigma_i}}$.
Hence, $e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{t_i}\right)^{\flat_2}\right)^{\varsigma_i}}}\leq e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left(\sin^2\theta\cdot\aleph^{\mathscr{I}}_{h_i}\right)^{\flat_2}\right)^{\varsigma_i}}}$.

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For any i, $\left(\mathcal{R}_{t_{ij}}^{\mathscr{F}}\right)^2 \geq \left(\mathcal{R}_{h_{ij}}^{\mathscr{F}}\right)^2$ and $\left(\mathcal{R}_{t_{ij}}^{\mathscr{F}}\right)^{\flat_3} \geq \left(\mathcal{R}_{h_{ij}}^{\mathscr{F}}\right)^{\flat_3}$.
Therefore, $1-\left(\bigodot_{i=1}^{n}\mathcal{R}_{t_{ij}}^{\mathscr{F}}\right)^{\flat_3} \leq 1-\left(\bigodot_{i=1}^{n}\mathcal{R}_{h_{ij}}^{\mathscr{F}}\right)^{\flat_3}$.
For any i, $\left(\sin^2\theta\cdot\aleph_{t_{ij}}^{\mathscr{F}}\right)^2 \geq \left(\sin^2\theta\cdot\aleph_{h_{ij}}^{\mathscr{F}}\right)^2$ and $\left(\sin^2\theta\cdot\aleph_{t_{ij}}^{\mathscr{F}}\right)^{\flat_3} \geq \left(\sin^2\theta\cdot\aleph_{h_{ij}}^{\mathscr{F}}\right)^{\flat_3}$.
Therefore, $-e^{i2\pi\left(\bigodot_{i=1}^{n}\sin^2\theta\cdot\aleph_{t_{ij}}^{\mathscr{F}}\right)^{\flat_3}} \leq -e^{i2\pi\left(\bigodot_{i=1}^{n}\sin^2\theta\cdot\aleph_{h_{ij}}^{\mathscr{F}}\right)^{\flat_3}}$.

$$= \frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_{ti}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\right)^2-\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_{ti}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}\right)^2\\ +1-\left(\bigodot_{i=1}^{n}(\mathcal{R}_{tij}^{\mathscr{F}})^{\flat_3}\right)^2\end{array}\right]\\ \left[\begin{array}{c}e^{i2\pi\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_{ti}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\right)^2}-e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_{ti}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}\right)^2}\\ -e^{i2\pi\left(\bigodot_{i=1}^{n}(\sin^2\theta\cdot\aleph_{tij}^{\mathscr{F}})^{\flat_3}\right)^2}\end{array}\right]\end{array}\right]$$

$$\leq \frac{1}{2}\times\left[\begin{array}{c}\left[\begin{array}{c}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_{hi}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\right)^2-\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_{hi}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}\right)^2\\ +1-\left(\bigodot_{i=1}^{n}(\mathcal{R}_{hij}^{\mathscr{F}})^{\flat_3}\right)^2\end{array}\right]\\ \left[\begin{array}{c}e^{i2\pi\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_{hi}^{\mathscr{T}})^{\flat_1}\right)^{\varsigma_i}}\right)^2}-e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_{hi}^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}\right)^2}\\ -e^{i2\pi\left(\bigodot_{i=1}^{n}(\sin^2\theta\cdot\aleph_{hij}^{\mathscr{F}})^{\flat_3}\right)^2}\end{array}\right]\end{array}\right]$$

Hence, $(\flat_1,\flat_2,\flat_3)ST-CNWA\left(\Psi_1,\Psi_2,...,\Psi_n\right) \leq (\flat_1,\flat_2,\flat_3)ST-CNWA\left(W_1,W_2,...,W_n\right)$.

4.2 $(\flat_1,\flat_2,\flat_3)$ST-CNWG

Definition 4.6. Let $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}}e^{i2\pi\aleph_i^{\mathscr{T}}},\mathcal{R}_i^{\mathscr{I}}e^{i2\pi\aleph_i^{\mathscr{I}}},\mathcal{R}_i^{\mathscr{F}}e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ be the $(\flat_1,\flat_2,\flat_3)$ST-CNNs. Then $(\flat_1,\flat_2,\flat_3)$ST-CNWG$(\Psi_1,\Psi_2,...,\Psi_n) = \bigodot_{i=1}^{n}\sin\Psi_i^{\varsigma_i}$.

Theorem 4.7. *Let* $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}}e^{i2\pi\aleph_i^{\mathscr{T}}},\mathcal{R}_i^{\mathscr{I}}e^{i2\pi\aleph_i^{\mathscr{I}}},\mathcal{R}_i^{\mathscr{F}}e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1,\flat_2,\flat_3)$*ST-CNNs. Then* $(\flat_1,\flat_2,\flat_3)$*ST-CNWG*$(\Psi_1,\Psi_2,...,\Psi_n)$

$$= \left[\begin{array}{c}\bigodot_{i=1}^{n}((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1})^{\varsigma_i}e^{i2\pi(\bigodot_{i=1}^{n}((\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1})^{\varsigma_i})},\\ \sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\varsigma_i}}},\\ \sqrt[\flat_3]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\varsigma_i}}e^{i2\pi\sqrt[\flat_3]{1-\bigodot_{i=1}^{n}\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\varsigma_i}}}\end{array}\right].$$

Proof. It follows from Theorem 4.2.

Theorem 4.8. *Let* $\sin\Psi_i = \langle(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}}e^{i2\pi\sin^2\theta\cdot\aleph_i^{\mathscr{T}}},\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}}e^{i2\pi\sin^2\theta\cdot\aleph_i^{\mathscr{I}}},\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}}e^{i2\pi\sin^2\theta\cdot\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1,\flat_2,\flat_3)$*ST-CNNs and all are equal. Then* $(\flat_1,\flat_2,\flat_3)$*ST-CNWG*$(\Psi_1,\Psi_2,...,\Psi_n) = \sin\Psi$.

Proof. It follows from Theorem 4.3.

Remark 4.9. It has other properties, including boundedness and monotonicity, as well as having $(\flat_1,\flat_2,\flat_3)$ST-CNWG.

Proof. It follows from Theorem 4.4 and Theorem 4.5.

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4.3 Generalized $(\flat_1, \flat_2, \flat_3)$ST-CNWA (G$(\flat_1, \flat_2, \flat_3)$ST-CNWA)

Definition 4.10. Let $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ be the $(\flat_1, \flat_2, \flat_3)$ST-CNN. Then G$(\flat_1, \flat_2, \flat_3)$ST-CNWA $(\Psi_1, \Psi_2, ..., \Psi_n) = \left(\biguplus_{i=1}^n \varsigma_i \sin^2\theta \cdot \Psi_i^{\Xi}\right)^{1/\Xi}$.

Theorem 4.11. *Let* $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1, \flat_2, \flat_3)$*ST-CNNs. Then G*$(\flat_1, \flat_2, \flat_3)$*ST-CNWA* $(\Psi_1, \Psi_2, ..., \Psi_n)$

$$= \begin{bmatrix} \left(\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{1/\flat_1} e^{i2\pi\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{1/\flat_1}}, \\ \left(\sqrt[\flat_2]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{1/\flat_2} e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{1/\flat_2}}, \\ \sqrt[\flat_3]{1-\left(1-\left(\bigodot_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}\right)^{\flat_3}\right)^{1/\flat_3}} \\ e^{i2\pi\sqrt[\flat_3]{1-\left(1-\left(\bigodot_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}\right)^{\flat_3}\right)^{1/\flat_3}}} \end{bmatrix}.$$

Proof. We can prove this first by demonstrating that,

$$\biguplus_{i=1}^n \varsigma_i \sin\Psi_i^{\flat_1} = \begin{bmatrix} \sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\; e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}}, \\ \sqrt[\flat_2]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\; e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^n\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}}, \\ \bigodot_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i} e^{i2\pi\bigodot_{i=1}^n\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}} \end{bmatrix}.$$

Put $n = 2$,

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$\varsigma_1 \sin \Psi_1 \uplus \varsigma_2 \sin \Psi_2$

$$
= \begin{bmatrix}
\sqrt[\flat_1]{\begin{array}{c}\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}+\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\\ -\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\end{array}}\; e^{i2\pi\sqrt[\flat_1]{\begin{array}{c}\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}+\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\aleph_2^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\\ -\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\aleph_2^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\end{array}}}, \\
\sqrt[\flat_2]{\begin{array}{c}\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}+\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\\ -\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\end{array}}\; e^{i2\pi\sqrt[\flat_2]{\begin{array}{c}\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}+\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\aleph_2^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\\ -\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\aleph_2^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\end{array}}} \\
\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_1}\cdot\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_2^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_1} \\
e^{i2\pi\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_1^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_1}\cdot\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_2^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_1}}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\sqrt[\flat_1]{1-\bigodot_{i=1}^{2}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\; e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{2}\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}}, \\
\sqrt[\flat_2]{1-\bigodot_{i=1}^{2}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\; e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{2}\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}}, \\
\bigodot_{i=1}^{2}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i} e^{i2\pi\bigodot_{i=1}^{2}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}}
\end{bmatrix}.
$$

Hence, $\biguplus_{i=1}^{l} \varsigma_i \sin \Psi_i^{\Xi}$

$$
= \begin{bmatrix}
\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\; e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}}, \\
\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\; e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\aleph_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}}, \\
\bigodot_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i} e^{i2\pi\bigodot_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}}
\end{bmatrix}.
$$

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If $n = l + 1$, then $\biguplus_{i=1}^{l} \varsigma_i \sin \Psi_i^{\Xi} + \varsigma_{l+1} \sin \Psi_{l+1}^{\Xi} = \biguplus_{i=1}^{l+1} \varsigma_i \sin \Psi_i^{\Xi}$.
Now, $\biguplus_{i=1}^{l} \varsigma_i \sin \Psi_i^{\Xi} + \varsigma_{l+1} \sin \Psi_{l+1}^{\Xi} = \varsigma_1 \sin \Psi_1^{\Xi} \uplus \varsigma_2 \sin \Psi_2^{\Xi} \uplus ... \uplus \varsigma_l \sin \Psi_l^{\Xi} \uplus \varsigma_{l+1} \sin \Psi_{l+1}^{\Xi}$

$$= \left[\begin{array}{c} \sqrt[\flat_1]{\begin{array}{l}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{\flat_1}+\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\\ -\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\end{array}} \\ e^{i2\pi\sqrt[\flat_1]{\begin{array}{l}\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{\flat_1}+\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\\ -\left(\sqrt[\flat_1]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{\flat_1}\cdot\left(\sqrt[\flat_1]{1-\left(1-\left((\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_1}}\right)^{\flat_1}\end{array}}}, \\ \sqrt[\flat_2]{\begin{array}{l}\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{\flat_2}+\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2},\\ -\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\end{array}} \\ e^{i2\pi\sqrt[\flat_2]{\begin{array}{l}\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{\flat_2}+\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2},\\ -\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{l}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{\flat_2}\cdot\left(\sqrt[\flat_2]{1-\left(1-\left((\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_1}}\right)^{\flat_2}\end{array}}} \\ \bigodot_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}\cdot\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_{l+1}^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_1} \\ e^{i2\pi\bigodot_{i=1}^{l}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}\cdot\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_{l+1}^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_1}} \end{array}\right]$$

$$\biguplus_{i=1}^{l+1} \varsigma_i \sin \Psi_i^{\flat_1} = \left[\begin{array}{c} \sqrt[\flat_1]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_1]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}}, \\ \sqrt[\flat_2]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\, e^{i2\pi\sqrt[\flat_2]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_1^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}}, \\ \bigodot_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i} e^{i2\pi\bigodot_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}} \end{array}\right].$$

$\left(\biguplus_{i=1}^{l+1} \varsigma_i \sin \Psi_i^{\Xi}\right)^{1/\Xi} =$

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$$\left[\begin{array}{c}\left(\sqrt[\Xi]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{1/\flat_1} e^{i2\pi\left(\sqrt[\Xi]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}\right)^{\varsigma_i}}\right)^{1/\flat_1}}, \\ \left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{1/\flat_2} e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{l+1}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{1/\flat_2}}, \\ \sqrt[\flat_3]{1-\left(1-\left(\bigodot_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}\right)^2\right)^{1/\flat_3}} \\ e^{i2\pi\sqrt[\flat_3]{1-\left(1-\left(\bigodot_{i=1}^{l+1}\left(\sqrt[\flat_3]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}}\right)^{\varsigma_i}\right)^2\right)^{1/\flat_3}}}\end{array}\right]$$

Remark 4.12. An operator modified from the G$(\flat_1,\flat_2,\flat_3)$ST-CNWA operator to the $(\flat_1,\flat_2,\flat_3)$ST-CNWA operator is performed if $\Xi = 1$.

Theorem 4.13. *If all* $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *and all are equal. Then G*$(\flat_1,\flat_2,\flat_3)$*ST-CNWA*$(\Psi_1,\Psi_2,...,\Psi_n) = \sin\Psi$.

Proof. There is a proof based on the Theorem 4.3.

Remark 4.14. In the G$(\flat_1,\flat_2,\flat_3)$ST-CNWA operator, boundedness and monotonicity are satisfied.

Proof. There is a proof based on the Theorem 4.4 and Theorem 4.5.

4.4 Generalized $(\flat_1,\flat_2,\flat_3)$ST-CNWG(G$(\flat_1,\flat_2,\flat_3)$ST-CNWG)

Definition 4.15. Let $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ be the $(\flat_1,\flat_2,\flat_3)$ST-CNNs. Then G$(\flat_1,\flat_2,\flat_3)$ST-CNWG$(\Psi_1,\Psi_2,...,\Psi_n) = \frac{1}{\Xi}\left(\bigodot_{i=1}^{n}(\Xi\sin\Psi_i)^{\varsigma_i}\right)$.

Theorem 4.16. *Let* $\sin\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *be the* $(\flat_1,\flat_2,\flat_3)$*ST-CNNs. Then G*$(\flat_1,\flat_2,\flat_3)$*ST-CNWG*$(\Psi_1,\Psi_2,...,\Psi_n)$

$$=\left[\begin{array}{c}\sqrt[\flat_1]{1-\left(1-\left(\bigodot_{i=1}^{n}\left(\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}}\right)^{\varsigma_i}\right)^{\flat_1}\right)^{1/\flat_1}} \\ e^{i2\pi\sqrt[\flat_1]{1-\left(1-\left(\bigodot_{i=1}^{n}\left(\sqrt[\flat_1]{1-\left(1-(\sin^2\theta\cdot\aleph_i^{\mathscr{T}})^{\flat_1}\right)^{\flat_1}}\right)^{\varsigma_i}\right)^{\flat_1}\right)^{1/\flat_1}}}, \\ \left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{1/\flat_2} e^{i2\pi\left(\sqrt[\flat_2]{1-\bigodot_{i=1}^{n}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{I}})^{\flat_2}\right)^{\flat_2}\right)^{\varsigma_i}}\right)^{1/\flat_2}}, \\ \left(\sqrt[\flat_3]{1-\bigodot_{i=1}^{n}\left(1-\left((\sin^2\theta\cdot\mathcal{R}_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}\right)^{\varsigma_i}}\right)^{1/\flat_3} e^{i2\pi\left(\sqrt[\flat_3]{1-\bigodot_{i=1}^{n}\left(1-\left((\sin^2\theta\cdot\aleph_i^{\mathscr{F}})^{\flat_3}\right)^{\flat_3}\right)^{\varsigma_i}}\right)^{1/\flat_3}}\end{array}\right].$$

Proof. As a conclusion, we can say that Theorem 4.11 is based on this proof.

Remark 4.17. There is a conversion that takes place when $\Xi = 1$, which converts the G$(\flat_1,\flat_2,\flat_3)$ST-CNWGinto the $(\flat_1,\flat_2,\flat_3)$ST-CNWG.

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Remark 4.18. Boundness and monotonicity properties that are satisfied by G$(\flat_1, \flat_2, \flat_3)$ST-CNWG operators.

Proof. The following proof builds on Theorem 4.4 and Theorem 4.5.

Theorem 4.19. *If all* $\sin \Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$ *are equal. Then G*$(\flat_1, \flat_2, \flat_3)$*ST-CNWG*$(\Psi_1, \Psi_2, ..., \Psi_n) = \sin \Psi$.

5 MADM approach based on $(\flat_1, \flat_2, \flat_3)$ST-CNN

L $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_n\}$ be the set of n-alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the set of m-attributes, $w = \{w_1, w_2, ..., w_m\}$ be the weights of attributes, for $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$
$\Psi_{ij} = \langle(\mathcal{R}_{ij}^{\mathscr{T}} e^{i2\pi\aleph_{ij}^{\mathscr{T}}}, \mathcal{R}_{ij}^{\mathscr{I}} e^{i2\pi\aleph_{ij}^{\mathscr{I}}}, \mathcal{R}_{ij}^{\mathscr{F}} e^{i2\pi\aleph_{ij}^{\mathscr{F}}})\rangle$ denote $(\flat_1, \flat_2, \flat_3)$ST-CNN of alternative Ψ_i in attribute C_j. Since $\mathcal{R}_i^{\mathscr{T}}, \mathcal{R}_i^{\mathscr{I}}, \mathcal{R}_i^{\mathscr{F}}, \aleph_i^{\mathscr{T}}, \aleph_i^{\mathscr{I}}, \aleph_i^{\mathscr{F}} \in [0, 1]$ and $0 \leq (\sin^2\theta \cdot \mathcal{R}_i^{\mathscr{T}})^{\flat_1} + (\sin^2\theta \cdot \mathcal{R}_i^{\mathscr{I}})^{\flat_2} + (\sin^2\theta \cdot \mathcal{R}_i^{\mathscr{F}})^{\flat_3} \leq 1$ and $0 \leq (\sin^2\theta \cdot \aleph_i^{\mathscr{T}})^{\flat_1} + (\sin^2\theta \cdot \aleph_i^{\mathscr{I}})^{\flat_2} + (\sin^2\theta \cdot \aleph_i^{\mathscr{F}})^{\flat_3} \leq 1$, where $\flat_1, \flat_2, \flat_3$ are a positive integers. By combining n-alternatives with m-attributes, we get $\mathscr{D} = (\widehat{\Psi}_{ij})_{n\times m}$. A decision is reached using the following algorithm.

5.1 Algorithm for $(\flat_1, \flat_2, \flat_3)$ST-CNN

Step-1: The decision values for $(\flat_1, \flat_2, \flat_3)$ST-CNN should be entered.
Step-2: The aggregate value of each alternative should be found. On the basis of $(\flat_1, \flat_2, \flat_3)$ST-CNN information aggregation operators, attribute C_j in $\widehat{\Psi}_i$, $\Psi_{ij} = \langle(\mathcal{R}_{ij}^{\mathscr{T}} e^{i2\pi\aleph_{ij}^{\mathscr{T}}}, \mathcal{R}_{ij}^{\mathscr{I}} e^{i2\pi\aleph_{ij}^{\mathscr{I}}}, \mathcal{R}_{ij}^{\mathscr{F}} e^{i2\pi\aleph_{ij}^{\mathscr{F}}})\rangle$ is aggregated into $\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$.
Step-3: For any ST-CNIVN $\Psi_i = \langle(\mathcal{R}_i^{\mathscr{T}} e^{i2\pi\aleph_i^{\mathscr{T}}}, \mathcal{R}_i^{\mathscr{I}} e^{i2\pi\aleph_i^{\mathscr{I}}}, \mathcal{R}_i^{\mathscr{F}} e^{i2\pi\aleph_i^{\mathscr{F}}})\rangle$, the score function of Ψ $S(\Psi) = \frac{C+D}{2}$, where $C = (\mathcal{R}^T)^2 - (\mathcal{R}^I)^2 + 1 - (\mathcal{R}^F)^2$ and $D = (\aleph^T)^2 - (\aleph^I)^2 - (\aleph^F)^2$, where $S(\Psi) \in [-1, 1]$.
Step-4: Based on the output, we can determine that a value of $\max S_i$ is optimal, and therefore we should choose that value as our optimal solution.

5.2 Real life applications

Personal computers are common in most homes today. There are many uses for personal computers outside of work, school, and home. These computers have large amounts of memory and storage. As with televisions, computers have glass monitors that provide users with a wider range of colors. Additionally, the resolution rate is higher, enhancing the clarity of the view. There are many ways to enhance a personal computer's capabilities. There are several devices that can be added, including printers, speakers, scanners, and hard drives. The ease of transporting a computer allows business travelers, vacationers, or anyone else to use it anywhere. Computers can be placed on flat surfaces such as desks, tables, or laps. There is a reduction in resolution rate due to the plastic screen on computers. This makes it difficult for people to see things on computers. Depending on where someone sits, the colors on the computer screen differ; this makes reading the screen difficult. A great many alternatives are considered, and the optimal alternative is selected on the basis of expert assessments against the criteria. Buying a computer may be motivated by a number of factors. There are several stages involved in the purchase of products and services. Let a customer decides to purchase a computer form the analysis five computer brands namely $\Psi = \{\Psi_a, \Psi_b, \Psi_c, \Psi_d, \Psi_e\}$. The differentiates attributes of the computer evaluated by the experts is represented by C_1 = battery life, C_2 = operating system, C_3 = storage capacity, C_4= speed of the processor and their corresponding weights are $w = \{0.4, 0.3, 0.2, 0.1\}$. A list of criteria is used to assess experts in order to determine which option is best. Several factors are considered when making a decision:

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	C_1	C_2
Ψ_a	$(0.85e^{i2\pi(0.75)}, 0.75e^{i2\pi(0.65)}, 0.8e^{i2\pi(0.75)})$	$(0.55e^{i2\pi(0.75)}, 0.75e^{i2\pi(0.85)}, 0.85e^{i2\pi(0.7)})$
Ψ_b	$(0.65e^{i2\pi(0.85)}, 0.8e^{i2\pi(0.4)}, 0.75e^{i2\pi(0.8)})$	$(0.65e^{i2\pi(0.85)}, 0.85e^{i2\pi(0.55)}, 0.75e^{i2\pi(0.8)})$
Ψ_c	$(0.75e^{i2\pi(0.65)}, 0.85e^{i2\pi(0.85)}, 0.4e^{i2\pi(0.75)})$	$(0.75e^{i2\pi(0.75)}, 0.8e^{i2\pi(0.65)}, 0.75e^{i2\pi(0.55)})$
Ψ_d	$(0.8e^{i2\pi(0.85)}, 0.85e^{i2\pi(0.45)}, 0.75e^{i2\pi(0.7)})$	$(0.85e^{i2\pi(0.65)}, 0.65e^{i2\pi(0.8)}, 0.85e^{i2\pi(0.65)})$
Ψ_e	$(0.85e^{i2\pi(0.75)}, 0.75e^{i2\pi(0.55)}, 0.8e^{i2\pi(0.65)})$	$(0.9e^{i2\pi(0.55)}, 0.75e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.55)})$

	C_3	C_4
Ψ_a	$(0.65e^{i2\pi(0.85)}, 0.7e^{i2\pi(0.75)}, 0.8e^{i2\pi(0.8)})$	$(0.55e^{i2\pi(0.65)}, 0.7e^{i2\pi(0.75)}, 0.65e^{i2\pi(0.85)})$
Ψ_a	$(0.4e^{i2\pi(0.75)}, 0.75e^{i2\pi(0.4)}, 0.85e^{i2\pi(0.8)})$	$(0.65e^{i2\pi(0.75)}, 0.75e^{i2\pi(0.85)}, 0.8e^{i2\pi(0.65)})$
Ψ_a	$(0.75e^{i2\pi(0.65)}, 0.85e^{i2\pi(0.85)}, 0.75e^{i2\pi(0.7)})$	$(0.8e^{i2\pi(0.85)}, 0.85e^{i2\pi(0.4)}, 0.75e^{i2\pi(0.65)})$
Ψ_a	$(0.85e^{i2\pi(0.75)}, 0.9e^{i2\pi(0.9)}, 0.8e^{i2\pi(0.4)})$	$(0.85e^{i2\pi(0.65)}, 0.65e^{i2\pi(0.75)}, 0.7e^{i2\pi(0.85)})$
Ψ_a	$(0.85e^{i2\pi(0.55)}, 0.65e^{i2\pi(0.8)}, 0.9e^{i2\pi(0.45)})$	$(0.75e^{i2\pi(0.85)}, 0.6e^{i2\pi(0.8)}, 0.85e^{i2\pi(0.8)})$

Aggregate information with $(\flat_1, \flat_2, \flat_3)$ST-CNWA operators are as follows:

	$(\flat_1, \flat_2, \flat_3)$ST-CNWA operator $(1,1,1)$
$\widehat{\Psi_a}$	$0.7256e^{i2\pi(0.9924)}), 0.9874e^{i2\pi(0.9567)}), 0.1775e^{i2\pi(0.1931)})$
$\widehat{\Psi_b}$	$0.9099e^{i2\pi(0.9493)}), 0.9069e^{i2\pi(0.9405)}), 0.7252e^{i2\pi(0.0835)})$
$\widehat{\Psi_c}$	$0.9964e^{i2\pi(0.9600)}), 0.6775e^{i2\pi(0.8589)}), 0.9923e^{i2\pi(0.3864)})$
$\widehat{\Psi_d}$	$0.6245e^{i2\pi(0.9311)}), 0.7832e^{i2\pi(0.5471)}), 0.3771e^{i2\pi(0.2230)})$
$\widehat{\Psi_e}$	$0.8224e^{i2\pi(0.9492)}), 0.9918e^{i2\pi(0.7802)}), 0.1192e^{i2\pi(0.3708)})$

S_1	S_2	S_3	S_4	S_5
0.2761	0.2446	0.2919	0.5762	0.4167

The following alternative rankings are provided:

$$\Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a \geqslant \Psi_b.$$

Consequently, Ψ_d is the best option to purchase.

5.3 Analysis and discussion

Our proposal is to apply the following strategies based on score values: ST-CNWA, ST-CNWG, ST-CGNWA, and ST-CGNWG. The following distance categories can be used to categorize distances. Change the $(\flat_1, \flat_2, \flat_3)$ values from ST-CNWA approach. The following values and orders of closeness are obtained:

$(\flat_1, \flat_2, \flat_3)$	S_1	S_2	S_3	S_4	S_5	$Order$
$(1,1,1)$	0.2761	0.2446	0.2919	0.5762	0.4167	$\Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a \geqslant \Psi_b$
$(2,1,1)$	0.2834	0.2481	0.2934	0.6101	0.4353	$\Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a \geqslant \Psi_b$
$(1,2,1)$	0.2723	0.2217	0.2671	0.4961	0.3815	$\Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a \geqslant \Psi_b$
$(1,1,2)$	0.3093	0.3728	0.3630	0.6608	0.4830	$\Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a \geqslant \Psi_b$
$(3,1,1)$	0.2904	0.2515	0.2948	0.6311	0.4512	$\Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a \geqslant \Psi_b$
$(1,3,1)$	0.2696	0.2077	0.2518	0.4397	0.3585	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$

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$(\flat_1, \flat_2, \flat_3)$	S_1	S_2	S_3	S_4	S_5	$Order$
$(1,1,3)$	0.3105	0.4383	0.3799	0.6706	0.4912	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(3,2,1)$	0.2865	0.2286	0.2700	0.5510	0.4160	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(2,3,1)$	0.2768	0.2112	0.2533	0.4736	0.3771	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(2,1,3)$	0.3177	0.4418	0.3815	0.7045	0.5099	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(3,1,2)$	0.3236	0.3797	0.3659	0.7157	0.5175	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(1,3,2)$	0.3028	0.3359	0.3229	0.5243	0.4248	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(1,2,3)$	0.3067	0.4154	0.3552	0.5906	0.4560	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(3,3,1)$	0.2838	0.2146	0.2547	0.4946	0.3930	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,3,3)$	0.3039	0.4015	0.3399	0.5341	0.4330	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(3,1,3)$	0.3247	0.4452	0.3828	0.7255	0.5258	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(4,1,1)$	0.2968	0.2548	0.2961	0.6454	0.4647	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,4,1)$	0.2674	0.1980	0.2412	0.4002	0.3421	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,1,4)$	0.3105	0.4728	0.3887	0.6719	0.4923	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(4,2,1)$	0.2930	0.2319	0.2713	0.5654	0.4295	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(2,4,1)$	0.2746	0.2015	0.2428	0.4341	0.3608	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(2,1,4)$	0.3178	0.4763	0.3902	0.7058	0.5110	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(4,1,2)$	0.3300	0.3829	0.3672	0.7300	0.5310	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(1,4,2)$	0.3006	0.3261	0.3123	0.4848	0.4084	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(1,2,4)$	0.3067	0.4499	0.3639	0.5919	0.4571	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(4,3,1)$	0.2903	0.2179	0.2560	0.5089	0.4065	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(3,4,1)$	0.2816	0.2049	0.2441	0.4551	0.3767	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(3,1,4)$	0.3247	0.4797	0.3916	0.7268	0.5269	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(4,1,3)$	0.3312	0.4485	0.3841	0.7399	0.5392	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(1,4,3)$	0.3017	0.3917	0.3293	0.4947	0.4166	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_{ba}$
$(1,3,4)$	0.3040	0.4359	0.3487	0.5354	0.4341	$\Psi_d \geqslant \Psi_b \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a$
$(4,4,1)$	0.2881	0.2082	0.2454	0.4695	0.3901	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(4,1,4)$	0.3312	0.4830	0.3929	0.7412	0.5404	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(1,4,4)$	0.3018	0.4262	0.3381	0.4960	0.4178	$\Psi_d \geqslant \Psi_b \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a$
$(5,1,1)$	0.3027	0.2580	0.2973	0.6561	0.4762	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,5,1)$	0.2654	0.1906	0.2331	0.3715	0.3298	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,1,5)$	0.3105	0.4910	0.3962	0.6721	0.4925	$\Psi_d \geqslant \Psi_e \geqslant \Psi_b \geqslant \Psi_c \geqslant \Psi_a$
$(6,1,1)$	0.3079	0.2611	0.2985	0.6646	0.4860	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,6,1)$	0.2637	0.1847	0.2264	0.3496	0.3201	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,1,6)$	0.3105	0.5005	0.4034	0.6721	0.4925	$\Psi_d \geqslant \Psi_b \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a$
$(7,1,1)$	0.3125	0.2640	0.2996	0.6716	0.4946	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,7,1)$	0.2621	0.1799	0.2207	0.3325	0.3124	$\Psi_d \geqslant \Psi_e \geqslant \Psi_a \geqslant \Psi_c \geqslant \Psi_b$
$(1,1,7)$	0.3105	0.5055	0.4104	0.6721	0.4925	$\Psi_d \geqslant \Psi_b \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a$
$(1,7,7)$	0.2965	0.4407	0.3392	0.4284	0.3882	$\Psi_b \geqslant \Psi_d \geqslant \Psi_e \geqslant \Psi_c \geqslant \Psi_a$

Therefore, Ψ_d should be changed to Ψ_b as the optimal alternative. As with the NWG operator, GNWA operator and GNWG operator, alternative rankings are determined according to $(\flat_1, \flat_2, \flat_3)$.

6 Conclusion:

The aim of this study was to establish score values for $(\flat_1, \flat_2, \flat_3)$ST-CNSs, which have the advantage of being mathematically simple. Utilizing appropriate methods results in superior score values. For $(\flat_1, \flat_2, \flat_3)$ST-CNWA, $(\flat_1, \flat_2, \flat_3)$ST-CNWG, G$(\flat_1, \flat_2, \flat_3)$ST-CNWA and G$(\flat_1, \flat_2, \flat_3)$ST-CNWG, we have proposed improved AO rules. In order to build these operators, several algebraic operations have also been discussed. Future research in this field, which is still very early in its development, can be greatly enhanced by reading this article. Future academics who are interested in this field will find the ideas presented here useful. Detailed discussions will be conducted on the following topics:

(1) There are three types of normal vague set, normal vague spherical set, and normal vague NS using complex interaction aggregation operators.

(2) Complex cubic NWAs, NWGs, GNWAs and GNWGs may be used to solve the problem.

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